Understanding the complex dynamics of financial markets through microsimulation

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Chapter 4

Understanding Stock Market Dynamics

In Chapter 2 and references therein, it was argued extensively that the complex dynamics of stock markets is characterized by some stylized facts which are common across many markets. As stated in Chapter 3, researchers in financial economics have not yet reached an agreement on the principal mechanisms underlying this complex dynamics. While many agent-based models published in the literature can reproduce these main stylized facts, some of them are so complicated that it is still difficult to identify the underlying mechanisms governing the dynamics. This has also been pointed out by Farmer (1999) and Cont (2005).

In view of these facts, we have developed a parsimonious cellular automaton (CA) model that can generate the empirically observed stylized facts in a robust and simple manner\(^1\). Our model is built on increasing levels of sophistication in order to identify the driving mechanisms underlying these stylized facts.

In Section 4.1, we firstly give a detailed description of our CA model. Next, the simulation results are presented in Section 4.2. Section 4.3 provides a thorough investigation of the simulated dynamics through computational experiments and mathematical analysis, and provides insights into the underlying mechanisms of the complex dynamics observed in stock markets. In the final section of this chap-

\(^1\)This chapter is based on Qiu et al. (2007).
4.1 A cellular automaton model of stock markets

We represent a stock market as a two-dimensional $L \times L$ lattice. Each vertex of the lattice denotes an agent (trader) who has interactions with other agents in a so-called Moore neighborhood. In the model, speculative traders of only two types are adopted: Fundamentalists and imitators. All the agents trade in a single stock.

Fundamentalists are those traders who are informed of the nature of the stock being traded and act according to its fundamental value. They believe that the price of the stock may temporarily deviate from, but will eventually return to the fundamental value. They therefore buy/sell the asset whenever its price is lower/higher than their perceived fundamental value. In stock markets, there are also some traders who do not know or do not care about fundamental values. Instead, they follow their acquaintances and adopt the trading opinions of the majority. An agent of this type is referred to as an imitator.

News influences both fundamentalists and imitators. However, the ways news affects them are distinct in many aspects. For example, fundamentalists pay relatively more attention to news about the specific company that has issued the stock, while imitators respond comparatively more frequently to news related to the stock market as a whole.

We can adopt other types of agent to model stock markets more realistically. However, we think that the two kinds of behavior discussed here are the most typical. The behavior of other speculative traders has no obvious characteristics. For example, we cannot find a general trait for chartists, because even using the

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1 A Moore neighborhood $\text{NB}_{ij}$ in our two-dimensional lattice is defined as a set whose members are the eight cells surrounding a given cell located at $(i, j)$, i.e., $\text{NB}_{ij} = \{(i-1,j-1), (i-1,j), (i-1,j+1), (i+1,j), (i+1,j+1), (i+1,j-1), (i,j-1), (i,j,1)\}$.
same data they may come to different conclusions due to differences in the techniques used. We can treat these agents as noise traders who randomly influence the price to different extents. However, because their adoption within our model does not fundamentally influence the dynamics characterized by the stylized facts, we choose to ignore them.

The real fundamental value of a stock is related to the current and prospective states of the company that has issued the stock, among many other factors. The modeling of its variations is beyond the scope of this work. Instead, we are more interested in the reason(s) for excess volatility, i.e., the extra factor(s) causing the price of a stock to be more volatile than its real fundamental value. For this reason, we assume that the real fundamental value of the asset $F$ is a constant.

(Tests showed that adding a drift to $F$ to model the time value of money does not influence the characteristics of the returns. Drifts are therefore excluded from our model.)

**4.1.1 Level I model**

**Fundamentalists**

Empirically, the larger the difference between the price of a stock and its fundamental value as perceived by a fundamentalist, the more likely he will trade it. We assume for the moment that the fundamentalists perceive the real fundamental value accurately. We can then adopt Equation (4.1) to express the transaction quantity based on the current price level at time $t+1$ of a fundamentalist when he is the $i$-th agent, $V_{i,Fu}^{t+1}$, and his actual transaction quantity at the same time, $q_{i,Fu}^{t+1}$:

$$q_{i,Fu}^{t+1} = V_{i,Fu}^{t+1} = F - P^t,$$

where $P^t$ is the price at time $t$. Notice that we have assumed for the moment that the two transaction quantities are equivalent. (The other factor determining (actual) transaction quantities will be introduced in Section 4.1.3.)
4.1 A cellular automaton model of stock markets

Imitators

We take the average transaction quantity based on the current price level of an imitator’s neighbors at the previous time step as his corresponding quantity at present, i.e., $V_{t+1}^{i,im} = \langle V_{t}^{i,nb} \rangle$. We can then use Equation (4.2) to express his (actual) transaction quantity at time $t+1$, $q_{t+1}^{i,im}$:

$$q_{t+1}^{i,im} = V_{t+1}^{i,im} = \langle V_{t}^{i,nb} \rangle.$$  \hspace{1cm} (4.2)

Imitations are ubiquitous in stock markets. However, they are carried out in many different manners. In the model introduced in Bak et al. (1997), noise traders imitate by adjusting their prices towards the current market price and mimicking other traders in the market. Similarly, Lux and Marchesi (1999) defined imitation as identifying price trends and patterns and mimicking other traders. In the model described in Cont and Bouchaud (2000), imitation is the process in which agents organize into coalitions and then trade identically. Iori (2002) defined imitation as the process in which traders receive signals from their neighborhoods. Bartolozzi and Thomas (2004) modeled imitation as a direct percolation process, in which active traders form a hierarchy of clusters and the agents in each group share information and interact with each other. Nevertheless, these specific imitating processes share the property that traders directly mimic the behavior of their acquaintances or copy the behavior of other traders that is implied by price trends.

To retain this property and in the meantime be consistent with our pursuit of simplicity, we have adopted a simple manner of imitation, i.e., traders adopt the opinions of their direct neighborhoods.

4.1.2 Level II model

Fundamentalists

News influences fundamentalists’ perceptions of fundamental values. Positive/negative news can cause them to overestimate/undervalue assets. Within our model, we assume that at each time step, all the fundamentalists perceive the fundamental
4.1 A cellular automaton model of stock markets

value identically. (We can alternatively assume that their perceived values at each time step are normally distributed, without fundamentally influencing the dynamics.)

We express the perceived fundamental value at time $t$ as $F\eta_{fu}$, in which $\eta_{fu}$ denotes the influence of the news at that time. We assume that $\eta_{fu} = 1 + c_{fu}\phi_{fu}$, where $\phi_{fu}$ is an independent Gaussian random variable with mean 0 and standard deviation 1 and $c_{fu}$ is a positive parameter indicating the fundamentalists’ sensitivity to news. At this point, we have a modified expression for the transaction quantity of a fundamentalist,

$$\begin{align*}
q_{i,fu}^{t+1} &= V_{i,fu}^{t+1} \\
&= F\eta_{fu}^{t+1} - P^t.
\end{align*}$$

(4.3)

**Imitators**

We assume that news influences all the imitators identically. (We can alternatively assume that the effects of news at each time step are normally distributed, without fundamentally influencing the dynamics.) Significant/unimportant news can make an imitator trade more/less than his neighbors, and vice versa. We reformulate the transaction quantity of an imitator as

$$\begin{align*}
q_{i,im}^{t+1} &= \langle V_{i,im}^{t+1} \rangle \\
&= \langle V_{i,nb}^{t+1} \rangle \eta_{im}^{t+1},
\end{align*}$$

(4.4)

in which $\eta_{im}^{t+1}$ indicates the influence of the news at time $t + 1$ and is equal to $1 + c_{im}\phi_{im}^{t+1}$, where $\phi_{im}^{t+1}$ is an independent Gaussian random variable with mean 0 and standard deviation 1 and $c_{im}$ is a positive parameter indicating the imitators’ sensitivity to news.

Due to the difference in characteristic between the two types of trader, $c_{im}$ is usually distinct from $c_{fu}$, so is $\phi_{im}$ from $\phi_{fu}$.

### 4.1.3 Level III model

A common strategy used by traders is buying low and selling high (BLASH). It aims for capital gains by taking advantage of changes in prices. Price fluctuations are therefore indispensable for this strategy.
4.1 A cellular automaton model of stock markets

Based on BLASH, capitals of traders move among different assets pursuing larger profits at lower risks. When the price fluctuation level of a stock is at the two extremes, i.e., very low and very high, the asset is the least desirable: If it is very low, traders who hold the asset will not be able to find an opportunity to sell it profitably and will not even be able to cover their opportunity costs\(^1\). If it is very high, traders will consider the investment in the asset too risky. Within the range between the two extremities, as the price fluctuation level rises, the asset will be first more favorable and then, after a certain level, less attractive.

When a stock is more favorable compared to other alternatives, traders will trade it more frequently. We therefore assume that the trading activity of the agents is equivalent to the desirability of the stock. However, BLASH is a risky approach itself, because there is no way to predict price changes accurately. Frequently, traders just end up selling at a loss. In order to reduce this risk, traders typically consider previous price changes of a stock for a longer period.

We represent the price fluctuation level of a stock at time \(t\) as

\[
L^t = \frac{1}{k} \sum_{i=t-k}^{t-1} |P^i - \bar{P}|/\bar{P},
\]

(4.5)

where \(k\) is the length of a period before \(t\), \(P^i\) is the price of the asset at time \(i\) in the period, and \(\bar{P}\) is the average price over the period. (We can alternatively assume that agents take different values of \(k\) that are normally distributed, without fundamentally influencing the dynamics.)

For the sake of simplicity, we adopt a straightforward linear function for the trading activity of the agents,

\[
M^t(L^t) = \begin{cases} 
    \epsilon^t L^t, & L^t \leq L_m \\
    \epsilon^t(-L^t + 2L_m), & L^t > L_m 
\end{cases}
\]

(4.6)

where \(L_m\) is the fluctuation level where the stock becomes less favorable and \(\epsilon^t\) is a positive parameter. (Simulations show that adopting other concave functions leads to similar results.)

\(^1\)Opportunity cost, or cost of capital, is the rate of return that a business could earn if it chose another investment with equivalent risk (Downes and Goodman (1998)).
4.1 A cellular automaton model of stock markets

Within the level III model we consider that the (actual) transaction quantity of an agent is the product of his transaction quantity based on the current price level and his current trading activity. The transaction quantity of a fundamentalist is therefore

\[ q_{t+1}^{fu} = V_{t+1}^{fu} M_{t+1}^{fu} = (F_{t+1}^{fu} - P_{t+1}^{fu}) M_{t+1}^{fu}, \quad (4.7) \]

whereas the transaction quantity of an imitator is

\[ q_{t+1}^{im} = V_{t+1}^{im} M_{t+1}^{im} = \langle V_{t+1}^{im} \rangle M_{t+1}^{im}, \quad (4.8) \]

Considering the fact that agents always have a number of exceptional reasons to transact, we adopt a lower bound for \( M \).

The BLASH behavior is the most basic feature of most speculators in stock markets, contrast with fundamentalists’ dividend-based behavior. In fact, making profits through BLASH is the very reason why many speculators are in stock markets in the first place. However, it is surprising that this typical speculative behavior has rarely been included in agent-based models.

We have also included another important feature of speculators, i.e. transferring capital among different assets or markets. It is the very fact that capital movements take place constantly. Unfortunately, this feature is also seldom be taken into consideration by most models of financial markets.

4.1.4 Rule of price updating

The price is updated according to the following rule:

\[ P_{t+1}^{s} = P_{t}^{s} + \epsilon_{p} \frac{Q_{t}}{N}, \quad (4.9) \]

where \( Q_{t} \) is the total transaction quantity or the excess demand for the asset at time \( t \) and \( N \) is the number of traders. Since \( Q_{t} \) is proportional to \( N \), we rescale it with \( N \). We adopt a positive parameter \( \epsilon_{p} \) to indicate the sensitivity of the price to the excess demand. Due to the fact that stock prices cannot be negative, the lower bound of \( P_{t}^{s} \) is 0.
4.1 A cellular automaton model of stock markets

Equation (4.9) can be explained as the action of market makers to balance the supply and the demand of the stock. In principle, however, it is merely the translation of the classic theory of supply and demand stating that price will move toward the point that equalizes supplied and demanded quantities.1

Another price updating rule often used in agent-based modeling, e.g., that of the model of Levy et al. (2000), is borrowed from that of a Walrasian auction. Introduced by Walras2. It is a type of simultaneous auction where each agent calculates its demand for the good at a hypothetical price and submits this to an auctioneer. The price is then set so that the total demand across all agents equals the total supply. The good is traded at this equilibrium price and no transactions take place at disequilibrium prices.

In principle, the Walrasian rule and the one adopted by us, expressed as Equation (4.9), are identical with regard to returns. The reason is that they are both based on the law of supply and demand: The supply/demand is an increasing/decreasing function of price and return is positively related to excess demand.

1In 1890, Alfred Marshall published Principles of Economics (Marshall (1890)), in which he discussed how both supply and demand interact to determine price. His supply-demand model has become one of the fundamental concepts of economics. According to the model, if all other factors remain equal, the higher the price, the lower the quantity demanded and the higher the quantity supplied, vice versa. In a price (ordinate) - quantity (abscissa) chart the curve of demand is a downward slope, the supply relationship shows an upward slope. Equilibrium occurs at the intersection point of the two curves. In the chart, if straight lines are drawn instead of the more general curves (the shapes of the curves do not change the general relationships), we immediately obtain Equation (4.9).

2Marie-Ésprit-Léon Walras was a mathematical economist. In 1874 and 1877 he published Elements of Pure Economics, a work that led him to be considered the father of the general equilibrium theory. This theory is a branch of theoretical neoclassical economics, pursuing the explanation of the behavior of supply, demand and prices in a whole economy with several or many markets, by seeking to prove that equilibrium prices for goods exist and that all prices are at equilibrium.
4.2 Simulation results

In this section, we present the simulation results of the model at different levels of complexification. We focus on the characteristics of the returns, among those of other variables such as the price, the trading volumes, and the activity level.

4.2.1 Simulation results of the Level I model

In simulations using the level I model, we set an initial price \( P^0 = 105 \) that deviates from the fundamental value \( F = 100 \). The number of agents is \( 1 \times 10^4 \). Figure 4.1 displays the price trajectories corresponding to two fractions of imitators: \( \alpha_{im} = 20\% \) and \( 80\% \) respectively.

The parameter \( c_p \) has an important impact on the price: When its value is increased up to 1 while other parameters are kept constant, the price process may start to switch from a convergent process to a divergent one, depending on the value of \( c_p \) itself and the value of \( \alpha_{im} \). We provide a theoretical analysis of this issue in Section 4.3. To model a stable market, we adopt only those values of \( c_p \) smaller than 1.

Figure 4.1: Price trajectories obtained through the simulation using the level I model when \( \alpha_{im} = 20\% \) and \( 80\% \) respectively. The curve that decays faster is of the first instance. The parameter setting used: \( N = 1 \times 10^4 \), \( c_p = 0.5 \), \( F = 100 \), and \( P^0 = 105 \).
4.2 Simulation results

As shown in Figure 4.1, although the level I model is not completely identical to the model of Bandini et al. (2004), it does generate similar price trajectories. Starting from an initial deviation from the fundamental value, the price either directly converges to it, or fluctuates around it for some time and eventually overlaps. Obviously, both models cannot produce sustained price movement. Since the price quickly dies out, we cannot obtain any stylized facts by using the level I model.

4.2.2 Simulation results of the Level II model

Within the level II model, we have added random factors \( \eta^a_t \) and \( \eta^m_t \), so that it can produce sustained price fluctuations. When the fraction of imitators is set to 70%, we obtain simulation results shown in Figure 4.2. In our simulations, return is represented by the difference between two successive natural logarithms of price, i.e., log-return.

We see that the level II model can generate a non-Gaussian (fat-tailed) distribution of return, but is not able to confirm another important stylized fact, namely volatility clustering. It therefore has the same problem as the model presented in Cont and Bouchaud (2000). Nevertheless, through simulations using this model, we can further study how the fraction of imitators influences the distribution of return. Figure 4.3 shows the results for different instances: \( \alpha_{im} = 20\% \), 50\%, and 80\% respectively. If the fraction is small, returns will follow a Gaussian distribution; increasing it enlarges the tails of the return distribution.

4.2.3 Simulation results of the level III model

In the level III model we have further added a mechanism through which agents’ activity is adjusted over time. Fixing \( \alpha_{im} \) to 70\%, we obtain simulation results shown in Figure 4.4, in which

a. Figure 4.4(a) records the price process. Some large ‘flights’ can be observed, which correspond to large (positive or negative) returns.

b. Figure 4.4(b) illustrates the time series of return. The effect of volatility clustering is clear.
4.2 Simulation results

Figure 4.2: Simulation results of the level II model when $\alpha_{im} = 70\%$. (a) Normalized return. (b) Distribution of return (the scale of the vertical axis is logarithmic). (c) Autocorrelation function of return and that of volatility. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.005$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, and $P^H = 100$. 
4.2 Simulation results

Figure 4.3: Return distributions of the level II model for different fractions of imitators. The ■ points, the ▲ points, and the + points refer to $\alpha_{im} = 20\%$, 50\%, and 80\% respectively. The scale of the vertical axis is logarithmic. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.005$, $F = 100$, $c_{fa} = 0.2$, $c_{cm} = 0.7$, and $P^B = 100$.

c. Figure 4.4(c) shows the probability distribution of return, together with a Gaussian PDF and a Lorentz PDF for comparison. The tails of the distribution are clearly heavier than those of a Gaussian PDF.

d. Figure 4.4(d) displays the autocorrelation function (ACF) of return (the lower curve) and that of volatility. The former converges quickly to the noise range, whereas the latter decays much more slowly.

e. Figure 4.4(e) shows the time evolution of trading volume$^1$.

f. Figure 4.4(f) illustrates the time evolution of trading activity. It is a slow process in comparison with the fast evolution of the influence of news$^2$.

These simulation results indicate that our CA model (level III) is able to reproduce the main stylized facts. In addition, as shown below, this model is robust with regard to the stylized facts for wide ranges of the parameters.

$^1$Volume is defined as the sum of absolute aggregate demand and absolute aggregate supply.

$^2$Here, we define the rate of time evolution of a variable $X$ as $(|\Delta X|/|X|)/\Delta t$, where $\Delta X$ is the change of $X$ within time increment $\Delta t$. 
4.2 Simulation results

Figure 4.4: Simulation results of the level III model when 70\% of the agents are imitators. (a) Price. (b) Normalized return. (c) Distribution of return (the scale of the vertical axis is logarithmic). (d) Autocorrelation function of return (the lower curve) and that of volatility. (e) Trading volume. (f) Trading activity. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, $P^0 = 100$, $k = 400$, $c_1 = 20$, and $L_m = 0.01$. The lower bound of $M_t$ is 0.05.
Table 4.1 compiles the kurtosis values of the return distributions for different values of $\alpha_m$, $c_m$, and $c_{fu}$ respectively. We see that imitators have a strong influence on kurtosis, while the relation between fundamentalists and kurtosis is not explicit. Specifically, the fraction of imitators $\alpha_m$ and the sensitivity of imitators to news $c_m$ are positively correlated with kurtosis. When either of them increases to a certain level, kurtosis suddenly becomes very large, implying that the system becomes unstable. For example, when $c_m = 0.9$, we obtain a price pattern with frequent dramatic ‘flights’ and a time series of return with many striking strokes. These are shown in Figure 4.5.

Table 4.1: Kurtosis values of the return distributions produced by the level III model for increasing values of $\alpha_m$, $c_m$, and $c_{fu}$ respectively. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_m = 0.7$, $P^b = 100$, $k = 400$, $a_l = 20$, and $L_m = 0.01$. The lower bound of $M'$ is 0.05.

<table>
<thead>
<tr>
<th>$\alpha_m$</th>
<th>Kurtosis</th>
<th>$c_m$</th>
<th>Kurtosis</th>
<th>$c_{fu}$</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>4.44</td>
<td>0.1</td>
<td>4.36</td>
<td>0.1</td>
<td>15.53</td>
</tr>
<tr>
<td>10%</td>
<td>4.36</td>
<td>0.2</td>
<td>5.37</td>
<td>0.2</td>
<td>17.22</td>
</tr>
<tr>
<td>20%</td>
<td>4.57</td>
<td>0.3</td>
<td>6.37</td>
<td>0.3</td>
<td>42.94</td>
</tr>
<tr>
<td>30%</td>
<td>5.33</td>
<td>0.4</td>
<td>7.44</td>
<td>0.4</td>
<td>37.28</td>
</tr>
<tr>
<td>40%</td>
<td>6.07</td>
<td>0.5</td>
<td>8.60</td>
<td>0.5</td>
<td>35.39</td>
</tr>
<tr>
<td>50%</td>
<td>6.64</td>
<td>0.6</td>
<td>10.55</td>
<td>0.6</td>
<td>46.71</td>
</tr>
<tr>
<td>60%</td>
<td>8.51</td>
<td>0.7</td>
<td>17.22</td>
<td>0.7</td>
<td>43.98</td>
</tr>
<tr>
<td>70%</td>
<td>17.22</td>
<td>0.8</td>
<td>32.57</td>
<td>0.8</td>
<td>42.11</td>
</tr>
<tr>
<td>80%</td>
<td>69.33</td>
<td>0.9</td>
<td>119.14</td>
<td>0.9</td>
<td>35.79</td>
</tr>
<tr>
<td>90%</td>
<td>190.46</td>
<td>1.0</td>
<td>422.77</td>
<td>1.0</td>
<td>30.72</td>
</tr>
</tbody>
</table>

Keeping other parameters constant and adopting different values of $k$, we obtain the autocorrelation functions of volatility shown in Figure 4.6(a). When $k$ is smaller than 50, ACFs of volatility quickly drop to the noise range and the effects of volatility clustering are correspondingly negligible. Volatility clustering becomes significant when $k$ is increased to around 100. The importance of $k$ to volatility clustering will further manifest itself in Section 4.3.4.
4.3 Discussion: The market dynamics revealed by the model

In this section, we provide a thorough investigation of the simulated dynamics through computational experiments and mathematical analysis. This study offers important insights into the mechanism governing the stock dynamics in reality.

Figure 4.5: Simulation results of the level III model when $\alpha_{in} = 70\%$ and $c_{in} = 0.9$. (a) Price. (b) Normalized return. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $P^0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of $M^t$ is 0.05.

Choosing three values for the number of agents (lattice sizes) and keeping other parameters constant, simulations give ACFs of volatility shown in Figure 4.6(b). All these ACFs are qualitatively similar to that of S&P 500 shown in Figure 2.1, indicating that the model can reproduce the stylized facts not only for markets with small numbers of agents, but also for markets with many agents. At this point, the model differs from some MS models that behave realistically only for limited numbers but not large numbers of traders (Egenter et al. (1999)).

4.3 Discussion: The market dynamics revealed by the model

In this section, we provide a thorough investigation of the simulated dynamics through computational experiments and mathematical analysis. This study offers important insights into the mechanism governing the stock dynamics in reality.
4.3 Discussion: The market dynamics revealed by the model

Figure 4.6: Autocorrelation functions of volatility produced by the level III model when $\alpha_{im} = 70\%$. (a) ACFs when $k = 10$ (the lowest curve), $k = 100$ (the second lowest), $k = 300$ (the highest), and $k = 500$ (the second highest) respectively. (b) ACFs when $N = 10 \times 10$ (the middle curve), 100 $\times$ 100 (the upper one), and 1000 $\times$ 1000 (the lower one) respectively. The parameter setting used: $N = 1 \times 10^4$, $c_p = 0.05$, $F = 100$, $c_{fu} = 0.2$, $c_{im} = 0.7$, $P_0 = 100$, $k = 400$, $c_l = 20$, and $L_m = 0.01$. The lower bound of $M_t$ is 0.05.
4.3 Discussion: The market dynamics revealed by the model

4.3.1 Long-range interactions can emerge from local interactions

In this section, for the sake of simplicity, we take a one-dimensional version of our CA model to derive analytical expressions. An agent located at \(i\) then has two neighbors at \(i-1\) and \(i+1\) respectively. Statistically, the total quantity at time \(t+1\) can be expressed as

\[
Q_{t+1} = \sum_{i=1}^{N} q_{i,im}^{t+1} = \sum_{i=1}^{N} (u_i q_{i,fu}^{t+1} + (1-u_i) q_{i,im}^{t+1}).
\]  

(4.10)

where \(u_i\) is determined in the following way: We sample a variable \(\gamma\) (0 \(\leq \gamma \leq 1\)) that is uniformly distributed. If 0 \(\leq \gamma \leq \alpha_{fu}\), \(u_i = 1\); else \(u_i = 0\). Here, \(\alpha_{fu}\) is the fraction of fundamentalists.

The terms \(q_{i,fu}^{t+1}\) and \(q_{i,im}^{t+1}\) in Equation (4.10) are determined by Equation (4.7) and Equation (4.8) respectively. However, because \(M^t\) changes much more slowly than \(Q^t\), we can consider the former as a constant to study the basic dynamics of the latter. We set \(M^{(t)} = 1\), then \(q_{i,fu}^{t+1}\) and \(q_{i,im}^{t+1}\) are respectively determined by Equation (4.3) and Equation (4.4). Therefore,

\[
q_{i,im}^{t+1} = \eta_{im}^{t+1} \left( \frac{1}{2} \right) V_{i-1,im}^t + V_{i+1,im}^t\]

(4.11)

Similarly, the terms \(V_{i-1,im}^t\) and \(V_{i+1,im}^t\) in Equation (4.11) can be respectively expressed as

\[
V_{i-1,im}^t = \eta_{im}^t \left( \frac{1}{2} \right) \left[ u_{i-1} V_{i-2,im}^{t-1} + (1-u_{i-1}) V_{i-2,im}^{t-1} \right] + \left[ u_i V_{i-1,im}^{t-1} + (1-u_i) V_{i-1,im}^{t-1} \right].
\]

(4.12)

and

\[
V_{i+1,im}^t = \eta_{im}^t \left( \frac{1}{2} \right) \left[ u_{i+1} V_{i+2,im}^{t-1} + (1-u_{i+1}) V_{i+2,im}^{t-1} \right] + \left[ u_{i+2} V_{i+1,im}^{t-1} + (1-u_{i+2}) V_{i+1,im}^{t-1} \right].
\]

(4.13)
4.3 Discussion: The market dynamics revealed by the model

Following the same scheme, we can further express the terms $V_{t-1,i-2,m}$, $V_{t,i,m}$, and $V_{t+2,i,m}$ in Equations (4.12) and (4.13) in terms of the corresponding quantities at time step $t-2$ of the neighbors of the agents located at $i-2$, $i$, and $i+2$ respectively, and so on. Basically, in this way, we can replace each imitator’s transaction quantity based on the current price level at each time step with the fundamentalists’ corresponding quantities at the preceding time steps, noting that $V_{t,i,\text{fu}} = V_{t,i,\text{fu}}$. After substitutions, we have

$$Q_{t+1} = \sum_{i=1}^{N} (A_{t+1,i}V_{t,i,\text{fu}} + A_{t,i}(\eta_{t+1})V_{t,i,\text{fu}} + A_{t-1,i}(\eta_{t+1})\eta_{t,i}V_{t-1,i,\text{fu}} + \ldots + A_{t-\tau,i}(\eta_{t+1})\eta_{t,i-1}\eta_{t-1,i}V_{t-\tau,i,\text{fu}} + \ldots).$$

(4.14)

where $\tau = -1, 0, 1, 2, \ldots$. The first few instances of $A_{t-\tau}$ are

$$A_{t+1} = a_i,$$

$$A_{t} = \frac{1}{2}[(1 - u_i)u_{i-1} + (1 - u_i)u_{i+1}],$$

$$A_{t-1} = \frac{1}{2^2}(1 - u_i)(1 - u_{i-1})u_{i-2} + (1 - u_i)(1 - u_{i-1})u_i + (1 - u_i)(1 - u_{i+1})u_i + (1 - u_i)(1 - u_{i+1})u_{i+2}. $$
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\[ A_{t-2} = \frac{1}{2N} [(1-u_i)(1-u_{i-1})(1-u_{i-2})u_{i-3} + (1-u_i)(1-u_{i-1})(1-u_{i-2})u_{i-1} + (1-u_i)(1-u_{i-1})(1-u_{i})u_{i-1} + (1-u_i)(1-u_{i-1})(1-u_{i})u_{i+1} + (1-u_i)(1-u_{i+1})(1-u_{i})u_{i-1} + (1-u_i)(1-u_{i+1})(1-u_{i})u_{i+1} + (1-u_i)(1-u_{i+1})(1-u_{i+2})u_{i+1} + (1-u_i)(1-u_{i+1})(1-u_{i+2})u_{i+3}] . \]

In each term within \( A_{t-\tau} \), the sequence of \( 1-u_i \) terms indicates the propagation of imitation over time (backwards) and space (agents). However, if at least one of the terms is equal to zero, which corresponds to a fundamentalist, the whole product will be zero. As the fraction of imitators/fundamentalists increases/decreases, some \( A_{t-\tau} \) terms with larger \( \tau \) values are greater than zero.

The imitation chains show that long-range interactions can form from local imitations. In the resultant networks, each agent is influenced, directly or indirectly, by some other near or remote agents. Here, the strengths and time lags of influence differ. In this respect, our CA model is different from the Cont-Bouchaud model, where any two agents can be directly linked, and agents in a group behave identically. It is also distinct from the model of Bartolozzi et al., within which agents in a cluster influence each other with an equivalent strength.

4.3.2 Price and volatility are mean-reverting

Fundamentalists behave according to price while imitators follow other agents but do not directly respond to price. We therefore argue that it is the fundamentalists’ behavior which determines price trend. This argument can be confirmed by our simulations: If \( \alpha_f = 0 \) (all the agents are imitators), price fluctuations die out; in other cases we obtain price trajectories similar in shape but distinct only in amplitude.

Therefore, for the sake of simplicity, we can take the special instance that all the agents are fundamentalists to study the basic dynamics of the price.
4.3 Discussion: The market dynamics revealed by the model

an instance, \( \alpha_{fu} = 1 \), hence \( u(\cdot) = 1 \). Then, Equation (4.14) gives

\[
Q^t = NV_{fu}^t = N(F\eta^t_{fu} - P^{t-1}).
\] (4.15)

The noise term \( \eta_{fu}^t \) is indispensable for a sustained price process, but is not responsible for any regularity in price trends. We therefore set \( \eta_{fu}^t = 1 \) for the sake of simplicity. Then, Equation (4.15) becomes

\[
Q^t = N(F - P^{t-1}).
\] (4.16)

Equation (4.9) gives,

\[
Q^t = \left( \frac{N}{c_p} \right) \left( P^{t+1} - P^t \right).
\] (4.17)

Substituting Equation (4.17) into Equation (4.16), we obtain

\[
P^{t+1} - P^t + c_pP^{t-1} = c_pF.
\] (4.18)

Equation (4.18) is a second-order difference equation. Depending on the value of \( c_p \), the price can follow a monotonically decaying process \( (c_p < 0.25) \), a damped fluctuating process \( (0.25 < c_p < 1) \), or an explosive fluctuating process \( (c_p > 1) \). Figure 4.7 shows the three typical price trajectories when the initial price is 105. In all these instances, the price is mean-reverting. Some researchers have studied the mean-reverting nature of price processes for different behavioral types, as well as different stabilizing-destabilizing endogenous mechanisms of financial markets (Baumol (1957), Beja and Goldman (1980), Day and Huang (1990)).

To analyze the process of volatility generated by our model, we need to consider the mechanism as well as the noise. First of all, we have assumed that the noise, which causes the volatility, follows an independent Gaussian random process. Within this process, those values more close to the mean have higher probabilities. Second, by examining Equations (4.5) through (4.9), we can recognize that when \( L' \) is smaller/greater than \( L_m \), a positive/negative feedback loop will form between \( L' \) and \( M' \). Namely, small/large values of \( L' \) tend to be enlarged/lessened. Therefore, the nature of the noise, the trading behavior of the agents, and the rule of price updating ensure that the volatility also follows a mean-reverting process.
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Figure 4.7: Price trajectories given by Equation (4.18) for different values of $c_p$. The monotonously decaying curve, the convergent fluctuating curve, and the divergent fluctuating curve correspond to $c_p = 0.1$, 0.9, and 1.02 respectively. The parameter setting used: $F = 100$ and $P_0 = 105$.

4.3.3 Heavy tails due to large price variations are caused by imitations

In this section, for the sake of simplicity, we adopt price change as return, i.e., $R_{t+1} = P_{t+1} - P_t$. According to Equation (4.9), we can then examine Equation (4.14) in order to investigate the cause of the resultant non-Gaussian return distributions.

In the simulations demonstrated in Section 4.2.2, if $\alpha_{fa} = 1$, we cannot generate fat tails. In this case, the total quantity is described by Equation (4.15), a special instance of Equation (4.14) when all the terms with a product of $\eta_{im}$ terms are equal to zero. Heavy tails are generated when $\alpha_{fa} < 1$ and some of these terms are present in Equation (4.14). We therefore suppose that it is the multiplication of the various $\phi_{im}$ terms in different $\eta_{im}$ terms that is responsible for the non-Gaussian distributions, although all these terms themselves follow a Gaussian distribution.

To confirm this supposition, we define a simple reference model:

$$H_t = \epsilon \phi_t + (1 - \epsilon) \phi_t \phi_{t-1} \phi_{t-2}, \quad (4.19)$$
where $\phi'$ is an independent Gaussian random variable with mean 0 and standard deviation 1 and $\epsilon$ is a parameter. Recall that, in Equation (4.14), $\eta'_{im} = 1 + \epsilon_{im} \phi'_{im}$. Since Equation (4.19), as with Equation (4.14), deals with the sum of products of Gaussian terms, it represents the basic structure of the latter.

Figure 4.8 presents the experimental probability distributions of $H'$ obtained when choosing different values of $\epsilon$ for comparison: 1, 0.5, and 0. In this figure we see that when $\epsilon$ decreases, the distribution of $H'$ gradually changes from being pure Gaussian to being very fat-tailed non-Gaussian. Thus, the more the product of $\phi'$ terms is weighted, the heavier the tails of the consequent distribution. From the discussion in Section 4.3.1 we know that, if $\alpha_{jm}$ is small, the products of more $\eta'_{im}$ factors in Equation (4.14) will have more weight. This experiment therefore explains the regularity discussed in Section 4.2.2 and Section 4.2.3: Larger fractions of imitators correspond to return distributions with heavier tails. In addition, products of $\eta'_{im}$ terms give rise to continued products of $c_{im}$. The multiplication of $c_{im}$ explains the exponential growth of kurtosis following the increase of $c_{im}$, as shown in Section 4.2.3.
4.3 Discussion: The market dynamics revealed by the model

4.3.4 Volatility clustering is related to the evolution of trading activity

According to Equation (4.9) and the definition of return adopted in this section,

\[ R_{t+1} \propto Q_t \]  \hfill (4.20)

Since \( M_t \) changes much more slowly than \( Q_t \), we have \( M_t \approx M_{t-1} \approx \ldots \approx M_{t-\tau} \) for small values of \( \tau \). (Note that the analysis here is by no means rigorous.) Then, for a small value of \( \tau \), according to Equations (4.7), (4.8), and (4.10), as well as the scheme conveyed by Equations (4.11) through (4.13), Equation (4.20) gives

\[ R_{t+1} \propto M_t U_t \]  \hfill (4.21)

where

\[ U_t = \sum_{i=1}^{N} \left[ A_i^l (F^l u_i - P^{l-1}) + A_i^{l-1} (\eta_{im}^l) (F^{l-1} u_i - P^{l-2}) + \ldots + A_i^{l-\tau} (\eta_{im}^{l-1} \eta_{im}^{l-2} \ldots \eta_{im}^{l-\tau}) (F^{l-\tau} u_i - P^{l-\tau-2}) \right] \]

In Equation (4.21), \( M_t \) is a factor that emerges from the agents’ trading and in turn reinforces it. Because it changes more slowly than \( U_t \), successive values of \( R_{t+1} \) are positively correlated with each other. However, consecutive values of \( R_{t+1} \) are only weakly correlated due to the fast variation in its sign, which is caused by the fast variation in the sign of \( U_t \) due to news and the mean-reverting nature of the price. These explain the stylized facts: long-term autocorrelation of volatility and short-term autocorrelation of return.

Thus, according to our simulations, three factors are indispensable for volatility clustering: a random component, a convergent mean-reverting mechanism and a factor that emerges from agents’ trading and in turn reinforces it. In comparison with the first factor, the third factor changes much more slowly, or on a longer
4.3 Discussion: The market dynamics revealed by the model

time scale. To show how the three factors contribute to volatility clustering, here we devise two reference models. The first one is

\[ H^t = \phi^t \sin(\lambda t), \]  

(4.22)

where \( \lambda \) is a constant, \( \phi^t \) is the same variable use in Equation (4.19). The second reference model is expressed as

\[ H^t = \xi \phi^t \sum_{i=t-k}^{t-1} H^i, \]  

(4.23)

where \( \xi \) is a constant, \( \phi^t \) is again the same variable use in Equation (4.19), \( k \) indicates the length of a past time period, and we denote \( \sum_{i=t-k}^{t-1} H^i \) as \( S_H^t \).

In both equations, \( H^t \) resembles total quantity or return in our CA model. Equation (4.22) includes a noise term \( \phi^t \) and an arbitrarily introduced factor \( \sin(\lambda t) \). We choose a small value for \( \lambda \) (0.01) so that the process of \( \sin(\lambda t) \) is much slower than that of \( \phi^t \). Equation (4.23) contains a noise term \( \phi^t \) and an emergent factor \( S_H^t \). We choose a large number for \( k \) (500) to ensure that the latter changes much more slowly than the former. Figure 4.9 shows the time series of these two reference models. The result of Equation (4.22) is shown in Figure 4.9(a). In this figure, large and small absolute values of \( H^t \) gather together and the periods of these clusters correlate with the (positive or negative) high and low periods of the \( \sin(\lambda t) \) curve. Although the curve is arbitrary instead of emergent, the result expresses the necessity of two factors for volatility clustering: A fast process of noise and a slower process that determines the amplitudes of the resultant time series. Figure 4.9(b) shows the time series of \( H^t \) defined by Equation (4.23). Here, \( S_H^t \) fluctuates over time but changes more slowly than \( H^t \). Emerging from \( H^t \), it in turn determines the amplitudes of \( H^t \). Volatility clustering exhibits in this time series. However, the process of \( H^t \) is very sensitive to \( \xi \), which can cause the process to quickly diverge. Obviously, this model lacks a balancing feedback mechanism to maintain the stability of the dynamics. Nevertheless, it shows how a fast noise factor and an emergent slow factor together induce volatility clustering.
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Figure 4.9: Time series of the reference models in Section 4.3.4. (a) The lower curve is that of $\sin(\lambda t)$, the upper one indicates the corresponding time series of $H^t$ defined by Equation (4.22). Here, $\lambda = 0.01$. (b) The lower curve and upper curve are, respectively, the time series of $S^t$ and $H^t$ defined by Equation (4.23). Here, $\xi = 0.1$, $k = 500$. In both graphs, the amplitudes of the lower curves and the longitudinal positions of the upper curves have been adjusted for the sake of easy comparison.

4.3.5 The regularity can be identified in some other microsimulation models

In a general sense, the MS models discussed in Section 3.2 that can confirm the stylized facts observed in stock markets agree with our CA model on the origins of large price variations and volatility clustering.

Although explaining imitation from different angles, all these MS models and our CA model show that the fraction of abnormally large price variations is much larger when agents imitate each other than when they are mutually independent. In the latter instance and in the limit of a large number of agents, returns follow a Gaussian distribution.

Within these MS models and our CA model, we can ultimately attribute volatility clustering to the evolution of agents’ activity, although the corresponding processes of the models that indicate activity are quite distinct. (Notice that all these processes are positively correlated with the evolution of trading volume.)
These processes are, respectively, the evolution of volatility (Bak et al. (1997)), the development of the fraction of noise traders (Lux and Marchesi (1999)), the evolution of agents’ activation thresholds (Iori (2002)), the percolation process (Bartolozzi and Thomas (2004)), and the progression of the desirability of an asset (our CA model). In addition, these processes are slower than their corresponding ‘source’ processes. Therefore, volatility clustering generated by these MS models and our CA model is the combined effect of two processes on different time scales.

In literature, on the one hand, there is not yet a common agreement on the origins of the stylized facts (Cont (2005)). On the other hand, various analytical models for describing the phenomena, e.g., GARCH models, stochastic volatility models\(^1\), and a recently published Itô-Langevin model described in Anteneodo and Riera (2005) do not provide explicit economic explanations for the underlying dynamics. The regularity discussed here can help us to achieve a better understanding of the complex dynamics of stock markets.

\(^{1}\)A class of stochastic volatility models considers volatility to be independent of return. Price is then assumed to follow a geometric Brownian motion with a time-dependent volatility: 
\[
dS(t) = \mu S(t)dt + \sigma(t)S(t)dZ_1.
\]
Here \(dZ_1\) describes a Wiener process. With \(\sigma(t) = \sigma^2(t)\), the time-dependent variance follows a different stochastic process 
\[
dv(t) = m(v(t))dt + s(v(t))dZ_2,
\]
where \(dZ_2\) is another Wiener process. Different forms of \(m(v(t))\) and \(s(v(t))\) correspond to some popular models of this type (Voit (2003)).
4.4 Conclusions

combined effect of a fast and a slow process: The evolution of the influence of news and the evolution of agents’ activity, respectively. In a general sense, these causes of heavy tails and volatility clustering appear to be common among some well-established microsimulation models that have confirmed the main characteristics of financial markets.