Appendix C

Levels of trading activity of different speculators

Here we describe the trading activity levels of the different types of speculator, by considering liquidity unbalancing and the number of portfolios in which the traders can invest.

In real markets, the liquidity of options is not balanced across strikes, as shown in Ederington and Guan (2002) and Rexhepi (2008). In general, OTM options are more liquid than ITM options, implying that at least some speculators trade the former more actively than the latter. This liquidity unbalancing might stem from the trading behavior of speculators and the price characteristics of options. Generally, speculators prefer cheap and liquid options in order to achieve high leverage and fast conversion. The potential for higher leverage provided by OTM options, which are cheaper than their ITM counterparts, attracts more speculators. Higher leverage thus leads to higher liquidity, which in turn pulls in even greater speculative trading volume. This positive feedback effect ensures the relatively higher/lower liquidity of OTM/ITM options. To reflect this fact, we assume that the activity levels of these speculators are strike dependent and follow the form

\[ \lambda_{SP}(K^n) = \eta_{SP}[\phi \tanh(\gamma(K^n - S^t)) + 1], \quad (C.1) \]

where \( \eta_{SP} \) and \( \gamma \) are positive parameters. Equation (C.1) is an increasing/decreasing function of strike for the call/put options. In the simulations, the values adopted
for η_{SP} and γ are 0.1 and 1.5 respectively.

SD and BS speculators can trade each option independently of other options. Activity unbalancing described by Equation (C.1) is therefore applicable to them. DSPR and BSPR strategies, however, usually involve ITM and OTM options at the same time and in specified proportions. Activity unbalancing is thus not applicable to these speculations.

In addition, the various types of speculative strategy have different numbers of potential portfolios (or combinations). We assume that the speculators are equally active in applying all the entire strategies, and that each type of speculators employ all the possible portfolios of call or/and put options prescribed by their strategies. The activity levels for individual portfolios assigned to the speculators are therefore inversely proportional to the numbers of the portfolios corresponding to the respective strategies. The activity levels of the different types of speculator are determined as follows:

- **SD and BS Speculations.** There are N_{STK} strike prices, therefore a SD or BS trader can establish 2N_{STK} (long and short) positions by using all the call options. Similarly, the trader can establish the same number of positions using all the put options. There are therefore totally 4N_{STK} different positions for the trader. We assume that the activity level of each SD or BS speculator for individual options, i.e. λ_{SP,SD} or λ_{SP,BS} respectively, is equal to λ_{SP}(K^*).

- **DSPR Speculation.** When a DSPR trader takes a long position in the call option with the smallest strike, there are N_{STK} − 1 ways to take a short position with another call option with lower strike price, i.e., the trader can establish N_{STK} − 1 portfolios. Similarly, using the call option with the second smallest strike, the trader has N_{STK} − 2 portfolios, and so on. Therefore, using the N_{STK} call options, a DSPR trader can establish \((1/2)N_{STK}(N_{STK} − 1)\) portfolios (the sum of the first N_{STK} terms of an arithmetic sequence, of which the first term is N_{STK} − 1 and the common difference is −1). Including the reverse combinations doubles the number of DSPR portfolios established with call options. Using put options, a DSPR trader can establish the same number of portfolios as that from using the call
options. There are therefore totally $2N_{STK}(N_{STK} - 1)$ different portfolios for a DSPR trader. The activity level of each DSPR trader, i.e. $\lambda_{SP_{DSPR}}$, is therefore equal to \[
\frac{(4N_{STK})}{(2N_{STK}(N_{STK} - 1))} \eta_{SP} = \frac{2}{(N_{STK} - 1)} \eta_{SP}
\] (strike-independent).

- **BSPR Speculation.**

By taking long positions in the pairs of call options with strikes symmetric to ATM, a BSPR trader can build $(N_{STK} - 1)/2$ portfolios. Including the reverse combinations doubles the number of BSPR portfolios. Using the put options, a BSPR trader can establish the same number of portfolios as that from using the call options. There are therefore totally $2(N_{STK} - 1)$ different portfolios for a BSPR trader. The activity level of each BSPR trader, i.e. $\lambda_{SP_{BSPR}}$, is therefore equal to \[
\frac{(4N_{STK})}{(2(N_{STK} - 1))} \eta_{SP} = \frac{(2N_{STK})}{(N_{STK} - 1)} \eta_{SP}
\] (strike-independent).