Understanding the complex dynamics of financial markets through microsimulation

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Publication date
2011

Appendix B

Analytical solution for a special case of the options market model

Here we show analytically that, if the mean of the speculators' expected prices M_{SP}^t coincides with the current price of the underlying S^t, the simulated option prices based on the S model converge to the corresponding BS prices.

The transaction quantity of a speculator is

$$Q_{SP}^{i,n,\phi,t} = \lambda_{SP}(\max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}). \tag{B.1}$$

In the S model, all the traders are speculators, so

$$\begin{aligned} Q_{SP}^{n,\phi,t} &= \sum_{i=1}^{N_{tr}} Q_{SP}^{i,n,\phi,t} \\ &= \lambda_{SP}[\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr} V^{n,\phi,t}]. \end{aligned} \tag{B.2}$$

The price updating rule is

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}. \tag{B.3}$$

The price of each option eventually converges, i.e., $V^{n,\phi,t+1} = V^{n,\phi,t}$. Then,

$$Q^{n,\phi,t} = 0. \tag{B.4}$$

Substituting Equation (B.4) in Equation (B.2), we have

$$\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - N_{tr} V^{n,\phi,t} = 0, \tag{B.5}$$

i.e. (law of large numbers),

$$\begin{aligned} V^{n,\phi,t} &= \frac{1}{N_{tr}} (\sum_{i=1}^{N_{tr}} \max(\phi(S_{SP}^{i,t} - K^n), 0) - V^{n,\phi,t}) \\ &\approx E[\max(\phi(S_{SP}^{i,t} - K^n), 0)]. \end{aligned} \tag{B.6}$$

Since we assume that $S_{SP}^{i,t} (i = 1, 2, \cdots, N_{tr})$ follow a lognormal distribution, the right hand side of Equation (B.6) is equal to the corresponding Black-Scholes solution.