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Chapter 1

Introduction

1.1 Complexity of financial markets

In today's financial markets huge volumes of interdependent assets are traded by a large number of interacting market participants in different locations and time zones. Their behavior is of unprecedented complexity and is exhibited in for example the unpredictable dynamics of asset prices characterized by strong cross-asset and intercontinental dependence. To introduce our work and in particular this thesis, here we briefly describe the components and the behavior of financial markets.

1.1.1 Financial markets as complex systems

Financial markets are (physical or virtual) places at which financial assets are traded. Connecting potential buyers and sellers and facilitating allocation of resources, they are crucial to the efficient operation of a market economy.

According to their functions, financial markets can be categorized into different types. Capital markets enable the raising of capital through stock (equity) or bond (debt) issuance and organize the trading of these assets. Money markets specializes in short-term cash borrowing and lending. Derivatives markets produce instruments for controlling financial risk and coordinate the trading of these products. Insurance markets provide financial protection against contingent losses. Foreign exchange markets facilitate the conversions between currencies. A detailed introduction to financial markets was presented in Melicher and Norton (2008).

The first stock market, the Amsterdam Stock Exchange, was invented by the Dutch East India Company (*'Verenigde Oostindische Compagnie'*) in 1602 for dealings in stocks and bonds (Melicher and Norton (2008)). Futures markets can be traced back to the Middle Ages and the first trading in options began as early as the eighteenth century (Hull (2004)). Financial markets have evolved significantly over several hundred years and there are now many financial exchanges in the world. With the vast increase in worldwide trading volumes in the last decades, financial markets are indispensable to the modern economy.

Usually, a financial market is composed of a large number of traders with different functions, motivations, and strategies. In order to achieve their respective objectives, they interact with each other in many distinct manners. The composition and mechanisms of financial markets will be briefly described below.

Types of assets and their interdependence

A financial asset is an intangible representation of, and a contractual claim to, the future cash flow arising from a physical item. We can classify financial assets in different ways, among which one is presented in Johnson et al. (2003). It categorizes financial assets into three groups corresponding to (1) whether the asset is a debt, equity, or foreign exchange, (2) whether the settlement should be made at the time the contract is agreed between the two parties of the contract, or settled at some time in the future from the time it was agreed, and (3) whether the contract has an associated obligation, or only the right but not the obligation to deliver another product. For simplicity, we divide financial assets into two groups: Those that are directly associated with some real assets, and those that whose values are dependent on or derives from the values of some other more basic financial assets known as underlying assets.

The first group of financial assets includes stocks, bonds, currencies, etc. Briefly, a stock is an equity claim, i.e., it represents the ownership of a portion of a corporation and a claim on its proportional share in the earnings and assets of the corporation. A bond is an interest-bearing or discounted debt instrument whose issuer has the obligation to repay the bondholders the principal along with interest (coupons) at maturity. A currency is a form of money that is legally designated as such by a governing body and is in public circulation. Detailed descriptions of these assets can be found in Melicher and Norton (2008). These assets have different return-risk profiles that require distinct modeling approaches. This is a nontrivial task because the payoff of each asset is determined by a few or many factors, such as interest rate, dividend rate, default rate, etc. To further complicate matters, these factors may fluctuate over time and are correlated with one another.

The most commonly used financial instruments of the second group are futures contracts and options. A futures contract is a standardized agreement to buy or sell a specified standardized commodity at a certain future date, for a certain price. An option is a contract between a buyer and a seller that conveys the buyer the right, but not the obligation, to buy (in case of a call option) or sell (in case of a put option) a particular asset termed the underlying asset (or simply the underlying), at a certain date known as the expiration date or maturity, and at a prescribed price termed the strike price. Detailed descriptions of futures, options, and other derivatives can be found in Melicher and Norton (2008) and Hull (2003). Derivatives can be used to hedge or mitigate risk in the underlying. In addition, they are ideal instruments of speculation because they are often leveraged, i.e. a small movement in the underlying value can cause a large difference in their values. Indeed, developing new derivatives is the focal point of modern financial innovations. New types of derivative have been created which are dependent on the payoffs of different underlying variables greatly beyond the traditional scope of underlying variables such as stock prices, exchange rates, commodities prices, etc. However, due to the complexity of the payoffs of underlying assets and the complicated dependence of derivatives on their underlying assets, the estimation of the values and inherent risk of investment portfolios becomes increasingly sophisticated.

Types of traders and their interactions

Financial traders are different in trading strategy, level of risk aversion, financial knowledge and information, etc. According to Hull (2003) and Johnson et al. (2003), three broad categories of traders can be identified: Hedgers, speculators, and arbitrageurs.

A hedger is a market participant who performs a security transaction with the goal of minimizing the exposure to the risk of an unfavorable price change of an asset. For example, a hedger who owns a bond and is concerned that its price might decline can sells a futures contract on the bond. If the price of the bond does fall, the profit on the transaction of the futures contract will cover the loss on the bond. There are many financial vehicles to accomplish a hedge, including insurance policies, forward and futures contracts, options, etc. A common aspect of all these vehicles is that, although reducing potential losses, they also tend to reduce potential profits. Continuing the example, if the price of the bond rise, the hedger gives up the potential profit.

A speculator is a trader who aims to make capital gains from risky transactions in anticipation of future price movements. For instance, a speculator who expects that the price of an equity will rise (fall) will buy (sell) the equity in order to make a profit if the price indeed moves in line with the expectation. By doing so, the trader bears the risk of a loss arising from a opposite price movement. Speculators can employ many different strategies by making use of different types of financial instrument. However, it has been realized in financial markets that speculations with high potential profits are usually associated with high potential risks. In general, speculators can in turn be categorized into the following few types. Fundamentalists are those traders who are informed of the nature of the asset being traded and act according to its fundamental value. They believe that the price of the asset may temporarily deviate from, but will eventually return to, the fundamental value. Imitators do not know or do not care about fundamental values. Instead, they follow their acquaintances and adopt the trading opinions of the majority. Chartists, also called technical analysts, forecast the future direction of some financial variables, primarily price and volume, through recognition of chart patterns in market data.

An arbitrageur is a market agent who attempts to profit from price disparities by simultaneously taking offsetting positions and capturing risk-free profits. For example, an arbitrageur would buy a currency which is cheaper in a market and then immediately sell it in another market, in which the price of the very same currency is higher, to lock-in a profit. Arbitrage opportunities can occur in various types of financial asset but cannot last for long. They will quickly be exploited by arbitrageurs. Therefore, the presence of arbitrageurs ensure that financial markets are generally arbitrage-free markets.

Traders of different types may interact with each other in different direct or indirect ways. The most typical direct interactions in financial markets involve imitative speculators who may copy the behavior of some other traders or do so by indirectly following the price trend that reflects the aggregate effect of the behavior of other traders in the whole market. Indirect interactions are, however, ubiquitous in financial markets. In fact, when estimating payoffs of financial investments, many traders refer to the market prices of the assets of interest, which is in turn determined by the supply and demand for the assets generated by the traders in the overall market. This behavior-price feedback, which embodies the indirect interactions of different traders, are realized through *market makers* — intermediations who always stand ready to facilitate trading in financial assets by offering competitive prices and act to ensure market liquidity.

Interactions in financial markets are highly nonlinear and can considerably impair our ability to identify cause-effect relations of market movements and predict consequence of investment transactions and market regulations.

1.1.2 The complex dynamics of financial markets

In literature, the study of the complexity or the complex dynamics of financial markets deals with a wide range of subjects and dates back to at least the sixties of the last century. For example, Simon (1962) reported the development of the concept of bounded rationality; Zeeman (1974) showed that stock market crashes can happen when there are too many chartists relative to fundamental traders; Grandmont (1985) suggested that unpredictability undermines the possibility of rational expectations in economics; and Brock and Hommes (1997) claimed that high rationality in an unstable market with information cost implies chaos.

In this thesis, we focus on those aspects of market dynamics that are characterized by some persistent patterns empirically observed in financial time series which can hardly be explained by the traditional economic theories. Specifically, we focus on stock and options markets, which are two of the most active types of financial market. We believe that these two types of market share many features with other types of financial market, and the understanding of their complex dynamics will be beneficial to, or even directly applicable to, the understanding of the complexity of any other types of financial market.

Stock markets

The complex dynamics of stock markets can be characterized by some 'stylized facts', which are common across many stocks, markets, and time horizons. Most of them are counterintuitive and contrary to the expectations of traditional financial theories.

On long time scales (typically a week or longer), empirical distributions of financial return (defined in Chapter 2) generally fit the Gaussian distribution. However, most financial returns over short time scales are described well by a non-Gaussian heavy-tailed or fat-tailed distribution, implying a greater frequency of extreme events than would be expected if they followed a normal distribution.

In addition, the autocorrelation function (ACF) (defined in Chapter 2) of the daily price changes quickly converges to the noise range, whereas the corresponding ACF of absolute price changes decays slowly. The long-term autocorrelation of volatility is the reflection of the phenomenon termed 'volatility clustering' — high positive or negative returns tend to group together. These stylized facts are discussed in detail in Chapter 2.

Option markets

The most widely used mathematical model for pricing options is the Black-Scholes (BS) model (Black and Scholes (1973); Merton (1973)). All parameters in the BS model other than the volatility (defined in Chapter 2) are observables, and according to the model the theoretical value of an option is a monotonic increasing function of the volatility. A unique volatility is therefore implied by the market price of an option, the so-called implied volatility (IV).

According to the BS model, IV should be independent of strike for a fixed time to maturity. Hence, plots of IV against strike should be flat. In reality, however, it is well known that IVs exhibit a remarkable curvature, which is commonly referred to as a volatility smile. Another important fact is that the volatility smile changes over time. This phenomenon obviously conflicts with the BS framework and understanding its origin has eluded the financial world for more than two decades. The volatility smile phenomenon is described in detail in Chapter 2.

1.2 Motivation and methodology

As discussed above, financial markets are among the most complex systems in reality and many phenomena observed in the dynamics of asset prices are still poorly understood. Specifically, markets often exhibit extraordinary or unexpected changes seemingly not induced by external causes, instead arising endogenously. This seriously challenges the neoclassical economics, which depicts markets as efficient machines that automatically seek out an equilibrium state and price changes are only caused by the arrival of news.

We aim to discover the mechanisms through which the complex market dynamics is generated. Specifically, we want to understand how the behavior of financial agents and their interactions, when trading different financial assets, give rise to the complex dynamics of financial markets characterized by some unexpected phenomena, i.e. how the complex macro-dynamics emerges from the ordinary and typical micro-behavior.

During the last few decades, behavioral approaches and agent-based methods have been widely applied to the study of market dynamics. They can explain many phenomena in a more plausible way than traditional financial theories. However, most of these theories or models either do not systematically examine the mechanisms underlying the unexpected phenomena or are too complicated to be helpful for clearly identifying the causal relations of the mechanisms. In addition, the majority of the agent-based models focus on stock markets, while very few center on derivatives markets.

In view of these facts, our general motivation is to apply a bottom-up approach for studying the mechanisms through which the complex market dynamics is generated. We study a financial market by realistically modeling its individual elements and their interactions. The macro-dynamics of the system will eventually emerge from the micro-behavior. In particular, we wish to develop microsimulation (MS) models with simple structures[1] that can reproduce the extraordinary patterns observed in the financial time series recorded in stock and options markets. Importantly, we present our models in the order of successive complexification so as to offer important insights into the origins of some puzzling phenomena observed in financial markets.

We focus on two of the most active markets and address the following research questions:

a. What are the principal mechanisms underlying stylized facts observed in stock markets? Are these mechanisms common across explanations provided by the well-established microsimulation models proposed in the literature?

b. What is the origin of the poorly understood volatility smile phenomenon observed in option markets and what are the driving factors determining the shape and the dynamic properties of the smile?

The outline of this thesis follows. In Chapter 2, we present the stylized facts observed in stock and options markets and discuss pitfalls and limitations of mainstream financial theories. These have stimulated the development of alternative theories, some of which are introduced in Chapter 3. Chapter 4 focuses on the first research question raised above, while Chapter 5 and Chapter 6 address the second question. In Chapter 7, the conclusions of this research and some directions for future research are presented.

[1] "Everything should be made as simple as possible, but no simpler"— Albert Einstein.

Chapter 2

Empirical Observations and Limitations of Traditional Economic Theories

Financial economists and practitioners have recognized certain statistical regularity in financial time series, which is common across markets and time horizons, and known as 'stylized facts'. Importantly, they are contrary to the expectations of traditional economic and financial theories and challenge our understanding of their origins. This chapter presents a brief review of these empirical observations and their disagreement with mainstream financial theories.

2.1 Stylized facts observed in financial markets

Here we describe the stylized facts observed in two types of financial market, i.e. stock markets and option markets. In fact, these markets are among the most active financial markets with respect to trading volumes. In addition, most of the empirical studies of market dynamics in the literature were performed on the transaction data obtained from these two types of market.

2.1.1 Stock markets

The stylized facts of stock markets have been observed and discussed by many researchers. See, among many others, Cont (2001); Ding et al. (1993); Guillaume et al. (1997); Mandelbrot (1963); Mantegna and Stanley (2000); Pagan (1996); Voit (2003).

On long time scales (typically a week or longer), empirical distributions of financial return[1] generally fit to the Gaussian distribution. However, most financial returns over short timescales are described well by a non-Gaussian (heavy-tailed or fat-tailed) distribution.

A commonly used criterion for the normality of the distribution of a variable X is its kurtosis (κ), defined as

$$\kappa = \frac{E[(X-\mu)^4]}{\sigma^4}, \tag{2.1}$$

where μ and σ are respectively the mean and standard deviation of X. $\kappa = 3$ corresponds to a Gaussian distribution, whereas $\kappa > 3$ indicates a so-called leptokurtic distribution with a sharp peak and heavy tails.

The kurtosis of financial returns is far from that of a Gaussian distribution. For instance, our estimate for the kurtosis of S&P 500[2] daily returns over the period June 1950 to June 2005 is around 38. Figure 2.1(a) shows the distribution of these returns, together with a Gaussian probability density function (PDF)

[1]Generally, return is defined as $R_1^{t+1} = \ln P^{t+1} - \ln P^t$, where R_1^{t+1} is the return at time $t+1$, P^t is the price at time t, and so on. The basic relation is $R_2^{t+1} = (P^{t+1} - P^t)/P^t$. Sometimes, return is defined as price change, $R_3^{t+1} = P^{t+1} - P^t$. For high-frequency data, $|R_3^{t+1}| \ll P^t$. Hence, $R_1^{t+1} = \ln[1 + R_3^{t+1}/P^t] \simeq R_3^{t+1}/P^t = R_2^{t+1}$. Since R_3^t is a fast variable and P^t is a slow variable, $R_2^t \simeq CR_3^t$, where the time dependence of C is negligible. (See p. 35-39 of Mantegna and Stanley (2000).) So, $R_1^t \simeq R_2^t \simeq CR_3^t$. For high-frequency data, these three indicators are therefore alternatives to each other in analyzing the regularity in return distributions.

[2]An index is a sample list of stocks that is representative of a whole stock market. It is used by investors to track the performance of the stock market. Different methods are being used for calculating the price of an index. For example, the Dow Jones Industrial Average (DJIA), which contains 30 of the most influential companies in the U.S., is the price-based weighted average of the prices of the included stocks. The Standard and Poor's 500 Index (S&P 500), which includes 500 large publicly held companies that trade on major U.S. stock exchanges, weights companies by market capitalization (the overall value of a company's stock on the market).

and a Lorentz PDF for comparison. Clearly, daily returns of S&P 500 follow a non-Gaussian (fat-tailed) distribution, implying a greater frequency of extreme events than would be expected if they followed a normal distribution. However, the variance of the distribution is finite, whereas that of a Lorentz distribution (or a stable Lévy distribution in general) is infinite.

In statistics, the autocorrelation function (ACF) of a time series describes the correlation between the values of the series at two different points in time. The autocorrelation coefficient of a process X_t as a function of time lag τ is defined as

$$R(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}. \tag{2.2}$$

Plotted against τ, $R(\tau)$ can be used for detecting the non-randomness of X_t.

The ACF of daily returns of financial assets quickly converges to the noise range, whereas the corresponding ACF of volatility[1] decays slowly (see Figure 2.1(b)). The long-term autocorrelation of volatility is the reflection of the phenomenon termed 'volatility clustering' — high (positive or negative) returns tend to group together. Figure 2.1(c) shows the time series of return over the period. In this figure, the effect of volatility clustering is clearly illustrated.

Based on a large amount of available transaction data, Mantegna and Stanley (1995) explored whether the scaling phenomena occur in financial markets. Specifically, they showed that the scaling of the probability distribution of S&P500 can be described by a non-Gaussian process. In addition, the scaling behavior can be observed for time intervals spanning three orders of magnitude (from 1,000 min to 1 min) and the scaling exponent is remarkably constant over the six-year period (1984-1989) of the data.

Remarkably, non-Gaussian (fat-tailed) distribution, volatility clustering, and the scaling law have also been found in the time series recorded in some other types of financial market, e.g., foreign exchange markets (Müller et al. (1990)).

[1]In the finance literature, volatility refers to the spread of asset returns measured as the standard deviation of a sample of returns over a period of time, i.e., $\sigma = \sqrt{(1/T)\sum_{t=1}^{T}(R^t - \overline{R})^2}$, where T is the length of the period, R^t is the return at time t, and $\overline{R}$ is the average return over the period. Substituting $T = 1$ and $\overline{R} = 0$ into this equation, we obtain the absolute value of the return over a period of one time unit, $|r|$, which is the most commonly used proxy for volatility in practice. The other commonly used proxy is r^2. We adopt $|r|$ as volatility.

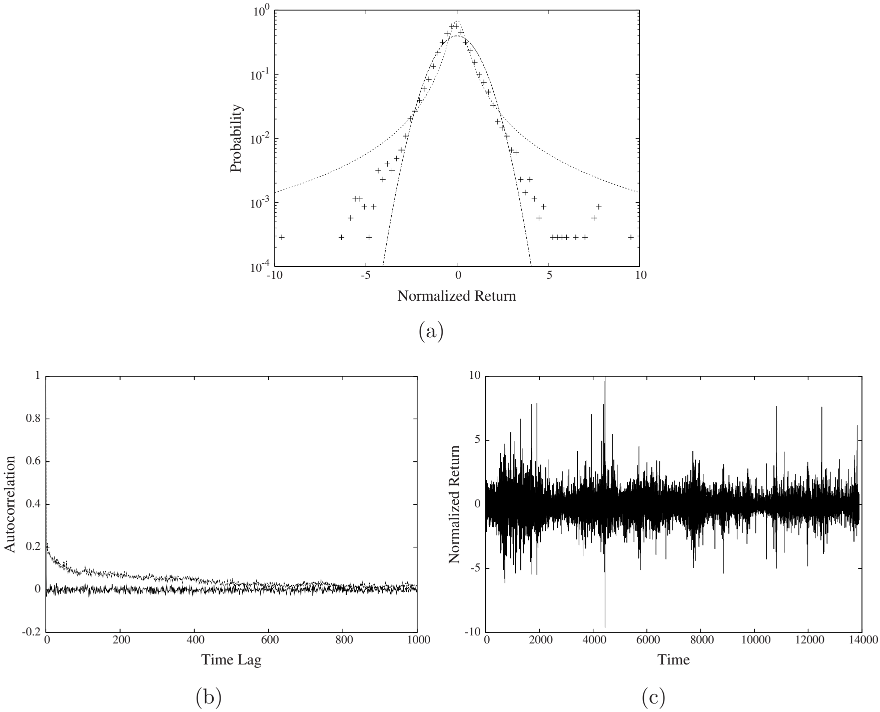


Figure 2.1: (a) Distribution of the daily returns of S&P 500 over the period from June 1950 to June 2005 (the points), compared with a Gaussian PDF (the curve that decays faster) and a Lorentz PDF. A logarithmic scale is used for the vertical axis. (b) Autocorrelation function of the daily returns (the lower line) and the corresponding ACF of volatility. (c) Time series of the daily returns.

In literature, there is not yet a common agreement on the origins of the stylized facts (Cont (2005)). On the other hand, the analytical models developed for describing these phenomena, some of which are described in Section 2.2.1, do not provide explicit economic explanations for the underlying mechanisms.

2.1.2 Options markets

The volatility smile phenomenon observed in options markets is a long-standing problem in financial economics and has been discussed by many researchers (Bakshi et al. (1997); Black (1975); Ciliberti et al. (2009); Cont and da Fonseca (2002); Ederington and Guan (2002); Fengler et al. (2003); Geman (2005); Johnson et al. (2003); Macbeth and Merville (1979); Rebonato (2004); Rubinstein (1985); Tompkins (2001)).

This phenomenon is intrinsically related to the widely used Black-Scholes (BS) model for option pricing (Black and Scholes (1973), Merton (1973)). The BS formula for pricing a European option[1] on a non-dividend-paying asset is

$$V_{BS}^{\phi,t}(S^t, K, r, \tau, \sigma) = \phi[S^t N(\phi d_1) - Ke^{-r\tau} N(\phi d_2)], \tag{2.3}$$

where S^t is the price of the underlying asset at time t, K the strike price of the option, r the risk-free interest rate, $\tau = T - t$ the time to maturity where T the expiration time of the option, σ the volatility of the asset, and $\phi = 1(-1)$ for a call (put) option. In Equation (2.3), d_1 and d_2 are defined as

$$d_1 = \frac{ln(S^t/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \tag{2.4}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{2.5}$$

and $N(x)$ is the standard normal cumulative distribution function.

All parameters in the BS model other than the volatility are observables and, according to the model, the theoretical value of an option is a monotonic increasing function of the volatility. A unique volatility is therefore implied by the market price of an option ($V^{\phi,t}$), the so-called *implied volatility*:

$$\sigma_{imp}^{\phi,t} : \quad V_{BS}^{\phi,t}(S^t, K, r, \tau, \sigma_{imp}^{\phi,t}) = V^{\phi,t}. \tag{2.6}$$

[1] A European option may be exercised only at the expiry date of the option; in contrast, an American option may be exercised at any time before the expiry date (Hull (2003)).

According to the BS model, implied volatility (IV) is independent of strike for a fixed time to maturity. Hence, plots of IV against strike should be flat. In reality, however, it is well known that IVs exhibit a remarkable curvature, which is commonly referred to as a *volatility smile*.

In particular, equity index options tend to have a downward sloping IV curve, i.e. a volatility skew. This skew has become much more pronounced after the stock market crash of October 1987. Foreign currency options, however, typically show a symmetric smile, especially if the currencies are of equal strength. Contrary to equity index options, some commodity options often show an upward sloping skew. Figure 2.2 depicts the IVs and the fitted curves of options derived from three underlying assets of different types: Equity index, currency, and commodity. The downward IV skew shown in Figure 2.2(a) is obtained from the European-style options on the S&P500 index with time to maturity 118 days, traded on April 7, 2004 in Chicago Mercantile Exchange. The symmetric IV smile shown in Figure 2.2(b) is from the British Pound options with time to maturity 43 days, traded on April 7, 2004 in Chicago Mercantile Exchange. The upward IV skew shown in Figure 2.2(c) is from the soybeans options with time to maturity 137 days, traded on January 15, 2004 in Chicago Board of Trade.

Another important fact is that the volatility smile changes over time. It has been revealed that three principal components can sufficiently account for the observed deformation of the smile: The first component reflects the shift in its overall level; the second one conveys the change of its slope; and the third component accounts for the adjustment in its convexity (Cont and da Fonseca (2002); Fengler et al. (2003)). Figure 2.3 shows the principal components obtained from some empirical time series of implied volatility and the proportions of the variances explained by these principal components. The results shown in Figures. 2.3(a) and 2.3(b) are based on those reported in Fengler et al. (2003), which were obtained from the daily IV time series of European style options on DAX index for the entire year 1999. The results shown in Figures. 2.3(c) and 2.3(d) are based on those reported in Rexhepi (2008), generated from the daily IV time series of options on S&P500 index over the period from December 1999 to October 2002.

There are other stylized facts or observed characteristics with respect to the IV dynamics. For detailed discussions, see Cont and da Fonseca (2002), Rebonato

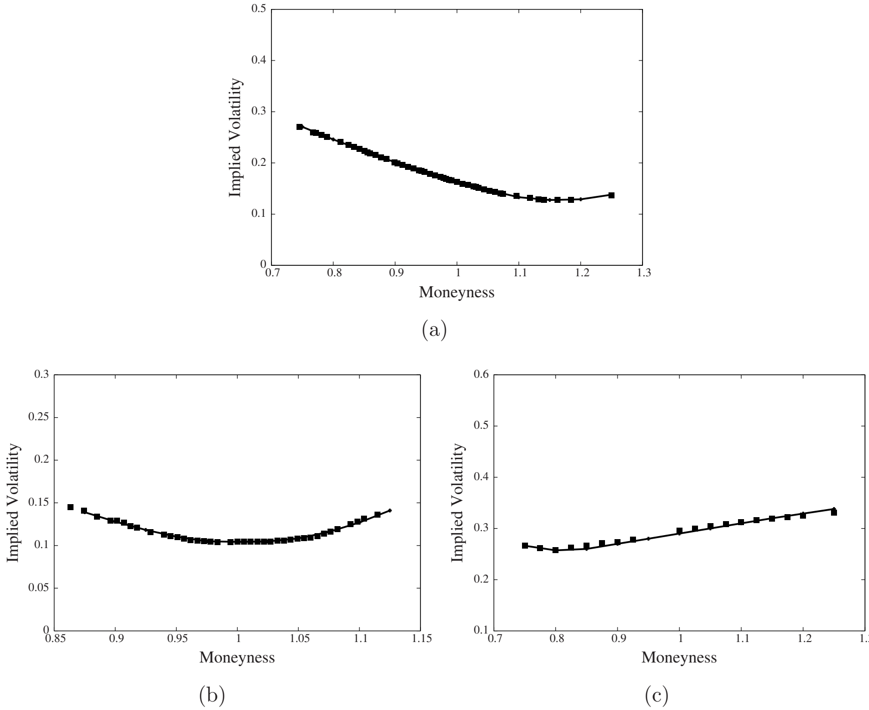


Figure 2.2: Empirical implied volatilities plotted against moneyness (K/S^t), for different types of underlying asset. (a) Implied volatilities (the points) and the fitted curve of the European-style options on the S&P500 index with time to maturity 118 days, traded on April 7, 2004 in Chicago Mercantile Exchange. (b) Implied volatilities (the points) and the fitted curve of the British Pound options with time to maturity 43 days, traded on April 7, 2004 in Chicago Mercantile Exchange. (c) Implied volatilities (the points) and the fitted curve of the soybeans options with time to maturity 137 days, traded on January 15, 2004 in Chicago Board of Trade. Notice that the three data sets are different in the range of moneyness and that of implied volatility.

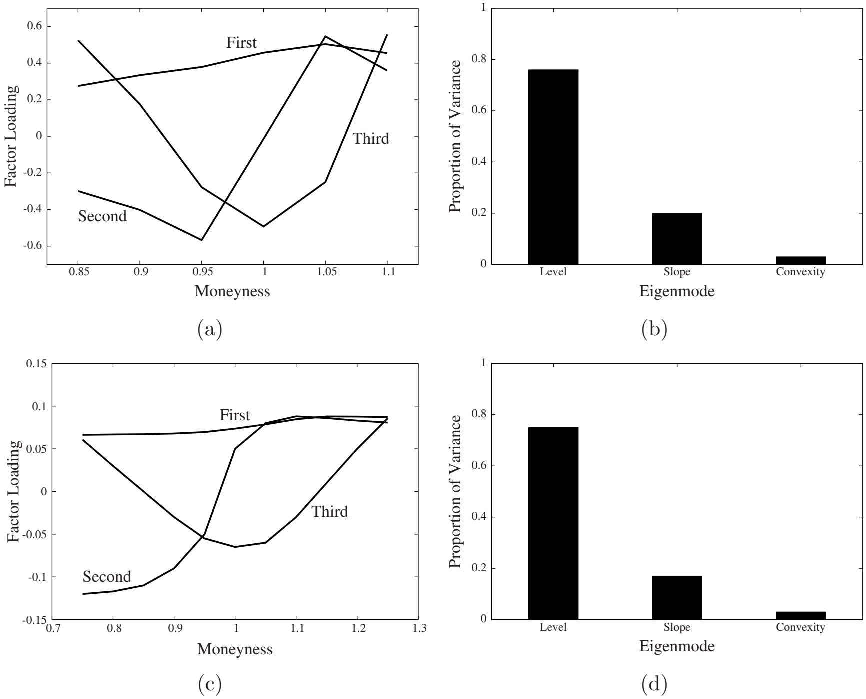


Figure 2.3: Empirical principal components and their proportions of variance. (a) Principal components calculated using the daily IV time series of European style options on DAX index for the entire year 1999, reported by Fengler et al. (2003); (b) Corresponding proportions of variance explained by the principal components. (c) Principal components obtained from the daily IV time series of options on S&P500 index over the period from December 1999 to October 2002, calculated by Rexhepi (2008); (d) Corresponding proportions of variance explained by the principal components. Notice that (a) and (c) are different in the range of moneyness and that of factor loading.

(2004), and Ederington and Guan (2002), among many others.

The existence of a volatility smile conflicts with the BS framework. To account for the associated deviations of option prices from the BS formula, models based on processes for underlying assets other than geometric Brownian motion[1] such as stochastic volatility and jump diffusion processes, have been proposed (Hull, 2003; Rebonato, 2004). Some models of this type are described in Section 2.2.2. They can include the smile effect on the valuation of options (Cont and Tankov, 2004; Gatheral, 2006). However, although these models may agree on option prices today, they can differ in the future prices (Bakshi et al., 1997). This disagreement on the so-called implied smile dynamics results in different prices of exotic options. In addition, it is important to realize that although traders may agree on the best model to use for a certain product, they can still disagree about the inputs of the model such as the volatility parameter mentioned above. This will lead to different prices for the same option contract where clients will most likely select the best offer, indicating that the market prices of options are ultimately determined by supply and demand. Finally, an important fact to note is that the volatility skew in the equity option market has only been observed after the stock market crash of October 1987 (Hull, 2003), while the non-Gaussian distribution of price changes[2] has been empirically identified in financial time series accumulated since at least the beginning of the 20th century (Mandelbrot, 1963). From these perspectives, the adoption of generalized stochastic processes in option pricing may not explain the main cause of the smile. Therefore, the origin of this phenomenon remains unclear.

During the last few decades, a series of formal studies have been undertaken to explain market anomalies based on the behavior of market participants and some of the work involves the volatility smile. Recent years have also witnessed active investigations of market dynamics through microsimulation, and a few MS-based studies address the smile phenomenon. Some of these models are introduced in

[1] In the BS model, a geometric Brownian motion is assumed for the price dynamics of the underlying asset.

[2] The geometric Brownian motion implies a Gaussian distribution of price changes, while stochastic volatility and jump diffusion processes indicate a non-Gaussian distribution.

Section 3. While these studies have clearly demonstrated the potential of behavioral and agent-based approaches for studying the origin of the smile phenomenon, they have not reproduced or realistically explained IV curves of options on different types of underlying asset, such as the upward sloping skew observed in commodity options markets. In addition, they have not confirmed the dynamical properties of the smile.

2.2 Empirically consistent models

In the literature, some analytical models have been developed in order to capture the complex dynamics observed in asset prices. In this section, we describe those that are most widely applied.

2.2.1 Models of price dynamics

The ubiquity of fat-tail distributions and volatility clustering has stimulated a great deal of theoretical work for developing models more consistent with empirical time series. Among the many models of this type, the ARCH model that was introduced by Engle (1982) and the GARCH model proposed by Bollerslev (1986) are the most widely used.

ARCH stands for 'autoregressive conditional heteroscedasticity'. To be specific, an ARCH(q) process is defined as

$$\sigma^2(t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2(t-i), \tag{2.7}$$

with

$$\varepsilon(t) \sim \mathcal{N}(0, \sigma^2(t)). \tag{2.8}$$

Here $\mathcal{N}(0, \sigma^2)$ represents a normal distribution with mean 0 and standard deviation σ, $\alpha_i (i = 0, 1, ..., q)$ are positive parameters, and the random variable $\varepsilon(t)$ is drawn from a normal distribution with mean 0 and time-dependent standard deviation $\sigma(t)$. Here $\sigma^2(t)$ is determined by the last q realizations of $\varepsilon(t)$.

GARCH represents 'generalized autoregressive conditional heteroscedasticity'. A GARCH(p, q) process is defined as

$$\sigma^2(t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2(t-i) + \sum_{i=1}^{p} \beta_i \sigma^2(t-i), \tag{2.9}$$

where $\sigma^2(t)$ is additionally dependent on its last p values.

By varying the value of q, we can control the memory effect. By including the memory of $\sigma^2(t)$ itself, the GARCH model can overcome the difficulties of the ARCH model in the optimal determination of the $q+1$ parameters (Mantegna and Stanley (2000)). Importantly, although the process of $\varepsilon(t)$ is chosen to be Gaussian, the asymptotic PDF presents a degree of leptokurtosis. Therefore, ARCH and GARCH models are simple models able to produce empirically consistent time series of asset price. Bera and Higgins (1993) remarked that a major contribution of the ARCH/GARCH literature is the finding that the changes in volatility may be predictable and result from a specific type of nonlinear dependence rather than changes in exogenous variables.

Through parameter estimations and variations on the basic model, ARCH and GARCH models can be effective for forecasting volatility. Therefore, they have widely and successfully applied in finance, of which the volatility is a central issue because financial decisions are generally based upon the tradeoff between risk and return.

However, these models are not compatible with all of the empirical properties of price fluctuations (Farmer (1999)). Conventional ARCH-type models are incompatible with the scaling properties of price fluctuations: They may fit at a given timescale but do not work well for explaining the volatility at a different timescale (Farmer (1999) and Mantegna and Stanley (2000)). In addition, conventional ARCH models do not have asymptotic power-law decay in the volatility autocorrelation function presented in empirical financial time series (Farmer (1999), Mantegna and Stanley (2000), Cont (2005)).

2.2.2 Option pricing models

The volatility smile phenomenon has attracted considerable attention in financial economics. In order to more realistically price options, many new alternatives to the Black-Scholes model have been proposed. In these models, some of the restrictive assumptions of the BS framework have been relaxed. For example, they allow volatilities, interest rates, or price jumps to be stochastic.

However, Bakshi et al. (1997) empirically examined several alternative models and concluded that, judged on consistency of implied parameters, all these models are misspecified. For example, the required level of volatility variation is implausibly too high. Rebonato (2004) reported that financially plausible models cannot produce good fit to the market prices of options, while models that do generate the smile are not financially convincing. In addition, various models are often combined in order to achieve a better fit to market data. In this case, models are no longer parsimonious, and it might be difficult to judge whether a good fit indicates a well-specified modeling approach, or is simply the result of the flexibility brought by the larger number of parameters.

Here we describe and examine some specific alternative efforts that focus on refining the volatility process. There are two broad approaches of this type. In *local volatility* models (Rebonato (2004)), the volatility process is represented by a deterministic function of the asset price and time,

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dz, \tag{2.10}$$

in which the terms $\mu(S_t, t)$ and $\sigma(S_t, t)$ are respectively the drift and the volatility of the stochastic process, and dz is a Wiener process or Brownian motion.

In *stochastic volatility* models, the volatility dynamics is described as a second stochastic process V_t in addition to the price process S_t,

$$dS_t = \mu_S(S_t, \sqrt{V_t}, t)dt + \sigma_S(S_t, \sqrt{V_t}, t)dz_1, \tag{2.11}$$

$$dV_t = \mu_V(S_t, V_t, t)dt + \sigma_V(S_t, V_t, t)dz_2, \tag{2.12}$$

$$E[dz_1 dz_2] = \rho dt \tag{2.13}$$

where dz_1 and dz_2 are two Wiener processes with constant correlation factor ρ.

We describe two typical local volatility models, i.e. the displaced diffusion model that was developed by Rubinstein (1983) and the constant elasticity of variance (CEV) model proposed by Cox and Ross (1976), and one typical stochastic volatility model, i.e. the SABR (Stochastic Alpha, Beta, Rho) model introduced by Hagan et al. (2002). In addition, we show the experimental results obtained by using these models, with regard to their in-sample and out-of-sample performances. The former shows how well they fit the empirical IV curve of each day; the latter indicates how well the models can fit the empirical curves of other days, by using the parameters obtained from the empirical data today.

Displaced diffusion model

Fundamental to the displaced diffusion model is the displaced-diffusion process requiring that the quantity $S_t + a$, rather than S_t, should follow a geometric Brownian motion:

$$\frac{d(S_t + a)}{S_t + a} = \mu_a dt + \sigma_a dz_t, \tag{2.14}$$

where μ_a and σ_a are respectively the percentage drift and volatility of $S_t + a$, in which a is a positive constant termed the displacement coefficient.

Due to the properties of geometric Brownian motion, only $S_t + a$, rather than S_t, is guaranteed to be positive. Since $a > 0$, we have $-a < S_t < +\infty$, indicating an obvious drawback of this model: S_t can be negative, in odd with the definition that S_t is the price of the underlying which should be strictly positive.

Leaving out the drift term, we start from

$$dS_t = \sigma_{abs} dz_t + \sigma_{log} S_t dz_t, \tag{2.15}$$

where σ_{abs} is the absolute responsiveness to the Brownian motion and σ_{log} is the responsiveness to the Brownian motion proportional to S_t. Equation (2.15) can be written as

$$dS_t = \sigma_{log}(S_t + \frac{\sigma_{abs}}{\sigma_{log}}) dz_t. \tag{2.16}$$

Since $d(S_t + a) = dS_t$, Equation (2.16) can be expressed as

$$\frac{d(S_t + \frac{\sigma_{abs}}{\sigma_{log}})}{S_t + \frac{\sigma_{abs}}{\sigma_{log}}} = \sigma_{log} dz_t. \tag{2.17}$$

Equation (2.14) is obtained by equating $\frac{\sigma_{abs}}{\sigma_{log}}$ with a and σ_{log} with σ_a. It can be seen that as a goes to zero, the process will behave similarly to a log-normal process and as a goes to infinity the process becomes a normal distribution. Option prices are given by the BS formula with the exception that the stock price is $S_t + a$ and the strike is $K + a$,

$$C_{BS}(S_t + a, t) = (S_t + a)N(d_1) - (K + a)e^{r\tau}N(d_2) \tag{2.18}$$

$$d_1 = \frac{\ln \frac{S_t+a}{K+a} + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \tag{2.19}$$

$$d_2 = d_1 + \sigma\sqrt{\tau} \tag{2.20}$$

where all the other parameters are identical to those in Equation (2.3).

As shown by Rebonato (2004) and Rexhepi (2008), the IV curves produced by the displaced diffusion model are downward sloping. To some extent, they agree with the empirical IV curves of options on equities. However, in order to fit the model to the empirical skews, the value of the displacement coefficient a is typically larger than the value of the underlying asset S_t, unrealistically indicating that S_t follows a Gaussian-like distribution with a large probability of taking a negative value. In addition, both the in-sample and out-of-sample performance of this model are poor (Rexhepi (2008)).

Constant elasticity of variance model

Inspired by the empirical fact that prices of underlying assets and corresponding implied volatilities display a strong inverse relationship, the so-called *leverage effect*, the constant elasticity of variance (CEV) model assumes that the underlying follows the following process,

$$dS_t = rS_t dt + \sigma S_t^{\alpha} dz_t, \tag{2.21}$$

where r is the risk-free interest rate, σ is a volatility parameter, α is a positive constant, and dz_t is a Wiener process.

The volatility of the underlying is therefore $\sigma S_t^{\alpha-1}$. When $\alpha = 1$, the process reduces to a geometric Brownian motion. When $\alpha < 1$, the volatility increases as the price of the underlying decrease. This creates a probability distribution similar to that observed in equity markets which is characterized by a heavy left tail and a less heavy right tail: As the price of the underling decreases, the volatility increases, making even lower prices more likely; when the price of the underling increases, the volatility decreases, making higher prices less likely (Hull (2003)). This produces the leverage effect observed in equity options.

The CEV option pricing formula can be expressed in terms of the non-central chi-square distribution. For $0 < \alpha < 1$, prices of call and put options are

$$c = S_0 e^{-qT}[1 - \chi^2(a, b+2, c)] - Ke^{-rT}\chi^2(c, b, a), \tag{2.22}$$
$$p = Ke^{-rT}[1 - \chi^2(c, b, a)] - S_0 e^{-qT}\chi^2(a, b+2, c), \tag{2.23}$$

where

$$a = \frac{K^{2(1-\alpha)}}{(1-\alpha)^2\sigma^2 T}, \quad b = \frac{1}{1-\alpha}, \quad c = \frac{(Se^{(r-q)T})^{2(1-\alpha)}}{(1-\alpha)^2\sigma^2 T}. \tag{2.24}$$

When $\alpha > 1$,

$$c = S_0 e^{-qT}[1 - \chi^2(c, -b, a)] - Ke^{-rT}\chi^2(a, 2-b, c), \tag{2.25}$$
$$p = Ke^{-rT}[1 - \chi^2(a, 2-b, c)] - S_0 e^{-qT}\chi^2(c, -b, a). \tag{2.26}$$

Here $\chi^2(z, v, k)$ is the cumulative probability that a variable with a non-central χ^2 distribution with non-centrality parameter v and k degrees of freedom is less than z.

As shown in Rexhepi (2008), the CEV model can generate a IV skew when the value of α is small. The skew flattens out as the α value increases. This model performs better than the displaced diffusion model in in-sample fitting, although still poorly because the skewness obtained is much smaller that the empirical one. In addition, the model performs poorly in out-of-sample fitting.

SABR model

The SABR model describes a forward price of the underlying F_t, of which the volatility is described by a parameter σ_t. The time evolution of F_t and σ_t is defined as the following system of stochastic differential equations,

$$dF = \omega_t F_t^\beta dz_1, \tag{2.27}$$
$$d\omega_t = \alpha\omega_t dz_2, \tag{2.28}$$
$$E[dz_1 dz_2] = \rho dt, \tag{2.29}$$

with

$$F_0 = f, \tag{2.30}$$

$$\omega_0 = \alpha, \tag{2.31}$$

where ω_t is the volatility process and ρ is the correlation coefficient of the two Wiener processes.

According to the SABR model, the price of a European option is given by the Black-Scholes formula

$$\begin{aligned} C_{BS}(S^t, t) &= S^t N(\phi d_1) - K e^{-r\tau} N(\phi d_2), \\ d_1 &= \frac{ln(S^t/K) + (r + \sigma_B^2/2)\tau}{\sigma_B\sqrt{\tau}}, \\ d_2 &= d_1 - \sigma_B\sqrt{\tau}, \end{aligned} \tag{2.32}$$

where S^t, K, r, τ, and $N(x)$ are identical to the corresponding terms in Equation (2.3). The implied volatility σ_B can be approximated as,

$$\begin{aligned} \sigma_B(f, K) = \frac{\alpha}{f^{1-\beta}}\{1 - \frac{1}{2}(1 - \beta - \rho\lambda)\ \log(\frac{K}{f}) + \frac{1}{12}[(1-\beta)^2 \\ +(2 - 3\rho^2)\lambda^2]\ \log^2\frac{K}{f} + \cdots\}, \end{aligned} \tag{2.33}$$

provided that the strike price K is not too far from the current forward (Hagan et al. (2002)). The parameter

$$\lambda = (\frac{\upsilon}{\alpha})f^{1-\beta} \tag{2.34}$$

is the ratio that measures the volatility of volatility at the current forward.

The SABR model has four parameters: α, β, υ, and ρ. α mainly controls the overall level of the IV curve, ρ and β determines the skewness of the curve, and υ controls the convexity.

As shown in Rexhepi (2008), the SABR model can produce strong IV skews and can fit in-sample IV curves better than the previously described local volatility models. The main reason for the better fit lies in the fact that the SABR have more parameters, enabling it to more flexibly fit complicated curves. With respect to out-of-sample fitting, the SABR model performs very well in fitting the empirical data of the days not so far from the sample day, but poorly in fitting the empirical IV curves of the days longer than a few days apart.

2.3 Problems of standard financial theories

Neoclassical economics stands for a general approach to economics that relates supply and demand to representative market participants with rational preferences and expectations. It defines itself as primarily the study of the allocation of resources and presumes that the market mechanism will lead to a general equilibrium. This approach was developed in the late-nineteenth century and represented the dominant tradition of economic theory throughout most of the twentieth century. Neoclassical economics has received widespread criticisms that mainly concern its adoption of many unrealistic assumptions. (See Farmer (1999), Mantegna and Stanley (2000), Tesfatsion (2002), and Hommes (2006)).

Inheriting the characteristics of the earlier neoclassical economics, the standard (mainstream) finance, which was shaped in the second half of the twentieth century, made great achievements evidenced by the births of the capital asset pricing model (CAPM) (Sharpe (1964)), efficient markets hypothesis (EMH) (Fama (1970)), and the Black-Scholes option pricing model (Black and Scholes (1973); Merton (1973)), among many others.

CAPM is an economic model for determining the required return rate of an asset, based on the idea that the investor demands the time value of the investment and a risk premium. The former is represented by the risk-free rate, while the latter is the product of a risk measure, i.e. the sensitivity of the returns of the asset to market returns, and a market premium, i.e. the difference between the expected market return and the risk-free rate. If the expected return rate is not equal or greater than the required rate, the investor will not perform the investment.

EMH states that at any given time, security prices already reflect all known information, whereas future price movements are determined entirely by new information and cannot be predicted by using past price movements. Therefore, prices must follow a Markov process. Because price movements do not follow any patterns or trends, it is impossible to consistently outperform the market, except through luck or taking riskier investments.

The BS model, as described in Section 2.1.2, is used for pricing European put and call options. The fundamental idea behind this model is to replicate an option using positions in the underlying stock and bonds. The fair value of the option is equal to the cost of setting up such a replicating portfolio. Two important assumptions of this model are (1) markets are efficient and arbitrage-free[1], and (2) the returns on the underlying stock are normally distributed. The first assumption implies that people cannot consistently predict the direction of each stock and share prices following a Markov process. Loosely speaking arbitrage-free means that there are no trade strategies that will result in a sure profit with a zero investment today.

Frequently, implications and expectations of standard financial theories do not agree with empirical observations. For instance, CAPM and EMH expect steady price movements corresponding to a Gaussian return distribution with time-independent variance (Sharpe (1964)), while empirical returns follow a non-Gaussian, leptokurtic distribution and their absolute values are with temporal correlations, as described in Section 2.1.1. Yet another example is that the BS model expects identical implied volatilities for all options on the same underlying, whereas real implied volatilities are different across strike and change over time, as depicted in Section 2.1.2.

Two widely accepted explanations, among numerous plausible ones, for the deviations of standard theories from empirical observations are discussed below.

2.3.1 Adoption of unrealistic assumptions

The mainstream financial economics relies on a top-down construction based on a number of unrealistic assumptions mainly for the sake of analytical tractability (Farmer (1999), Mantegna and Stanley (2000), Tesfatsion (2002), Hommes (2006)).

Standard theories typically assume that the aggregate effect of the participants in a market can be replaced with that of a so-called *representative* agent who maximizes an expected utility function[2]. A representative agent is therefore an

[1]The no arbitrage condition states that in a financial market it should not be possible to make a profit with zero net investment and without bearing any risk.

[2]Briefly, utility is a synonym for individual welfare, i.e., satisfaction from consumption of goods and services; a utility function is a representation expressing the relationship between the utility and the consumption (Black et al. (2002))

idealized trader whose behavior represents the aggregate action of all agents. Through this aggregation, economists avoid the difficulties of modeling a group of traders and wish to derive the behavior of a whole market.

They also assume that traders are rational, i.e. they have clear preferences, make identical unbiased forecasts about the future, and perform actions to maximize their chances of success. In addition, traders are considered as efficient machines who have unlimited computing abilities enabling them to process any inflow of new information and make optimal choices instantaneously.

There are some other assumptions commonly made in standard financial theories. For example, the returns of traded assets follow a normal distribution, traders have identical beliefs, all relevant information is known to all parties involved, and markets are frictionless, etc. (Farmer (1999), Mantegna and Stanley (2000), Tesfatsion (2002), Hommes (2006)).

All these assumptions have received severe criticisms. For example, under the assumption that traders are rational and with homogeneous beliefs, all investors will hold the same market portfolio and there will be no trade, contradicting the large trading volumes recorded frequently in real markets. Another important point is that, it is very difficult to obtain analytical results if even one of the assumptions of a model is relaxed, and the more is a model adjusted to represent the real-life situation, the less is it analytically tractable. In addition, it is hard to figure out analytically the effect of the relaxation of each of the unrealistic assumptions of a model on its equilibrium results (Levy et al. (2000)).

Here we again take the BS model as an example to indicate that some of its assumptions are indeed unrealistic and how they affect real world financial operations, in particular, risk management. A detailed explanation of this point can be found in Johnson et al. (2003). The BS model is widely used for creating delta-neutral portfolios, i.e., portfolios that are insensitive to changes in the value of the underlying. For example, option writers are exposed to large potential losses, so they usually hedge their position by buying a certain quantity of the underlying asset. By keeping this portfolio neutral to changes in the price of the underlying, an option writer can hedge away the risk associated with the short position in the option. The quantity of underlying is determined by *delta*, which is represented as the partial derivative of the option's fair value with respect to the price of the underlying and usually calculated using the BS formula. Importantly, delta is a function of the price of the underlying and therefore changes over time. Consequently, the option writer's position remains delta neutral only for a short period of time and the hedge has to be adjusted periodically.

According to the BS model, it is therefore possible that, through this dynamic-hedging scheme, the variation of the option writer's wealth always remains zero. Some of the main assumptions adopted by the BS model that guarantee zero risk are (1) there are no transaction cost, (2) the trading is continuous, and (3) the price of the underlying follows a Geometric Brownian motion. In reality, however, all these assumption are questionable. Cost is always present and it gives rise to a barrier to high-frequency trading: The greater the frequency of re-hedging, the greater the cost. In addition, as discussed in Section 2.1, returns of the underlying do not follow a Gaussian distribution.

Bouchaud and Potters (2000) and Johnson et al. (2003) examined the variation of the option writer's wealth when the price of the underlying changes over time, by studying the basic components of the wealth. They focused on the two basic components, i.e., the payoff, which the option writer must give to the option holder at the expiry of the option, and the hedging profit, which comes from the profit or loss realized on the purchased quantity of the underlying asset. In order to investigate the effects of discrete hedging on the risk of writing an option, they simulated repeatedly the process of writing and hedging an option, under different schemes of hedging, different underlying asset movements, and different option types. At each re-hedging time, the hedge quantity of the underlying is calculated using the BS delta-hedging recipe. In the case that the price of the underlying follow random walk, the simulation results showed that as the frequency of re-hedging increases, the spread in the variation of wealth decreased, meaning less risk for the option writer. Specifically, as the trading time reduces to zero, the spread in the distribution of wealth variation also reduces to zero, recovering the BS result. However, by using a slightly more realistic model for the price movement of the underlying, e.g., a process with stochastic volatility, the simulation results showed a marked increase of risk for all trading times. Importantly, reducing the trading time to zero no longer gets the zero-risk result.

Johnson et al. (2003) further addressed the crucial issue of managing portfolio in the presence of non-zero transaction costs. The more frequently the option is hedged, the more risk can be eliminated, however, the more cost is paid. Hence, the option price can only be minimized by balancing the reduction of risk with the increase in transaction cost, but not by neglecting both of them as carried out in the deriving of the the BS model.

2.3.2 Evasion of endogenous mechanisms

By adopting a representative agent whose behavior represents the actions of all the traders in a market, standard economists have implicitly adopted the method of reductionism, through which the dynamics of a system is described simply as the sum of the dynamics of its components (Tesfatsion (2002), Hommes (2006)).

However, the integrated behavior of a complex system generally cannot be deduced by simply summing the behavior of the components. In fact, the most complex behavior of a system usually arises from the interactions among its components, not from the complexity of extraneous factors or that of the components themselves (Mantegna and Stanley (2000), Voit (2003)).

Financial markets are characterized by the completely lack of linearity. If we break down a market into individual traders who do not interact, the market ceases to exist. In addition, traders of distinct types are very different in trading interest and trading behavior, the aggregate effect of their actions can hardly be translated to that of any sensible types of representative agent. In studying the complex dynamics of financial markets, the approach of reductionism can hardly work.

In fact, some mainstream financial economists have realized the importance of the interactions between market participants and the feedback between micro and macro market structures. However, they have avoided modeling them explicitly. A reasonable explanation for this evasion is that, for a long period of time, economists lacked the means to handle the modeling of trading behavior in real markets (Tesfatsion (2002)). Economists have therefore turned to model a financial market based on its macro-level manifestations. However, economists in this track have realized that the same data might lead to wildly divergent models performing equally well and, after all, the conclusions based on such models are not robust (Sterman (2000)).

As a typical and most successful neoclassical model, the BS model is a *principle theory*, in that the various principles going into their model were given the status of postulates, and no underlying mechanisms for the phenomena were elucidated. By contrast, a *constructive theory* is derived from the data and are physically well-founded by providing basic mechanisms for the phenomena. Rickles (2008) explained the distinction between these two types of approach by citing thermodynamics as well as neoclassical economics (on the 'principle' side), and statistical mechanics as well as econophysics (the 'constructive' side).

In deriving principle theories, some general principles that were assumed to be universally valid are taken and conclusions are drawn about the nature of the object under study based on these universal assumptions. In physics, principle theories, such as Einstein's theory of relativity, were not hypothesis built on data reached through experimentation, but were universal principles intended to impact all of physics. However, the conclusions have to be compared with empirical data and, if they are incompatible, the theories must be trashed or amended.

Many neoclassical economists have taken a principle-theory-type approach but have not been aware of some serious pitfalls. Here we again take the BS model as an example. The model takes some plausible principles which the markets are expected to follow, e.g., the no-arbitrage condition. In the meantime, however, it adopts some unrealistic assumptions, such as zero transaction cost, continuous trading, and the normal distribution of returns. More importantly, results calculated by using this model do not agree with empirical data, as illustrated in the volatility smile phenomenon described in Section 2.1.2 and the non-zero risk discussed in Section 2.3.1. Due to these flaws, it is highly doubtful whether the current application of the principle-theory-type approach in the main stream theories is valid.

The current (2008-2009) financial crisis highlights the shortcomings of standard financial theories. Buchanan (2008) pointed out that the very reason of the financial turmoil is that economists still try to understand markets by using ideas from traditional economics, especially the so-called equilibrium theory. This theory views markets as reflecting a balance of forces and changing only in response to new information, but totally neglects the internal dynamics of the markets themselves. Lohr (2008) argued that risk management models failed to keep pace with the explosive growth in complex securities. Farmer and Foley (2009) stated that the best models the policy makers have are all with fatal flaws: They assume a perfect world and by their very nature rule out crises. The policy makers are basing their decisions on common sense, but do not understand how the economy really works.

Chapter 3

Modern Approaches to Financial Modeling: Heterogeneity, Irrationality and Interactions

As discussed in Chapter 2, the mainstream financial theory has encountered difficulties in explaining many phenomena in real markets. The disagreements between the standard theories and empirical data have stimulated researchers to re-examine the foundation of the traditional theories. This has enabled the development of some alternative theories related to economic or financial systems and phenomena. In the meantime, a few new research fields have accordingly emerged: Behavioral finance, agent-based simulation, and econophysics. In this chapter, we will present a brief overview of these developments.

3.1 Behavioral finance

Many researchers believe that the problems of the mainstream theories discussed above stem from the unrealistic assumptions adopted in the representative agent paradigm, i.e., agents are homogeneous and rational, in the sense that they make unbiased forecasts about the future in response to new information and correctly make decisions to maximize their expected utilities (Hommes and Wagener (2009)). This has tremendously promoted the development of a new paradigm in finance, i.e. behavioral finance.

Behavioral finance proposes psychology-based theories and concerns bounded rationality as well as behavioral heterogeneity. It attempts to understand investors' reasoning patterns and psychological influences on decision-making processes. Behavioral finance argues that people act rationally only to a limited extent and some financial phenomena can be better understood by using models in which agents are not fully rational. It therefore helps in explaining why and how markets might be inefficient. Barberis and Thaler (2003) contended that the field has two building blocks: Psychology and limits to arbitrage.

Psychology

Based on extensive experimental evidence compiled by cognitive psychologists, researchers have identified some specific forms of irrationality, characterized by some systematic biases that arise when people form beliefs and preferences.

Psychologists have learned that, when forming beliefs, people are overconfident in their abilities and judgments (Weinstein (1980), Alpert and Raiffa (1982), Fischhoff et al. (1977)). In addition, once people have formed opinions, they adhere to them persistently, being reluctant to search for evidence that contradicts their beliefs or treating such evidence with excessive skepticism (Lord et al. (1979)).

Traditional models for understanding asset prices are based on the assumption that investors evaluate investments according to their expected utility (EU) values, the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities. Unfortunately, experiments have shown that people systematically violate the EU theory when choosing among risky investments. Researchers in behavioral finance have further argued that some of the lessons we learn from these violations are central to understanding many financial phenomena. Among all the non-EU theories that have been developed, prospect theory is the most successful one in the sense that it can capture experimental results and is promising for financial applications (Barberis and Thaler (2003)). In fact, it won the Nobel price in economics in 2002.

Developed by Kahneman and Tversky (1979), prospect theory describes how people make choices when facing alternatives with uncertain outcomes, of which the probabilities are known. It considers preferences as a function of 'decision weights', and states that people do not always behave rationally, so that the weights do not always match the probabilities. Specifically, people tend to over-weight small probability events but under-react to those with moderate and high probabilities. It also proposes that agents value gains and losses differently and are more sensitive to losses than gains (loss aversion). In addition, agents assign values to gains and losses rather than final assets. The curve of values against gains is normally concave (risk aversion), while the value curve for losses is commonly convex (risk seeking)

Limits to arbitrage

Another debate between the traditional framework and behavioral finance lies in market efficiency. The former asserts that prices already reflect all known information related to the fundamental values of assets and it is impossible to consistently outperform the market except through luck, while the latter argues that asset prices often deviate from their fundamental values, caused by traders who are not fully rational.

Defenders of the traditional theory claim that rational traders can quickly bring prices back to fundamental values through arbitrage (Friedman (1953)). This is refuted by advocates of behavioral finance who argue that even if an asset has been wildly mispriced, strategies designed to correct the mispricing can be both risky and costly, allowing the mispricing to persist. Therefore, the presence of mispricing does not imply that of a profitable investment strategy, i.e., prices can be very wrong without creating profit opportunities.

A phenomenon related to limits to arbitrage is documented by Harris and Gurel (1986) and Shleifer (1986). When stocks were added to indexes, their prices jumped, even seemingly permanently. In this example, on the one hand, obviously there is mispricing because the prices changed even though the fundamental values did not. On the other hand, the mispricing appears to be persistent, threatening arbitrageurs. For more examples from real markets about this issue, see Barberis and Thaler (2003).

Behavioral finance has been applied to explain many regularly occurring anomalies that are inconsistent with standard economic theories. For example, Siegel and Thaler (1997) tried to resolve the equity premium puzzle[1] by employing prospect theory.

Proponents of behavioral finance has responded to some objections offered by supporters of EMH (efficient market hypothesis). Discussions of this issue can be found in, e.g., Subrahmanyam (2007) and Rabin (1998). One of the objections is that behavioral models are designed to explain specific stylized facts. The response is that behavioral models are based on how people actually behave and explain market phenomena in a better way than traditional ones.

Another criticism is about the experimental and survey based techniques which are used extensively in behavioral economics. Many traditional economists are distrustful of results obtained in this manner due to the difficulty of avoiding systemic biases. The response is that the results are reproduced in various situations and markets and they can help reveal some hidden regularities.

There is also an important objection that behavioral finance presents no unified theory unlike the mainstream one. The reaction is that this may well be true at this point, but models of bounded rationality are both possible and much more accurate in describing behavior than purely rational models. In fact, there has already been a burst of theoretical work modeling financial markets with less-than-fully-rational agents (Thaler (1999)). In addition, traditional theories should not be superior to behavioral approaches because the former are not supported by empirical data; specifically, the former discuss how people *should* rather then *actually* behave. Thaler even predicted that in the not-too-distant future, the term 'behavioral' will be a redundant phrase and economists will routinely incorporate as much behavioral factors into their models (Thaler (1999)).

Some of the work of behavioral finance deals with the understanding of the unexpected phenomena observed in financial markets.

[1]Coined in 1985 by Mehra and Prescott (1985), the equity premium puzzle states that historical real returns from stocks are much higher than the real returns from government bonds, and the differences cannot be well explained by the general utility-based theories. This puzzle has attracted extensive research effort in economics and finance, and is still inspiring ongoing debates since a generally accepted solution remains elusive.

3.1.1 Stock markets

Maymin (2009) demonstrated that investors who evaluate risky assets based on prospect theory will often induce high kurtosis, negative skewness, and persistent autocorrelation in return distributions even when the underlying business risk follows a random process and has no extremes. The mere assumptions are loss aversion, i.e., losses are about twice as painful as gains are pleasant, and mental accounting, i.e., evaluating assets based on their past performance. The author therefore suggested that it is investors' trading characterized by psychological bias that is causing the extreme events, not the underlying business risk. Specifically, traders' incorporation of prior gains and losses into evaluations of future prospects may be part of the explanation for the excess-volatility phenomenon observed in real markets, i.e., markets tend to move too much relative to the volatility of the underlying earnings.

McQueen and Vorkink (2004) pointed out that, while much effort has been undertaken to analytically model volatility clustering and the current statistical knowledge of this phenomenon is impressive, our theoretical knowledge of why volatility clusters is very poor. They proposed a preference-based asset pricing model which is claimed to be able to explain many empirical facts in finance. In the model, agents care about wealth changes, experience loss aversion, and keep a mental scorecard that affects their level of risk aversion. They become temporarily more sensitive to news when perturbed by unexpected returns. In addition, their utility increase from gains are smaller than their utility decrease from losses, i.e. loss aversion. It is claimed that the state-dependent sensitivity to news creates volatility clustering and is empirically supported.

3.1.2 Options markets

In option pricing theory, the future value of any security is obtained by integrating its payoff function with respect to the risk-neutral density function. Shefrin (2008) pointed out that the traditional risk-neutral approach to option pricing has led to the view that option prices are independent of investors' beliefs. However, as options are naturally structured as contingent payoffs, they are inevitably impacted by investors' beliefs. In particular, since sentiment measures the degree of bias in the representative investor's probability density function, it acts as a proxy to explain how investors' behavioral and psychological factors influence option prices in equilibrium. In particular, pessimists overestimate volatility and underestimate expected returns, while optimists underestimate the former and overestimate the latter. In addition, the overestimate of volatility associated with pessimists dominates prices of out-of-the-money puts, and the underestimate of volatility associated with optimists dominates prices of out-of-the-money calls. Importantly, Shefrin (2008) presented a behavioral counterpart to the BS formula with a closed-form solution, and illustrates that the volatility smile is a feature of the behavioral framework.

Shefrin (1999) and Shefrin (2008) took a specific example to illustrate how investors' sentiment can influence option prices and induce smile patterns. The implied volatilities of the options traded in the period from November to December 1996 were studied. In this period, the investors' sentiment, which is represented by some indexes, experienced a process from being highly positive to being very negative.

This study stated that disagreement among investors is pervasive. Having seen a price trend, some investors predict continuation, while others predict reversal. Some investors overreact, while others underreact. The disagreement causes markets to be inefficient. Options markets are particularly vulnerable in this respect and the volatility smile is one manifestation of the inefficiency. Through an event investigation by combining information collected from option prices with other information about market sentiment, the study assessed the impact of heterogeneous beliefs on market efficiency. The conclusion was that, when investors are sharply different in the level of optimism, bullish ones tend to take long positions in calls and bearish ones tend to take long positions in puts. This difference gives rise to the volatility smile; the sharper the difference, the stronger the smile.

3.2 Agent-based simulation

Traditional analytical approaches in finance and economics to study aggregate phenomena either are purely macroscopic, or rely on top-down construction based on a number of unrealistic assumptions mainly for the sake of analytical tractability. Interactions between traders play no role in the explanation of the phenomena (Tesfatsion (2002), Hommes (2006)). In fact, financial markets are decentralized systems comprising of large numbers of autonomous, adaptive and interacting agents. Their behavior and interactions give rise to macroeconomic regularities, including the patterns observed in financial time series. The macro-dynamics in turn feeds back to influence or determine the microscopic behavior and interactions. A market is therefore a network of many highly nonlinear causal chains, and its operation and behavior can hardly be studied analytically. The avoidance or omission of treating economic systems as complex systems may mainly account for the failure or deficiency of the mainstream economic theory's explanations of many market phenomena.

As stated by Tesfatsion (2002) and Hommes (2006), economics and finance are witnessing an important paradigm shift from the neoclassical modeling approach towards an agent-based approach based on computational models where markets are viewed as complex evolving systems with many autonomous, heterogeneous, interacting agents. Researchers of agent-based simulation (ABS) rely on computational laboratories to study the evolution of economic systems under controlled experimental conditions. In this paradigm, a financial market is constructed as a multi-agent system, of which the components, i.e. the agents are realistically modeled. In experiments, the initial attributes of the agents are first specified and the artificial market is then left to evolve over time without further intervention. Time series of some variables of the model, which may represent certain economic factors such as price and volume, are generated in this process. According to Hommes (2006), the following observations and developments have contributed to this paradigm shift,

- No trade theorem: If all agents are rational and each agent knows that other agents are rational, there will be no trade. The reason is that rational agents will not sell assets to any other rational agents when the latter want to buy, because the former think that the latter have superior private information. This is in odd with the large daily trading volume observed in real markets.

- Stock prices exhibit excess volatility, i.e., movements in stock prices are much larger than those in underlying economic fundamentals. In particular, it is difficult to explain the stock market crash in October 1987 by models with representative, rational agents.

- The traditional financial theories concern agents with perfect information about the environment and with unlimited computing abilities. This cannot be realistic because, in a heterogeneous world, a rational agent cannot know the beliefs of all other non-rational agents.

- A evolutionary approach has been advocated in the ABS paradigm, which can plausibly represent the process in which bounded-rational agents select from a large class of possible forecasting and trading strategies. The story is that agents select strategies according to how well they perform and how much they are used by others.

- Economists have applied new developments in nonlinear dynamics and complex system theory to study financial markets, which are almost always highly nonlinear and adaptive systems.

- Laboratory experiments have shown that individuals often do not behave rationally and this type of experiments have reinforced theoretical studies of markets with heterogeneous, non-rational agents.

- Empirical data showed that financial practitioners use different trading and forecasting strategies. In addition, some techniques can generate significantly positive returns, suggesting extra structure above and beyond the benchmark of efficient market hypothesis.

- Both in research and teaching, computational tools have become widely available and tremendously stimulated the development of numerical simulation of financial markets.

By nature, ABS is suitable for studying financial markets which are inherently multi-agent systems. ABS researchers can investigate how large-scale effects in financial markets arise from the behavior and interactions among many agents.

Specifically, as stated by Axelrod and Tesfatsion (2006), ABS researchers pursue four specific goals: Empirical, normative, heuristic, and methodological.

- The goal of empirical understanding focuses on the explanation of the emergence and persistence of global regularities despite the absence of top-down planning and control, specifically the governing causal mechanisms grounded in the repeated agent interactions.
- The normative goal centers on the application of agent-based models in the designs of desirable economic or financial systems, processes, and policies, in order to achieve higher efficiency of the entire systems despite continuous attempts by privately motivated agents to gain individual advantages.
- The heuristic goal is for the exploration of the deeper insights attained about the fundamental causal mechanisms in a wide range of economic systems.
- The methodological goal focuses on the discovery and development of general methods and tools need to undertake the rigorous study of economic systems through controlled computational experiments.

At present, however, most ABS research in finance focuses on understanding the empirically observed characteristics of financial markets. To achieve this objective, many agent-based models of financial markets have been developed during the last decade.

3.2.1 Stock markets

In view of the fact that financial prices exhibit some universal attributes that resemble the scaling laws characterizing physical systems in which large numbers of units interact, Lux and Marchesi (1999) questioned whether scaling in finance emerges in a similar way — from the interactions of a large ensemble of market participants. For this purpose, they developed a multi-agent model of financial markets consisting of two groups of traders: Fundamentalists and noise traders. Fundamentalists buy (sell) the asset when its market price is below (above) its fundamental value. Noise traders attempt to identify price trends and patterns, and imitate the behavior of other traders. The group of noise traders are further differentiated into optimists and pessimists. The former believe in a rising market and buy the asset, whereas the latter believe in a declining market and sell. The crucial point of this model is that agents may move to the other group or subgroup when they believe that the traders in that (sub)group are more successful. Price changes are modeled as endogenous responses of the market to imbalances between demand and supply. Although the news-arrival process in the model lacks both power-law scaling and any temporal dependence in volatility, fat tails and volatility clustering are generated through the interactions between the agents. Based on the results, the authors challenge the prevalent efficient market hypothesis which assumes that the movement of financial prices are an immediate and unbiased reflection of incoming news about future earning prospects, and support the idea that scaling arises from mutual interactions of market participants. A detailed description of this model and its simulation results can be found in Wang (2005).

Cont and Bouchaud (2000) presented a model of a speculative market to examine how the existence of herd behavior among market participants gives rise to large fluctuations in the aggregate excess demand reflected by a heavy-tailed non-Gaussian distribution, and how the excess kurtosis of returns and the average order flow are related. The agents in the model face three alternatives at each time period: To buy a unit of a financial asset, to sell a unit of the asset, or not to trade. They form coalitions through independent binary links between each other. The resulting market structure is then described by a random graph whose connected components (or clusters) correspond to groups of investors who act together, independently from other groups, to buy or sell. Each cluster of agents decides, independently from other clusters, whether to buy, to sell, or not to trade. Albeit its simplicity, the model can generate a probability distribution with heavy tails and finite variance, similar to empirical distributions of asset returns. In addition, analytical results based on the model indicate that the volumes of the order flow and the fluctuations of the asset price are negatively related, in line with the empirical facts that large price fluctuations are more likely to occur in less active markets and larger order flows enable market makers to more easily balance supply and demand. In brief, the model demonstrates a quantitative link between the two issues observed in financial markets: The heavy tails of return distributions and the herd behavior of financial traders. A detailed description of this model and its simulation results can be found in Wang (2005).

Cellular automata (CA) have been widely applied to study complex phenomena in different fields such as physics, chemistry, biology, and social and economic sciences, etc. (Talia and Sloot (1999)). Recently, some researchers have used cellular automata to model financial markets for studying their complex dynamics.

Iori (2002) employed a random field Ising model to describe the trading behavior of agents in a stock market. In the model, the agents are represented as the nodes of a square lattice connected to four nearest neighbors. At each time step, a given trader can take one of the three actions: Buying one unit of the stock, selling one unit, or remaining inactive. The decision making is driven by idiosyncratic noise and the influence of the nearest neighbors. A trade friction is further assumed which can be interpreted as a transaction cost. This friction is modeled as an individual activation threshold, only if exceeding which an agent's signal can trigger a trade. There is also a market maker who clear the orders and adjust prices. Three ingredients: Imitation, adjusting thresholds and variable rate of price adjustment, play a crucial role in generating volatility clustering. While the first ingredient can generate a large spike in trading volume, the second and third help propagate it through time. They together creates volatility clustering along with a positive volatility-volume correlation. This work supported the idea that power law fluctuations of asset prices are caused by the inherent interaction among market players and the trading process, rather than merely reflecting the probability distributions of exogenous shocks hitting the market.

Some agent-based models have been developed to explain the similar characteristics observed in some other types of financial market. For example, Chatagny and Chopard (1997) presented a microscopic agent-based model of the foreign exchange market. Two types of agent are included in the model: Market makers and speculative traders (chartists). The model is promising because it captures many of the qualitative features of the real markets.

However, researchers in this field have not yet reached an agreement on explaining the complex dynamics of financial markets. In addition, as recently pointed out by Cont (2005), due to the complexity of the existing (agent-based) models, it is often not clear which aspects of the models are responsible for generating the stylized facts and whether all their ingredients are indeed required for explaining empirical observations.

3.2.2 Options markets

Recently some agent-based studies have been undertaken to investigate the origin of the volatility smile phenomenon observed in option markets. Levy et al. (2000) pointed out the drawback of the BS model and most of its extensions is that the volatility of the underlying is assumed to be known and agreed about by all investors. They argued that if this is the case, there will not be any trading in options and that, in reality, however, the volatility is not known and investors have different and uncertain estimations of its value. Based on these ideas, the authors developed a MS model in which investors have some uncertainty about the volatility and may disagree about its distribution. Through the BS formula, this translates to uncertainty and disagreement about the values of options. Each investor believes that the volatility is distributed according to a probability distribution and thus the option price at the future time is a random variable. In order to maximize utility based on their distributions, the investors hold different numbers of an option and the equilibrium price of the option is the price for which the excess demand is zero. Due to the convexity of the plot of BS price against volatility, uncertainty of estimated volatility will induce higher option prices. The higher the convexity, the more significant the overpricing. The levels of convexity of in-the-money (ITM) and out-of-the-money (OTM) options are higher than those of at-the-money (ATM) and near-the-money options[1], therefore ITM and OTM options will be overpriced and their IVs are higher, indicating a volatility smile.

[1] An in-the-money, at-the-money, and out-of-the-money option would give the holder respectively a positive, zero, and negative cash flow if it were exercised immediately. Therefore, a call option is in the money, at the money, and out of the money when the price of the underlying asset is respectively greater than, equal to, and smaller than the strike price, while a put option is in the money, at the money, and out of the money when the former is respectively smaller than, equal to, and greater than the latter.

Platen and Schweizer (1998) presented a method for constructing diffusion models for stock prices explicitly incorporating the technical demand induced by hedging strategies. The price processes are with a stochastic volatility and imply option price distortions. Consequently, a U-shaped IV surface is obtained from the expected payoffs of the options with different strike prices. The important argument of this work is that the existing literature has predominantly focused on directly modeling volatility as a stochastic process in order to explain such option price distortions. But the description of the volatility process is typically ad hoc, i.e. based on the assumption that the stock or option prices cannot be affected by the trading of agents. It excludes the feedback process in which a substantial use of hedging strategies affects the dynamics of the underlying stock. In other words, the demand for a stock can be determined by the evolution of the stock price itself.

Very recently, Buraschi and Jiltsov (2006) reported both theoretical and empirical studies of a simple general equilibrium economy with two sets of risk-averse agents that are with rational learning capabilities but have different beliefs on expected returns and are uncertain about the stochastic drifts of some risk factors. In maximizing utility, their difference in expectation creates natural demands for insurance: Optimistic traders act as insurers of pessimistic agents writing out-of-the-money (OTM) puts in exchange for OTM calls. The model derives the optimal portfolio holdings in the underlying asset and the options as a function of the differences in belief. Then, in general equilibrium, a downward sloping IV skew is generated. Specifically, among many other findings, they conclude that changes in the difference of belief affect the level and steepness of the smile: The greater the difference, the higher and steeper the IV smile. In addition, the extent of most arbitrage violations are correlated with abnormal changes in the difference of belief. Furthermore, current levels of belief difference have positive and statistically significant predictive power for the future realized volatility

While these studies have clearly demonstrated the potential of MS-based approaches for studying the origin of the smile phenomenon, they have not reproduced or realistically explained IV curves of options on underlying assets of different types, such as the upward sloping skew observed in the commodity options market. In addition, they have not confirmed the dynamical properties of the smile.

3.3 Econophysics

The traditional view in economics characterized by agent rationality and market equilibrium has been challenged severely in recent years. Behavioral economists have advocated bounded rationality and presented evidence of frequent market anomalies; agent-based approaches have manifested the importance of agent heterogeneity and interactions for the dynamics of financial markets. However, as posed by Farmer (1999), is there a statistical mechanics that can explain some of the statistical properties of the market?

The complexity of financial markets has attracted a growing number of physicists and a new research community has emerged (Mantegna and Stanley (2000)). This field of research is known as *econophysics*, coined in 1995 by Eugene Stanley. Econophysicists argue that the empirical data can be studied by using the tools and methodologies developed in statistical physics and the stylized facts of financial markets are best understood as emergent properties of a complex system (Rickles (2008)). Specifically, the research activity in this field puts its emphasis on the empirical analysis of economic data. It is characterized by the concepts developed in statistical physics, such as scaling, universality, and self-organization, etc.

The minority game (Challet and Zhang (1997)) is one of the most successful econophysics models (Gallegati et al. (2006)). It was originally motivated by the El Farol bar problem (Arthur (1994)), in which a fixed number of agents face the question of whether or not to attend the bar. If less than half the agents turn up, the agents attended win. If the bar is too crowded, those agents who stayed home win. Agents make decisions based on the recent record of total attendance at the bar. The bar is analogous to a financial market in which those wining traders are generally in minority; in addition, traders try to forecast the aggregate because if everyone uses the same strategy it is guaranteed to lose. The minority game is a mathematical formulation of the El Farol problem.

At each time step, a fixed number of agents independently choose between two possibilities and the attendance is recorded. Each agent has a set of strategies, and at any given time plays the one that has been most successful up until that point in time, representing a simple learning mechanism. The minority game is characterized by agents' heterogeneity, bounded rationality, and adaptation. In addition, the minority rule forces the agents to choose the strategies that makes them singular in the community. Driven by switching between strategies, the attendance record fluctuates aperiodically. The standard deviation of the historical attendance record serves as a measure of the efficiency of the market: Higher variance corresponds to lower predictability hence higher efficiency, vice versa. When the memory of the agents (measured by the number of previous time steps) is low, the market is efficient. However, as the memory increases, the market gradually becomes inefficient.

Briefly, in contrast to the standard view in economics which considers that markets are efficient and prices only change due to the arrival of news regarding the fundamental value of the asset, the minority game points out that a market with many traders is inefficient by nature and the price fluctuates even in the absence of any new external information (Challet and Zhang (1997)).

The EL Farol market model was also formulated and parametrized by Johnson et al. (2003). Through simulations, they showed that the statistical features of the model are consistent with the stylized facts observed in real financial markets. The authors also qualitatively and quantitatively described the essential interactions driving the variation in volatility in the minority game.

Some researchers in econophysics deal with the statistical characterization of the stochastic dynamics of asset prices, empirically-consistent derivatives pricing, portfolio optimization, and income distribution of firms and statistical properties of their growth rates, etc. (Mantegna and Stanley (2000)). In addition, a group of econophysicists concerns the development of models that are able to encompass all the essential features of real financial markets. Below we summarize some models that have clear links to statistical physics.

Bak et al. (1997) reported an extremely simple, but completely defined, economic model of many agents. There is only one type of stock and each agent can own at most one share. The agents are of two types: (1) noise traders whose choice of price to buy or sell may imitate choices of other traders and whose current volatility may depend on recent changes in the market; (2) rational agents who optimize their own utility functions, based on the expected dividends of the stock, and their degrees of risk aversion. In the simplest case, a single agent is chosen who updates price, randomly looks into the market, and chooses the most favorite price (if there is one) advertised by another trader to transact. If the noise traders are independent of other traders and the stock price, the model is equivalent to a reaction-diffusion model where two types of particle, A particles and B particles, are injected at different ends of a tube. These systems have been studied extensively by physicists. Surprisingly, the resulting price variations based on the diffusive behavior of the independent noise traders follow the power law. If the noise traders are allowed to adjust their current price towards the current market price and mimic other traders in the market, the price variations become much more dramatic and qualitatively look a lot more like the variations observed in a real stock market. When the information on the volatility of the markets is fed back to the agents, i.e., the strength of the drifting of each noise trader is proportional to recent price variation, the effect of volatility clustering is generated. This work was motivated by the observations in statistical physics that distributions with fat tails naturally occur in systems with very many interacting components, and economics clearly deals with many interacting agents. A detailed description of this model and its simulation results can also be found in Wang (2005).

A stochastic CA model for simulating the dynamics of stock markets was introduced in Bartolozzi and Thomas (2004). The agents of the market are represented by cells on a two-dimensional grid and each agent at each discrete time step is characterized by three possible states or spin orientations: 1 is associated with the purchasing of a stock, -1 with the selling, and 0 with being inactive. Through a direct percolation process, active traders form a hierarchy of clusters and the traders in each group share information and have identical spin states, representing traders' herding behavior. The trading dynamics is governed by the synchronous updating of the spins of the active traders. The simulation shows that the returns show a type of intermittent behavior and their distribution is characterized by power-law tails. In addition, long time correlations are present in the volatility. The key element of this model is the formation of the hierarchy of clusters of active traders, mimicking the process in which traders are associated with professional investors. Large price changes, including crashes and bubbles, can be interpreted as a chronization of the spin orientation of the more influential clusters.

The boundaries between these new research fields: Behavioral finance, agent-based simulation, and econophysics are by no means clear cut. Many models of behavioral finance are comprised of numerous heterogeneous agents; many agent-based models deal with human intelligence and psychology; and many models in econophysics are based on agent behavior and interactions. Nevertheless, through theoretical and experimental explorations from different angles, the research activity in these fields have greatly promoted our understanding of the complexity of financial markets and they have become complementary to the traditional approaches of economics and finance.

The current financial turmoil evokes the strong advice that policy makers should adopt behavioral and agent-based approaches. As stated by Buchanan (2008), none of the economic models being used by government regulators anticipated the crisis, while the academic economics profession remains reluctant to embrace new computational approaches. If we want to avoid crises, we need something more imaginative, starting with a more open-minded attitude to how science can help us understand how markets really work. Computer simulation can enable us to discover relationships that the unaided human mind, or even the human mind aided with the best mathematical analysis, would never grasp. Farmer and Foley (2009) pointed out that agent-based simulations potentially present a way to model the financial economy as a complex system and can handle a far wider range of nonlinear behavior than conventional equilibrium models. Policy makers can thus simulate an artificial economy under different policy scenarios and quantitatively explore their consequences. Buchanan (2009) went further to suggest that agent-based computer models, when applied properly, could prevent another financial crisis.

Chapter 4

Understanding Stock Market Dynamics

In Chapter 2 and references therein, it was argued extensively that the complex dynamics of stock markets is characterized by some stylized facts which are common across many markets. As stated in Chapter 3, researchers in financial economics have not yet reached an agreement on the principal mechanisms underlying this complex dynamics. While many agent-based models published in the literature can reproduce these main stylized facts, some of them are so complicated that it is still difficult to identify the underlying mechanisms governing the dynamics. This has also been pointed out by Farmer (1999) and Cont (2005).

In view of these facts, we have developed a parsimonious cellular automaton (CA) model that can generate the empirically observed stylized facts in a robust and simple manner[1]. Our model is built on increasing levels of sophistication in order to identify the driving mechanisms underlying these stylized facts.

In Section 4.1, we firstly give a detailed description of our CA model. Next, the simulation results are presented in Section 4.2. Section 4.3 provides a thorough investigation of the simulated dynamics through computational experiments and mathematical analysis, and provides insights into the underlying mechanisms of the complex dynamics observed in stock markets. In the final section of this chap-

[1] This chapter is based on Qiu et al. (2007).

ter, we argue that indeed these mechanisms are shared by other well established models proposed in the literature.

4.1 A cellular automaton model of stock markets

We represent a stock market as a two-dimensional $L \times L$ lattice. Each vertex of the lattice denotes an agent (trader) who has interactions with other agents in a so-called Moore neighborhood[1]. In the model, speculative traders of only two types are adopted: Fundamentalists and imitators. All the agents trade in a single stock.

Fundamentalists are those traders who are informed of the nature of the stock being traded and act according to its fundamental value. They believe that the price of the stock may temporarily deviate from, but will eventually return to the fundamental value. They therefore buy/sell the asset whenever its price is lower/higher than their perceived fundamental value. In stock markets, there are also some traders who do not know or do not care about fundamental values. Instead, they follow their acquaintances and adopt the trading opinions of the majority. An agent of this type is referred to as an imitator.

News influences both fundamentalists and imitators. However, the ways news affects them are distinct in many aspects. For example, fundamentalists pay relatively more attention to news about the specific company that has issued the stock, while imitators respond comparatively more frequently to news related to the stock market as a whole.

We can adopt other types of agent to model stock markets more realistically. However, we think that the two kinds of behavior discussed here are the most typical. The behavior of other speculative traders has no obvious characteristics. For example, we cannot find a general trait for chartists, because even using the

[1] A Moore neighborhood $\mathrm{NB_{i,j}}$ in our two-dimensional lattice is defined as a set whose members are the eight cells surrounding a given cell located at (i, j), i.e., $\mathrm{NB_{i,j}} = \{(\mathrm{i}-1,\mathrm{j}-1),(\mathrm{i},\mathrm{j}-1),(\mathrm{i}+1,\mathrm{j}-1),(\mathrm{i}-1,\mathrm{j}),(\mathrm{i}+1,\mathrm{j}),(\mathrm{i}-1,\mathrm{j}+1),(\mathrm{i},\mathrm{j}+1),(\mathrm{i}+1,\mathrm{j}+1)\}$.

same data they may come to different conclusions due to differences in the techniques used. We can treat these agents as noise traders who randomly influence the price to different extents. However, because their adoption within our model does not fundamentally influence the dynamics characterized by the stylized facts, we choose to ignore them.

The real fundamental value of a stock is related to the current and prospective states of the company that has issued the stock, among many other factors. The modeling of its variations is beyond the scope of this work. Instead, we are more interested in the reason(s) for excess volatility, i.e., the extra factor(s) causing the price of a stock to be more volatile than its real fundamental value. For this reason, we assume that the real fundamental value of the asset F is a constant. (Tests showed that adding a drift to F to model the time value of money does not influence the characteristics of the returns. Drifts are therefore excluded from our model.)

4.1.1 Level I model

Fundamentalists

Empirically, the larger the difference between the price of a stock and its fundamental value as perceived by a fundamentalist, the more likely he will trade it. We assume for the moment that the fundamentalists perceive the real fundamental value accurately. We can then adopt Equation (4.1) to express the *transaction quantity based on the current price level* at time $t+1$ of a fundamentalist when he is the i-th agent, $V_{i,fu}^{t+1}$, and his *actual transaction quantity* at the same time, $q_{i,fu}^{t+1}$:

$$\begin{aligned} q_{i,fu}^{t+1} &= V_{i,fu}^{t+1} \\ &= F - P^t, \end{aligned} \tag{4.1}$$

where P^t is the price at time t. Notice that we have assumed for the moment that the two transaction quantities are equivalent. (The other factor determining (actual) transaction quantities will be introduced in Section 4.1.3.)

Imitators

We take the average transaction quantity based on the current price level of an imitator's neighbors at the previous time step as his corresponding quantity at present, i.e., $V_{i,im}^{t+1} = \langle V_{i,nb}^{t} \rangle$. We can then use Equation (4.2) to express his (actual) transaction quantity at time $t+1$, $q_{i,im}^{t+1}$:

$$\begin{aligned} q_{i,im}^{t+1} &= V_{i,im}^{t+1} \\ &= \langle V_{i,nb}^{t} \rangle. \end{aligned} \tag{4.2}$$

Imitations are ubiquitous in stock markets. However, they are carried out in many different manners. In the model introduced in Bak et al. (1997), noise traders imitate by adjusting their prices towards the current market price and mimicking other traders in the market. Similarly, Lux and Marchesi (1999) defined imitation as identifying price trends and patterns and mimicking other traders. In the model described in Cont and Bouchaud (2000), imitation is the process in which agents organize into coalitions and then trade identically. Iori (2002) defined imitation as the process in which traders receive signals from their neighborhoods. Bartolozzi and Thomas (2004) modeled imitation as a direct percolation process, in which active traders form a hierarchy of clusters and the agents in each group share information and interact with each other. Nevertheless, these specific imitating processes share the property that traders directly mimic the behavior of their acquaintances or copy the behavior of other traders that is implied by price trends.

To retain this property and in the meantime be consistent with our pursuit of simplicity, we have adopted a simple manner of imitation, i.e., traders adopt the opinions of their direct neighborhoods.

4.1.2 Level II model

Fundamentalists

News influences fundamentalists' perceptions of fundamental values. Positive/negative news can cause them to overestimate/undervalue assets. Within our model, we assume that at each time step, all the fundamentalists perceive the fundamental value identically. (We can alternatively assume that their perceived values at each time step are normally distributed, without fundamentally influencing the dynamics.)

We express the perceived fundamental value at time t as $F\eta^t_{fu}$, in which η^t_{fu} denotes the influence of the news at that time. We assume that $\eta^t_{fu} = 1 + c_{fu}\phi^t_{fu}$, where ϕ^t_{fu} is an independent Gaussian random variable with mean 0 and standard deviation 1 and c_{fu} is a positive parameter indicating the fundamentalists' sensitivity to news. At this point, we have a modified expression for the transaction quantity of a fundamentalist,

$$\begin{aligned} q^{t+1}_{i,fu} &= V^{t+1}_{i,fu} \\ &= F\eta^{t+1}_{fu} - P^t. \end{aligned} \tag{4.3}$$

Imitators

We assume that news influences all the imitators identically. (We can alternatively assume that the effects of news at each time step are normally distributed, without fundamentally influencing the dynamics.) Significant/unimportant news can make an imitator trade more/less than his neighbors, and vice versa. We reformulate the transaction quantity of an imitator as

$$\begin{aligned} q^{t+1}_{i,im} &= V^{t+1}_{i,im} \\ &= \langle V^t_{i,nb}\rangle \eta^{t+1}_{im}, \end{aligned} \tag{4.4}$$

in which η^{t+1}_{im} indicates the influence of the news at time $t+1$ and is equal to $1 + c_{im}\phi^{t+1}_{im}$, where ϕ^t_{im} is an independent Gaussian random variable with mean 0 and standard deviation 1 and c_{im} is a positive parameter indicating the imitators' sensitivity to news.

Due to the difference in characteristic between the two types of trader, c_{im} is usually distinct from c_{fu}, so is ϕ^t_{im} from ϕ^t_{fu}.

4.1.3 Level III model

A common strategy used by traders is buying low and selling high (BLASH). It aims for capital gains by taking advantage of changes in prices. Price fluctuations are therefore indispensable for this strategy.

Based on BLASH, capitals of traders move among different assets pursuing larger profits at lower risks. When the price fluctuation level of a stock is at the two extremes, i.e., very low and very high, the asset is the least desirable: If it is very low, traders who hold the asset will not be able to find an opportunity to sell it profitably and will not even be able to cover their opportunity costs[1]. If it is very high, traders will consider the investment in the asset too risky. Within the range between the two extremities, as the price fluctuation level rises, the asset will be first more favorable and then, after a certain level, less attractive.

When a stock is more favorable compared to other alternatives, traders will trade it more frequently. We therefore assume that the trading activity of the agents is equivalent to the desirability of the stock. However, BLASH is a risky approach itself, because there is no way to predict price changes accurately. Frequently, traders just end up selling at a loss. In order to reduce this risk, traders typically consider previous price changes of a stock for a longer period.

We represent the price fluctuation level of a stock at time t as

$$L^t = \frac{1}{k} \sum_{i=t-k}^{t-1} |P^i - \bar{P}|/\bar{P}, \tag{4.5}$$

where k is the length of a period before t, P^i is the price of the asset at time i in the period, and $\bar{P}$ is the average price over the period. (We can alternatively assume that agents take different values of k that are normally distributed, without fundamentally influencing the dynamics.)

For the sake of simplicity, we adopt a straightforward linear function for the trading activity of the agents,

$$M^t(L^t) = \begin{cases} c_l L^t, & L^t \leq L_m \\ c_l(-L^t + 2L_m), & L^t > L_m \end{cases} \tag{4.6}$$

where L_m is the fluctuation level where the stock becomes less favorable and c_l is a positive parameter. (Simulations show that adopting other concave functions leads to similar results.)

[1]Opportunity cost, or cost of capital, is the rate of return that a business could earn if it chose another investment with equivalent risk (Downes and Goodman (1998)).

Within the level III model we consider that the (actual) transaction quantity of an agent is the product of his transaction quantity based on the current price level and his current trading activity. The transaction quantity of a fundamentalist is therefore

$$\begin{aligned} q_{i,fu}^{t+1} &= V_{i,fu}^{t+1} M^{t+1} \\ &= (F\eta_{fu}^{t+1} - P^t) M^{t+1}, \end{aligned} \tag{4.7}$$

whereas the transaction quantity of an imitator is

$$\begin{aligned} q_{i,im}^{t+1} &= V_{i,im}^{t+1} M^{t+1} \\ &= \langle V_{i,nb}^{t} \rangle \eta_{im}^{t+1} M^{t+1}. \end{aligned} \tag{4.8}$$

Considering the fact that agents always have a number of exceptional reasons to transact, we adopt a lower bound for M^t.

The BLASH behavior is the most basic feature of most speculators in stock markets, contrast with fundamentalists' dividend-based behavior. In fact, making profits through BLASH is the very reason why many speculators are in stock markets in the first place. However, it is surprising that this typical speculative behavior has rarely been included in agent-based models.

We have also included another important feature of speculators, i.e. transferring capital among different assets or markets. It is the very fact that capital movements take place constantly. Unfortunately, this feature is also seldom be taken into consideration by most models of financial markets.

4.1.4 Rule of price updating

The price is updated according to the following rule:

$$P^{t+1} = P^t + \frac{c_p Q^t}{N}, \tag{4.9}$$

where Q^t is the total transaction quantity or the excess demand for the asset at time t and N is the number of traders. Since Q^t is proportional to N, we rescale it with N. We adopt a positive parameter c_p to indicate the sensitivity of the price to the excess demand. Due to the fact that stock prices cannot be negative, the lower bound of P^t is 0.

Equation (4.9) can be explained as the action of market makers to balance the supply and the demand of the stock. In principle, however, it is merely the translation of the classic theory of supply and demand stating that price will move toward the point that equalizes supplied and demanded quantities[1].

Another price updating rule often used in agent-based modeling, e.g., that of the model of Levy et al. (2000), is borrowed from that of a Walrasian auction. Introduced by Walras[2]. It is a type of simultaneous auction where each agent calculates its demand for the good at a hypothetical price and submits this to an auctioneer. The price is then set so that the total demand across all agents equals the total supply. The good is traded at this equilibrium price and no transactions take place at disequilibrium prices.

In principle, the Walrasian rule and the one adopted by us, expressed as Equation (4.9), are identical with regard to returns. The reason is that they are both based on the law of supply and demand: The supply/demand) is an increasing/decreasing function of price and return is positively related to excess demand.

[1]In 1890, Alfred Marshall published *Principles of Economics* (Marshall (1890)), in which he discussed how both supply and demand interact to determine price. His supply-demand model has become one of the fundamental concepts of economics. According to the model, if all other factors remain equal, the higher the price, the lower the quantity demanded and the higher the quantity supplied, vice versa. In a price (ordinate) - quantity (abscissa) chart the curve of demand is a downward slope, the supply relationship shows an upward slope. Equilibrium occurs at the intersection point of the two curves. In the chart, if straight lines are drawn instead of the more general curves (the shapes of the curves do not change the general relationships), we immediately obtain Equation (4.9).

[2]Marie-Esprit-Léon Walras was a mathematical economist. In 1874 and 1877 he published *Elements of Pure Economics*, a work that led him to be considered the father of the general equilibrium theory. This theory is a branch of theoretical neoclassical economics, pursuing the explanation of the behavior of supply, demand and prices in a whole economy with several or many markets, by seeking to prove that equilibrium prices for goods exist and that all prices are at equilibrium.

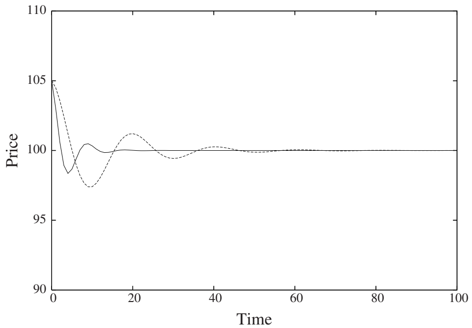


Figure 4.1: Price trajectories obtained through the simulation using the level I model when $\alpha_{im} = 20\%$ and 80% respectively. The curve that decays faster is of the first instance. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.5$, $F = 100$, and $P^0 = 105$.

4.2 Simulation results

In this section, we present the simulation results of the model at different levels of complexification. We focus on the characteristics of the returns, among those of other variables such as the price, the trading volumes, and the activity level.

4.2.1 Simulation results of the Level I model

In simulations using the level I model, we set an initial price ($P^0 = 105$) that deviates from the fundamental value ($F = 100$). The number of agents is 1×10^4. Figure 4.1 displays the price trajectories corresponding to two fractions of imitators: $\alpha_{im} = 20\%$ and 80% respectively.

The parameter c_p has an important impact on the price: When its value is increased up to 1 while other parameters are kept constant, the price process may start to switch from a convergent process to a divergent one, depending on the value of c_p itself and the value of α_{im}. We provide a theoretical analysis of this issue in Section 4.3. To model a stable market, we adopt only those values of c_p smaller than 1.

As shown in Figure 4.1, although the level I model is not completely identical to the model of Bandini et al. (2004), it does generate similar price trajectories. Starting from an initial deviation from the fundamental value, the price either directly converges to it, or fluctuates around it for some time and eventually overlaps. Obviously, both models cannot produce sustained price movement. Since the price quickly dies out, we cannot obtain any stylized facts by using the level I model.

4.2.2 Simulation results of the Level II model

Within the level II model, we have added random factors η_{fu}^t and η_{im}^t, so that it can produce sustained price fluctuations. When the fraction of imitators is set to 70%, we obtain simulation results shown in Figure 4.2. In our simulations, return is represented by the difference between two successive natural logarithms of price, i.e., log-return.

We see that the level II model can generate a non-Gaussian (fat-tailed) distribution of return, but is not able to confirm another important stylized fact, namely volatility clustering. It therefore has the same problem as the model presented in Cont and Bouchaud (2000). Nevertheless, through simulations using this model, we can further study how the fraction of imitators influences the distribution of return. Figure 4.3 shows the results for different instances: $\alpha_{im} = 20\%$, 50%, and 80% respectively. If the fraction is small, returns will follow a Gaussian distribution; increasing it enlarges the tails of the return distribution.

4.2.3 Simulation results of the level III model

In the level III model we have further added a mechanism through which agents' activity is adjusted over time. Fixing α_{im} to 70%, we obtain simulation results shown in Figure 4.4, in which

a. Figure 4.4(a) records the price process. Some large 'flights' can be observed, which correspond to large (positive or negative) returns.

b. Figure 4.4(b) illustrates the time series of return. The effect of volatility clustering is clear.

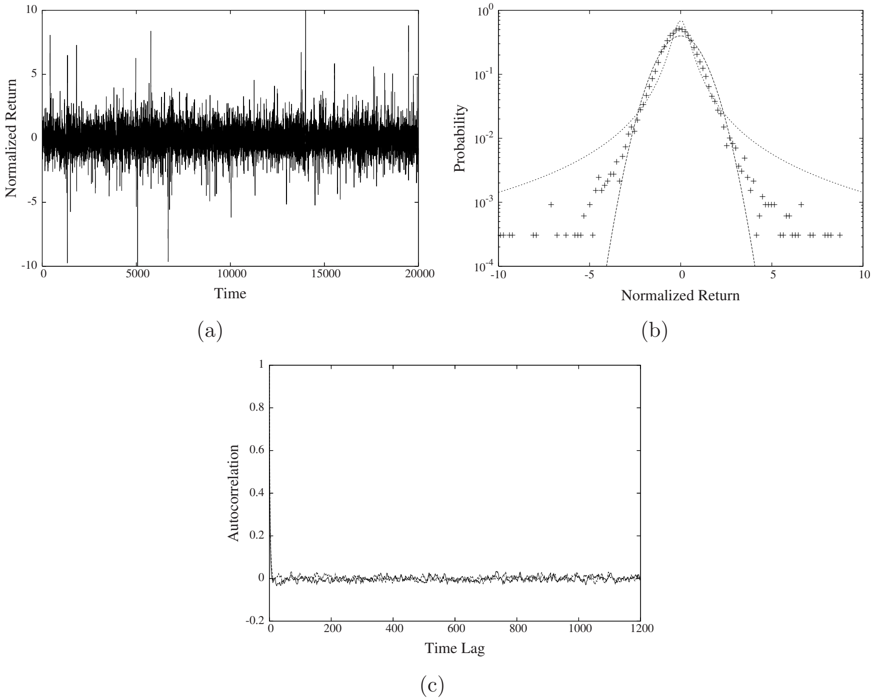


Figure 4.2: Simulation results of the level II model when $\alpha_{im} = 70\%$. (a) Normalized return. (b) Distribution of return (the scale of the vertical axis is logarithmic). (c) Autocorrelation function of return and that of volatility. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.005$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, and $P^0 = 100$.

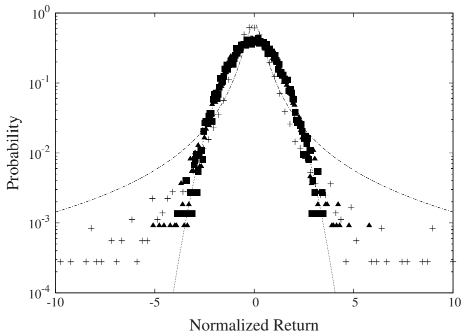


Figure 4.3: Return distributions of the level II model for different fractions of imitators. The ■ points, the ▲ points, and the + points refer to $\alpha_{im} = 20\%$, 50%, and 80% respectively. The scale of the vertical axis is logarithmic. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.005$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, and $P^0 = 100$.

c. Figure 4.4(c) shows the probability distribution of return, together with a Gaussian PDF and a Lorentz PDF for comparison. The tails of the distribution are clearly heavier than those of a Gaussian PDF.

d. Figure 4.4(d) displays the autocorrelation function (ACF) of return (the lower curve) and that of volatility. The former converges quickly to the noise range, whereas the latter decays much more slowly.

e. Figure 4.4(e) shows the time evolution of trading volume[1].

f. Figure 4.4(f) illustrates the time evolution of trading activity. It is a slow process in comparison with the fast evolution of the influence of news[2].

These simulation results indicate that our CA model (level III) is able to reproduce the main stylized facts. In addition, as shown below, this model is robust with regard to the stylized facts for wide ranges of the parameters.

[1] Volume is defined as the sum of absolute aggregate demand and absolute aggregate supply.

[2] Here, we define the rate of time evolution of a variable X as $(|\Delta X|/|X|)/\Delta t$, where ΔX is the change of X within time increment Δt.

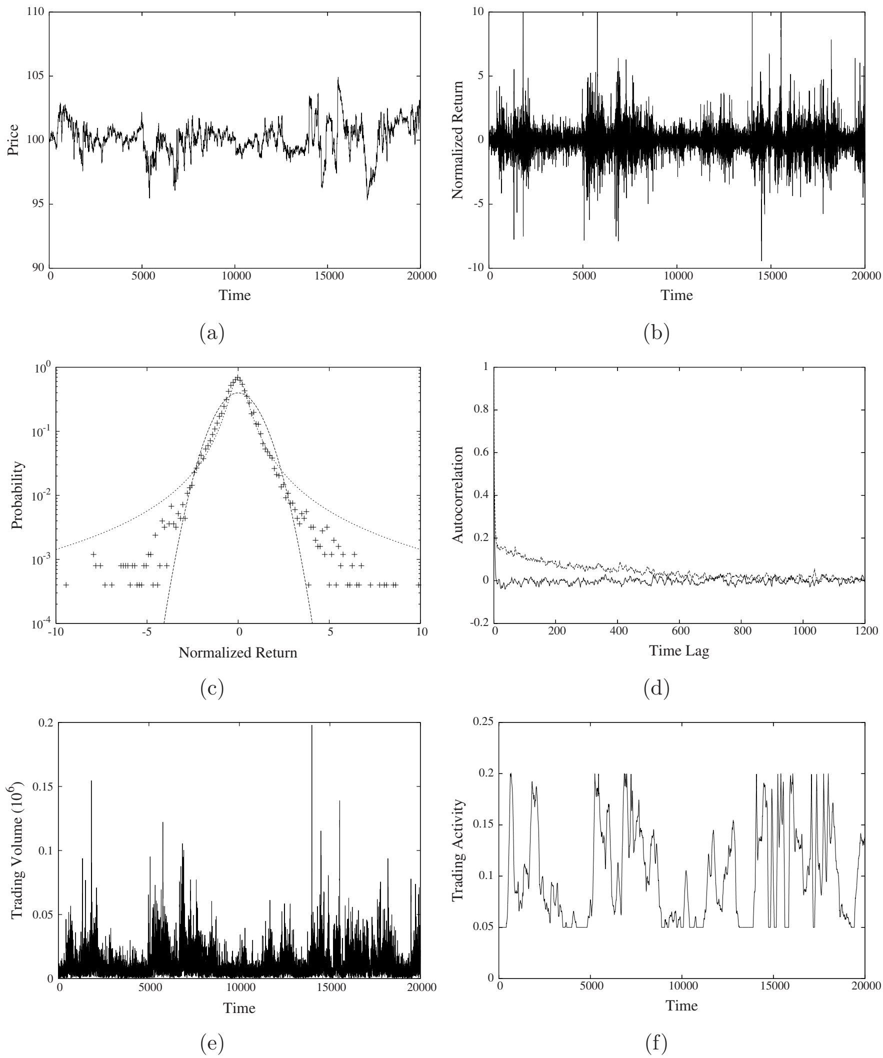


Figure 4.4: Simulation results of the level III model when 70% of the agents are imitators. (a) Price. (b) Normalized return. (c) Distribution of return (the scale of the vertical axis is logarithmic). (d) Autocorrelation function of return (the lower curve) and that of volatility. (e) Trading volume. (f) Trading activity. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, $P^0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of M^t is 0.05.

Table 4.1 compiles the kurtosis values of the return distributions for different values of α_{im}, c_{im}, and c_{fu} respectively. We see that imitators have a strong influence on kurtosis, while the relation between fundamentalists and kurtosis is not explicit. Specifically, the fraction of imitators α_{im} and the sensitivity of imitators to news c_{im} are positively correlated with kurtosis. When either of them increases to a certain level, kurtosis suddenly becomes very large, implying that the system becomes unstable. For example, when $c_{im} = 0.9$, we obtain a price pattern with frequent dramatic 'flights' and a time series of return with many striking strokes. These are shown in Figure 4.5.

Table 4.1: Kurtosis values of the return distributions produced by the level III model for increasing values of α_{im}, c_{im}, and c_{fu} respectively. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, $P^0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of M^t is 0.05.

α_{im}	Kurtosis	c_{im}	Kurtosis	c_{fu}	Kurtosis
0%	4.44	0.1	4.36	0.1	15.53
10%	4.36	0.2	5.37	0.2	17.22
20%	4.57	0.3	6.37	0.3	42.94
30%	5.33	0.4	7.44	0.4	37.28
40%	6.07	0.5	8.60	0.5	35.39
50%	6.64	0.6	10.55	0.6	46.71
60%	8.51	0.7	17.22	0.7	43.98
70%	17.22	0.8	32.57	0.8	42.11
80%	69.33	0.9	119.14	0.9	35.79
90%	190.46	1.0	422.77	1.0	30.72

Keeping other parameters constant and adopting different values of k, we obtain the autocorrelation functions of volatility shown in Figure 4.6(a). When k is smaller than 50, ACFs of volatility quickly drop to the noise range and the effects of volatility clustering are correspondingly negligible. Volatility clustering becomes significant when k is increased to around 100. The importance of k to volatility clustering will further manifest itself in Section 4.3.4.

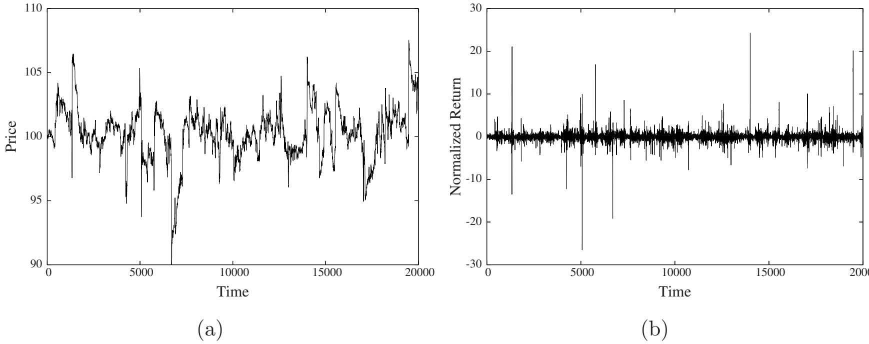
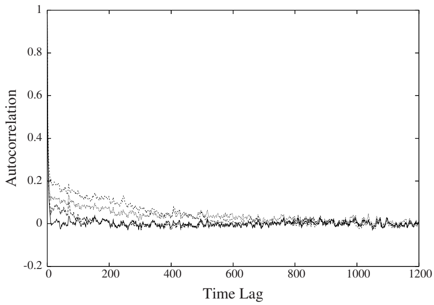


Figure 4.5: Simulation results of the level III model when $\alpha_{im} = 70\%$ and $c_{im} = 0.9$. (a) Price. (b) Normalized return. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $P^0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of M^t is 0.05.

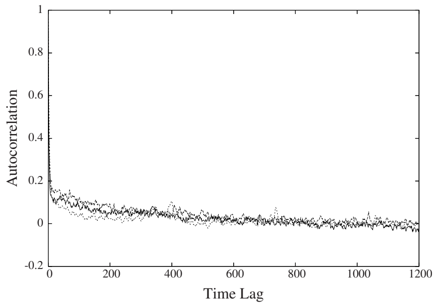
Choosing three values for the number of agents (lattice sizes) and keeping other parameters constant, simulations give ACFs of volatility shown in Figure 4.6(b). All these ACFs are qualitatively similar to that of S&P 500 shown in Figure 2.1, indicating that the model can reproduce the stylized facts not only for markets with small numbers of agents, but also for markets with many agents. At this point, the model differs from some MS models that behave realistically only for limited numbers but not large numbers of traders (Egenter et al. (1999)).

4.3 Discussion: The market dynamics revealed by the model

In this section, we provides a thorough investigation of the simulated dynamics through computational experiments and mathematical analysis. This study offers important insights into the mechanism governing the stock dynamics in reality.



(a)



(b)

Figure 4.6: Autocorrelation functions of volatility produced by the level III model when $\alpha_{im} = 70\%$. (a) ACFs when $k = 10$ (the lowest curve), $k = 100$ (the second lowest), $k = 300$ (the highest), and $k = 500$ (the second highest) respectively. (b) ACFs when $N = 10 \times 10$ (the middle curve), 100×100 (the upper one), and 1000×1000 (the lower one) respectively. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, $P^0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of M^t is 0.05.

4.3.1 Long-range interactions can emerge from local interactions

In this section, for the sake of simplicity, we take a one-dimensional version of our CA model to derive analytical expressions. An agent located at i then has two neighbors at $i-1$ and $i+1$ respectively. Statistically, the total quantity at time $t+1$ can be expressed as

$$Q^{t+1} = \sum_{i=1}^{N} q_{i,(\cdot)}^{t+1} = \sum_{i=1}^{N} [u_i q_{i,fu}^{t+1} + (1-u_i) q_{i,im}^{t+1}], \tag{4.10}$$

where u_i is determined in the following way: We sample a variable γ $(0 \leq \gamma \leq 1)$ that is uniformly distributed. If $0 \leq \gamma \leq \alpha_{fu}$, $u_i = 1$, else $u_i = 0$. Here, α_{fu} is the fraction of fundamentalists.

The terms $q_{i,fu}^{t+1}$ and $q_{i,im}^{t+1}$ in Equation (4.10) are determined by Equation (4.7) and Equation (4.8) respectively. However, because M^t changes much more slowly than Q^t, we can consider the former as a constant to study the basic dynamics of the latter. We set $M^{(\cdot)} = 1$, then $q_{i,fu}^{t+1}$ and $q_{i,im}^{t+1}$ are respectively determined by Equation (4.3) and Equation (4.4). Therefore,

$$\begin{aligned} q_{i,im}^{t+1} &= \eta_{im}^{t+1}(\frac{1}{2})[V_{i-1,(\cdot)}^{t} + V_{i+1,(\cdot)}^{t}] \\ &= \eta_{im}^{t+1}(\frac{1}{2})\{[u_{i-1}V_{i-1,fu}^{t} + (1-u_{i-1})V_{i-1,im}^{t}] \\ &\quad +[u_{i+1}V_{i+1,fu}^{t} + (1-u_{i+1})V_{i+1,im}^{t}]\}. \end{aligned} \tag{4.11}$$

Similarly, the terms $V_{i-1,im}^{t}$ and $V_{i+1,im}^{t}$ in Equation (4.11) can be respectively expressed as

$$\begin{aligned} V_{i-1,im}^{t} &= \eta_{im}^{t}(\frac{1}{2})\{[u_{i-2}V_{i-2,fu}^{t-1} + (1-u_{i-2})V_{i-2,im}^{t-1}] \\ &\quad +[u_{i}V_{i,fu}^{t-1} + (1-u_{i})V_{i,im}^{t-1}]\}, \end{aligned} \tag{4.12}$$

and

$$\begin{aligned} V_{i+1,im}^{t} &= \eta_{im}^{t}(\frac{1}{2})\{[u_{i}V_{i,fu}^{t-1} + (1-u_{i})V_{i,im}^{t-1}] \\ &\quad +[u_{i+2}V_{i+2,fu}^{t-1} + (1-u_{i+2})V_{i+2,im}^{t-1}]\}. \end{aligned} \tag{4.13}$$

Following the same scheme, we can further express the terms $V^{t-1}_{i-2,im}$, $V^{t-1}_{i,im}$, and $V^{t-1}_{i+2,im}$ in Equations (4.12) and (4.13) in terms of the corresponding quantities at time step $t-2$ of the neighbors of the agents located at $i-2$, i, and $i+2$ respectively, and so on. Basically, in this way, we can replace each imitator's transaction quantity based on the current price level at each time step with the fundamentalists' corresponding quantities at the preceding time steps, noting that $V^t_{(\cdot),fu} = V^t_{i,fu}$. After substitutions, we have

$$
\begin{aligned}
Q^{t+1} \quad = \quad & \sum_{i=1}^{N} [A_i^{t+1} V_{i,fu}^{t+1} + A_i^t(\eta_{im}^{t+1}) V_{i,fu}^t \\
& + A_i^{t-1}(\eta_{im}^{t+1}\eta_{im}^t) V_{i,fu}^{t-1} \\
& + A_i^{t-2}(\eta_{im}^{t+1}\eta_{im}^t\eta_{im}^{t-1}) V_{i,fu}^{t-2} + \ldots \\
& + A_i^{t-\tau}(\eta_{im}^{t+1}\eta_{im}^t\eta_{im}^{t-1} \ldots \eta_{im}^{t-\tau+1}) V_{i,fu}^{t-\tau} + \ldots],
\end{aligned}
\tag{4.14}
$$

where $\tau = -1, 0, 1, 2, \ldots$. The first few instances of $A_i^{t-\tau}$ are

$$A_i^{t+1} = u_i,$$

$$A_i^t = \frac{1}{2}[(1-u_i)u_{i-1} + (1-u_i)u_{i+1}],$$

$$
\begin{aligned}
A_i^{t-1} \quad = \quad & \frac{1}{2^2}[(1-u_i)(1-u_{i-1})u_{i-2} \\
& +(1-u_i)(1-u_{i-1})u_i \\
& +(1-u_i)(1-u_{i+1})u_i \\
& +(1-u_i)(1-u_{i+1})u_{i+2}],
\end{aligned}
$$

$$
\begin{aligned}
A_i^{t-2} = \frac{1}{2^3}[&(1-u_i)(1-u_{i-1})(1-u_{i-2})u_{i-3} \\
&+(1-u_i)(1-u_{i-1})(1-u_{i-2})u_{i-1} \\
&+(1-u_i)(1-u_{i-1})(1-u_i)u_{i-1} \\
&+(1-u_i)(1-u_{i-1})(1-u_i)u_{i+1} \\
&+(1-u_i)(1-u_{i+1})(1-u_i)u_{i-1} \\
&+(1-u_i)(1-u_{i+1})(1-u_i)u_{i+1} \\
&+(1-u_i)(1-u_{i+1})(1-u_{i+2})u_{i+1} \\
&+(1-u_i)(1-u_{i+1})(1-u_{i+2})u_{i+3}].
\end{aligned}
$$

In each term within $A_i^{t-\tau}$, the sequence of $1-u_i$ terms indicates the propagation of imitation over time (backwards) and space (agents). However, if at least one of the terms is equal to zero, which corresponds to a fundamentalist, the whole product will be zero. As the fraction of imitators/fundamentalists increases/decreases, some $A_i^{t-\tau}$ terms with larger τ values are greater than zero.

The imitation chains show that long-range interactions can form from local imitations. In the resultant networks, each agent is influenced, directly or indirectly, by some other near or remote agents. Here, the strengths and time lags of influence differ. In this respect, our CA model is different from the Cont-Bouchaud model, where any two agents can be directly linked, and agents in a group behave identically. It is also distinct from the model of Bartolozzi *et al.*, within which agents in a cluster influence each other with an equivalent strength.

4.3.2 Price and volatility are mean-reverting

Fundamentalists behave according to price while imitators follow other agents but do not directly respond to price. We therefore argue that it is the fundamentalists' behavior which determines price trend. This argument can be confirmed by our simulations: If $\alpha_{fu} = 0$ (all the agents are imitators), price fluctuations die out; in other cases we obtain price trajectories similar in shape but distinct only in amplitude.

Therefore, for the sake of simplicity, we can take the special instance that all the agents are fundamentalists to study the basic dynamics of the price. In such an instance, $\alpha_{fu} = 1$, hence $u_{(\cdot)} = 1$. Then, Equation (4.14) gives

$$\begin{aligned} Q^t &= NV^t_{i,fu} \\ &= N(F\eta^t_{fu} - P^{t-1}). \end{aligned} \tag{4.15}$$

The noise term η^t_{fu} is indispensable for a sustained price process, but is not responsible for any regularity in price trends. We therefore set $\eta^{(\cdot)}_{fu} = 1$ for the sake of simplicity. Then, Equation (4.15) becomes

$$Q^t = N(F - P^{t-1}). \tag{4.16}$$

Equation (4.9) gives,

$$Q^t = (\frac{N}{c_p})(P^{t+1} - P^t). \tag{4.17}$$

Substituting Equation (4.17) into Equation (4.16), we obtain

$$P^{t+1} - P^t + c_p P^{t-1} = c_p F. \tag{4.18}$$

Equation (4.18) is a second-order difference equation. Depending on the value of c_p, the price can follow a monotonically decaying process ($c_p < 0.25$), a damped fluctuating process ($0.25 < c_p < 1$), or an explosive fluctuating process ($c_p > 1$). Figure 4.7 shows the three typical price trajectories when the initial price is 105. In all these instances, the price is mean-reverting. Some researchers have studied the mean-reverting nature of price processes for different behavioral types, as well as different stabilizing-destabilizing endogenous mechanisms of financial markets (Baumol (1957), Beja and Goldman (1980), Day and Huang (1990)).

To analyze the process of volatility generated by our model, we need to consider the mechanism as well as the noise. First of all, we have assumed that the noise, which causes the volatility, follows an independent Gaussian random process. Within this process, those values more close to the mean have higher probabilities. Second, by examining Equations (4.5) through (4.9), we can recognize that when L^t is smaller/greater than L_m, a positive/negative feedback loop will form between L^t and M^t. Namely, small/large values of L^t tend to be enlarged/lessened. Therefore, the nature of the noise, the trading behavior of the agents, and the rule of price updating ensure that the volatility also follows a mean-reverting process.

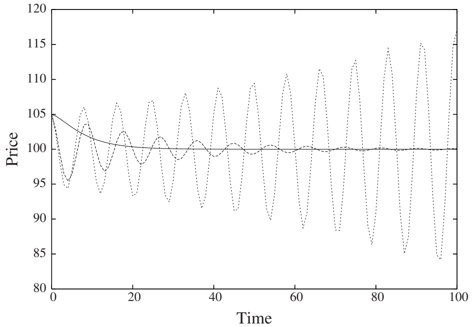


Figure 4.7: Price trajectories given by Equation (4.18) for different values of c_p. The monotonously decaying curve, the convergent fluctuating curve, and the divergent fluctuating curve correspond to $c_p = 0.1$, 0.9, and 1.02 respectively. The parameter setting used: $F = 100$ and $P^0 = 105$.

4.3.3 Heavy tails due to large price variations are caused by imitations

In this section, for the sake of simplicity, we adopt price change as return, i.e., $R^{t+1} = P^{t+1} - P^t$. According to Equation (4.9), we can then examine Equation (4.14) in order to investigate the cause of the resultant non-Gaussian return distributions.

In the simulations demonstrated in Section 4.2.2, if $\alpha_{fu} = 1$, we cannot generate fat tails. In this case, the total quantity is described by Equation (4.15), a special instance of Equation (4.14) when all the terms with a product of η_{im}^t terms are equal to zero. Heavy tails are generated when $\alpha_{fu} < 1$ and some of these terms are present in Equation (4.14). We therefore suppose that it is the multiplication of the various ϕ_{im}^t terms in different η_{im}^t terms that is responsible for the non-Gaussian distributions, although all these terms themselves follow a Gaussian distribution.

To confirm this supposition, we define a simple reference model:

$$H^t = \epsilon\phi^t + (1-\epsilon)\phi^t\phi^{t-1}\phi^{t-2}, \tag{4.19}$$

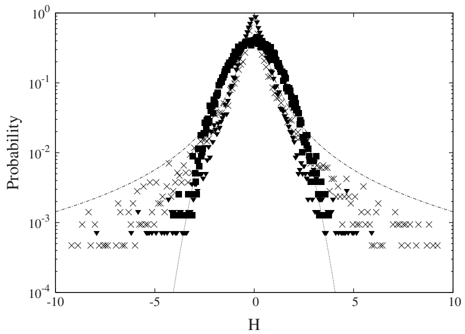


Figure 4.8: Probability distributions of H^t defined by Equation (4.19). Different values of ϵ are taken for comparison. The ■ points, the ▼ points, and the $\times$ points refer to the instances of $\epsilon = 1$, $\epsilon = 0.5$, and $\epsilon = 0$ respectively. The scale of the vertical axis is logarithmic.

where ϕ^t is an independent Gaussian random variable with mean 0 and standard deviation 1 and ϵ is a parameter. Recall that, in Equation (4.14), $\eta^t_{im} = 1 + c_{im}\phi^t_{im}$. Since Equation (4.19), as with Equation (4.14), deals with the sum of products of Gaussian terms, it represents the basic structure of the latter.

Figure 4.8 presents the experimental probability distributions of H^t obtained when choosing different values of ϵ for comparison: 1, 0.5, and 0. In this figure we see that when ϵ decreases, the distribution of H^t gradually changes from being pure Gaussian to being very fat-tailed non-Gaussian. Thus, the more the product of ϕ^t terms is weighted, the heavier the tails of the consequent distribution. From the discussion in Section 4.3.1 we know that, if α_{fu} is small, the products of more η^t_{im} factors in Equation (4.14) will have more weight. This experiment therefore explains the regularity discussed in Section 4.2.2 and Section 4.2.3: Larger fractions of imitators correspond to return distributions with heavier tails. In addition, products of η^t_{im} terms give rise to continued products of c_{im}. The multiplication of c_{im} explains the exponential growth of kurtosis following the increase of c_{im}, as shown in Section 4.2.3.

4.3.4 Volatility clustering is related to the evolution of trading activity

According to Equation (4.9) and the definition of return adopted in this section,

$$R^{t+1} \propto Q^t \tag{4.20}$$

Since M^t changes much more slowly than Q^t, we have $M^t \simeq M^{t-1} \simeq \ldots \simeq M^{t-\tau}$ for small values of τ. (Note that the analysis here is by no means rigorous.) Then, for a small value of τ, according to Equations (4.7), (4.8), and (4.10), as well as the scheme conveyed by Equations (4.11) through (4.13), Equation (4.20) gives

$$R^{t+1} \quad \propto \quad M^t U^t \tag{4.21}$$

where

$$\begin{aligned} U^t \quad = \quad & \sum_{i=1}^{N} [A_i^t (F\eta_{fu}^t - P^{t-1}) \\ & + A_i^{t-1}(\eta_{im}^t)(F\eta_{fu}^{t-1} - P^{t-2}) \\ & + \ldots \\ & + A_i^{t-\tau-1}(\eta_{im}^t \eta_{im}^{t-1} \ldots \eta_{im}^{t-\tau})(F\eta_{fu}^{t-\tau-1} - P^{t-\tau-2})] \end{aligned}$$

In Equation (4.21), M^t is a factor that emerges from the agents' trading and in turn reinforces it. Because it changes more slowly than U^t, successive values of $|R^{t+1}|$ are positively correlated with each other. However, consecutive values of R^{t+1} are only weakly correlated due to the fast variation in its sign, which is caused by the fast variation in the sign of U^t due to news and the mean-reverting nature of the price. These explain the stylized facts: long-term autocorrelation of volatility and short-term autocorrelation of return.

Thus, according to our simulations, three factors are indispensable for volatility clustering: a random component, a convergent mean-reverting mechanism and a factor that emerges from agents' trading and in turn reinforces it. In comparison with the first factor, the third factor changes much more slowly, or on a longer time scale. To show how the three factors contribute to volatility clustering, here we devise two reference models. The first one is

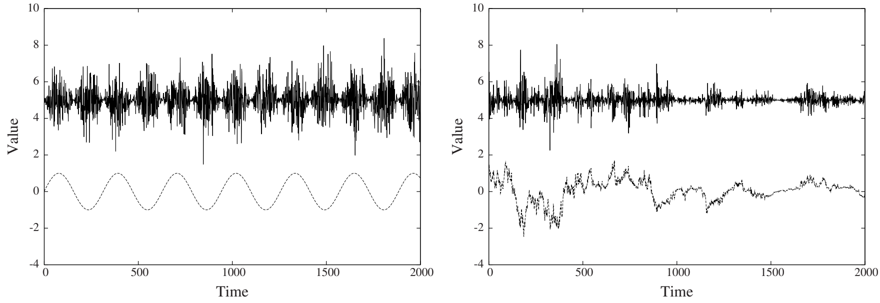
$$H^t = \phi^t sin(\lambda t), \tag{4.22}$$

where λ is a constant, ϕ^t is the same variable use in Equation (4.19). The second reference model is expressed as

$$H^t \;=\; \xi\phi^t \sum_{i=t-k-1}^{t-1} H^i, \tag{4.23}$$

where ξ is a constant, ϕ^t is again the same variable use in Equation (4.19), k indicates the length of a past time period, and we denote $\sum_{i=t-k-1}^{t-1} H^i$ as S_H^t.

In both equations, H^t resembles total quantity or return in our CA model. Equation (4.22) includes a noise term ϕ^t and an arbitrarily introduced factor $sin(\lambda t)$. We choose a small value for λ (0.01) so that the process of $sin(\lambda t)$ is much slower than that of ϕ^t. Equation (4.23) contains a noise term ϕ^t and an emergent factor S_H^t. We choose a large number for k (500) to ensure that the latter changes much more slowly than the former. Figure 4.9 shows the time series of these two reference models. The result of Equation (4.22) is shown in Figure 4.9(a). In this figure, large and small absolute values of H^t gather together and the periods of these clusters correlate with the (positive or negative) high and low periods of the $sin(\lambda t)$ curve. Although the curve is arbitrary instead of emergent, the result expresses the necessity of two factors for volatility clustering: A fast process of noise and a slower process that determines the amplitudes of the resultant time series. Figure 4.9(b) shows the time series of H^t defined by Equation (4.23). Here, S_H^t fluctuates over time but changes more slowly than H^t. Emerging from H^t, it in turn determines the amplitudes of H^t. Volatility clustering exhibits in this time series. However, the process of H^t is very sensitive to ξ, which can cause the process to quickly diverge. Obviously, this model lacks a balancing feedback mechanism to maintain the stability of the dynamics. Nevertheless, it shows how a fast noise factor and an emergent slow factor together induce volatility clustering.



(a) (b)

Figure 4.9: Time series of the reference models in Section 4.3.4. (a) The lower curve is that of $sin(\lambda t)$, the upper one indicates the corresponding time series of H^t defined by Equation (4.22). Here, $\lambda = 0.01$. (b) The lower curve and upper curve are, respectively, the time series of S^t and H^t defined by Equation (4.23). Here, $\xi = 0.1$, $k = 500$. In both graphs, the amplitudes of the lower curves and the longitudinal positions of the upper curves have been adjusted for the sake of easy comparison.

4.3.5 The regularity can be identified in some other microsimulation models

In a general sense, the MS models discussed in Section 3.2 that can confirm the stylized facts observed in stock markets agree with our CA model on the origins of large price variations and volatility clustering.

Although explaining imitation from different angles, all these MS models and our CA model show that the fraction of abnormally large price variations is much larger when agents imitate each other than when they are mutually independent. In the latter instance and in the limit of a large number of agents, returns follow a Gaussian distribution.

Within these MS models and our CA model, we can ultimately attribute volatility clustering to the evolution of agents' activity, although the corresponding processes of the models that indicate activity are quite distinct. (Notice that all these processes are positively correlated with the evolution of trading volume.)

These processes are, respectively, the evolution of volatility (Bak et al. (1997)), the development of the fraction of noise traders (Lux and Marchesi (1999)), the evolution of agents' activation thresholds (Iori (2002)), the percolation process (Bartolozzi and Thomas (2004)), and the progression of the desirability of an asset (our CA model). In addition, these processes are slower than their corresponding 'source' processes. Therefore, volatility clustering generated by these MS models and our CA model is the combined effect of two processes on different time scales.

In literature, on the one hand, there is not yet a common agreement on the origins of the stylized facts (Cont (2005)). On the other hand, various analytical models for describing the phenomena, e.g., GARCH models, stochastic volatility models[1], and a recently published Itô-Langevin model described in Anteneodo and Riera (2005) do not provide explicit economic explanations for the underlying dynamics. The regularity discussed here can help us to achieve a better understanding of the complex dynamics of stock markets.

4.4 Conclusions

In this chapter, a CA model for simulating the complex dynamics of stock markets has been described. This model represents a stock market as a two-dimensional lattice in which each vertex stands for a trader who is either a fundamentalist or an imitator. Our CA model is based on local interactions, adopting simple rules for representing traders' behavior and a simple rule for price updating. This model can reproduce, in a simple and robust manner, the main characteristics observed in empirical financial time series. Heavy-tailed return distributions due to large price variations can be generated through the imitating behavior of agents. Volatility clustering, which also leads to heavy tails, seems to be related to the

[1] A class of stochastic volatility models considers volatility to be independent of return. Price is then assumed to follow a geometric Brownian motion with a time-dependent volatility: $dS(t) = \mu S(t)dt + \sigma(t)S(t)dz_1$. Here dz_1 describes a Wiener process. With $v(t) = \sigma^2(t)$, the time-dependent variance follows a different stochastic process $dv(t) = m(v(t))dt + s(v(t))dz_2$, where dz_2 is another Wiener process. Different forms of $m(v(t))$ and $s(v(t))$ correspond to some popular models of this type (Voit (2003)).

combined effect of a fast and a slow process: The evolution of the influence of news and the evolution of agents' activity, respectively. In a general sense, these causes of heavy tails and volatility clustering appear to be common among some well-established microsimulation models that have confirmed the main characteristics of financial markets.

Chapter 5

Why Do Options Markets Smile?

As remarked in Chapter 3, many agent-based models for studying market dynamics have been developed in the last few decades, however, most of them focus on stock markets (see Chapter 4) and very few center on options markets. More importantly, one of the most challenging and puzzling problems in financial economics stems from options markets, namely the understanding of the origin of the volatility smile phenomenon. For a detailed discussion of this problem, we refer the reader to Chapter 2. Briefly, up till now, the reason for the volatility smile phenomenon remains unclear.

Moreover, across the globe, financial markets are currently in turmoil, in part due to speculative trading in derivatives. With current trading volumes exceeding $530 trillion, derivatives have been labeled 'financial weapons of mass destruction' (Goodman (2008)). In an ideal market, derivative contracts are priced to eliminate risk (Black and Scholes (1973), Merton (1973)). In practice, however, many derivatives traders correct for market imperfections using ad-hoc, perturbative pricing schemes. The volatility smile is a key consequence of these imperfections in real markets. Ironically, this lack of understanding at a microscopic level turns the heuristics which had been developed to control risk, into new sources of unquantifiable risk, in particular at moments of abnormal market behavior, as evidenced by the current (2008-2009) financial crisis.

This fact has stimulated us to develop a microsimulation (MS) model of options markets capable of revealing the mechanisms underlying the volatility smile[1].

Section 5.1 exhibits our MS model of options markets. In Section 5.2 the simulation results are presented. More detailed discussion on the mechanisms governing the shape and dynamic properties are presented in Section 5.3.

5.1 A microsimulation model of options markets

In options markets, there are three main types of trader: Hedgers, speculators, and arbitrageurs. Typically, hedgers use options as insurance to protect their financial interests, but do not attempt to gain profit from options markets; speculators trade options to bet on the future prices in the hope of making capital gains; and arbitrageurs involve making riskless profits by taking advantage of price disparities.

Traders are also different in their trading activity. Typically, hedgers only need to trade once in a certain period. In contrast, speculators are much more active due to the benefits of option trading, i.e., its power of leverage, namely gaining exposure to larger amounts of assets for a much smaller investment, and its potential to profit whether the underlying asset moves up or down. Arbitrageurs are even more active. There are many traders of this type in options markets and their trading will quickly eliminate any detected arbitrage opportunity. Consequently, price disparities are usually small and transient, arbitrageurs therefore need to trade frequently in order to lock-in significant profits.

The smile dynamics exhibits apparent regularity even on short (e.g., daily) time scales, as shown in Cont and da Fonseca (2002) and Fengler et al. (2003). We therefore consider that, whereas the smile is inevitably influenced by the less active traders, its basic dynamics should be accounted for by the behavior of the more active market participants. In fact, this is supported by some empirical studies: Natenberg (1994) stated that, in most options markets, arbitrageurs and speculators typically outnumber hedgers. Lakonishok et al. (2007) reported that little option volume can be attributed to hedging; instead, a significant fraction

[1] This chapter is based on Qiu et al. (2010a) and Qiu et al. (2010b)

of option trading is speculative in nature and mainly motivated by views about the direction of future stock price movements. In modeling options markets, we hence focus on active traders, namely (directional) speculators and arbitrageurs.

In our model, the market participants trade in European call/put options on a single underlying asset with the identical time to maturity but with different strikes. There are N_{tr} traders and N_{op} call/put options. The strike price of the n-th call/put option is represented by K^n, and its market price at time t is denoted as $V^{n,\phi,t}$. Speculators make decisions based on their expectations of the asset price at the option expiration time. In addition, their expected prices are influenced by news over time. Arbitrageurs trade in response to different arbitrage opportunities. Inspired by the empirical fact that trading in out-of-the-money (OTM) options is more liquid than in-the-money (ITM) options (Ederington and Guan (2002)), a differential liquidity mechanism is introduced in the model. Change of option price is modeled as being proportional to excess demand, mimicking the action of market makers to balance supply and demand (see Appendix A.1). For the sake of simplicity, we ignore interest rate, dividends, transactions costs, taxes, and keep the time to maturity fixed. In the remaining part of this section, we describe the model in detail. In order to offer important insight into the smile phenomenon, we adopt an approach of successive complexification of the basic model, which is similar to that taken in Johnson et al. (2003) and Qiu et al. (2007).

5.1.1 S model: The market consists of only speculators

We start with the case that all the traders are speculators (SP). Our model at this stage is named 'S model'. Each speculator has an idiosyncratic belief about the future price of the underlying asset. We denote the price of the underlying asset at the option expiration time as perceived by the i-th speculator at time t as $S_{SP}^{i,t}$. The current price of the underlying asset S^t, the fraction of speculators F_{SP}^t, and $S_{SP}^{i,t}$ are all influenced by news and vary over time. We denote the long-term mean of $S_{SP}^{i,t}$ as $\bar{S}_{SP}^i$ and assume that, for the entire group of speculators, $\bar{S}_{SP}^i$ follows a lognormal distribution (for the simple reason that prices cannot be negative). The mean of the distribution of $S_{SP}^{i,t}$ for the entire group of speculators at time t is represented by M_{SP}^t and the corresponding standard deviation is denoted as D_{SP}^t. The difference between M_{SP}^t and S^t reflects the level of the speculators' anxiety about the price movement, while D_{SP}^t conveys the level of the speculators' disagreement about the future. For a more detailed description, see Appendix A.2.

At each time-step, based on their expected asset prices, the speculators estimate the profitability of trading each option. We assume that the transaction quantity of the i-th speculator for the n-th option, denoted as $Q_{SP}^{i,n,\phi,t}$, is proportional to the trader's expected profit of the deal:

$$Q_{SP}^{i,n,\phi,t} = \lambda_{SP}[\max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}], \tag{5.1}$$

where λ_{SP} is a positive parameter indicating the activity level of the speculators and, at this stage of modeling, it is independent of strike. Here, $\max(\phi(S_{SP}^{i,t} - K^n), 0)$ is the payoff as estimated by the trader that can be gained from buying the option, while the price of the option $V^{n,\phi,t}$ represents the cost for establishing this long position. Therefore, if the former is greater/smaller than the latter, the trader will purchase/write the option.

5.1.2 SA model: The market consists of both speculators and arbitrageurs

Next, arbitrageurs (AR) are included. We name the model at this stage 'SA model'. The arbitrageurs monitor the option prices and trade those options found to violate the arbitrage relations, e.g., put-call parity (PCP) and butterfly spread (BFS) (Cox and Rubinstein (1985)). If these relations are violated, arbitrageurs will sell/buy the relatively overvalued/undervalued option(s). We assume that their transaction quantities are proportional to the strength with which the relations are violated.

PCP is an arbitrage relationship between the European call and put options with the identical strike price: $V^{n,1,t} - V^{n,-1,t} = S^t - K^n$ (Cox and Rubinstein, 1985). If it is violated, an arbitrageur's transaction quantity is

$$Q_{AR_{PCP}}^{i,n,\phi,t} = -\phi\, \lambda_{AR_{PCP}}\, (V^{n,1,t} - V^{n,-1,t} - S^t + K^n), \tag{5.2}$$

where $\lambda_{AR_{PCP}}$ indicates the activity level of the arbitrageurs for the PCP strategy and $V^{n,1,t} - V^{n,-1,t} - S^t + K^n$ expresses the strength of the PCP violation related to the n-th call and put options. By transacting the relatively under- and overvalued claims in equal amounts, a sure profit is expected to be guaranteed.

BFS stands for an arbitrage relationship involving three call/put options with strike prices that are separated by an equal distance: $V^{n,\phi,t} < (1/2)(V^{n-h,\phi,t} + V^{n+h,\phi,t})$ (Cox and Rubinstein (1985)). If this relation is violated, an arbitrageur's transaction quantity is

$$\begin{pmatrix} Q^{i,n-h,\phi,t}_{AR_{BFS}} \\ Q^{i,n,\phi,t}_{AR_{BFS}} \\ Q^{i,n+h,\phi,t}_{AR_{BFS}} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \lambda_{AR_{BFS}} \max(V^{n,\phi,t} - \frac{1}{2}(V^{n-h,\phi,t} + V^{n+h,\phi,t}) + \varepsilon^h, 0), \quad (5.3)$$

in which $\lambda_{SP_{BFS}}$ is a parameter indicating the activity level of the arbitrageurs for the BFS strategy. Here a portfolio consisting of three options with strikes K^{n-h}, K^n, and K^{n+h} and relative weights 1:2:1 is expected to lead to a sure profit. The value of h varies, meaning that the arbitrageurs apply this rule to all the possible butterfly spreads of which K^n is the common middle strike. $\varepsilon^h \geq 0$ denotes the convexity of the curve considered by the arbitrageurs. For a typical convex IV curve, ε^h increases with increasing h. For simplicity, we adopt $\varepsilon^h = \alpha h$ where $\alpha \geq 0$. The term $\max(V^{n,\phi,t} - \frac{1}{2}(V^{n-h,\phi,t} + V^{n+h,\phi,t}) + \varepsilon^h, 0)$ expresses the strength of the BFS violation related to the three call/put options.

5.1.3 SAL model: Difference in liquidity is included

In real markets, the liquidity of options is not balanced across strikes (Ederington and Guan (2002)). In general, OTM options are more liquid than ITM options, implying that speculators trade in the former more actively than the latter. This difference in liquidity might stem from the trading behavior of speculators and the price characteristics of options. Generally, speculators prefer cheap and liquid options in order to achieve high leverage and fast conversion. The potential for higher leverage provided by OTM options, which are cheaper than their ITM counterparts, attracts more speculators. Higher leverage thus leads to higher liquidity, which in turn pulls in even more speculative trading volume. This positive feedback effect ensures the relatively higher/lower liquidity of OTM/ITM options.

We therefore further include the difference in liquidity by introducing a strike dependence to the speculators' trading activity, which is an increasing/decreasing function of strike price for the call/put options:

$$\lambda_{SP}(K^n) = \eta_{SP}[\phi \tanh(\gamma(K^n - S^t)) + 1], \tag{5.4}$$

where η_{SP} and γ are positive parameters.

Notice that in constructing arbitrage portfolios, all the options involved must be traded simultaneously and in the specified proportions. Liquidity unbalancing is therefore not applicable to the arbitrageurs in our model. This complete version of the model is termed 'SAL model'.

5.2 Simulation results

In simulations a wide range of values are considered for the model parameters[1]. Our main findings are robust in the sense that all the main characteristics of the resulting IV smile remain qualitatively unchanged. The parameter values corresponding to the results shown in this paper are discussed in Appendix A.3.

5.2.1 Shapes of the implied volatility curves

Here we present the option prices and the corresponding IV curves generated by our model at different levels, by keeping S^t, F^t_{SP}, and $S^{i,t}_{SP}$ (hence M^t_{SP} and D^t_{SP}) constant and equal to their respective long-term means.

[1] In microsimulations, we often face the situation as described in Lux and Marchesi (2000): "In order to choose appropriate sets of parameter values, one would ideally like to calibrate the model by using relevant empirical observations on its various components. Unfortunately, we lack empirical estimates for all those parameters ..."

5.2.1.1 Implied volatility curves from the S model

Using the S model, if M_{SP}^t coincides with S^t, the option prices overlap with the corresponding BS prices and the resultant IV curve against strike is flat (data not shown). This can also be shown analytically (see Appendix B).

We then examine the IV smile in the case that $M_{SP}^t < S^t$. In this situation, the price of the underlying asset is considered by the speculators more likely to suddenly drop than to suddenly rise, in line with the nature of stock indices. (Notice that we have assumed a zero interest rate and ignored dividends.) As shown in Figures 5.1(a) and 5.1(b), the resultant prices of all call/put options are lower/higher than the corresponding BS prices. Consequently, the IVs of the call options are lower than the corresponding IVs for the put options. Obviously, the simulated option prices do not satisfy put-call parity.

5.2.1.2 Implied volatility curves from the SA model

As illustrated in Figure 5.1(c), the simulated option prices from the SA model satisfy both put-call parity and the butterfly spread relation. It can be observed that the speculators' trading and that of the arbitrageurs tend to move the option prices in opposite directions. Hence, the different strategies followed by the speculators and the arbitrageurs lead to a competing effect on the simulated option prices.

5.2.1.3 Implied volatility curves from the SAL model

In the simulations based on the SAL model, if the arbitrageurs only act on violation of put-call parity, we have obtained results as shown in Figures 5.1(e) and 5.1(f). Here we see that most of the simulated option prices satisfy put-call parity. In addition, the IVs of the options with strikes lower than S^t are higher than those IVs corresponding to the options with strikes higher than S^t. Here we can observe that the competition becomes unbalanced due to the difference in speculators' activity between OTM options and ITM options. However, the options do not satisfy the butterfly spread relation. Consequently, the IV curves are not convex as shown in empirical data.

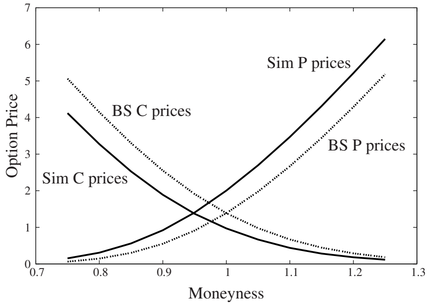
If the arbitrageurs also trade in response to violations of the butterfly spread relation, new competition is induced at some call and put options between the speculation and the butterfly spread arbitrage and also between the two types of arbitrage. At this point, as displayed in Figure 5.1(h), we have an IV skew similar to those observed in real markets for equity index options.

5.2.2 Dynamical properties of the implied volatility curves

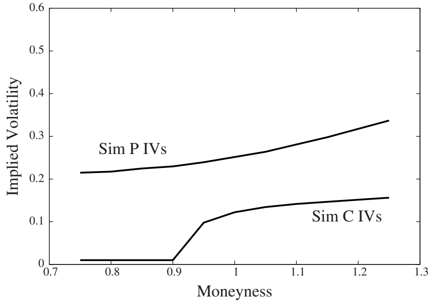
Based on the complete (SAL) model, we now consider the case that S^t, $S^{i,t}_{SP}$, and F^t_{SP} are all influenced by news and change over time. Figure 5.2(a) depicts the evolution of the IVs of three put options with different strike prices. Notice that the IVs are generally more volatile for small than large moneyness (K^n/S^t), in line with the empirical analysis reported in Cont and da Fonseca (2002). The transaction volumes for the call and put options are shown in Figure 5.2(b). Although we have adopted a simple activity function for the speculators, i.e. Equation (5.4), the resultant volume distribution is similar to the corresponding empirical distribution reported in Ederington and Guan (2002).

We then perform principal component analysis (PCA) on the resulting IV time series. Figure 5.2(c) exhibits the eigenvectors corresponding to the three largest eigenvalues. These principal components are in line with the empirical results reported in Cont and da Fonseca (2002) and Fengler et al. (2003). Figure 5.2(d) displays the proportions of variance explained by these eigenvectors. They agree with the empirical data from skew-dominated markets on the total variance explained by the first three eigenmodes and approximately on their relative values.

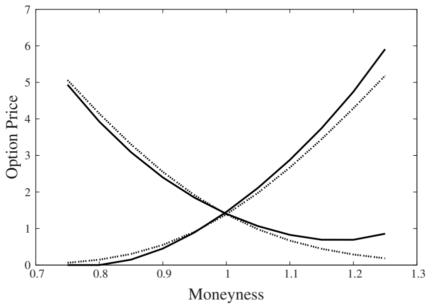
We have performed a sensitivity analysis of our model by varying the values chosen for some parameters that are critical to the simulated IV dynamics (see Section 5.4). It suggests that our results are qualitatively robust.


Option Price
Moneyness
Sim P prices
BS C prices
BS P prices
Sim C prices

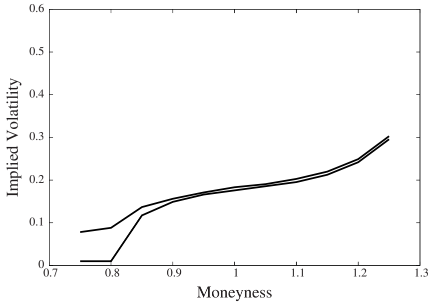
(a)


Implied Volatility
Moneyness
Sim P IVs
Sim C IVs

(b)


Option Price
Moneyness

(c)


Implied Volatility
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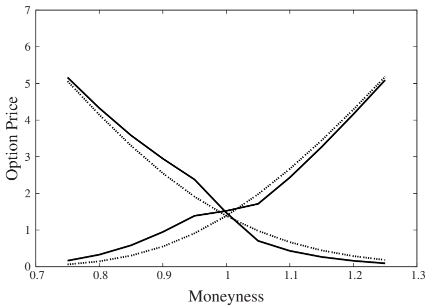
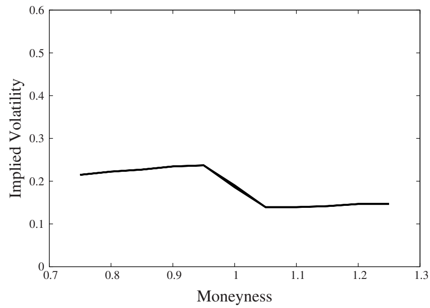
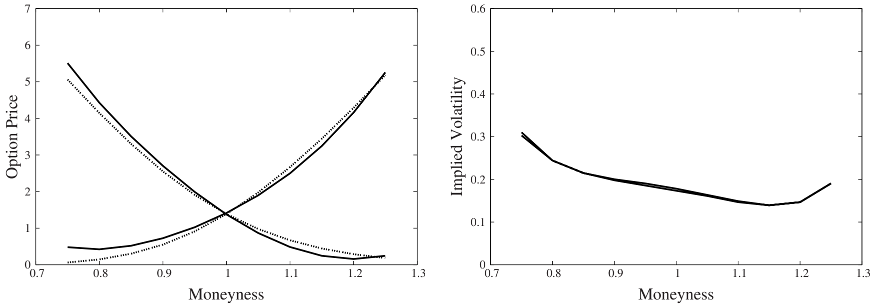
(d)


Option Price
Moneyness

(e)


Implied Volatility
Moneyness

(f)



(g) (h)

Figure 5.1: Simulated option prices and implied volatilities. They are plotted against moneyness (K^n/S^t) as solid lines (Sim is for 'simulated', C for 'call', and P for 'put'). The corresponding BS prices (dashed lines), which satisfy all the arbitrage relations, are also plotted here for comparison. (a) Option prices obtained from the S model; (b) corresponding IVs. (c) Option prices obtained from the SA model; (d) corresponding IVs. (e) Option prices obtained from the SAL model if the arbitrageurs only act on violation of put-call parity; (f) corresponding IVs. (g) Option prices obtained from the SAL model if the arbitrageurs also act on violations of the butterfly spread relation; (f) corresponding IVs. A lower bound of 0.01 is taken for the IVs.

5.3 Discussion: The mechanism underlying the smile phenomenon

5.3.1 Competing effect induced by heterogeneous trading behavior

Figure 5.3 clarifies the competing types of trading behavior and their effects on the shape of the IV curves. It illustrates that, although the traders have distinct trading interests, their behavior leads to contests regarding the option prices and eventually the shape of the IV smile.

In the following parts of this section, we again keep S^t, $S_{SP}^{i,t}$, $F_{SP}^{i,t}$ constant

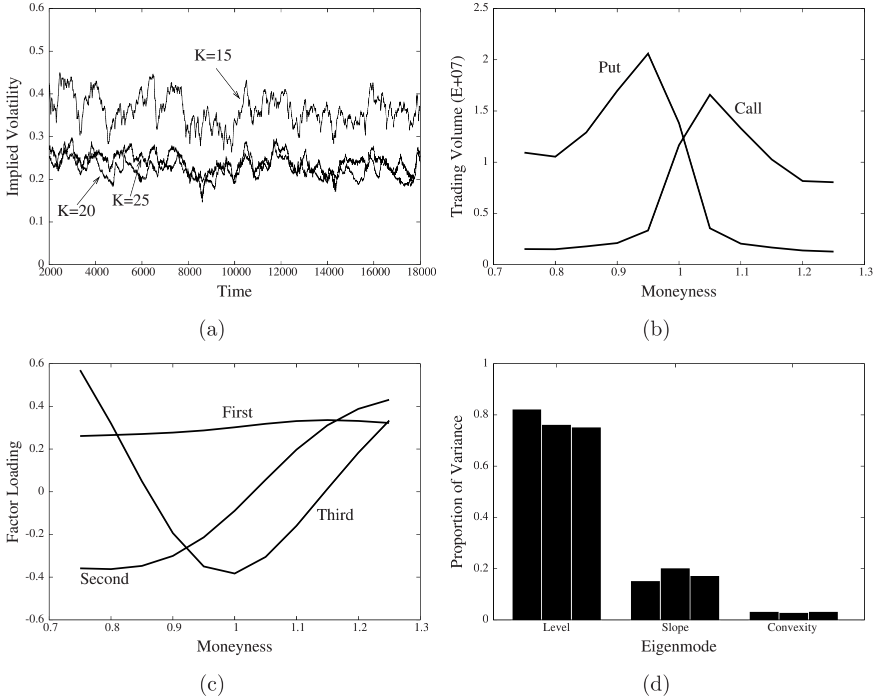


Figure 5.2: Dynamic results. (a) IV time series of the put options with different strike prices: $K = 15$, 20, and 25. (b) Trading volumes of the call and put options over the selected time period. (c) The eigenvectors corresponding to the three largest eigenvalues. (d) The proportions of variance explained by the principal components (first bar of each eigenmode): 82%, 15%, and 3%, respectively, in comparison with the empirical results shown in Fengler et al. (2003) (second bar) and our empirical findings.

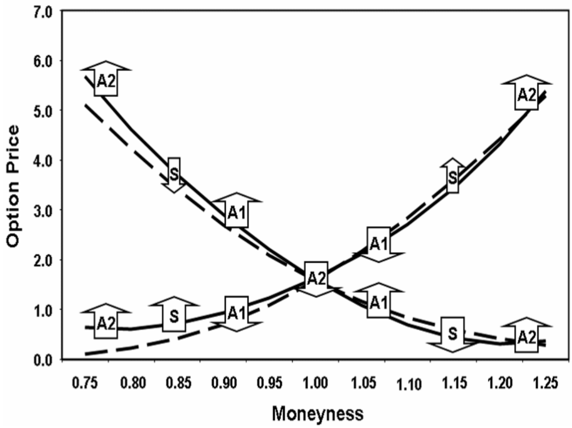


Figure 5.3: Competition. This figure exhibits the competition between the different types of trading behavior regarding the option prices. The S, A1, and A2 arrows represent, respectively, the effects of speculation, put-call parity arbitrage, and butterfly spread arbitrage. The larger/smaller S arrow denotes high/low speculators' activity level.

in order to investigate the influence of the variations in some individual free parameters of the model on the characteristics of the smile curve.

5.3.2 Shapes of implied volatility curves corresponding to different types of uncertainty

In financial markets, speculators have realized that large sudden positive and negative price changes are not equally likely to happen. Correspondingly, they will shift their expectations of the future price, to a certain extent, to the direction with higher possibility. Therefore the mean of the overall distribution of the expected prices will be generally away from the current price to that direction.

As shown in Figure 5.4, when $M^t_{SP} < S^t$, we obtain a downward sloping IV curve; conversely, if $M^t_{SP} > S^t$, we have an upward sloping one; if M^t_{SP} is close to S^t, the resultant curve is symmetric and turns upwards at both ends. The case

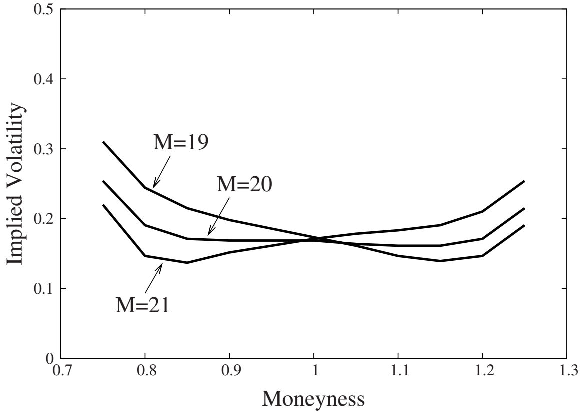


Figure 5.4: Shapes of IV curves. This figure presents the simulated IV curves corresponding to different values chosen for M_{SP}^t: 19, 20, and 21 respectively, while S_t is fixed at 20.

$M_{SP}^t < S^t$ indicates that the speculators believe that unexpected large price drops are more likely to happen than sudden large price rising. It reminds us of the nature of stock indices. This characteristic might only be realized by traders after a few large stock market crashes, e.g., the one happened in 1987. This explains why the smile has existed only since that crash. The situation $M_{SP}^t > S^t$ is similar to certain circumstances of some commodities, in which traders think that the prices might suddenly rise due to, eventually, the general shortage of resources. The case $M_{SP}^t \approx S^t$ resembles the situation of options on foreign exchange rates between currencies with equal strength, in which traders believe that the relative values of these currencies will change in a more symmetric way. The deviation of M_{SP}^t from S^t is therefore crucial to the shape of the IV curve.

5.3.3 Driving factors in the fluctuations of implied volatility curves.

As expressed in Figure 5.5(a), the smaller is M_{SP}^t with respect to S^t, the steeper is the skew. This may confirm the empirical observation that, whenever the market declines/increases, the skew tends to become more/less pronounced (Hull (2003)). Our explanation is that, the more S^t falls, the further M^t_{SP} deviates from S^t due to 'crashophobia' (Rubinstein (1994)) and therefore the steeper the IV curve becomes. As shown in Figure 5.5(b), adopting larger/smaller values for D^t_{SP}, the IV curves are similar in shape but with higher/lower levels. This may confirm the empirical observation that scheduled news announcements generally lead to a drop of IVs; conversely, most major unscheduled announcements cause a rise (Ederington and Lee (1996)). Our explanation is that a scheduled/unscheduled announcement decreases/increases the variance of speculators' expected prices, hence causes IVs to fall/rise. Figure 5.5(c) exhibits the effect of the competition between the speculators and the arbitrageurs: Speculation/(butterfly spread arbitrage) tends to decrease/increase the convexity.

We can further consider the heterogeneity of speculators with regard to the fear of large sudden price changes. In the case of equity index options, speculators are different in the degree of 'crashophobia' and in general pessimistic traders are more sensitive to negative news than optimistic traders. Then, the more the price of the underlying falls due to negative news, the larger D^t_{SP} becomes and consequently the more the IV curve rises. This can explain the so-called 'leverage effect' described in, e.g., Cont and da Fonseca (2002), that shifts in global level of IVs are negatively correlated with returns of the underlying. Contrary to index options, some speculators trading in commodity options are more sensitive than other traders to the news that triggers price rises. In this case, the more the price rise, the larger D^t_{SP} becomes and consequently the more the IV curve rises. This can confirm the 'inverse leverage effect' observed in commodity options markets, i.e., shifts in IV level are positively correlated with returns of the underlying (Geman (2005)).

When D^t_{SP}, M^t_{SP}, and F^t_{SP} vary together over time, the first two factors cause the most variation of the IV curve, as shown in Figure 5.2(d). We therefore conclude that the overall simulated dynamics is mainly driven by fluctuations in the level of speculators' disagreement, as well as the level of their general anxiety about the price movement of the underlying asset.

In the next section, we will discuss some additional analysis that we have performed to demonstrate the robustness of our model.

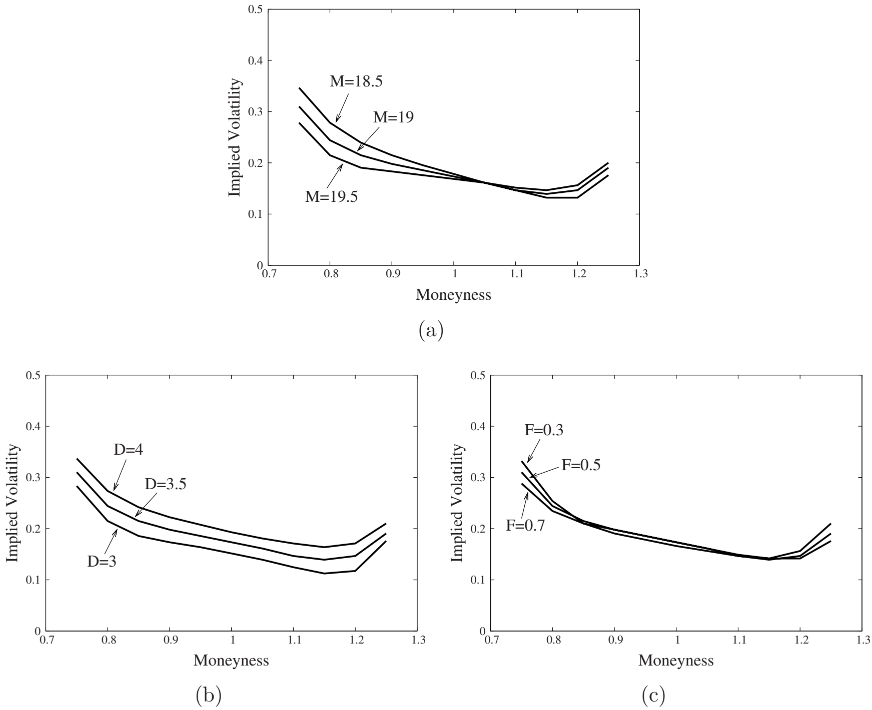


Figure 5.5: Driving factors of the IV dynamics. This figure shows the simulated IV curves corresponding to different values for M_{SP}^t, D_{SP}^t, and F_{SP}^t. Here, $S^t = 20$ and, unless otherwise indicated, the values for M_{SP}^t, D_{SP}^t, and F_{SP}^t are fixed at 19, 3.5, and 0.5 respectively. (a) $M_{SP}^t = 18.5$, 19, and 19.5 respectively. (b) $D_{SP}^t = 3$, 3.5, and 4 respectively. (c) $F_{SP}^t = 0.3$, 0.5, and 0.7 respectively.

5.4 Robustness study

We have studied the robustness of our model by (1) investigating the variation in the shape of the IV smile due to speculative traders who use the Black-Scholes formula in assessing the option prices, and (2) examining the sensitivity of the IV dynamics to the variations in the model parameters.

5.4.1 Reactions of the implied volatility curve to speculations using the Black-Scholes model

In options markets, there are some speculators who employ (analytical) option pricing models, typically the BS formula. In valuing options, BS traders need to estimate the volatility of the underlying asset, which is the only unobservable parameter in the formula. For analyzing the robustness of our results, here we include some BS speculators in our model. For simplicity, we assume that these traders adopt strike-independent volatilities.

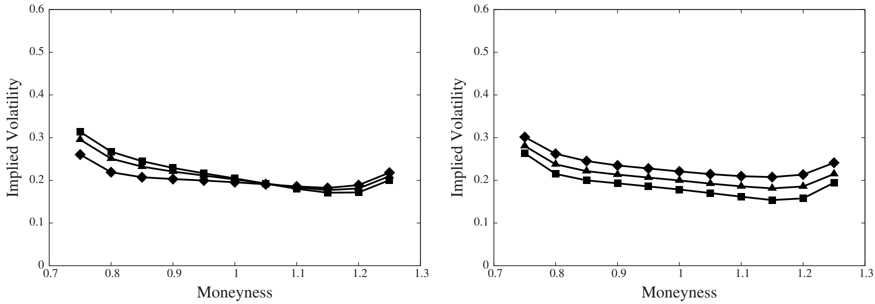
In real markets, BS speculators have different perceptions of the future volatility of the underlying asset. We express the volatility expected by the i-th speculator (if this trader is a BS speculator) as $\Sigma_{SP\_BS}^{i,t}$ with a long-term average value $\overline{\Sigma}_{SP\_BS}^{i}$. We assume that, for the entire group of BS speculators, $\overline{\Sigma}_{SP\_BS}^{i}$ follow a lognormal distribution (for the simple reason that volatility cannot be negative).

At each time-step, based on their perceived volatilities, the BS speculators estimate the profitability of trading each option. We assume that the transaction quantity of the i-th speculator (who is a BS trader) for the n-th option is proportional to the trader's expected profit of the deal:

$$Q_{SP\_BS}^{i,n,\phi,t+1} = \lambda_{SP\_BS}(K^n)[V_{BS}^{\phi,t}(S^t, K^n, r, T, \Sigma_{SP\_BS}^{i,t}) - V^{n,\phi,t}], \tag{5.5}$$

where $\lambda_{SP\_BS}(K^n)$ is a positive parameter indicating the activity level of the BS speculators and, for simplicity, we assume $\lambda_{SP\_BS(K^n)} = \lambda_{SP}(K^n)$;
$V_{BS}^{\phi,t}(S^t, K^n, r, T, \Sigma_{SP\_BS}^{i,t})$ is the price of the option as expected by the BS trader;
$V_{BS}^{\phi,t}(S^t, K^n, r, T, \Sigma_{SP\_BS}^{i,t}) - V^{n,\phi,t}$ is therefore the payoff that can be gained from buying the option, while the price of the option $V^{n,\phi,t}$ represents the cost for establishing this long position. If the former is greater/smaller than the latter, the trader will purchase/write the option.

As shown in Figure 5.6(a), the BS speculators' trading can decrease the skewness of the IV curve produced by the directional speculators (through interactions with arbitrageurs). As the relative number of the BS traders is increased the steepness of the skew is further reduced. The flattening effect is expected because the BS speculators adopt strike-independent volatilities. Moreover, as shown in Figure 5.6(b), the higher the average perceived volatility of the BS traders, the higher the IV curve, vice versa.



(a) (b)

Figure 5.6: IV curves if the speculators using the BS model trade together with the simple directional (SD) speculators. The fraction of arbitrageurs is 30%. (a) The fractions of the BS speculators are 0% (the ■ line), 20% (▲ line), and 50% (the ♦ line) respectively. Here, the mean of the BS speculators' volatilities is 0.2. (b) The mean of the BS speculators' volatilities are 0.15 (the ■ line), 0.2 (the ▲ line), and 0.25 (the ♦ line) respectively. Here, the fraction of the BS speculators and that of the SD speculators are both 35%.

However, although the BS speculators can vary the skewness and level of the IV curve, the general shape of the curve remains unchanged. Further taking into consideration the fact that in reality directional speculators outnumber other types of speculator (Lakonishok et al. (2007)), the mechanism underlying the volatility smile revealed by our model is robust.

5.4.2 Sensitivity analysis of the implied volatility dynamics

Next, we vary the values chosen for some parameters that are critical to the simulated IV dynamics, i.e., ν_ψ, ν_ω, and ν_F, and examine the sensitivity of the IV dynamics to the variations in these parameters.

Table 5.1: Proportions of variance explained by the principal components corresponding to different values chosen for ν_ψ. All other parameters are kept fixed with $\nu_\omega = 0.02$ and $\nu_F = 0.05$.

ν_ψ	0.001	0.0015	0.002	0.0025	0.003
Level	0.87	0.85	0.82	0.80	0.77
Slope	0.09	0.12	0.15	0.17	0.20
Convexity	0.03	0.03	0.03	0.03	0.03

Table 5.2: Proportions of variance explained by the principal components corresponding to different values chosen for ν_ω. All other parameters are kept fixed with $\nu_\psi = 0.002$ and $\nu_F = 0.05$.

ν_ω	0.01	0.015	0.02	0.025	0.03
Level	0.56	0.72	0.82	0.88	0.92
Slope	0.36	0.22	0.15	0.10	0.07
Convexity	0.08	0.05	0.03	0.02	0.02

The results in Tables 5.1, 5.2, and 5.3 demonstrate that ν_ψ, ν_ω, and ν_F are positively correlated with the variance proportions explained by slope, level, and convexity respectively. This is in line with the mechanism discussed in Section 5.3.

Empirical studies have shown that the smile dynamics is characterized by three eigenmodes, among which level explains most of the variation of the smile, followed by slope and convexity (Cont and da Fonseca (2002), Fengler et al. (2003)). Our model robustly confirms this finding for a wide range of parameter settings.

Table 5.3: Proportions of variance explained by the principal components corresponding to different values chosen for ν_F. All other parameters are kept fixed with $\nu_\psi = 0.002$ and $\nu_\omega = 0.02$.

ν_F	0.025	0.038	0.05	0.063	0.075
Level	0.84	0.82	0.82	0.82	0.82
Slope	0.15	0.14	0.15	0.14	0.14
Convexity	0.01	0.03	0.03	0.04	0.04

5.5 Conclusions

In today's financial markets derivatives on different underlying assets with complex payoffs are traded in huge volumes. Standard financial practice employs mathematical risk models such as the Black-Scholes price formula which are built upon oversimplified stochastic models for price dynamics, and then corrected in an ad hoc way to reduce inaccuracies. These imperfections are highlighted by the volatility smile phenomenon. Importantly, in all these models the individual trading behavior is assumed to be irrelevant.

In this chapter, we present a microsimulation model of options markets to explain the origin of the smile. In our model, the typical behavior of the most active options traders, i.e., speculators and arbitrageurs, is adopted and the prices of the options are determined by demand and supply. Notwithstanding its simplicity, the model reproduces the empirical smile curve in terms of its shape and dynamics. Our results suggest that the volatility smile is a natural consequence of traders' heterogeneous behavior and expectations about the future.

In fact, the importance of heterogeneous beliefs for derivatives markets has been empirically disclosed by studies in behavioral finance. See, for example, Shefrin (1999). Our results confirm that individual trading preferences indeed play a critical role in the behavior of the implied volatility curve. These insights will improve the understanding and quantification of the inherent risk of derivatives trading for institutions and investors alike.

Chapter 6

Effects of Heterogeneous Speculative Strategies on the Volatility Smile

We have studied the market mechanism underlying the volatility smile phenomenon through a microsimulation (MS) model of options markets, described in Chapter 5. The model is able to reproduce the volatility smile and its dynamic properties in a simple and robust manner, and can explain the related stylized facts observed in real markets. It mainly adopts one type of speculative strategy, i.e. simple directional (SD) speculation, supported by the finding of a recent empirical study reported in Lakonishok et al. (2007).

However, it is still not clear how other commonly-used speculative strategies, which can be generally classified into directional strategies and volatility strategies, influence the smile. We hence investigate the effects of these strategies on the shape the IV curve[1]. Since the strategies coexist with the SD strategy in real markets, this study is indispensable for the comprehensive understanding of the volatility smile phenomenon.

We first describe the main heterogeneous speculative strategies included in our model in Section 6.1. Through simulations we study the effects of these strategies on the IV curve. The simulation results and our conclusions are discussed in

[1]This chapter is based on Qiu et al. (2010c)

Sections 6.2, 6.3 and 6.4, respectively.

6.1 Modeling heterogeneous speculators

The trading behavior of different types of speculator is based on their views on certain determinant factors that control the expected profits from employing the specific trading strategies. Rudimentary speculative strategies can be classified into two types: Directional and volatility. Directional strategies profit from either rising or falling price movements of the underlying, while volatility speculations rely on absolute price movements regardless of the direction.

Speculators are generally heterogeneous with respect to their judgments about the values of the determinant factors. The judgments are influenced by news and change overtime. However, since we are investigating the general effects of the strategies on the shape of smile, we assume that they are constant for simplicity. Directional spread speculators have different expectations regarding the future price of the underlying, denoted as $S^i_{SP\_D}$. Volatility spread speculators expect different levels of the fluctuations in the future price, represented by $W^i_{SP\_V}$, which is the absolute difference between two prices symmetric to the spot price, denoted respectively as $S^i_{SP\_V,S}$ (S for 'small') and $S^i_{SP\_V,L}$ (L for 'large').

Directional spread (DSPR) traders are the most typical directional speculators, apart from the simple directional (SD) traders. A typical volatility trading strategy is the so-call butterfly spread (BSPR) speculation. Detailed discussions about the payoffs of these strategies can be found in, among many others, Natenberg (1994) and Hull (2003). The corresponding profit diagrams are shown in Figure 6.1. Briefly, these two strategies involve combinations of a few options and the profit of each portfolio is the sum of the profits of the constituent options. In the next section, we will discuss these strategies in more detail.

6.1.1 Directional spread speculation

A typical DSPR strategy involves a position in two call/put options. A bull spread is created by buying a call/put option with a certain strike K^{n_1} and selling another call/put option with a higher strike K^{n_2}. It restrains the investor's upside potential. In return, the investor limits the downside risk and finances the purchasing through the selling. Bull spreads benefit from an increase in the price of the underlying. By contrast, a bear spread is constructed by reversing the positions, i.e. buying a call/put option with a certain strike and selling another call/put option with a lower strike. It also limits the investor's upside potential as well as downside risk, and receives premiums through selling options. Bear spread benefits from an price decrease of the underlying.

Based on their assessments of the price of the underlying, the DSPR speculators can estimate the profits from trading the individual options, denoted as $E_{SP\_D}^{i,n,\phi,t}$.

$$E_{SP\_D}^{i,n,\phi,t+1} = max(\phi(S_{SP\_D}^{i,t} - K^n), 0) - V^{n,\phi,t}, \tag{6.1}$$

where $max(\phi(S_{SP\_D}^{i,t} - K^n), 0)$ is the payoff as estimated by the trader that can be gained from buying the option; $V^{n,\phi,t}$, which is the market price of the option, represents the cost for establishing this long position. The speculator's expected profit from selling the option is $-E_{SP\_D}^{i,n,\phi,t+1}$.

We assume that the transaction quantities of each DSPR trader are determined by the trader's activity level and expected profit of the portfolio, namely

$$\begin{pmatrix} Q_{SP\_DSPR}^{i,n_1,\phi,t+1} \\ \\ Q_{SP\_DSPR}^{i,n_2,\phi,t+1} \end{pmatrix} = \begin{pmatrix} 1 \\ \\ -1 \end{pmatrix} \lambda_{SP\_DSPR} \, (E_{SP\_D}^{i,n_1,\phi,t+1} - E_{SP\_D}^{i,n_2,\phi,t+1}), \tag{6.2}$$

in which λ_{SP_DSPR} is a positive parameter which reflects the activity level of the DSPR speculators.

6.1.2 Butterfly spread speculation

A BSPR portfolio is created by buying a call/put option with a relatively low strike K^{n_1}, buying a call/put option with a relatively high strike K^{n_3}, and selling two call/put options with a medium strike K^{n_2} equally distant from the other two strikes. Generally the middle strike is close to the current price of the underlying. A butterfly spread leads to a profit/loss if the underlying stays close to/(move significantly to either direction from) its current price. This spread can be sold

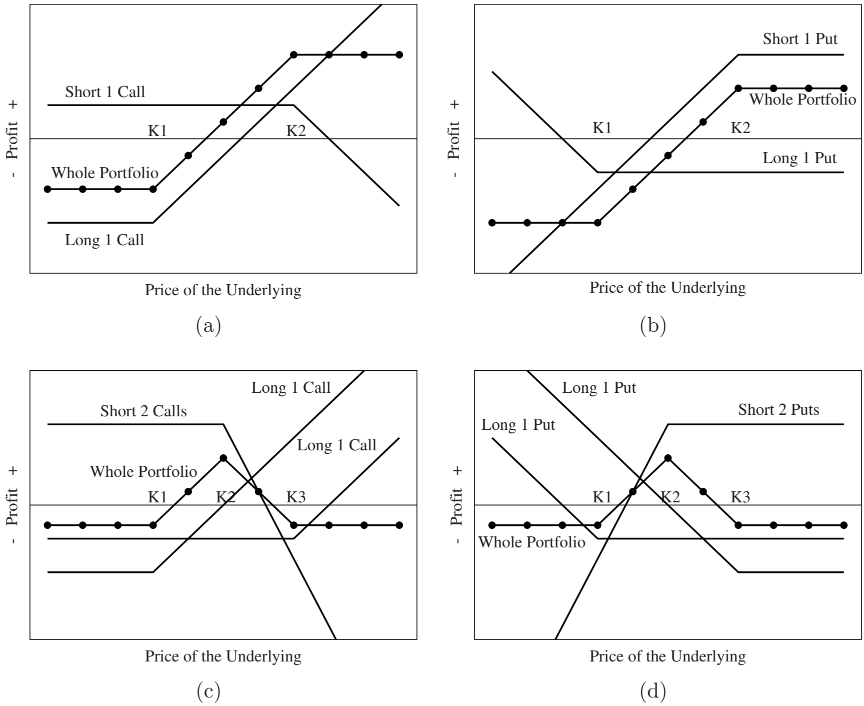


Figure 6.1: Profit diagrams of directional spread (DSPR) and butterfly spread (BSPR) speculative strategies. (a) A DSPR portfolio composed of call options. (b) A DSPR portfolio composed of put options. (c) A BSPR portfolio composed of call options. (d) A BSPR portfolio composed of put options.

by reversing the positions. BSPR speculations limit the investors' profits as well as risk.

Based on their assessments of the price of the underlying, the BSPR speculators can estimate the profits from trading the individual options, denoted as $E_{SP\_V}^{i,n,\phi,t}$. It is the average of the profit estimated according to the trader's lower expected price ($S_{SP\_V,S}^{i,t}$) and that according to the higher expected price ($S_{SP\_V,L}^{i,t}$),

$$\begin{aligned} E_{SP\_V}^{i,n,\phi,t+1} &= \frac{1}{2}[(max(\phi(S_{SP\_V,S}^{i,t} - K^n), 0) - V^{n,\phi,t} \\ &\quad + max(\phi(S_{SP\_V,L}^{i,t} - K^n), 0) - V^{n,\phi,t})], \end{aligned} \tag{6.3}$$

where $max(\phi(S_{SP\_V,(\cdot)}^{i,t} - K^n), 0)$ and $V^{n,\phi,t}$ are respectively the payoff as estimated by the volatility trader and the cost of the long position. The trader's expected profit from selling the option is $-E_{SP\_V}^{i,n,\phi,t+1}$.

We assume that the transaction quantities of each BSPR trader for the options are determined by the trader's activity level and expected profit of the portfolio, namely

$$\begin{pmatrix} Q_{SP\_BSPR}^{i,n_1,\phi,t+1} \\ Q_{SP\_BSPR}^{i,n_2,\phi,t+1} \\ Q_{SP\_BSPR}^{i,n_3,\phi,t+1} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \lambda_{SP\_BSPR} \, (E_{SP\_V}^{i,n_1,\phi,t+1} - 2E_{SP\_V}^{i,n_2,\phi,t+1} + E_{SP\_V}^{i,n_3,\phi,t+1}), \tag{6.4}$$

in which λ_{SP_BSPR} is a positive parameter indicating the activity level of the BSPR speculators.

We further assume that the speculators are equally active in applying all possible strategies. However, the various strategies differ in the number of potential portfolios in which traders can invest. In addition, different from SD speculators' trading activity which is strike dependent, DSPR and BSPR speculators' trading activity is identical across strikes. We take these differences into consideration in choosing values for λ_{SP_DSPR} and λ_{SP_BSPR}. The detailed explanations, functional forms, and values adopted for these activity parameters are described in Appendix C.

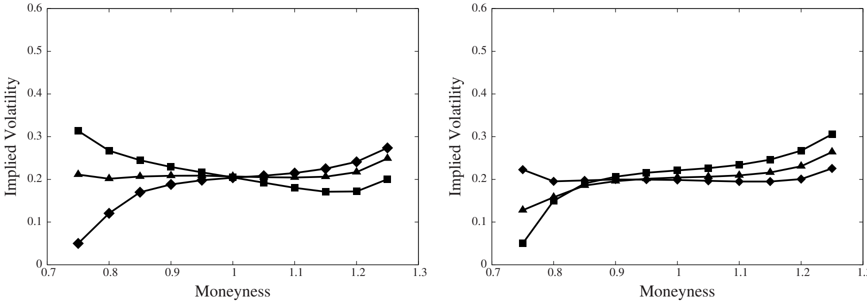
6.2 Effects on the volatility smile

Here we study the influences of the DSPR and BSPR speculators on the IV curve produced by the SD traders, by examining the shape of the IV curve when they separately trade together with the SD traders. Firstly, we adopt different fractions of the various types of speculator. Secondly, we fix the fraction of each type of traders and adopt different values for the parameters of the distributional properties of the relevant determinant factor. In explaining the mechanisms underlying the effects, we refer to the profit diagrams of the speculative strategies displayed in Figure 6.1. If the profit corresponding to an speculator's expected price is positive/negative according to the diagram, the trader will buy/sell the portfolio.

6.2.1 Directional spread speculators

We again consider the situation that the mean of the expected prices of the directional speculators is smaller than the market price of the underlying. In this case, as shown in Figure 6.1(a) and Figure 6.1(b), more traders will expect negative profits from purchasing the DSPR call and put portfolios, in comparison with the situation that the mean is equal to the market price which gives rise to a symmetric IV curve. Therefore, more traders will sell/buy call and put options with lower/higher strikes, leading to a upward sloping IV curve, and vice versa.

In Chapter 5, we have shown that in the same situation, the SD speculators instead produce a downward sloping IV curve. When trading together with SD speculators, the DSPR speculators hence tend to reverse the skewness generated by the former. The more DSPR traders are in the market, the more the skewness is reversed, as shown in Figure 6.2(a). In addition, if the difference between the directional speculators' expected prices and the spot price of the underlying is increased, the slope of the IV curve will increase, as is displayed in Figure 6.2(b).



(a) (b)

Figure 6.2: IV curves if the directional spread (DSPR) speculators trade together with the simple directional (SD) speculators. The fraction of arbitrageurs is 30%, the mean of the SD speculators' expected prices is 19, and the current price of the underlying asset is 20. (a) The fractions of the DSPR speculators are 0% (the ■ line), 20% (▲ line), and 50% (the ♦ line) respectively. Here, the mean of the DSPR speculators' expected prices is 19. (b) The mean of the DSPR speculators' expected prices are 18 (the ■ line), 19 (the ▲ line), and 20 (the ♦ line) respectively. Here, the fraction of the SD speculators and that of the DSPR speculators are both 35%.

6.2.2 Butterfly spread speculators

Firstly, as shown in Figure 6.3(a), BSPR traders can change the level of the smile. Secondly, when the average price fluctuation level of the BSPR speculators becomes higher, as shown in Figure 6.1(c) and Figure 6.1(d), more traders will expect negative profits from purchasing the BSPR call and put portfolios, and vice versa. Consequently more ITM and OTM call and put options will be sold and in the meantime more ATM call and put options will be bought. This leads to a decrease of the prices of the ITM and OTM options and an increase of the price of the ATM option. Therefore the IV curve becomes less convex, as shown in Figure 6.3(b).

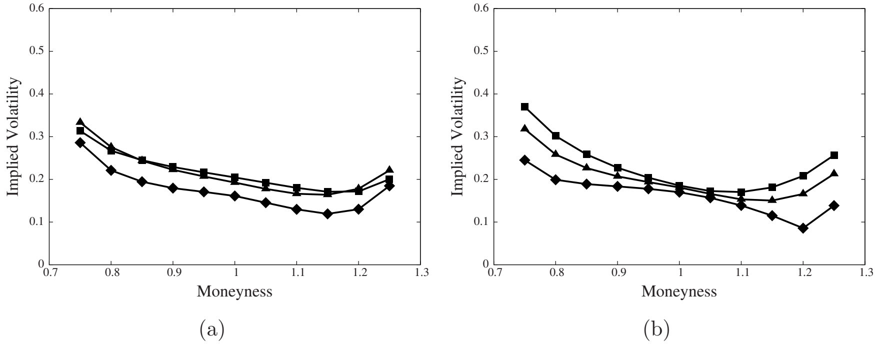
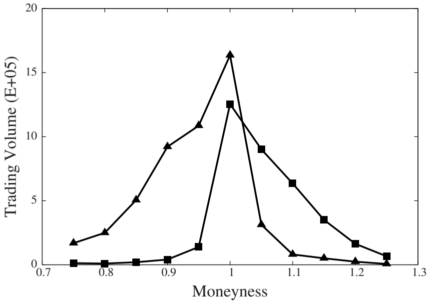


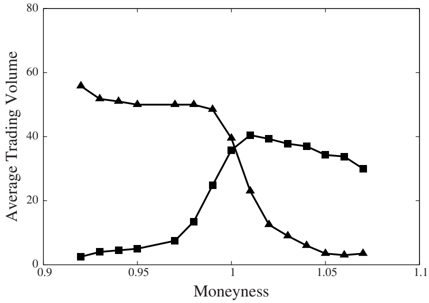
Figure 6.3: IV curves if the butterfly spread strategies (BSPR) speculators trade together with the simple directional (SD) speculators. The fraction of arbitrageurs is 30%, the mean of the SD speculators' expected prices is 19, and the current price of the underlying asset is 20. (a) The fractions of the BSPR speculators are 0% (the ■ line), 20% (▲ line), and 50% (the ♦ line) respectively. Here, the mean of the BSPR speculators' expected variance are 4. (b) The mean of the BSPR speculators' expected variance are 3 (the ■ line), 4 (the ▲ line), and 5 (the ♦ line) respectively. Here,the fraction of the BSPR speculators is 35%

6.3 Analysis of the trading volumes

Although DSPR and BSPR speculators can change the IV curve to a certain extent, they are not the dominant speculators in real markets. This can be justified by comparing the trading volumes produced by the different types of speculator (together with arbitrageurs) in our model with those recorded in real markets. Two examples of empirical volume distributions are shown in Figure 6.4. Figure 6.4(a) displays the trading volumes of the call and put options on the S&P500 index over the period from March 2000 to February 2001, while Figure 6.4(b), of which the data is taken from Reference Ederington and Guan (2002), shows the trading volumes of options on S&P500 Futures over the period from January 1988 to April 1998. They are common in three aspects: (1) The call and put volumes are higher close to ATM. (2) The total volume of put options is larger



(a)



(b)

Figure 6.4: Empirical trading volumes plotted against moneyness (K/S^t). Call and put volumes are denoted by a ■ line and a ▲ line respectively. (a) Volumes of options on the S&P500 index over the period from March 2000 to February 2001. The minimum time to maturity is 0.1 year. (b) Average daily volumes of options on S&P500 Futures over the period from January 1988 to April 1998 and with time to maturity 13 to 26 weeks. Notice that the two data sets are different in moneyness range.

than that of call options. (3) The total volumes of ITM options are much smaller than those of OTM options, with those of deep ITM options being negligible.

To the best of our knowledge, little is known about the fraction of the different types of speculator in real markets. In this section, we analyze the volume distributions generated by the different types of speculator adopted in our microsimulation model. In each numerical experiment we consider only one type of speculator who trades together with arbitrageurs in equal fractions. We let all these directional speculators' expected prices and the volatility speculators' price fluctuation levels change over time, following the general process represented by Equation (A.2) and Equation (A.3) in Appendix A (for the specific forms and corresponding parameter values, see Qiu et al. (2010c)).

Figure 6.5 displays the trading volumes obtained if all the speculators in the simulations are of a specific type. The volume distributions in the case that all the speculators are SD traders are displayed in Figure 6.5(a), which have the same characteristics as the empirical distributions shown in Figure 6.4. If all the speculators are DSPR traders, we obtain trading volumes shown in Figure 6.5(b). They are low at ATM and high at ITM and OTM, and therefore not in line with the empirical observations. In the case that all the speculators are BSPR traders, the volumes are low at ITM and OTM, but high at ATM as well as deep ITM and OTM, as shown in Figure 6.5(c). They also do not agree with the empirical distributions. Only the SD speculators' trading volumes are in line with those observed in real markets. This suggests that the SD traders, rather than other types of speculator, are indeed the dominant speculative traders in real markets. This is in agreement with the findings reported in the empirical study by Ederington and Guan (2002) and further confirms the robustness of our findings regarding the mechanism underlying the volatility smile phenomenon, discussed in the previous chapter.

6.4 Conclusions

Heterogeneous speculative traders, such as DSPR and BSPR speculators, when trading together with SD speculators, induce competing effects with regard to the shape of the volatility smile. In particular, SD and DSPR traders have opposite effect with regard to the skewness of the IV curve, while SD and BSPR traders have opposite effect with regard to the level and convexity of the IV curve. Our analysis of trading volumes suggest that these traders are not the dominant speculators. This is in agreement with empirical findings.

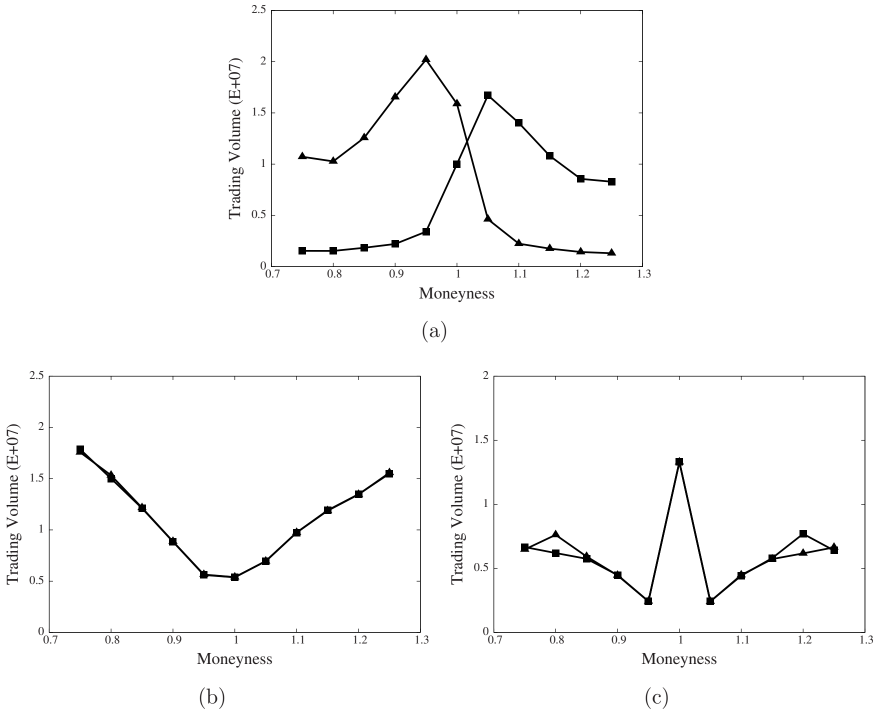


Figure 6.5: Trading volumes obtained if all the speculators in the simulation are of a specific type. Call and put volumes are denoted by a ■ line and a ▲ line respectively. (a) Simple directional traders. (b) Directional spread traders. (c) Butterfly spread traders. The fractions of each specific type of speculators and arbitrageurs are both 50%.

Chapter 7

Conclusions and Future Prospects

7.1 Concluding discussion

In this work, we apply microsimulation to study the complex dynamics of financial markets. We focus on those aspects of market dynamics that are characterized by some persistent patterns empirically observed in financial time series of which the root cause can hardly be explained by the traditional economic theories, as discussed extensively in Chapters 2 and 3. This chapter summarizes the research that has been presented in this thesis and provides some ideas for future work.

7.1.1 Answers to the research questions

As stated in Chapter 1, this research focuses on two most active types of market, i.e., stock markets and options markets, to address two research questions. These questions and the corresponding findings that have been obtained from this research can be summarized as follows:

a. What are the principal mechanisms underlying stylized facts observed in stock markets? Are these mechanisms common across explanations provided by the well-established microsimulation models proposed in the literature?

We have developed a cellular automaton (CA) model and have performed a detailed investigation regarding the mechanism underlying the dynamics of stock markets (see Chapter 4). The model can reproduce, in a simple and robust manner, the main characteristics observed in empirical financial time series. Our simulations and analysis suggest that long-range agent interactions, which are responsible for large price variations, can form from local interactions. Volatility clustering is associated with the variation in agents' trading activity, a slow process in comparison to the variation in the influence of news. Heavy-tailed distributions of return are related to both large price variations and volatility clustering. Finally, these non-Gaussian distributions are produced by agents' behavior in response to the arrival of news, even though the influence of news is assumed to follow a Gaussian random process. In a general sense, these causes of heavy tails and volatility clustering appear to be common among some well-established microsimulation models that have confirmed the main characteristics of financial markets.

b. What is the origin of the poorly understood volatility smile phenomenon observed in option markets and what are the driving factors determining the shape and the dynamic properties of the smile?

We have developed a microsimulation model to investigate the market mechanism governing the volatility smile (see Chapters 5 and 6). In our model, the typical behavior of the most active options traders, i.e., speculators and arbitrageurs, is adopted and the prices of the options are determined by demand and supply. Our results agree with empirical studies in respect to the shape and dynamic properties of the smile. The detailed analysis shows that, although traders have distinct trading interests, their behavior leads to contests regarding the option prices and eventually the shape of the smile. In addition, the level of the smile is positively related to the variance of speculators' expected prices. Moreover, the deviation of the mean of speculators' expected prices from the current price of the underlying is crucial to the shape of the smile. The cases that the former is smaller than, equal to, and larger than the latter, will give rise to respectively a downward sloping, a symmetric, and a upward sloping curve. In Chapters 5 and 6, we demonstrate rigorously that these findings are robust with respect to the types of trading behavior included in the microsimulations and the actual values employed for the different model parameters. These results strongly suggest that the volatility smile is a natural consequence of traders' heterogeneous behavior and expectations about the future and confirm that individual trading preferences indeed play a critical role in the dynamics of the implied volatility curve.

While the above discussion briefly summarizes the main findings of the research, in the following sections we discuss some general insights obtained from this research and our extensive literature study. We will focus on aspects related to the characteristics of complex system dynamics and the methodology employed in this work.

7.1.2 Complex dynamics can emerge from simple behavior

Many phenomena which are difficult to explain with the traditional economic framework and considered irregular can be conveniently reproduced in microsimulations. In contrast to the standard models, which view the stylized facts of financial markets as the result of exogenous factors, the microsimulation approach considers these features as emergent properties resulting from the internal dynamics, i.e., traders' behavior and their interactions. Importantly, as shown by our models and many other microsimulation models, the behavior does not need to be complicated, i.e., complex market dynamics can emerges from *simple* and ordinary trading behavior.

Specifically, interaction is a key factor in generating the complex dynamics. Interactions in financial markets can be categorized into two broad types: Those among traders and those between prices and traders. Imitators' behavior is of the first type, while speculators' behavior is of the second one.

Imitation is commonly exhibited in financial markets. Although imitative behavior is associated with diverse activities in financial markets and researchers have focused on different kinds of imitation, all types of imitation described in the various agent-based models share a single feature: The imitators mimick the transactions that have been performed by other traders. From this perspective, we adopt a very simple but representative form of imitation in our cellular automaton model of stock markets: The imitators take the average transaction quantities of their local neighbors at the previous time step as their current quantities. Interestingly, such a simple framework can reproduce the large price jumps observed in real markets.

Speculators typically rely on their expectations about the price in the future. In options markets, this is particularly true because options are contingent claims by nature: The payoffs of options are eventually determined by the future prices of the underlying assets. Speculators trade according to their expectations about the price and their trading will in turn change the price. These micro-macro interactions are typical of financial markets. In our options market model, the speculators' expected prices play a crucial role. They determine the traders' expected profits and ultimately their actions. Surprisingly, by examining the effects of the changes in only the mean and variance of the expected prices, all the main stylized facts related to the volatility smile can be convincingly explained.

7.1.3 Microsimulation: Necessity and difficulty

7.1.3.1 Microscopic modeling is necessary for understanding market dynamics

Financial markets consist of a large number of heterogeneous agents trading in diverse financial instruments. Traders' behavior and interactions give rise to the macroscopic price dynamics, which in turn influences the microscopic behavior and interactions. This process is highly nonlinear and difficult to describe analytically. Traditional approaches in finance and economics to study aggregative phenomena either are purely macroscopic, or rely on top-down construction based on a number of unrealistic assumptions mainly for the sake of analytical tractability. Heterogeneity is neglected and interactions between traders play no role in the explanation of the phenomena. This may account for the inability of the mainstream economic theories to explain the complex market dynamics characterized by the stylized facts.

On the other hand, many studies of financial market dynamics are difficult to carry out experimentally. An obvious reason is that experiments with regard to large scale markets using real people are usually too expensive to conduct. Moreover, in experiments the subjects no longer behave naturally, so that the resulting conclusions might not be convincing.

Microsimulation enables us to explore complex economic dynamics from the bottom up. With microsimulations, we study a complex system by directly modeling its individual elements and their interactions. The macroscopic behavior of the system will eventually emerge from the microdynamics. Microsimulation has shown great potential for more realistically modeling complex dynamical systems in economics and finance (Levy et al. (2000)). In addition, it facilitates the testing of existing economic or financial models and theories, and the development of new theories and models (Tesfatsion (2002)). Moreover, microsimulations are much more cost feasible for investigating phenomena observed in large scale financial markets. In brief, microsimulation is an indispensable method for deeply studying market dynamics.

7.1.3.2 Difficulty in implementing microsimulations

Although the necessity of microsimulation is being recognized, the implementation of this method is by no means easy. Here we list a few difficult points identified from our own experience and literature study.

A difficult point lies in the identification of the types of trader and trading behavior relevant to the phenomenon under investigation. Financial markets are comprised of a large number of heterogeneous traders with distinct interests and strategies, apart from different amounts of capital, channels of information, restrictions, etc. However, only some specific types of trader and trading behavior are directly related to certain stylized facts and usually the main difficulty lies in accurately identifying them. A obvious reason for this difficulty stems from the fact that we can hardly carry out large scale experiments with real traders to study the complicated cause-effect relations among different objects in real markets. (In fact, this is the very reason for our employment of microsimulation.) Therefore, to explain the same phenomenon, microsimulation modelers might select different types of trader and/or trading behavior and develop different models. Certainly, models are different in explanatory power. In our opinion, apart from other criteria, a microsimulation model is considered good if it can qualitatively and quantitatively explain most relevant stylized facts.

Another difficult point is encountered in the simplification of the selected traders. In microsimulations, we face the dilemma of modeling traders as realistically as possible or exhibiting the causal relations of the market dynamics as clearly as we can. Unrealistic modeling is unacceptable, whereas models with a host of details can conceal the mechanisms of interest and will be of no use. A model is just a representation of a system for addressing a certain problem. It should be built by including just the essential features necessary for addressing a certain problem under study and excluding aspects that are considered secondary. However, it is difficult to determine the boundary that separates the the most relevant elements from secondary details. In overcoming this difficulty, we only adopt the most typical heterogeneous trading behavior in real markets which is complex enough to cover traders' main characteristics, in the meantime simple enough for revealing the underlying market mechanisms.

Yet another difficult point arises in the adoption of parameter values. Because microsimulation models are just concise representations of real markets, parameters are unavoidable. However, since microsimulation is fairly new in economics and finance, no empirical or experimental studies related to many parameters in MS models have been carried out. Consequently, we lack empirical estimates that can be used to calibrate the model parameters. In the absence of relevant empirical observations for some parameters, we have performed qualitative and quantitative analyzes to ensure that the findings obtained from our models are robust.

7.2 Future work

The main insight we have obtained is that the complex market dynamics is generated by heterogeneous traders' behavior and interactions. Based on and guided by this understanding, we suggest two directions for future work: Studying the multiscale nature of the complex dynamics of financial markets and exploring the applications of our research results to financial practice.

7.2.1 Multiscale modeling of financial markets

We have developed microsimulation models for understanding the dynamics of stock markets and that of option markets separately. In these models, the market participants only trade either stocks or options. However, in reality, many traders deal with portfolios, which are collections of different financial instruments and typically contain derivatives and their corresponding underlying securities. They may adjust their positions in some instruments based on the prices of some other instruments. This will generate links among different markets and further increase the complexity of financial economy.

For example, delta hedging is a strategy widely used by derivative dealers to reduce or eliminate the risk of their portfolios associated with price movements in the underlying assets. (For a detailed description of this strategy, see Hull (2003).) Importantly, because the price of the underlying asset changes over time, the dealers' positions remain delta neutral only for a short period of time and the hedges have to be adjusted periodically. Delta hedging is therefore a dynamic hedging scheme and it connect the demand and supply of the underlying asset to the prices of the options, and eventually the demand and supply of the options. In this manner, the price dynamics of the underlying asset is explicitly linked to that of the corresponding derivatives, although the degree of correlation is certainly dependent on the relative trading volumes. What can be expected if the trading volume of dynamic hedging is comparable to those of other traditional trading strategies? Considering the fact that in reality traders use advanced hedging strategies which connect markets of different types of underlying or derivative security, this is an important issue to address.

7.2.2 Applications to practice

Financial practitioners should benefit from the better understanding of the complex dynamics of financial markets achieved through microsimulations. However, there is still a large gap between the understanding of the underlying mechanisms and its implementation in day-to-day practice. Future research should endeavor to cover this gap. Our microsimulation models illustrate the importance of heterogeneous traders' beliefs and behavior for the market dynamics. In developing new approaches to risk management or other financial practices, collective human behavior should somehow be incorporated. Surely, this is one of the greatest challenges of this field which in our opinion can only be addressed by combining insights and principles from mathematical finance, agent-based simulations and behavioral finance.

Appendix A

A detailed description of the options market model

A.1 The rule of option price updating

The prices of the options are updated according to the following rule, which can be understood as the effect of market makers' action to balance the supply and demand:

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}, \tag{A.1}$$

where $Q^{n,\phi,t}$ is the total transaction quantity or the excess demand of the n-th option at time t. Since the excess demand is proportional to N_{tr}, we rescale it with the latter. Here β is a positive parameter that reflects the sensitivity of the option price to the excess demand. Due to the fact that option prices cannot be negative, the lower bound of $V^{n,\phi,t}$ is 0.

A.2 The dynamic aspect of speculators' behavior

We define $S^{i,t}_{SP}$ as a function of the directional trader's general level of optimism, the price of the underlying asset, the psychological effect of the news on the speculators, and the level of ambiguity of the news:

$$S_{SP}^{i,t} = (\bar{S}_{SP}^{i} + dS^{t})(1 + \psi^{t})(1 + \omega^{t}\xi), \tag{A.2}$$

where $\bar{S}_{SP}^{i}$ is the long-term mean of the trader's expected price, dS^t is equal to $S^t - \bar{S}$, in which S^t is the price of the underlying and $\bar{S}$ is the long-term mean of S^t; ψ^t expresses the psychological impact of the news on the trader's expected price; ω^t reflects the ambiguity level of news, and ξ is a random number sampled from a uniform distribution in the range $[-1, 1]$. The term dS^t is included since it is reasonable to expect that speculators will adjust their expected prices when the price of the underlying changes.

The processes S^t, ψ^t, ω^t, and F_{SP}^t are all influenced by news and vary simultaneously over time and we assume that they all follow the Ornstein-Uhlenbeck process (a mean-reverting process):

$$U^{t+1} = U^t + \theta_U(\mu_U - U^t) + \nu_U dz \tag{A.3}$$

where U^t denotes the specific factor of interest, θ_U the mean reversion rate, μ_U the mean reversion level, ν_U the volatility of the random fluctuations, and dz the Wiener process. The reason for adopting a mean-reverting process is that all these factors are influenced by news and therefore fluctuate around their long-term average levels.

A.3 The parameter values

The units of the variables or parameters in our model are 'year' for t, T; 'monetary unit' for S^t, $S_{SP}^{i,t}$, S_{BS}, K^n, $V^{n,\phi,t}$, M_{SP}^t, D_{SP}^t, μ_U, ε^h, h, and β; 'number/(monetary unit)' for η_{SP}, λ_{SP}, λ_{AR}, and γ; and 'number' for N_{op}, N_{tr}, and $Q^{n,\phi,t}$.

We adopt $T - t = 1$, $N_{tr} = 5000$, $N_{op} = 11$ (15, 16, $\cdots$, 25), $\gamma = 1.5$, $\alpha = 0.1$[1], and $\beta = 0.1$. We assume that the long-term means of S^t, M_{SP}^t, D_{SP}^t, and F_{SP}^t are respectively $\bar{S} = 20$ and $\bar{M}_{SP} = 19$, $\bar{D}_{SP} = 4$, and $\bar{F}_{SP} = 0.5$. In Section 5.3 we

[1] Solely for reference, the average alpha value of the corresponding BS prices where the common middle strike is 20 and the volatility is between 0.1 and 0.3 is around 0.08.

assign different values to these three factors to analyze the mechanism underlying the smile phenomenon.

To study whether the option prices satisfy the arbitrage relations, we display them together with the corresponding prices obtained by the BS model with spot price S_{BS} and volatility σ_{BS}. The BS prices satisfy all the arbitrage relations. We set $S_{BS} = 20$ and $\sigma_{BS} = 0.198$[1]. The initial price of the underlying and that of the options are 20 and 1 respectively. In simulating the IV dynamics, there are 18000 time-steps in each simulation run and we take an IV curve after every 150 time-steps for performing principal components analysis (PCA).

The parameters η_{SP}, $\lambda_{AR_{PCP}}$, and $\lambda_{AR_{BFS}}$ represent the traders' activity for employing the corresponding strategies. We assume that their values are proportional to the traders' confidence levels regarding the profitability of the strategies. For example, speculative profits are much more uncertain than arbitrage gains, so the value for η_{SP} should be smaller than that for $\lambda_{AR_{PCP}}$, and $\lambda_{AR_{BFS}}$. In the simulations, $\eta_{SP} = 0.1$, $\lambda_{AR_{PCP}} = 1.0$, and $\lambda_{AR_{BFS}} = 1.5$.

In modeling the dynamic behavior of the relevant factors we use a mean-reverting Gaussian process (Equation (A.3)). This stochastic process U^t consists of three parameters, the long-term mean μ_U, the mean-reversion speed θ_U and the volatility ν_U. The long-term mean of S^t, ψ^t, ω^t, and F^t_{SP} is $\mu_S = 20$, $\mu_\psi = 0$, $\mu_\omega = 0.2$, and $F^t_{SP} = 0.5$ respectively. The adoption of the values for μ_ψ and μ_ω respectively is based on the assumption that the psychological effect of news is neutral on average, while there is always some ambiguity in news. To mimic high-frequency time series of underlying assets, we adopt 0.001 and 0.01 for θ_S and ν_S respectively. In addition, it is reasonable to assume that the values θ_U and ν_U are proportional to the strength in which news influence the corresponding processes. For example, in real markets, trading in underlying assets is mediated by market makers who dampen the volatility by enhancing liquidity and increasing market depth[2]. Therefore S^t is expected to be less volatile compared to ψ^t, ω^t, and F^t_{SP}

[1]Here we adopt $\sigma_{BS} = \sqrt{ln((\bar{D}_{SP}/S_{BS})^2 + 1)}$, so that a log-normal distribution with the mean equal to S_{BS} and the standard deviation equal to $\bar{D}_{SP}$ is assumed for the price of the underlying when using the BS model. However, in principle, the volatility does not influence the comparison of the simulated option prices and the corresponding BS prices because all the relevant arbitrage relations are independent of volatility (see Cox and Rubinstein (1985)).

[2]Market depth is the size of an order needed to move the price a given amount.

which are related to traders' mentality and behavior. The values chosen for ψ^t, ω^t, and F_{SP}^t are therefore greater than that for S^t. In our simulations, unless otherwise stated, $\theta_\psi = 0.01$, $\nu_\psi = 0.002$ for directional speculators and 0.02 for volatility speculators; $\theta_\omega = 0.01$, $\nu_\omega = 0.02$; and $\theta_F = 0.01$, $\nu_F = 0.05$. The lower bounds of S^t, ω^t, and F_{SP}^t are all 0, and the upper bound of F_{SP}^t is 1. It should be noted that the stochastic process U^t can be solved analytically: The specific factor follows a Gaussian distribution with long-term variance equal to ${\nu_U}^2/(2\theta_U)$. Accordingly, the parameter values yield reasonable long-term variances in the order from 0.01 to 0.2. To further demonstrate the robustness of our results, we have performed a sensitivity analysis with respect to these parameters (see Section 5.4).

Appendix B

Analytical solution for a special case of the options market model

Here we show analytically that, if the mean of the speculators' expected prices M_{SP}^t coincides with the current price of the underlying S^t, the simulated option prices based on the S model converge to the corresponding BS prices.

The transaction quantity of a speculator is

$$Q_{SP}^{i,n,\phi,t} = \lambda_{SP}(\max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}). \tag{B.1}$$

In the S model, all the traders are speculators, so

$$\begin{aligned} Q_{SP}^{n,\phi,t} &= \sum_{i=1}^{N_{tr}} Q_{SP}^{i,n,\phi,t} \\ &= \lambda_{SP}[\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr}V^{n,\phi,t}]. \end{aligned} \tag{B.2}$$

The price updating rule is

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}. \tag{B.3}$$

The price of each option eventually converges, i.e., $V^{n,\phi,t+1} = V^{n,\phi,t}$. Then,

$$Q^{n,\phi,t} = 0. \tag{B.4}$$

Substituting Equation (B.4) in Equation (B.2), we have

$$\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr}V^{n,\phi,t} = 0, \tag{B.5}$$

i.e. (law of large numbers),

$$\begin{aligned} V^{n,\phi,t} &= \frac{1}{N_{tr}}(\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}) \\ &\approx E[\max(\phi(S_{SP}^{i,t} - K^n), 0)]. \end{aligned} \tag{B.6}$$

Since we assume that $S_{SP}^{i,t}(i = 1, 2, \cdots, N_{tr})$ follow a lognormal distribution, the right hand side of Equation (B.6) is equal to the corresponding Black-Scholes solution.

Appendix C

Levels of trading activity of different speculators

Here we describe the trading activity levels of the different types of speculator, by considering liquidity unbalancing and the number of portfolios in which the traders can invest.

In real markets, the liquidity of options is not balanced across strikes, as shown in Ederington and Guan (2002) and Rexhepi (2008). In general, OTM options are more liquid than ITM options, implying that at least some speculators trade the former more actively than the latter. This liquidity unbalancing might stem from the trading behavior of speculators and the price characteristics of options. Generally, speculators prefer cheap and liquid options in order to achieve high leverage and fast conversion. The potential for higher leverage provided by OTM options, which are cheaper than their ITM counterparts, attracts more speculators. Higher leverage thus leads to higher liquidity, which in turn pulls in even greater speculative trading volume. This positive feedback effect ensures the relatively higher/lower liquidity of OTM/ITM options. To reflect this fact, we assume that the activity levels of these speculators are strike dependent and follow the form

$$\lambda_{SP}(K^n) = \eta_{SP}[\phi \tanh(\gamma(K^n - S^t)) + 1], \tag{C.1}$$

where η_{SP} and γ are positive parameters. Equation (C.1) is an increasing/decreasing function of strike for the call/put options. In the simulations, the values adopted for η_{SP} and γ are 0.1 and 1.5 respectively.

SD and BS speculators can trade each option independently of other options. Activity unbalancing described by Equation (C.1) is therefore applicable to them. DSPR and BSPR strategies, however, usually involve ITM and OTM options at the same time and in specified proportions. Activity unbalancing is thus not applicable to these speculations.

In addition, the various types of speculative strategy have different numbers of potential portfolios (or combinations). We assume that the speculators are equally active in applying all the entire strategies, and that each type of speculators employ all the possible portfolios of call or/and put options prescribed by their strategies. The activity levels for individual portfolios assigned to the speculators are therefore inversely proportional to the numbers of the portfolios corresponding to the respective strategies. The activity levels of the different types of speculator are determined as follows:

- **SD and BS Speculations.** There are N_{STK} strike prices, therefore a SD or BS trader can establish $2N_{STK}$ (long and short) positions by using all the call options. Similarly, the trader can establish the same number of positions using all the put options. There are therefore totally $4N_{STK}$ different positions for the trader. We assume that the activity level of each SD or BS speculator for individual options, i.e. λ_{SP_SD} or λ_{SP_BS} respectively, is equal to $\lambda_{SP}(K^n)$.

- **DSPR Speculation.** When a DSPR trader takes a long position in the call option with the smallest strike, there are $N_{STK} - 1$ ways to take a short position with another call option with lower strike price, i.e., the trader can establish $N_{STK} - 1$ portfolios. Similarly, using the call option with the second smallest strike, the trader has $N_{STK} - 2$ portfolios, and so on. Therefore, using the N_{STK} call options, a DSPR trader can establish $(1/2)N_{STK}(N_{STK} - 1)$ portfolios (the sum of the first N_{STK} terms of an arithmetic sequence, of which the first term is $N_{STK} - 1$ and the common difference is -1). Including the reverse combinations doubles the number of DSPR portfolios established with call options. Using put options, a DSPR trader can establish the same number of portfolios as that from using the call options. There are therefore totally $2N_{STK}(N_{STK}-1)$ different portfolios for a DSPR trader. The activity level of each DSPR trader, i.e λ_{SP_DSPR}, is therefore equal to $[(4N_{STK})/(2N_{STK}(N_{STK}-1))]\eta_{SP} = [2/(N_{STK}-1)]\eta_{SP}$ (strike independent).

- **BSPR Speculation.**

  By taking long positions in the pairs of call options with strikes symmetric to ATM, a BSPR trader can build $(N_{STK}-1)/2$ portfolios. Including the reverse combinations doubles the number of BSPR portfolios. Using the put options, a BSPR trader can establish the same number of portfolios as that from using the call options. There are therefore totally $2(N_{STK}-1)$ different portfolios for a BSPR trader. The activity level of each BSPR trader, i.e. λ_{SP_BSPR}, is therefore equal to $[(4N_{STK})/(2(N_{STK}-1))]\eta_{SP} = [(2N_{STK})/(N_{STK}-1)]\eta_{SP}$ (strike-independent).

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Glossary

ABS	Agent-based simulation
ACF	Autocorrelation function
AR	Arbitrageur
ARCH	Autoregressive conditional heteroscedasticity
ATM	At-the-money
BFS	Butterfly spread (an arbitrage condition)
BS	Black-Scholes
BSPR	Butterfly spread (a speculative strategy)
CA	Cellular automaton
CAPM	Capital asset pricing model
CEV	Constant elasticity of variance
DAX	Deutscher Aktien Index (a German stock index)
DSPR	Directional spread
EMH	Efficient markets hypothesis
EU	Expected utility
GARCH	Generalized autoregressive conditional heteroscedasticity
ITM	In-the-money
IV	Implied volatility
MS	Microsimulation
OTM	Out-of-the-money
PCA	Principal component analysis
PCP	Put-call parity
PDF	Probability density function
SABR	Stochastic alpha, beta, rho
SD	Simple directional
SP	Speculator
S&P 500	The Standard and Poor's 500 (a stock market index)

Publications

Journal Publications

- G. Qiu, D. Kandhai, and P. M. A. Sloot, Understanding the Complex Dynamics of Stock Markets through Cellular Automata, Physical Review E, 75, 2007, 046116.
- G. Qiu, D. Kandhai, N. F. Johnson, and P. M. A. Sloot, Why Do Options Markets Smile? Quantitative Finance (submitted).
- G. Qiu, D. Kandhai, and P. M. A. Sloot, Effects of Heterogeneous Speculative Strategies on the Volatility Smile: A Microsimulation Study (in preparation).
- G. Qiu, D. Kandhai, and P. M. A. Sloot, Exploring Financial Market Complexity through Microsimulation: A Case Study of Option Markets (in preparation).

Conference Proceedings

- G. Qiu, D. Kandhai, and P. M. A. Sloot, Understanding the Volatility Smile through Microsimulation, presented in Applications of Physics in Financial Analysis, 2007, Lisbon.
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Summary

Financial markets are among the most complex systems in reality and many phenomena observed in real markets are still poorly understood. Specifically, prices of traded products often exhibit extraordinary changes or unexpected patterns seemingly not induced by external causes, instead arising endogenously. This seriously challenges neoclassical economics which depicts markets as efficient machines that automatically seek out an equilibrium state in which irregularities in prices are mainly caused by external factors.

We focused on two of the most active financial markets, which share many features with other types of markets. The dynamics of the financial assets in these markets is characterized by some 'stylized facts', which are counterintuitive and contrary to the expectations of traditional financial theories. In stock markets, high frequency returns follow a non-Gaussian heavy-tailed distribution. In addition, high and low absolute returns tend to group together, a phenomenon termed 'volatility clustering'.

In options markets, there is a phenomenon called 'volatility smile', which is intrinsically related to the well-known and widely used Black-Scholes model for options pricing. The only unobservable parameter of this Nobel Prize winning model is the volatility of the underlying asset which by nature should be independent of the strike of the option contract, i.e. the price at which the option can be exercised at expiry. However, the volatilities required to match option prices quoted in real markets, i.e. implied volatilities, exhibit a remarkable curvature against strikes and may change strongly over time. Understanding the origin of this phenomenon has eluded the financial world for more than two decades.

The ubiquity of the stylized facts has stimulated a great deal of academic work for developing models more consistent with empirical time series. For ex-

ample, ARCH and GARCH models have been developed to reflect the changes in volatility. Similarly, in the derivative finance literature, many new alternatives to the Black-Scholes model have been proposed, mainly through relaxing some of the restrictive assumptions of the Black-Scholes framework. However, although these models can reproduce the stylized facts to some extent, they do not explain the origin of the complex dynamics of financial markets.

During the last few decades, behavioral approaches and agent-based methods have been widely applied to the study of market dynamics. They can explain many phenomena in a more plausible way than traditional financial theories do. However, most of these theories or models either do not systematically examine the mechanisms underlying the phenomena or are too complicated to be helpful for clearly identifying the causal relations of the mechanisms. In addition, the majority of the existing agent-based models focus on stock markets, while very few center on derivatives markets.

In view of these facts, our general motivation was to apply a bottom-up approach for studying the mechanisms through which the complex market dynamics is generated. We studied a financial market by modeling its individual elements and their interactions. The macro-dynamics of the system would ultimately emerge from the micro-behavior. In particular, we aimed to develop microsimulation models with simple structures that can reproduce the extraordinary patterns observed in financial time series. In order to offer important insights into the complex dynamics, we adopted an approach of successive complexification of the basic models.

Our stock-market model can reproduce, in a simple and robust manner, the main characteristics observed in empirical financial time series. Heavy-tailed return distributions due to large price variations can be generated through agents' imitating behavior. Volatility clustering is related to the combined effect of a fast and a slow process: The evolution of the influence of external facts such as news and the evolution of agents' trading activity respectively. In a general sense, these explanations appear to be common among the most well-established microsimulation models which have confirmed the main characteristics of financial markets.

Our options-market model agrees with empirical studies in respect to the shape and dynamic properties of the volatility smile. It suggests that the smile phenomenon is a natural consequence of speculative and arbitrage trading behavior and the heterogeneity and variation of speculative traders' expectations about the future. Specifically, the variance of directional speculators' expected prices determines the level of the implied volatility curve and the corresponding mean controls the skewness. Other heterogeneous speculators such as spread traders and traders using the Black-Scholes model can alter the shape of the smile. However, our simulation results with regard to trading volumes suggest that these speculators are not the dominant traders in real markets. Overall, these results confirm that individual trading preferences indeed play a critical role in the formations of options prices.

The general insight we have obtained from this work is that the complex dynamics of financial markets can emerge naturally from market participants' simple and ordinary behavior and their interactions. Our results confirm that microsimulation is an indispensable method for deeply studying financial market complexity. The remaining challenge lies in applying our findings to the day-to-day practice of derivative valuation and financial risk management.

Samenvatting

Financiële markten behoren tot de meest complexe systemen uit de hedendaagse praktijk en veel van de waargenomen verschijnselen in echte markten zijn nog steeds slecht begrepen. Vooral prijzen van verhandelde producten vertonen vaak buitengewone veranderingen of onverwachte patronen die niet veroorzaakt lijken te worden door externe oorzaken, maar in plaats daarvan endogeen ontstaan. Dit vormt een grote uitdaging voor de Neoklassieke Economie die de markten als efficiënte machines ziet die automatisch zoeken naar een staat van evenwicht waarin onregelmatigheden in de prijzen met name worden veroorzaakt door externe factoren.

We richten ons op twee van de meest actieve financiële markten, die veel overeenkomsten vertonen met andere type markten. De dynamiek van de financiële producten die verhandeld worden in deze markten wordt gekenmerkt door een aantal 'gestileerde feiten', die contra-intuïtief zijn en in strijd met de verwachtingen van de traditionele financiële theorieën. In de aandelenmarkten volgen hoog frequente rendementen een niet-normale verdeling met dikke staarten. Bovendien hebben hoge en lage absolute rendementen de neiging om zich te groeperen, een fenomeen genoemd 'volatility clustering'.

In de optiemarkten komt het fenomeen genaamd 'volatility smile' voor, dat intrinsiek is gerelateerd aan het bekende en veel gebruikte Black-Scholes model voor de waardering van opties. De enige onwaarneembare parameter van dit Nobelprijswinnende model is de volatiliteit van de onderliggende aandeel, hetgeen per definitie niet afhankelijk hoort te zijn van de uitoefenprijs van de optie. Echter de volatiliteiten die nodig zijn om optieprijzen geobserveerd in echte markten te beschrijven, de zogenaamde geïmpliceerde volatiliteiten, vertonen een opmerkelijke kromming als functie van de uitoefenprijs en deze kromming kan sterk veranderen in de tijd. Het ontrafelen van de oorsprong van dit fenomeen is reeds twee decennia lang een raadsel voor de financiële wereld.

De alomtegenwoordigheid van de gestileerde feiten heeft geleid tot veel wetenschappelijk werk gericht op de ontwikkeling van modellen die meer in overeenstemming zijn met empirische tijdreeksen. Zo zijn ARCH- en GARCH-modellen ontwikkeld om de veranderingen in de volatiliteit weer te geven. Ook zijn in de financiële derivaten literatuur veel nieuwe alternatieven voor het Black-Scholes-model voorgesteld, met name door het minder stringent toepassen van de beperkende veronderstellingen binnen het kader van het Black-Scholes model. Hoewel deze modellen tot op zekere hoogte de gestileerde feiten kunnen reproduceren, geven ze uiteindelijk geen verklaring voor de oorsprong van de complexe dynamiek van de financiële markten.

Gedurende de laatste decennia zijn benaderingen uit de gedragseconomie en agent-gebaseerde methoden op grote schaal toegepast bij het bestuderen van marktdynamieken. Deze kunnen veel verschijnselen op een meer plausibele manier verklaren dan de traditionele financiële theorieën. De meeste van deze theorieën of modellen onderzoeken echter niet systematisch de onderliggende mechanismen van de verschijnselen of ze zijn te ingewikkeld om causale relaties van de mechanismen duidelijk te identificeren. Bovendien richt het merendeel van de bestaande agent-gebaseerde modellen zich op de aandelenmarkten, terwijl slechts enkele zich concentreren op derivatenmarkten.

In het licht van deze feiten is onze algemene motivatie om een bottom-up benadering toe te passen voor het bestuderen van de mechanismen waarmee de complexe marktdynamiek wordt gegenereerd. We bestuderen een financiële markt door de individuele elementen en hun interacties te modelleren. De macrodynamiek van het systeem vloeit uiteindelijk voort uit het microgedrag. We willen in het bijzonder microsimulatiemodellen met eenvoudige structuren ontwikkelen welke buitengewone patronen kunnen reproduceren die zijn waargenomen in financiële tijdreeksen. Met het oog op het bieden van belangrijke inzichten in de complexe dynamiek, hanteren we een aanpak van opeenvolgende complexificatie van de basismodellen.

Ons aandelenmarktmodel kan op eenvoudige en robuuste wijze de belangrijkste kenmerken, waargenomen in de empirische financiële tijdreeksen, reproduceren. Dikke-staart distributies van rendementen als gevolg van grote prijsverschillen kunnen worden gegenereerd door middel van het imiterende gedrag van agenten. Volatility clustering is gerelateerd aan het gecombineerde effect van een snel en een langzaam proces: Respectievelijk de evolutie van de invloed van externe feiten zoals nieuws en de evolutie van de handel activiteiten van de agenten. Over het algemeen blijken deze verklaringen over een te stemmen met de bevindingen van de meest gerenommeerde microsimulatiemodellen die de belangrijkste kenmerken van de financiële markten hebben kunnen bevestigen.

De resultaten verkregen met ons model voor optiemarkten is in lijn met empirische studies met betrekking tot de vorm en de dynamische eigenschappen van de volatility smile. Het suggereert dat het smile-fenomeen een natuurlijk gevolg is van speculatief en arbitrage trading gedrag en van de heterogeniteit en variatie van de toekomstverwachtingen van speculerende traders. In het bijzonder, de variantie in de door directionele speculanten verwachte prijzen voor de onliggende, bepaalt het niveau van de implied volatiliteitscurve, terwijl de gemiddelde verwachte prijs de steilheid van de curve beïnvloedt. Andere heterogene speculanten, zoals spread traders en traders die gebruikmaken van het Black-Scholes model, kunnen de vorm van de smile beïnvloeden. Onze simulatieresultaten betreffende de omvang van de verhandelde instrumenten voor variërende uitoefenprijzen, suggereren dat deze speculanten niet de dominante traders zijn in echte markten. Over het algemeen bevestigen deze resultaten dat gedrag van individuen inderdaad een cruciale rol spelen in de formatie van optieprijzen.

Het algemene inzicht dat wij verkregen hebben uit dit onderzoek is dat de complexe dynamiek van de financiële markten op natuurlijke wijze kan voortkomen uit het eenvoudige en simpele gedrag van marktdeelnemers en hun interacties. Onze resultaten bevestigen dat microsimulatie een onmisbare methode is voor het diepgaand bestuderen van de complexiteit van de financiële markt. De grote uitdaging ligt in het toepassen van onze bevindingen in de dagelijkse praktijk van waardering van derivaten en financiële risico management.

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