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de Boer, J.; Shigemori, M.

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Exotic Branes and Nongeometric Backgrounds

Jan de Boer and Masaki Shigemori

Institute for Theoretical Physics, University of Amsterdam Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
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When string or M theory is compactified to lower dimensions, the U -duality symmetry predicts so-called exotic branes whose higher-dimensional origin cannot be explained by the standard string or M -theory branes. We argue that exotic branes can be understood in higher dimensions as nongeometric backgrounds or U folds, and that they are important for the physics of systems which originally contain no exotic charges, since the supertube effect generically produces such exotic charges. We discuss the implications of exotic backgrounds for black hole microstate (non-)geometries.

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Introduction.—String theory includes various extended objects as collective excitations, such as D -branes. The U -duality symmetry [1] which maps these objects into one another has played a pivotal role in the development of string theory and provided crucial insights into its non-perturbative behavior. When string or M theory is compactified to lower dimensions, the U -duality group gets enhanced, relating objects that were not related in higher dimensions. For example, M theory compactified on T^k has a discrete U -duality group known as $E_{k(k)}(\mathbb{Z})$ [1].

In the lower ($d = 11 - k$) dimensional theory, if we start from a codimension-two object obtained by partially wrapping a known 11D object and act by U duality on it, we start to produce objects whose higher-dimensional origin is unknown; they are called *exotic branes* [2]. In type II language, some of them have a tension proportional to g_s^{-3} or g_s^{-4} . For example, in type II string compactified on T^2 , consider an NS5-brane extending along six of the eight remaining noncompact directions, not wrapping the internal T^2 . If we perform a T duality along both T^2 directions, we obtain an exotic brane called 5_2^2 . We will see later that this is a nongeometric background known as a T fold [3]; as we go around the exotic brane, the internal T^2 is nontrivially fibered and does not come back to itself, but rather to a T -dual version.

One may think that such codimension-two objects are problematic due to logarithmic divergences [4], and that we do not need them if we are concerned with the physics of nonexotic states. However, this is not true because of the supertube effect [5]—the spontaneous polarization phenomenon that occurs when we bring a particular combination of charges together. A basic example is

$$D0 + F1(1) \rightarrow D2(1\psi) \quad (1)$$

in which $D0$ -branes and fundamental strings along x^1 polarize into a $D2$ -brane extending along x^1 and a closed curve in the transverse directions parametrized by ψ . Note that the $D2$ charge did not exist in the original configuration. Since the $D2$ is along a closed curve, there is no net $D2$ charge, but only a $D2$ dipole charge. The microscopic

entropy of the $D0$ - $F1$ system can be recovered by counting the possible ψ curves that the system can polarize into [6].

Even if we start with a configuration of nonexotic charges, the supertube effect can produce exotic charges. Because the exotic charges thus produced are dipole charges, there is no net exotic charge at infinity and the problem of log divergences does not arise. This implies that exotic states are relevant even for the physics of systems which do not originally contain exotic charges.

This is especially interesting in the context of black hole physics where one typically considers a configuration of multiple (nonexotic) charges. We will argue later that the supertube effect and exotic charges are relevant for the understanding of the physics of such black holes.

Exotic states and their higher-dimensional origin.—If we compactify M theory on T^8 or type IIA/B string theory on T^7 down to 3D, we obtain $\mathcal{N} = 16$ supergravity [7] with 128 scalars (note that gauge fields can be dualized into scalars in 3D). This theory has $E_{8(8)}$ as the U -duality group which is broken to the discrete subgroup $E_{8(8)}(\mathbb{Z})$ in string theory [1]. This $E_{8(8)}(\mathbb{Z})$ is generated by S and T dualities along the internal torus. For example, in type IIB, a $D7$ -brane wrapped on the T^7 yields a point particle in three dimensions. Acting with S and T dualities, we can obtain all other states in the “particle multiplet” of the U -duality group as explained in [2].

In Table I, we list the states in the particle multiplet, including the exotic ones. The notation for nonexotic states is standard; e.g., P denotes a gravitational wave and KKM denotes a Kaluza-Klein monopole. For type II exotic states, we follow [2] and denote them by how their mass depends on the T^7 radii. The mass M of b_n^c depends linearly on b radii and quadratically on c radii. For $b_n^{(d,c)}$, M also depends cubically on d radii. Moreover, M is proportional to g_s^{-n} . For example, the mass of 5_2^2 depends on the radii R_i , $i = 3, \dots, 9$ of T^7 as $M = R_3 \cdots R_7 (R_8 R_9)^2 / g_s^2 l_s^9$. We often display how the state “wraps” the internal T^7 as $5_2^2(34567, 89)$. In this notation, the KK monopole is denoted by 5_1^1 . In M theory, we use a similar notation except that we drop the subscript n . Using the transformation rules

TABLE I. The point particle states and their multiplicities (boldface numbers) in string or M theory compactified to 3D.

Type IIA	$P(7)$, $F1(7)$, $D0(1)$, $D2(\mathbf{21})$, $D4(\mathbf{35})$, $D6(7)$, $NS5(\mathbf{21})$, $KKM(\mathbf{42})$, $5_2^2(\mathbf{21})$, $0_3^7(\mathbf{1})$, $2_3^5(\mathbf{21})$, $4_3^2(\mathbf{35})$, $6_3^1(7)$, $0_4^{(1,6)}(7)$, $1_4^6(7)$
Type IIB	$P(7)$, $F1(7)$, $D1(7)$, $D3(\mathbf{35})$, $D5(\mathbf{21})$, $D7(1)$, $NS5(\mathbf{21})$, $KKM(\mathbf{42})$, $5_2^2(\mathbf{21})$, $1_3^6(7)$, $3_3^4(\mathbf{35})$, $5_3^2(\mathbf{21})$, $7_3(1)$, $0_4^{(1,6)}(7)$, $1_4^6(7)$
M theory	$P(8)$, $M2(\mathbf{28})$, $M5(\mathbf{56})$, $KKM(\mathbf{56})$, $5^3(\mathbf{56})$, $2^6(\mathbf{28})$, $0^{(1,7)}(\mathbf{8})$

for the radii R_i and g_s under S and T dualities, we can read off how those states transform into one another [2].

In the 3D theory, we would have 128 gauge fields if we could dualize all the scalars into gauge fields [2]. However, as we can see from Table I, there are as many as 240 charged particles [8], and this discrepancy (240 versus 128) in the 3D theory is not understood [2]. For $d \geq 4$, this issue does not arise because we obtain just as many charged particles as gauge fields [2]. Here, we argue that the higher-dimensional origin of exotic states consists of nongeometric backgrounds or U folds [3,9].

The argument is simple. For example, consider a $D7$ -brane wrapping T^7 , which is (magnetically) coupled to the RR 0-form $C^{(0)}$. From the 3D viewpoint, the $D7$ -brane is a point particle and, as we go around it, the 3D scalar $\phi = C^{(0)}$ shifts as $\phi \rightarrow \phi + 1$. Namely, in 3D, the “charge” of the point particle is nothing but the monodromy of the scalar ϕ around it. This symmetry of shifting ϕ by one gets combined with other dualities such as S and T dualities to form the U -duality group $G(\mathbb{Z}) = E_{8(8)}(\mathbb{Z})$, and the scalar ϕ gets combined with other scalars into a matrix M parametrizing the moduli space $\mathcal{M} = SO(16) \backslash E_{8(8)}(\mathbb{R}) / E_{8(8)}(\mathbb{Z})$. U duality means that we can more generally consider a 3D particle around which M jumps by a general U -duality transformation. Thus, the charge of a 3D particle is defined by the U -duality monodromy around it. This can be regarded as a non-Abelian generalization of the usual notion of $U(1)$ charges for which the monodromy is an additive shift. Clearly, the number of different charges thus generalized is not in general equal to that of gauge fields, which resolves the above puzzle.

If we lift such a monodromy to 10D/11D, we obtain a configuration in which the internal space is nontrivially fibered as we go around the particle and glued together by a U -duality transformation. So, exotic states correspond in 10D/11D to nongeometric backgrounds, or “ U folds” [3]. To our knowledge, the interpretation of exotic states as U folds has not appeared in the literature. Note that this construction differs from the more familiar U folds in the context of string compactifications [9], where U duality is nontrivially fibered over a noncontractible circle in the internal manifold, not over a contractible circle in the noncompact directions.

Let us discuss how to classify “charges” defined by the monodromies around them. First, assume the existence of a charge with monodromy q . Namely, as we go around the particle in three dimensions, the moduli matrix M undergoes the monodromy transformation $M \rightarrow Mq$, $q \in G(\mathbb{Z})$.

If we go to another U duality frame by a U -duality transformation $U \in G(\mathbb{Z})$, then this becomes $\tilde{M} \rightarrow \tilde{M} \tilde{q}$ with $\tilde{M} = MU$, $\tilde{q} = U^{-1}qU$. So, in the dual frame, there exists a charge with monodromy \tilde{q} . Now, let us change the moduli \tilde{M} adiabatically to the original value M . If the charge is BPS, an object with monodromy \tilde{q} continues to exist, implying the existence of the charge \tilde{q} even for the original value of the moduli M (assuming that there is no line of marginal stability). So, starting from a charge q , we can generate other possible charges by conjugation $\tilde{q} = U^{-1}qU$. Note that this does not mean that we can generate all charges that exist in the theory by conjugation; there can be many conjugacy classes in the group $G(\mathbb{Z})$ and we cannot generate charges in different conjugacy classes. Also, there can be non-BPS charges for which the above argument (of changing moduli adiabatically) does not apply.

As a simple example, consider a $D7$ -brane. Around it, there is an $SL(2, \mathbb{Z})$ monodromy given by

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Let us conjugate this with a general $SL(2, \mathbb{Z})$ matrix

$$U = \begin{pmatrix} s & r \\ q & p \end{pmatrix}.$$

The conjugated charge is

$$\tilde{T} = U^{-1}TU = \begin{pmatrix} 1 + pq & p^2 \\ -q^2 & 1 - pq \end{pmatrix},$$

which is the monodromy of the standard (p, q) 7-brane. Note that, although U has 3 independent parameters, the resulting charge \tilde{T} has only 2 parameters. In this sense, there exist only two different charges.

So, the set of all possible charges we can obtain from a given one q by U duality is its conjugation orbit. This orbit is a subset of the discrete non-Abelian “lattice” $G(\mathbb{Z})$, and it makes no sense to ask how many different charges there are in it. However, to get a qualitative idea of the size of the orbit, we can replace $G(\mathbb{Z})$ by the continuous group $G(\mathbb{R})$ and count the dimension of the (now continuous) orbit. Using standard results on the dimensions of conjugacy classes in noncompact groups and their relation to $sl(2)$ -embeddings, one can show that, e.g., for $G(\mathbb{Z}) = E_{8(8)}(\mathbb{Z})$, the dimension of the orbit generated by a 1/2-BPS object such as the $D7$ -brane is 58 [10]. The 240 states in Table I represent 240 particular points in this orbit, which can be obtained by U dualities preserving the rectangularity of the internal torus [2].

Supergravity description of exotic states.—To demonstrate the above idea, let us present the supergravity metric

for 5_2^2 as an example. This can be obtained by T dualizing the KK monopole metric transverse to its worldvolume. The KK monopole ($5_2^1(56789, 4)$) metric is

$$ds^2 = dx_{056789}^2 + H dx_{123}^2 + H^{-1}(dx^4 + \omega)^2, \quad e^{2\Phi} = 1, \\ d\omega = *_3 dH, \quad H = 1 + \sum_p H_p, \quad H_p = R_4/(2|\vec{x} - \vec{x}_p|), \quad (2)$$

where \vec{x}_p are the positions of the centers in \mathbb{R}_{123}^3 . Now compactify x^3 , which is the same as arraying centers at intervals of $2\pi\tilde{R}_3$ along x^3 . So,

$$H = 1 + \sum_{n \in \mathbb{Z}} R_4/[2(r^2 + (x^3 - 2\pi\tilde{R}_3 n)^2)^{1/2}] \\ \approx 1 + \sigma \log[(\Lambda + \sqrt{r^2 + \Lambda^2})/r], \quad (3)$$

where $\sigma = R_4/2\pi\tilde{R}_3$ and we took a cylindrical coordinate system $ds_{123}^2 = dr^2 + r^2 d\theta^2 + (dx^3)^2$. We approximated the sum by an integral and introduced a cutoff Λ to make it convergent (see [11, 12]). H in (3) diverges as we send $\Lambda \rightarrow \infty$, but this can be formally shifted away by introducing a “renormalization scale” μ and writing

$$H(r) = h + \sigma \log(\mu/r) \Rightarrow \omega = -\sigma\theta dx^3, \quad (4)$$

where h is a “bare” quantity which diverges in the $\Lambda \rightarrow \infty$ limit. The log divergence of H implies that such an infinitely long codimension-two object is ill-defined by itself. In physically sensible configurations, this must be regularized either by taking a suitable superposition of codimension-two objects [4] or, as we will do later, by considering instead a configuration which is of higher codimension at long distance.

Now let us do a T duality along x^3 . By the standard Buscher rule, we obtain the metric for $5_2^2(56789, 34)$:

$$ds^2 = H(dr^2 + r^2 d\theta^2) + HK^{-1} dx_{34}^2 + dx_{056789}^2, \quad (5) \\ B_{34}^{(2)} = -K^{-1}\theta\sigma, \quad e^{2\Phi} = HK^{-1}, \quad K \equiv H^2 + \sigma^2\theta^2.$$

In terms of the radii in this frame, $\sigma = R_3 R_4/2\pi\alpha'$. Similar metrics of exotic states have been written down (e.g., [12] considered 6_3^1), but they do not appear to have been discussed in the context of U folds. As can be seen from (5), as we go around $r = 0$ from $\theta = 0$ to 2π , the size of the 3–4 torus does not come back to itself:

$$\theta = 0: G_{33} = G_{44} = H^{-1}, \\ \theta = 2\pi: G_{33} = G_{44} = H/[H^2 + (2\pi\sigma)^2]. \quad (6)$$

This can be understood as a T fold. If we package the 3–4 part of the metric and B field in a 4×4 matrix [13]

$$M = \begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G - BG^{-1}B \end{pmatrix} \quad (12)$$

then the $SO(2, 2, \mathbb{R})$ T -duality transformation matrix Ω satisfying $\Omega^t \eta \Omega = \eta$,

$$\eta = \begin{pmatrix} 0 & \mathbf{1}_2 \\ \mathbf{1}_2 & 0 \end{pmatrix},$$

acts on M as $M \rightarrow M' = \Omega^t M \Omega$. It is easy to see that the matrix

$$\Omega = \begin{pmatrix} \mathbf{1}_2 & 0 \\ 2\pi\sigma & \mathbf{1}_2 \end{pmatrix}$$

relates the $\theta = 0, 2\pi$ configurations in (6). Namely, 5_2^2 is a nongeometric T fold with the monodromy Ω .

Although the mass of such a codimension-two object is not strictly well-defined, we can still compute it by the following *ad hoc* procedure. The Einstein metric in 3D is given by $ds_3^2 = -dt^2 + H dx_{12}^2$. If γ_{ij} is the spatial metric for constant t slices and $G_{\mu\nu}$ is the Einstein tensor, we can compute $\sqrt{\gamma} G_0^0 = \frac{1}{2} \partial_i^2 \log H$. So, the energy is

$$M = -\frac{1}{8\pi G_3} \int d^2 x \sqrt{\gamma} G_0^0 = -\frac{1}{16\pi G_3} \int dS \cdot \nabla \log H.$$

If we use (4) and assume that $H(r = \infty) = 1$, then

$$M = \frac{1}{16\pi G_3} \left[\frac{2\pi\sigma}{H(r)} \right] \Big|_{r \rightarrow \infty} = \frac{(R_3 R_4)^2 R_5 \cdots R_9}{g_s^2 l_s^9},$$

as expected of a $5_2^2(56789, 34)$. Here, we used $16\pi G_3 = g_s^2 l_s^8 / R_3 \cdots R_9$. Although the 5_2^2 changes the asymptotics, setting $H(r = \infty) = 1$ effectively puts it in an asymptotically flat space and allows us to compute its mass.

Similarly, one can derive the metric for other exotic states appearing in Table I. The metric provides an approximate description, just as for ordinary branes, unless the tension of the exotic branes is proportional to g_s^{-3} or g_s^{-4} and the metric description breaks down.

Supertube effect and exotic states.—The above exotic 5_2^2 brane appears in $d = 3$ dimensions, but exotic states are relevant to physics in $d \geq 4$ dimensions as well. By dualizing the basic supertube effect (1), we can derive the following spontaneous polarization:

$$D4(6789) + D4(4589) \rightarrow 5_2^2(4567\psi, 89). \quad (8)$$

The configuration on the left can be thought of as a configuration in 4D, which puffs up into an extended configuration of an exotic dipole charge along a curve in \mathbb{R}_{123}^3 . Such exotic *dipole* charges do not change the asymptotics of spacetime. Note that the original configuration of $D4$ -branes is part of the standard $D0$ – $D4$ configuration used for the black hole microstate counting in four dimensions [14]. So, to understand the physics of such black holes, it is unavoidable to consider exotic charges.

The supergravity solution for the configuration (8) can be obtained by dualizing the solution for the supertube [15] and is given by

$$ds^2 = -f_1^{-(1/2)} f_2^{-(1/2)} (dt - A)^2 + f_1^{1/2} f_2^{1/2} dx_{123}^2 \\ + f_1^{1/2} f_2^{-(1/2)} dx_{45}^2 + f_1^{-1/2} f_2^{1/2} dx_{67}^2 \\ + f_1^{1/2} f_2^{1/2} h^{-1} dx_{89}^2, \\ e^{2\Phi} = f_1^{1/2} f_2^{1/2} h^{-1}, \quad B_{89}^{(2)} = \gamma h^{-1}, \quad C^{(3)} = -\gamma\rho + \sigma, \quad (14)$$

where $h = f_1 f_2 + \gamma^2$ and ρ, σ are 3-forms given by

$$\begin{aligned}\rho &= (f_2^{-1} \tilde{d}t - dt) \wedge dx^4 \wedge dx^5 + (f_1^{-1} \tilde{d}t - dt) \wedge dx^6 \wedge dx^7 \\ \sigma &= (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7\end{aligned}$$

with $\tilde{d}t = dt - A$. The ψ curve in (8) is an arbitrary closed curve in \mathbb{R}_{123}^3 , and $f_{i=1,2}, A$ are harmonic functions sourced along the curve [15]; see, e.g., [16] for their explicit expressions. The 1-form β_i and scalar γ are related to f_i, A by $d\beta_i = *_3 df_i, d\gamma = *_3 dA$. In particular, for a circular curve, they can be explicitly written down [15,16], including γ, β_i [10]. As one goes around the curve, γ undergoes a shift $\gamma \rightarrow \gamma + q$ with q a constant proportional to the 5_2^2 dipole charge. This gives rise to a monodromic structure in the metric and the B field, similar to the one in (5). Because the exotic 5_2^2 charge in (8) is merely a dipole charge, the 4D spacetime is still asymptotically flat.

The $D0$ – $D4$ system studied in the context of 4D black hole microstate counting [14] involves more stacks than (8): $D0, D4(6789), D4(4589), D4(4567)$. If we bring these four stacks together, each pair is expected to undergo the supertube effect:

$$\begin{array}{ll} D4(6789) & \text{NS5}(6789\psi) \quad 5_2^2(6789, 45\psi) \\ D0 \quad D4(4589) & \rightarrow \text{NS5}(4589\psi) \quad 5_2^2(4589, 67\psi). \quad (10) \\ D4(4567) & \text{NS5}(4567\psi) \quad 5_2^2(4567, 89\psi) \end{array}$$

However, the charges on the right of (10) include combinations of charges which can puff up again. *A priori*, there is no reason to exclude such further puff-ups which will produce all kinds of exotic charges appearing in Table I, assuming that such puff-ups do not break supersymmetry. As a different example, take the 3-charge $M2$ system [17] which is a well studied configuration in the context of 5D black hole microstate counting [18]. In this case, even if we restrict to codimension-two puff ups, the following sequence seems logically possible:

$$\begin{array}{lll} M2(56) & M5(\psi 789A) & 5^3(\phi 789A, \psi 56) \\ M2(78) & \rightarrow M5(\psi 569A) & \rightarrow 5^3(\phi 569A, \psi 78) \rightarrow \dots \\ M2(9A) & M5(\psi 5678) & 5^3(\phi 5678, \psi 9A) \end{array}$$

where “A” denotes the x^{10} direction. Namely, the system can polarize into exotic branes extended along a two-dimensional surface parametrized by ψ, ϕ in \mathbb{R}_{1234}^4 . In the two-charge system [19], entropy comes from the Higgs branch of the worldvolume theory associated with the intersection of two stacks of branes. In gravity, the same entropy is explained by the degrees of freedom coming from the fluctuations of the one-dimensional geometric object which is the result of puffing up the intersection [20]. In the three-charge system, the triple intersection of three stacks of branes leads to a more complicated Higgs branch and larger entropy. It is conceivable that the fluctuations of the above two-dimensional exotic object that naturally appears, with its larger number of degrees of freedom, account for the entropy of the 3-charge system. It would hence be very interesting to construct nongeomet-

ric solutions involving such exotic charges to see if they can really reproduce the expected entropy. The fact that the three-charge supergravity microstates constructed thus far (see, e.g., [21,22]) do not seem enough to account for the entropy of the three-charge black hole [22] may be related to the nongeometric nature of exotic branes that have been overlooked.

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