Understanding the complex dynamics of financial markets through microsimulation

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Appendix A

A detailed description of the options market model

A.1 The rule of option price updating

The prices of the options are updated according to the following rule, which can be understood as the effect of market makers' action to balance the supply and demand:

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}, \tag{A.1}$$

where $Q^{n,\phi,t}$ is the total transaction quantity or the excess demand of the n-th option at time t. Since the excess demand is proportional to N_{tr}, we rescale it with the latter. Here β is a positive parameter that reflects the sensitivity of the option price to the excess demand. Due to the fact that option prices cannot be negative, the lower bound of $V^{n,\phi,t}$ is 0.

A.2 The dynamic aspect of speculators' behavior

We define $S^{i,t}_{SP}$ as a function of the directional trader's general level of optimism, the price of the underlying asset, the psychological effect of the news on the speculators, and the level of ambiguity of the news:

$$S_{SP}^{i,t} = (\bar{S}_{SP}^{i} + dS^t)(1 + \psi^t)(1 + \omega^t \xi), \tag{A.2}$$

where $\bar{S}_{SP}^{i}$ is the long-term mean of the trader's expected price, dS^t is equal to $S^t - \bar{S}$, in which S^t is the price of the underlying and $\bar{S}$ is the long-term mean of S^t; ψ^t expresses the psychological impact of the news on the trader's expected price; ω^t reflects the ambiguity level of news, and ξ is a random number sampled from a uniform distribution in the range $[-1, 1]$. The term dS^t is included since it is reasonable to expect that speculators will adjust their expected prices when the price of the underlying changes.

The processes S^t, ψ^t, ω^t, and F_{SP}^t are all influenced by news and vary simultaneously over time and we assume that they all follow the Ornstein-Uhlenbeck process (a mean-reverting process):

$$U^{t+1} = U^t + \theta_U(\mu_U - U^t) + \nu_U dz \tag{A.3}$$

where U^t denotes the specific factor of interest, θ_U the mean reversion rate, μ_U the mean reversion level, ν_U the volatility of the random fluctuations, and dz the Wiener process. The reason for adopting a mean-reverting process is that all these factors are influenced by news and therefore fluctuate around their long-term average levels.

A.3 The parameter values

The units of the variables or parameters in our model are 'year' for t, T; 'monetary unit' for S^t, $S_{SP}^{i,t}$, S_{BS}, K^n, $V^{n,\phi,t}$, M_{SP}^t, D_{SP}^t, μ_U, ε^h, h, and β; 'number/(monetary unit)' for η_{SP}, λ_{SP}, λ_{AR}, and γ; and 'number' for N_{op}, N_{tr}, and $Q^{n,\phi,t}$.

We adopt $T - t = 1$, $N_{tr} = 5000$, $N_{op} = 11$ (15, 16, $\cdots$, 25), $\gamma = 1.5$, $\alpha = 0.1$[1], and $\beta = 0.1$. We assume that the long-term means of S^t, M_{SP}^t, D_{SP}^t, and F_{SP}^t are respectively $\bar{S} = 20$ and $\bar{M}_{SP} = 19$, $\bar{D}_{SP} = 4$, and $\bar{F}_{SP} = 0.5$. In Section 5.3 we

[1] Solely for reference, the average alpha value of the corresponding BS prices where the common middle strike is 20 and the volatility is between 0.1 and 0.3 is around 0.08.

assign different values to these three factors to analyze the mechanism underlying the smile phenomenon.

To study whether the option prices satisfy the arbitrage relations, we display them together with the corresponding prices obtained by the BS model with spot price S_{BS} and volatility σ_{BS}. The BS prices satisfy all the arbitrage relations. We set $S_{BS} = 20$ and $\sigma_{BS} = 0.198$[1]. The initial price of the underlying and that of the options are 20 and 1 respectively. In simulating the IV dynamics, there are 18000 time-steps in each simulation run and we take an IV curve after every 150 time-steps for performing principal components analysis (PCA).

The parameters η_{SP}, $\lambda_{AR_{PCP}}$, and $\lambda_{AR_{BFS}}$ represent the traders' activity for employing the corresponding strategies. We assume that their values are proportional to the traders' confidence levels regarding the profitability of the strategies. For example, speculative profits are much more uncertain than arbitrage gains, so the value for η_{SP} should be smaller than that for $\lambda_{AR_{PCP}}$, and $\lambda_{AR_{BFS}}$. In the simulations, $\eta_{SP} = 0.1$, $\lambda_{AR_{PCP}} = 1.0$, and $\lambda_{AR_{BFS}} = 1.5$.

In modeling the dynamic behavior of the relevant factors we use a mean-reverting Gaussian process (Equation (A.3)). This stochastic process U^t consists of three parameters, the long-term mean μ_U, the mean-reversion speed θ_U and the volatility ν_U. The long-term mean of S^t, ψ^t, ω^t, and F^t_{SP} is $\mu_S = 20$, $\mu_\psi = 0$, $\mu_\omega = 0.2$, and $F^t_{SP} = 0.5$ respectively. The adoption of the values for μ_ψ and μ_ω respectively is based on the assumption that the psychological effect of news is neutral on average, while there is always some ambiguity in news. To mimic high-frequency time series of underlying assets, we adopt 0.001 and 0.01 for θ_S and ν_S respectively. In addition, it is reasonable to assume that the values θ_U and ν_U are proportional to the strength in which news influence the corresponding processes. For example, in real markets, trading in underlying assets is mediated by market makers who dampen the volatility by enhancing liquidity and increasing market depth[2]. Therefore S^t is expected to be less volatile compared to ψ^t, ω^t, and F^t_{SP}

[1]Here we adopt $\sigma_{BS} = \sqrt{ln((\bar{D}_{SP}/S_{BS})^2 + 1)}$, so that a log-normal distribution with the mean equal to S_{BS} and the standard deviation equal to $\bar{D}_{SP}$ is assumed for the price of the underlying when using the BS model. However, in principle, the volatility does not influence the comparison of the simulated option prices and the corresponding BS prices because all the relevant arbitrage relations are independent of volatility (see Cox and Rubinstein (1985)).

[2]Market depth is the size of an order needed to move the price a given amount.

which are related to traders' mentality and behavior. The values chosen for ψ^t, ω^t, and F^t_{SP} are therefore greater than that for S^t. In our simulations, unless otherwise stated, $\theta_\psi = 0.01$, $\nu_\psi = 0.002$ for directional speculators and 0.02 for volatility speculators; $\theta_\omega = 0.01$, $\nu_\omega = 0.02$; and $\theta_F = 0.01$, $\nu_F = 0.05$. The lower bounds of S^t, ω^t, and F^t_{SP} are all 0, and the upper bound of F^t_{SP} is 1. It should be noted that the stochastic process U^t can be solved analytically: The specific factor follows a Gaussian distribution with long-term variance equal to ${\nu_U}^2/(2\theta_U)$. Accordingly, the parameter values yield reasonable long-term variances in the order from 0.01 to 0.2. To further demonstrate the robustness of our results, we have performed a sensitivity analysis with respect to these parameters (see Section 5.4).

Appendix B

Analytical solution for a special case of the options market model

Here we show analytically that, if the mean of the speculators' expected prices M_{SP}^t coincides with the current price of the underlying S^t, the simulated option prices based on the S model converge to the corresponding BS prices.

The transaction quantity of a speculator is

$$Q_{SP}^{i,n,\phi,t} = \lambda_{SP}(\max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}). \tag{B.1}$$

In the S model, all the traders are speculators, so

$$\begin{aligned} Q_{SP}^{n,\phi,t} &= \sum_{i=1}^{N_{tr}} Q_{SP}^{i,n,\phi,t} \\ &= \lambda_{SP}[\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr} V^{n,\phi,t}]. \end{aligned} \tag{B.2}$$

The price updating rule is

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}. \tag{B.3}$$

The price of each option eventually converges, i.e., $V^{n,\phi,t+1} = V^{n,\phi,t}$. Then,

$$Q^{n,\phi,t} = 0. \tag{B.4}$$

Substituting Equation (B.4) in Equation (B.2), we have

$$\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr} V^{n,\phi,t} = 0, \tag{B.5}$$

i.e. (law of large numbers),

$$\begin{aligned} V^{n,\phi,t} &= \frac{1}{N_{tr}}(\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}) \\ &\approx E[\max(\phi(S_{SP}^{i,t} - K^n), 0)]. \end{aligned} \tag{B.6}$$

Since we assume that $S_{SP}^{i,t}(i = 1, 2, \cdots, N_{tr})$ follow a lognormal distribution, the right hand side of Equation (B.6) is equal to the corresponding Black-Scholes solution.