Exotic Branes and Nongeometric Backgrounds

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When string or _M_ theory is compactified to lower dimensions, the _U_-duality symmetry predicts so-called exotic branes whose higher-dimensional origin cannot be explained by the standard string or _M_-theory branes. We argue that exotic branes can be understood in higher dimensions as nongeometric backgrounds or _U_ folds, and that they are important for the physics of systems which originally contain no exotic charges, since the supertube effect generically produces such exotic charges. We discuss the implications of exotic backgrounds for black hole microstate (non-)geometries.

**Introduction.**—String theory includes various extended objects as collective excitations, such as _D_-branes. The _U_-duality symmetry [1] which maps these objects into one another has played a pivotal role in the development of string theory and provided crucial insights into its non-perturbative behavior. When string or _M_ theory is compactified to lower dimensions, the _U_-duality group gets enhanced, relating objects that were not related in higher dimensions. For example, _M_ theory compactified on _T^2_ has a discrete _U_-duality group known as _E_4(2) [1].

In the lower (_d_ = 11 – _k_) dimensional theory, if we start from a codimension-two object obtained by partially wrapping a known 11D object and act by _U_ duality on it, we start to produce objects whose higher-dimensional origin is unknown; they are called _exotic branes_ [2]. In type II language, some of them have a tension proportional to _g_−3 or _g_−4. For example, in type II string compactified on _T^2_, consider an NS5-brane extending along six of the eight remaining noncompact directions, not wrapping the internal _T^2_. If we perform a _T_ duality along both _T^2_ directions, we obtain an exotic brane called _S_5. We will see later that this is a nongeometric background known as a _T_ fold [3]; as we go around the exotic brane, the internal _T^2_ is nontrivially fibered and does not come back to itself, but rather to a _T_-dual version.

One may think that such codimension-two objects are problematic due to logarithmic divergences [4], and that we do not need them if we are concerned with the physics of nonexotic states. However, this is not true because of the supertube effect [5]—the spontaneous polarization phenomenon that occurs when we bring a particular combination of charges together. A basic example is

\[
D0 + F1(1) \rightarrow D2(1\psi)
\]

in which _D0_-branes and fundamental strings along _x^1_ polarize into a _D2_-brane extending along _x^1_ and a closed curve in the transverse directions parametrized by _ψ_. Note that the _D2_ charge did not exist in the original configuration. Since the _D2_ is along a closed curve, there is no net _D2_ charge, but only a _D2_ dipole charge. The microscopic entropy of the _D0_-F1 system can be recovered by counting the possible _ψ_ curves that the system can polarize into [6].

Even if we start with a configuration of nonexotic charges, the supertube effect can produce exotic charges. Because the exotic charges thus produced are dipole charges, there is no net exotic charge at infinity and the problem of log divergences does not arise. This implies that exotic states are relevant even for the physics of systems which do not originally contain exotic charges.

This is especially interesting in the context of black hole physics where one typically considers a configuration of multiple (nonexotic) charges. We will argue later that the supertube effect and exotic charges are relevant for the understanding of the physics of such black holes.

**Exotic states and their higher-dimensional origin.**—If we compactify _M_ theory on _T^8_ or type IIA/B string theory on _T^7_ down to 3D, we obtain _N_ = 16 supergravity [7] with 128 scalars (note that gauge fields can be dualized into scalars in 3D). This theory has _E_8(8) as the _U_-duality group which is broken to the discrete subgroup _E_8(4) in string theory [1]. This _E_8(8)(Z) is generated by _S_ and _T_ dualities along the internal torus. For example, in type IIB, a _D7_-brane wrapped on the _T^3_ yields a point particle in three dimensions. Acting with _S_ and _T_ dualities, we can obtain all other states in the “particle multiplet” of the _U_-duality group as explained in [2].

In Table I, we list the states in the particle multiplet, including the exotic ones. The notation for nonexotic states is standard; e.g., _P_ denotes a gravitational wave and KKM denotes a Kaluza-Klein monopole. For type II exotic states, we follow [2] and denote them by how their mass depends on the _T^3_ radii. The mass _M_ of _b_(_p_,_q_,_s_,_t_,_i_,_j_) depends linearly on _p_ radii and quadratically on _c_ radii. For _b_(_p_,_q_,_s_,_t_) , _M_ also depends cubically on _d_ radii. Moreover, _M_ is proportional to _g_−1. For example, the mass of _S_2 depends on the radii _R_(_i_), _i_ = 3, . . . , 9 of _T^3_ as _M_ = _R_3 · · · _R_9(_R_3 _R_9)^2 _g_−2. We often display how the state “wraps” the _T^3_ as _S_2(34567, 89). In this notation, the KK monopole is denoted by _S_3. In _M_ theory, we use a similar notation except that we drop the subscript _n_. Using the transformation rules
for the radii \( R_s \) and \( s \) under \( S \) and \( T \) dualities, we can read off how those states transform into one another [2].

In the 3D theory, we would have 128 gauge fields if we could dualize all the scalars into gauge fields [2]. However, as we can see from Table I, there are as many as 240 charged particles [8], and this discrepancy (240 versus 128) in the 3D theory is not understood [2]. For \( d = 4 \), this issue does not arise because we obtain just as many charged particles as gauge fields [2]. Here, we argue that the higher-dimensional origin of exotic states consists of nongeometric backgrounds or \( U \) folds [3,9].

The argument is simple. For example, consider a \( D7 \)-brane wrapping \( T^7 \), which is (magnetically) coupled to the RR 0-form \( C(0) \). From the 3D viewpoint, the \( D7 \)-brane is a point particle and, as we go around it, the 3D scalar \( \phi = C(0) \) shifts as \( \phi \rightarrow \phi + 1 \). Namely, in 3D, the “charge” of the point particle is nothing but the monodromy of the scalar \( \phi \) around it. This symmetry of shifting \( \phi \) by one gets combined with other dualities such as \( S \) and \( T \) dualities to form the \( U \)-duality group \( G(\mathbb{Z}) = E_{8(8)}(\mathbb{Z}) \), and the scalar \( \phi \) gets combined with other scalars into a matrix \( M \) parametrizing the moduli space \( \mathcal{M} = SO(16) \langle E_{8(8)}(\mathbb{R}) / E_{8(8)}(\mathbb{Z}) \rangle \). \( U \)-duality means that we can more generally consider a 3D particle around which \( M \) jumps by a general \( U \)-duality transformation. Thus, the charge of a 3D particle is defined by the \( U \)-duality monodromy around it. This can be regarded as a non-Abelian generalization of the usual notion of \( U(1) \) charges for which the monodromy is an additive shift. Clearly, the number of different charges thus generalized is not in general equal to that of gauge fields, which resolves the above puzzle.

If we lift such a monodromy to 10D/11D, we obtain a configuration in which the internal space is nontrivially fibered as we go around the particle and glued together by a \( U \)-duality transformation. So, exotic states correspond in 10D/11D to nongeometric backgrounds, or “\( U \) folds” [3]. To our knowledge, the interpretation of exotic states as \( U \) folds has not appeared in the literature. Note that this construction differs from the more familiar \( U \) folds in the context of string compactifications [9], where \( U \) duality is nontrivially fibered over a noncontractible circle in the internal manifold, not over a contractible circle in the noncompact directions.

Let us discuss how to classify “charges” defined by the monodromies around them. First, assume the existence of a charge with monodromy \( q \). Namely, as we go around the particle in three dimensions, the moduli matrix \( M \) undergoes the monodromy transformation \( M \rightarrow Mq, q \in G(\mathbb{Z}) \).

If we go to another \( U \) duality frame by a \( U \)-duality transformation \( U \in G(\mathbb{Z}) \), then this becomes \( \tilde{M} \rightarrow \tilde{M} \tilde{q} \) with \( \tilde{M} = MU, \tilde{q} = U^{-1}q U \). So, in the dual frame, there exists a charge with monodromy \( \tilde{q} \). Now, let us change the moduli \( \tilde{M} \) adiabatically to the original value \( M \). If the charge is BPS, an object with monodromy \( \tilde{q} \) continues to exist, implying the existence of the charge \( q \) even for the original value of the moduli \( M \) (assuming that there is no line of marginal stability). So, starting from a charge \( q \), we can generate other possible charges by conjugation \( \tilde{q} = U^{-1}q U \). Note that this does not mean that we can generate all charges that exist in the theory by conjugation; there can be many conjugacy classes in the group \( G(\mathbb{Z}) \) and we cannot generate charges in different conjugacy classes. Also, there can be non-BPS charges for which the above argument (of changing moduli adiabatically) does not apply.

As a simple example, consider a \( D7 \)-brane. Around it, there is an \( SL(2, \mathbb{Z}) \) monodromy given by

\[
T = \begin{pmatrix}
1 & 1 \\
0 & 1 
\end{pmatrix}.
\]

Let us conjugate this with a general \( SL(2, \mathbb{Z}) \) matrix

\[
U = \begin{pmatrix}
s & r \\
q & p 
\end{pmatrix}.
\]

The conjugated charge is

\[
\tilde{T} = U^{-1}TU = \begin{pmatrix}
1 + pq & p^2 \\
-q^2 & 1 - pq 
\end{pmatrix},
\]

which is the monodromy of the standard \((p, q)\) 7-brane. Note that, although \( U \) has 3 independent parameters, the resulting charge \( \tilde{T} \) has only 2 parameters. In this sense, there exist only two different charges.

So, the set of all possible charges we can obtain from a given one \( q \) by \( U \) duality is its conjugation orbit. This orbit is a subset of the discrete non-Abelian “lattice” \( G(\mathbb{Z}) \), and it makes no sense to ask how many different charges there are in it. However, to get a qualitative idea of the size of the orbit, we can replace \( G(\mathbb{Z}) \) by the continuous group \( G(\mathbb{R}) \) and count the dimension of the (now continuous) orbit. Using standard results on the dimensions of conjugacy classes in noncompact groups and their relation to \( sl(2) \)-embeddings, one can show that, e.g., for \( G(\mathbb{Z}) = E_{8(8)}(\mathbb{Z}) \), the dimension of the orbit generated by a 1/2-BPS object such as the \( D7 \)-brane is 58 [10]. The 240 states in Table I represent 240 particular points in this orbit, which can be obtained by \( U \) dualities preserving the rectangularity of the internal torus [2].

Supergravity description of exotic states.—To demonstrate the above idea, let us present the supergravity metric

<table>
<thead>
<tr>
<th>TABLE I. The point particle states and their multiplicities (boldface numbers) in string or ( M ) theory compactified to 3D.</th>
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<tbody>
<tr>
<td>Type IIA</td>
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<td>Type IIB</td>
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for $S^3_2$ as an example. This can be obtained by $T$ duality of
the KK monopole metric transverse to its worldvolume.

The KK monopole ($S^3_2(56789, 4)$) metric is

$$ds^2 = dx_1^2 + dx_2^2 + H dx_{12}^2 + H^{-1} (dx^4 + o^2), \quad e^{2\Phi} = 1,$$

$$d\omega = *_3 dH, \quad H = \frac{1 + \sum_p H_p}{2} = R_4(2|\vec{x} - \vec{x}_p|), \quad (2)$$

where $\vec{x}_p$ are the positions of the centers in $\mathbb{R}^3_{123}$. Now
compactify $x^3$, which is the same as arraying centers at
intervals of $2\pi R_3$ along $x^3$. So,

$$H = 1 + \sum_{n \in \mathbb{Z}} R_4/[2(r^2 + (x^3 - 2\pi R_3 n)^2)^{1/2}]$$

$$= 1 + \alpha \log((\Lambda + \sqrt{r^2 + \Lambda^2})/r), \quad (3)$$

where $\alpha = R_4/2\pi R_3$ and we took a cylindrical coordinate
system $dx_{12}^2 = dr^2 + r^2 d\theta^2 + (dx^3)^2$. We approximated
the sum by an integral and introduced a cutoff $\Lambda$ to make it
convergent (see [11,12]). $H$ in (3) diverges as we send $\Lambda \to \infty$, but this can be formally shifted away by introducing a
"renormalization scale" $\mu$ and writing

$$H(r) = h + \sigma \log(\mu/r) \Rightarrow \omega = -\sigma \theta dx^3, \quad (4)$$

where $h$ is a "bare" quantity which diverges in the $\Lambda \to \infty$
limit. The log divergence of $H$ implies that such an infinitely
long codimension-two object is ill-defined by itself. In
physically sensible configurations, this must be regularized
either by taking a suitable superposition of
codimension-two objects [4] or, as we will do later, by
considering instead a configuration which is of higher
codimension at long distance.

Now let us do a $T$ duality along $x^3$. By the standard
Buscher rule, we obtain the metric for $S^3_2(56789, 34)$:

$$ds^2 = H(dr^2 + r^2 d\theta^2) + H K^{-1} dx_{34}^2 + dx_{056789}^2$$

$$B_{12}^{(2)} = -K^{-1} \theta \sigma, \quad e^{2\Phi} = HK^{-1}, \quad K = H^2 + \alpha^2 \sigma^2. \quad (5)$$

In terms of the radii in this frame, $\sigma = R_5 R_4 / 2\pi \alpha'$. Similar
metrics of exotic states have been written down
(e.g., [12] considered $6_1$), but they do not appear to have
been discussed in the context of $U$ folds. As can be seen from
(5), as we go around $r = 0$ from $\theta = 0$ to $2\pi$, the size of
the 3–4 torus does not come back to itself:

$$\theta = 0: G_{33} = G_{44} = H^{-1},$$

$$\theta = 2\pi: G_{33} = G_{44} = H/[H^2 + (2\pi \sigma)^2]. \quad (6)$$

This can be understood as a $T$ fold. If we package the 3–4
part of the metric and $B$ field in a $4 \times 4$ matrix [13]

$$M = \begin{pmatrix}
G^{-1} & G^{-1} B \\
BG^{-1} & G - BG^{-1} B
\end{pmatrix} \quad (12)$$

then the $SO(2, 2, \mathbb{R})$ $T$-duality transformation matrix $\Omega$

$$\eta = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix};$$

acts on $M$ as $M \to M' = \Omega'M\Omega$. It is easy to see that the
matrix

$$\Omega = \begin{pmatrix}
1 & 0 \\
2\pi \sigma & 1
\end{pmatrix}$$

relates the $\theta = 0, 2\pi$ configurations in (6). Namely, $S^3_2$ is a
nongeometric $T$ fold with the monodromy $\Omega$.

Although the mass of such a codimension-two object is
not strictly well-defined, we can still compute it by
the following $ad$ hoc procedure. The Einstein metric in 3D is
given by $ds_3^2 = -dt^2 + H dx_{12}^2$. If $\gamma_{ij}$ is the spatial metric
for constant $t$ slices and $G_{ij}$ is the Einstein tensor, we can compute
$\sqrt{\gamma} G_{00}^0 = \frac{1}{2} d\psi^2 log H$. So, the energy is

$$M = -\frac{1}{8\pi G_3} \int d^2 x \sqrt{\gamma} G_{00}^0 = -\frac{1}{16\pi G_3} \int dS \cdot \nabla \log H.$$

If we use (4) and assume that $H(r = \infty) = 1$, then

$$M = \frac{1}{16\pi G_3} \left[ 2\pi \sigma \right] \Big|_{r = \infty} = (R_5 R_4)^2 R_3 \cdots R_9 \frac{g_{s}^2}{g_{s}^2},$$

as expected of a $S^3_2(56789, 34)$. Here, we used $16\pi G_3 = g_{s}^2 / R_3 \cdots R_9$. Although the $S^3_2$ changes the asymptotics,
setting $H(r = \infty) = 1$ effectively puts it in an asympto-
ically flat space and allows us to compute its mass.

Similarly, one can derive the metric for other exotic
states appearing in Table I. The metric provides an
approximate description, just as for ordinary branes, unless
the tension of the exotic branes is proportional to $g_s^{-3}$ or
$g_s^{-4}$ and the metric description breaks down.

Supertube effect and exotic states.—The above exotic $S^3_2$
brane appears in $d = 3$ dimensions, but exotic states are
relevant to physics in $d \geq 4$ dimensions as well. By dual-
izing the basic supertube effect (1), we can derive the
following spontaneous polarization:

$$D_{4(6789)} + D_{4(4589)} \to S^3_2(4567, \psi), \quad (8)$$

The configuration on the left can be thought of as a
configuration in 4D, which passes up into an extended
configuration of an exotic dipole charge along a curve in
$\mathbb{R}^3_{123}$. Such exotic dipole charges do not change the
asymptotics of spacetime. Note that the original configuration
of $D_4$-branes is part of the standard $D0$–$D4$ configuration
used for the black hole microstate counting in four
dimensions [14]. So, to understand the physics of such black
holes, it is unavoidable to consider exotic charges.

The supergravity solution for the configuration (8) can be
obtained by dualizing the solution for the supertube [15] and
is given by

$$ds^2 = -f_1^{-1/2} f_2^{-1/2} (dt - A)^2 + f_1^{1/2} f_2^{1/2} dx_{12}^2$$

$$+ f_1^{1/2} f_2^{1/2} dx_{45}^2 + f_1^{-1/2} f_2^{1/2} dx_{67}^2$$

$$+ f_1^{1/2} f_2^{1/2} h^{-1} dx_{89}^2,$$

$$e^{2\Phi} = f_1^{1/2} f_2^{1/2} h^{-1}, \quad B_{89}^{(2)} = \gamma h^{-1}, \quad C^{(3)} = -\gamma \rho + \sigma, \quad (14)$$

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where $h = f_1 f_2 + \gamma^2$ and $\rho$, $\sigma$ are 3-forms given by

$$\rho = (f_2^{-1} \dot{d}t - dt) \wedge dx^4 \wedge dx^5 + (f_1^{-1} \dot{d}t - dt) \wedge dx^6 \wedge dx^7$$

$$\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7$$

with $\dot{d}t = dt - A$. The $\psi$ curve in (8) is an arbitrary closed curve in $\mathbb{R}^{1,3,4,6,7}$, and $f_1, f_2, A$ are harmonic functions sourced along the curve [15]; see, e.g., [16] for their explicit expressions. The 1-form $\beta_1$, and scalar $\gamma$ are related to $f_1$, $A$ by $d\beta_1 = * d f_1$, $d \gamma = * d A$. In particular, for a circular curve, they can be explicitly written down [15,16], including $\gamma$, $\beta_1$. As one goes around the curve, $\gamma$ undergoes a shift $\gamma \rightarrow \gamma + q$ with $q$ a constant proportional to the $5_2^2$ dipole charge. This gives rise to a monodromic structure in the metric and the $B$ field, similar to the one in (5). Because the exotic $5_2^2$ charge in (8) is merely a dipole charge, the 4D system studied in the context of 4D black hole microstate counting [14] involves more stacks than (8): $D_0$, $4D(6789)$, $4D(4589)$, $4D(4567)$. If we bring these four stacks together, each pair is expected to undergo the supertube effect:

$$D4(6789) \rightarrow NS5(6789\psi) \rightarrow 5_2^2(6789,45\psi)$$

$$D0 \rightarrow 4D(4589) \rightarrow NS5(4589\psi) \rightarrow 5_2^2(4589,67\psi). \quad (10)$$

$$D4(4567) \rightarrow NS5(4567\psi) \rightarrow 5_2^2(4567,89\psi)$$

However, the charges on the right of (10) include combinations of charges which can puff up again. A priori, there is no reason to exclude such further puff-ups which will produce all kinds of exotic charges appearing in Table I, assuming that such puff-ups do not break supersymmetry. As a different example, take the 3-charge $M_2$ system [17] which is a well studied configuration in the context of 5D black hole microstate counting [18]. In this case, even if we restrict to codimension-two puff ups, the following sequence seems logically possible:

$$M2(56) \rightarrow M5(\psi 789A) \rightarrow 5_2^3(\phi 789A, \psi 56)$$

$$M2(78) \rightarrow M5(\psi 569A) \rightarrow 5_3^3(\phi 569A, \psi 78) \rightarrow \ldots ,$$

$$M2(9A) \rightarrow M5(\psi 5678) \rightarrow 5_3^3(\phi 5678, \psi 9A)$$

where “$A$” denotes the $x^{10}$ direction. Namely, the system can polarize into exotic branes extended along a two-dimensional surface parametrized by $\psi$, $\phi$ in $\mathbb{R}^{1,2,3,4,5}$. In the two-charge system [19], entropy comes from the Higgs branch of the worldvolume theory associated with the intersection of two stacks of branes. In gravity, the same entropy is explained by the degrees of freedom coming from the fluctuations of the one-dimensional geometric object which is the result of puffing up the intersection [20]. In the three-charge system, the triple intersection of three stacks of branes leads to a more complicated Higgs branch and larger entropy. It is conceivable that the fluctuations of the above two-dimensional exotic object that naturally appears, with its larger number of degrees of freedom, account for the entropy of the 3-charge system. It would hence be very interesting to construct nongeomet-ric solutions involving such exotic charges to see if they can really reproduce the expected entropy. The fact that the three-charge supergravity microstates constructed thus far (see, e.g., [21,22]) do not seem enough to account for the entropy of the three-charge black hole [22] may be related to the nongeometric nature of exotic branes that have been overlooked.

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[8] References [2] suggest that there are 8 more states (7 in type II and 8 in $M$ theory) in addition to the ones in Table I. It would be interesting to study how they can fit within the arguments of the current note.