The physics of line-driven winds of hot massive stars

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Predictions of the effect of clumping on the wind properties of O-type stars

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Abstract

Both empirical evidence and theoretical findings indicate that the stellar winds of massive early-type stars are inhomogeneous, i.e. porous and clumpy. For relatively dense winds, empirically derived mass-loss rates might be reconciled with predictions if these empirical rates are corrected for the presence of clumping. The predictions, however, do not account for structure in the wind. To allow for a consistent comparison we investigate and quantify the effect of clumpiness and porosity of the outflow on the predicted wind energy and the maximal effect on the mass-loss rate of O-type stars.

Combining non-LTE model atmospheres and a Monte Carlo method to compute the transfer of momentum from the photons to the gas, the effect of clumping and porosity on the energy transferred from the radiation field to the wind is computed in outflows in which the clumping and porosity stratification is parameterized by heuristic prescriptions.

The impact of structure in the outflow on the wind energy is complex and is a function of stellar temperature, the density of gas in the clumps and the physical scale of the clumps. If the medium is already clumped in the photosphere the emergent radiation field will be softer, slightly increasing the wind energy of relatively cool O stars (30 000 K) but slightly decreasing it for relatively hot O stars (40 000 K). More important is that as a result of recombination of the gas in a clumped wind the line force increases. However, due to porosity the line force decreases, simply because
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photons may travel in between the clumps, avoiding interactions with the gas. If the changes in the wind energy only affect the mass-loss rate and not the terminal velocity of the flow, we find that the combined effect of clumpiness and porosity is a small reduction in the mass-loss rate if the clumps are smaller than 1/100th the local density scale height \( H_\rho \). For this case empirical mass-loss determinations based on \( H\alpha \) fitting and theory match for stars with dense winds \( \dot{M} \gtrsim 10^{-7}M_\odot\text{yr}^{-1} \) if the over-density of gas in the clumps, relative to the case of a smooth wind, is modest. For clumps larger than 1/100th \( H_\rho \), the predicted mass-loss rates show about the same dependence on clumpiness as do empirical rates. We show that this implies that empirical and predicted mass-loss rates can no longer be matched. Very large overdensities of gas in clumps of such large size may cause the predicted \( \dot{M} \) to decrease by a factor 10 to 100. This type of structure is likely not the cause for the “weak wind problem” in early-type stars, unless a mechanism can be identified that causes extreme structure to develop in winds that have \( \dot{M} \lesssim 10^{-7}M_\odot\text{yr}^{-1} \) (weak winds) that is not active in denser winds.

2.1 Introduction

In the last decade studies of the mass-loss rate of early type massive stars have focussed, for an important part, on the role of structure or inhomogeneities in the stellar outflow (for a review, see e.g. Puls et al. 2008). One reason is that recombination-based processes are very sensitive to the presence of structure in the wind and that therefore key mass-loss diagnostics, such as \( H\alpha \) and \( \text{He}\ h\alpha \lambda4686 \) line radiation and infrared and radio continuum radiation, being sensitive to the square of the density, are affected. As a result, analyses based on these diagnostics assuming a smooth outflow will lead to an overestimate of the mass-loss rate if in reality the wind has a clumpy structure. One can show that if the typical clumping factor, expressing the ratio of the actual density in clumps relative to the mean density, is given by \( C_C > 1 \), the empirical mass-loss rate \( \dot{M} \) needs to be scaled down by a factor \( 1/\sqrt{C_C} \) (see section 2.2.2 for more details).

Attempts to empirically quantify the clumping factor in O stars, Luminous Blue Variables and Wolf-Rayet stars yield a rather broad spectrum of \( C_C \) values, from a factor of a few up to 100 (see e.g. Figer et al. 2002; Crowther et al. 2002; Hillier et al. 2003; Bouret et al. 2003; Repolust et al. 2004; Markova et al. 2004; Bouret et al. 2005; Fullerton et al. 2006). This implies that empirical mass-loss rates may have to be scaled down by factors 2–10. To give one explicit example, intended to serve as a frame of reference, empirical mass loss rates of O-type stars brighter than 175 000 \( L_\odot \) based on the analysis of \( H\alpha \) and assuming smooth flows are brought into agreement with predictions (Vink et al. 2001) if clumping is modest \((C_C \sim 3 – 4; \text{Mokiem et al. 2007; de Koter et al. 2008})\). Notice that recent stellar evolution calculations adopt
these predictions, therefore, if these predictions are correct, they implicitly account for a modest amount of clumping.

A second reason for the attention to this topic is that empirical studies of the radial stratification of the clumping factor throughout the wind sketch a picture that is discrepant from what is expected. Hydrodynamical modeling of the time-dependent structure of line-driven winds (for a review, see e.g. Owocki 1994; Feldmeier 1999) reveals the wind to be quite stable in the inner parts, but predicts that extensive structure – both in terms of density and velocity – develops further out (at $r \gtrsim 1.3$ times the stellar radius $R_*$ or $v \gtrsim 0.4$ times the terminal velocity) and can survive out to very large distances ($r \gtrsim 1000 R_*$; see Runacres & Owocki 2005). Empirical studies, however, show that O stars develop clumping already close to the stellar surface (Markova et al. 2004; Repolust et al. 2004; Puls et al. 2006). Apparently, the line-driven instability – first proposed by Lucy & Solomon (1970) – is not the only mechanism at work and other effects, acting in different parts of the wind, may also cause structure.

In this paper we address the following question: does the presence of structure impact the rate at which massive stars lose mass? If, on the one hand, structure would cause the mass loss to increase, only modest clumping would be required to bring the above mentioned Hα based and predicted mass-loss rates into agreement. If, on the other hand, structure would cause a strong decrease in $\dot{M}$, strong clumping may be required to bring this agreement or no agreement may be reached at all.

The hydrodynamic models of line-driven winds (see again Owocki 1994) predict that the time-averaged terminal flow velocity and time-averaged mass-loss rate agree well with those following from a stationary approach, i.e. there is no significant impact of clumping on $\dot{M}$ or the terminal flow velocity $v_\infty$. These predictions, however, are based on one-dimensional calculations (but see Dessart & Owocki 2003, 2005, for first results on two-dimensional flows), i.e. neglecting a possible porous structure of the wind. Also, the impact of the density and velocity perturbations on the excitation and ionization state of the gas, therefore on the local line force, is not treated.

In studying aspects of this problem we will take a heuristic approach. We feel this is justified given the discrepancies between the empirical clumping stratification and that predicted by the line-driven instability and the complex nature of the problem. We focus on effects of clumping and porosity on changes in the state of the gas and the continuum radiation field, and on the impact of these changes on the line force. These effects are computed self-consistently. The clumping and porosity is, however, described by simple empirical laws. In section 2.2 the method is described in detail. The results are presented in section 2.3 and discussed in section 2.4. We end with conclusions.
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2.2 Method

2.2.1 NLTE hydrodynamic wind models

To determine the momentum transfer from the radiation field to the wind in O-stars, we employ the model atmospheres of de Koter et al. (1993) in combination with a Monte Carlo code for determining the line force as described by de Koter et al. (1997). The Monte-Carlo approach, though extensively modified, is essentially based on that developed by Abbott & Lucy (1985). This methodology of determining the properties of stellar winds has been used extensively to predict the mass-loss behavior of massive early-type stars, including O and B stars (Vink et al. 1999, 2000, 2001), Luminous Blue Variables (Vink & de Koter 2002) (LBVs) and selected Wolf-Rayet stars (de Koter et al. 1997; Vink & de Koter 2005). For details on the method we refer the reader to the above references. Here, we only give a very brief overview of essential aspects.

The atmospheric model extends from the base of the photosphere (at a Rosseland optical depth of about 25) to 20 stellar radii, and assumes that the outflow is homogeneous, spherically symmetric and stationary. To calculate the radiative transfer in spectral lines, the Sobolev method is used. The occupation numbers of (excited) levels and the ionization conditions are solved assuming statistical equilibrium. Model atoms for hydrogen, helium, carbon, nitrogen, oxygen and silicon are explicitly treated. Other atoms are accounted for using a modified nebular approximation.

It is important to realize that in our method the equation of motion for gas streaming out from the star is not solved explicitly (but see Müller & Vink 2008 and Muijres et al. in preparation). Instead, we adopt a $\beta$-type velocity law. For this velocity structure we compute, by means of Monte Carlo, the total radial momentum that is transferred from photons to the gas on their way from the photosphere to the interstellar medium. The cumulative effect of this process also yields the total rate at which the wind extracts energy from the radiation field. By requiring that this energy is used to accelerate the wind and to let the gas escape from the stellar potential well (so assuming no non-radiative forces are at work) we can iteratively derive a mass-loss rate, given by:

$$\Delta L = \frac{1}{2} M \left( v_{\infty}^2 + v_{\text{esc}}^2 \right),$$  \hspace{1cm} (2.1)

where $\Delta L$ is the energy extracted from the radiation field, $v_{\infty}$ is the terminal wind velocity and $v_{\text{esc}}$ the escape velocity from the stellar surface.

The advantages of this method are that mass-loss rates can be derived with a modest computational effort, therefore relatively large fractions of parameter space can be explored. From a physical point the strong point of the method is that effects of multiple photon scattering (that are already important for O stars) are self-consistently accounted for and that changes in the line force due to excitation/ionization processes
are included. What is actually predicted in our method is the gain in total kinetic energy of the outflow due to transfer of momentum in the radiation field to the gas. This technique requires a pre-specified velocity law $v(r)$. Therefore, we do not predict the velocity stratification (nor for that matter the terminal flow velocity) but assume an empirically motivated $v(r)$. This allows to extract the predicted mass-loss rate. In this study we investigate the effects of clumping and porosity on the transferred energy $\Delta L$, therefore on the wind energy. As our velocity stratification is pre-specified, we cannot investigate the effects of structure on $v_\infty$. Therefore, if one contributes the full effect of clumping and porosity to a change in the mass-loss rate, one obtains the maximal effect of such processes on $\dot{M}$.

2.2.2 The implementation of clumping

A self-consistent treatment of clumping in a stellar outflow would in any case require a hydrodynamical simulation of the line-driven instability (see e.g. Owocki & Puls 1999, and references therein) subject to a non-local thermodynamic (NLTE) treatment of the gas. From a computation point this is extremely challenging. It is, moreover, currently unclear whether or not the line-driven instability is the only process causing inhomogeneities in the outflow (see Sect. 2.2.2). For these reasons we argue that a more heuristic approach to this problem is justified.

In our model we prescribe the radial behavior of clumping assuming: i) all the gas is concentrated in clumps, i.e. the space in between the clumps is void; ii) each clump is homogeneous, iii) clumps are distributed randomly on small spatial scales and follow a prescribed radial behavior on large spacial scales, and iv) the velocity law (of the clumpy medium) is a smooth function of radius.

The radial behavior of the clumping is prescribed in terms of the clumping factor, specifying the over-density in the clump relative to a smooth medium, and the porosity length, essentially specifying a physical scale of the clumps. We will first introduce these two concepts in more detail.

Clumping factor

Following e.g. Owocki & Cohen (2006), we introduce the clumping factor as

$$C_c(r) = \langle \rho(r)^2 \rangle / \langle \rho(r) \rangle^2,$$

(2.2)

where the angle brackets denote volume averaging. Empirical arguments have motivated this definition. The strength of both the free-free continuum and of spectral lines formed through the process of recombination depend on the square of the density, while other processes, for instance that of electron scattering, show a linear dependence (see e.g. Hillier 1991).
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![Figure 2.1: Schematic representation of the different clumping stratifications investigated in this study. Each stratification has been given an index number (at the right side of the figure) which is used in Figure 2.4. They represent: (1) constant clumping; (2) clumping starting at the sonic point; (3) clumping starting at 0.4v_{\infty}; (4) clumping up to the sonic point, and (5) clumping up to 0.4v_{\infty}.](image)

In case of an interclump medium that is void (see above) the density in the clump is given by

$$\rho_c(r) = C_c(r) \langle \rho(r) \rangle.$$  \hfill (2.3)

The simplest possible assumption on the behavior of clumping is that it is constant throughout the photosphere and wind and equal to $C_c > 1$. A clumping factor $C_c = 1$ implies a smooth wind. To investigate a radial dependence in the clumping we introduce additional clumping prescriptions. A schematic representation of all clumping stratifications adopted in this study is given in Fig. 2.1. The top drawing (labeled 1) depicts the case of a constant clumping factor.

Clumping in the outer wind. In this prescription we assume that the onset of clumping occurs at some prescribed radius $r_p$, i.e.

$$C_c(r) = \begin{cases} 
1.0 & \text{for } r < r_p \\
C_c & \text{for } r \geq r_p.
\end{cases} \hfill (2.4)$$
The onset and development of stochastic structure in the acceleration zone of the outflow is a natural consequence of a line-driven wind. It is the result of a positive feedback in which a small increase in velocity of a fluid parcel exposes the parcel to a more intense (read: unattenuated) radiation from the star and causes it to be further accelerated (Owocki et al. 1988; Feldmeier 1995; Owocki & Puls 1996, 1999). Simulations of this self-excited wind instability show that the compression of gas in clumps typically starts at about 0.3–0.4 \( v_\infty \) and that it may extend to very large radii (Runacres & Owocki 2002). These simulations, however, do not account for (transonic) velocity curvature terms. In stars with relatively weak winds it has been shown that these terms may lead to gradient terms in the source function and modifications of the line acceleration (Puls et al. 1998b) causing a highly structured wind in the lower parts of the outflow (Owocki & Puls 1999). The theory of line driven winds dictates that the mass-loss rate is set by conditions at or below a critical point that is very roughly at 0.2\( v_\infty \).

Based on the above arguments for structure formation in line-driven winds we define two new clumping prescriptions, where we opt to initiate the clumping: 2) at the sonic velocity (about 15 km/s for the models of 30 kK and about 18 km/s for the models of 40 kK), and 3) at 0.4\( v_\infty \) (see Table 2.1 for values of \( v_\infty \)). The former clumping stratification explores a potential effect of clumping on the mass-loss rate because clumping sets in before the mass-loss rate is formally fixed. The latter prescription focusses on the effect of clumping on \( v_\infty \) (see again Fig. 2.1). The density in the clumps produced by the line-driven instability relative to the ambient medium can reach one to two orders of magnitude (Owocki & Puls 1999). We will adopt clumping factors \( C_c \) of unity through 100, in steps of 0.5 dex.

**Clumping in the inner wind.** In this prescription we assume that clumping occurs in the inner wind and that the outer wind is smooth, i.e.

\[
C_c(r) = \begin{cases} 
C_c & \text{for } r < r_p \\
1.0 & \text{for } r \geq r_p,
\end{cases}
\]

(2.5)

where \( r_p \) again refers to the prescribed radius defining the boundary between the two regimes. This clumping prescription is “opposite” to the one described in the previous paragraph and which was motivated by theoretical expectations. The clumping prescription of Eq. 2.5 is motivated by observational arguments. Puls et al. (2006) present empirical evidence for a radial dependence of clumping in hot star winds. These authors use Hα, infrared and radio diagnostics to investigate the clumping behavior of the inner wind (inside about two stellar radii) relative to the clumping in the outer wind (beyond tens of stellar radii) of a large sample of giant and supergiant stars. They find a qualitative difference in the radial behavior of clumping in stars with strong winds compared to stars with weak winds. In the case of dense winds
the inner wind is more strongly clumped than the outer wind, whereas in the case of thin winds the inner and outer region have similar clumping properties. Puls et al. speculate that the cause for this difference between strong and weak winds may be connected to photospheric instabilities and/or pulsations as their strong wind stars are usually supergiants with low gravity. Interestingly, Cantiello et al. (2009) show that the stars in the sample of Puls et al. that show stronger clumping in the inner wind compared to the outer wind have sub-surface convective layers due to iron opacity peaks. They suggest that this convection may (indirectly) trigger stochastic velocities and clumping in the photosphere and lower part of the wind.

Based on these arguments we define again two clumping stratifications: 4) clumping up to the sonic point, and 5) clumping up to 0.4\(v_\infty\) (see also Fig. 2.1).

Porosity length

If the physical scale of a clump is given by \(\ell\) and the separation of clumps by \(L\) (see Fig. 2.2 for a visualization of the definition of \(L\)) a fraction \(f = \ell^3/L^3\) of the medium will be filled with gas, again assuming that the inter-clumped medium is void. This filling fraction \(f\) relates to the clumping factor as \(C_c = 1/f\). In Fig. 2.2 two possible configurations of regularly stacked cubic clumps having identical filling fractions \(f\) (and clumping factors \(C_c\)) are shown. The way in which these configurations differ is in the physical size and separation of the clumps. If in both configurations the individual clumps are optically thin for radiation the effects of clumping will be the same and will only occur through an adjustment of the excitation and ionization properties of the gas. If, however, the physical size of the large clumps is such that the individual clumps become optically thick, they will suffer from local self-shielding.
In that case the medium becomes porous, i.e. radiation will be able to travel more efficiently through inter-clump channels. The bigger the scale length $L$ the more efficient this mechanism will be (imagine for instance that all material in the outflow is concentrated in a single clump). Porosity in stellar winds, sometimes referred to as macro-clumping, is discussed by (Feldmeier et al. 2003; Owocki et al. 2004; Owocki & Cohen 2006; Oskinova et al. 2007).

The effective opacity of a clump is given by

$$
\kappa_{\text{eff}} = \kappa_c(\rho_c) \frac{1 - \exp(\tau_c)}{\tau_c},
$$

where $\kappa_{\text{eff}}$ is the effective mass absorption coefficient (e.g. in cm$^2$ g$^{-1}$) considering an ensemble of clumps. The mass absorption coefficient of material in the clumps, $\kappa_c$, is thus reduced because of the porous nature of the medium. The clump optical thickness $\tau_c = \kappa_c \rho_c \ell = \kappa_c C_c(\rho) \ell = \kappa_c \langle \rho \rangle \ell / f$.

We introduce a radial dependence of the scale of the clumps, i.e.

$$
L(r) = \begin{cases} \text{negligible} & \text{for } r < r_p \\ H_\rho / D & \text{for } r \geq r_p, \end{cases}
$$

where $H_\rho$ is the local density scale height and $D$ is a constant, which we choose to be 10, 100 and 1000. The fact that $L$ is negligible below $r_*$ implies that we do not account for porosity in the subsonic part of the outflow. The increase of the physical scale $\ell = L C_c^{-1/3}$ with radial distance reflects the likely case that the clumps expand and possibly merge while receding from the star.

To provide a quantitative feeling for the number of clumps we compute the number of clumps $N$ passing a radial shell at $r$ in a typical flow time $\tau = R_* / v_\infty$. We define this as the total volume associated with one wind flow time divided by the volume in which there is one clump. One finds

$$
N = 4\pi r^2 \left( \frac{D}{H_\rho} \right)^3 \frac{R_*}{v_\infty} v(r).
$$

In our dwarf model of 30,000 K the scale height has increased to about half a stellar radius at $r = 2 R_*$. At this point the flow velocity is 1100 km s$^{-1}$. For $N$ we obtain $\sim 200 D^3$. Notice that the number of clumps is not conserved, but decreases with distance. For instance, in the same model $H_\rho \sim 0.1 R_*$ at $r = 1.1 R_*$, therefore the number of clumps passing this point in one flow time is about 3.5 times as large as that at $r = 2R_*$. Physically, this implies that in our description clumps merge as they move away from the surface.

**Porosity**

The velocity law of the structured medium is treated as a monotonic function of radius. We therefore do not assume that on a local scale (where the flow speed is about
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![Graph showing the increase of optical depth with distance.](image)

**Figure 2.3:** Schematic representation of the increase of the optical depth with distance. The latter is given in arbitrary units. The slope of the lines thus represent the linear extinction coefficient $\kappa$. The solid line depicts the case of a smooth medium; the dashed line that of a clumpy medium. In the smooth flow, the slanted regions represent continuum extinction, while at the points where the line becomes vertical a line interaction occurs. The line interaction region is assumed to be infinitely narrow. In the case of a clumpy flow, the flat parts of the curve reflect the inter-clump medium. Within a clump the continuum extinction is relatively large, therefore the slope is relatively steep. For reasons explained in the text, we have assumed the line interaction region to have a finite width, determined by the Sobolev length. The line that can interact at about distance 1 is missed as it is associated with the inter-clump medium. Notice that in a clumpy medium the photon needs to travel a larger geometrical depth to cross a given optical depth.

Constant) an ensemble of clumps may be distributed over a range in velocities, nor do we account for shape changes of individual clumps caused by internal velocity gradients. The overlaps and/or gaps in velocity space that may be the result of such motions are termed "vorosity" (which is short for velocity porosity). A general treatment of vorosity is beyond the scope of this study. We do point out that the intrinsic instability of the line-driving mechanism is expected to lead to velocity structure. Assuming the internal velocity dispersion in a clump is small, and that clump velocities sample the smooth outflow (matter being concentrated in the clumps), Owocki (2008) using one-dimensional dynamical simulations of the wind instability finds a reduction in the over-all line absorption of about 10-20%. In our simulations the velocity change inside a single clump is essentially treated in a similar way as in Owocki. It is assumed to be monotonic and amounts to $\delta v \sim (dv/ds) \times \ell$, where $s$ is geometrical distance in the arbitrary direction $s$ and $v$ is the smooth flow velocity. The value of $\delta v$ is typically small.

Radiation hydrodynamical simulations (in 2D) show, however, a large velocity
dispersion in the clumps (Sundqvist et al. 2010). Interestingly, Sundqvist et al. point out that this structure prevents a desaturation of lines (of intermediate strength) also implying only a modest reduction in the line absorption.

### 2.2.3 Inclusion of clumps in the Monte Carlo Code

The mass-loss prediction consists of two parts. First, an \texttt{isa-wind} non-LTE spherically symmetric model atmosphere with prescribed outflow properties is computed. The atomic models and non-LTE treatment are identical to Vink et al. (2000). The density, \textit{casu quo} velocity stratification, in the photosphere and the onset region of the wind is computed accounting for the force due to the gradient in gas pressure and continuum radiation pressure. Near the sonic point the velocity stratification is smoothly connected to a $\beta$-law. For details, see also Vink et al. (2000). The treatment of clumping in the model atmosphere is through implementation of the clumping factor $C_c(r)$ in the description of the density, Eq. 2.3. In the description of optical depth, the clumping is treated in the effective opacity, Eq. 2.6, and the scale length $L$, Eq. 2.7.

By neglecting the porosity correction in describing the opacity (in both \texttt{isa-wind} and \texttt{mc-wind}), we can single out the effect of clumping on the excitation and ionization structure. We will study the impact of clumping on the state of the gas in Sect. 2.3.1.

Second, the Monte Carlo simulation program \texttt{mc-wind} is used to trace the momentum transfer of photons to the gas, from which a mass-loss rate can be derived following the method of Abbott & Lucy (1985). In order to study the full effect of clumping the porous nature of the medium needs to be accounted for. In a homogeneous medium without spectral lines a photon traveling an optical depth $\Delta \tau_v$ will cross a geometrical distance $s = \Delta \tau_v / \kappa_v \rho$. In a spherically symmetric stellar wind with a monotonically increasing wind velocity in which both continuum and line absorption may occur, the photon will experience a local barrier in optical depth at regions where its frequency matches that of a spectral line. If we map the distance in optical depth to a distance in physical space one may get a behavior as is schematically shown in Fig. 2.3. The figure depicts the situation in a spherical shell in which the density is assumed constant. The optical depth increases linearly with distance due to free-free processes, bound-free processes and Thomson scattering. These are the slanted parts of the line. In a rapidly expanding spherical outflow, a photon that is emitted at a wavelength that is slightly blue relative to the wavelengths at which a spectral line may absorb, may interact with the spectral line once it encounters particles that move with the proper relative Doppler (red)shift. Assuming the typical width of the line is given by the Doppler width, $v_D = \sqrt{kT/m}$, where $T$ is the temperature, $k$ the Boltzmann constant and $m$ the mass of the particle, the geometrical length of a line absorption region is $L_{\text{Sob}} = v_D / (dv/dz)$, where $z$ is measuring the di-
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Prediction in which the photon is propagating. This length is referred to as the Sobolev length (Sobolev 1960). If the direction measured by \( z \) is at an angle \( \theta \) with the radial direction, such that \( \hat{z}/\hat{r} = \cos \theta = \mu \), then

\[
\frac{dv}{dz} = \left(1 - \mu^2\right) \frac{v}{r} + \mu^2 \frac{dv}{dr}.
\]  

(2.9)

In computing the radiation field in spectral lines the Sobolev approximation is adopted in \textsc{isa-wind}, i.e. it is assumed that the velocity gradient is so large that the properties of the medium do not change within a length interval \( L_{\text{Sob}} \). In \textsc{mc-wind} models without clumping we assume that line interactions take place at line center, i.e. the Sobolev absorption region is assumed to be infinitely narrow. The vertical solid lines in Fig. 2.3 reflect such line interactions, and represent an optical depth \( \tau_{lu} = \kappa_{lu} \rho \lambda_{lu} L_{\text{Sob}} / v_\lambda \). In this equation the mass absorption coefficient of the transition at wavelength \( \lambda_{lu} \) between lower level \( l \) and upper level \( u \) is given by

\[
\kappa_{lu} = \frac{\pi e^2}{m_e c f_{lu}} \frac{n_l}{\rho} \left(1 - \frac{n_u g_l}{n_l g_u}\right),
\]  

(2.10)

where \( e \) and \( m_e \) are the charge and mass of the electron, \( c \) the speed of light, \( f_{lu} \) is the oscillator strength, \( n_l \) and \( n_u \) the number density of the lower and upper level and \( g_l \) and \( g_u \) the statistical weight of the lower and upper level. In the Monte Carlo simulation the optical depth at which the photon should interact is randomly drawn and given by \( \tau_\nu = -\ln p \), where \( p \in (0, 1) \) is a random number.

In a clumpy wind, with a clumping factor \( C_c \) and porosity length \( L \), photons will travel alternatively through clumps or vacuum. If we assume the Sobolev length \( L_{\text{Sob}} \) to be infinitely small, photons would "miss" spectral lines for which the interaction point is in a void region. If the interaction region would be the actual Sobolev length, a fraction of these ineffective lines could still contribute to the opacity as part of the interaction region may coincide with the location of one or more nearby clumps. Accounting for the extent of the Sobolev interaction region yields a more representative sampling of the spectral lines contributing to the line force. For this reason we account for the actual \( L_{\text{Sob}} \) by introducing a mean line opacity in the line interaction region

\[
\chi_{lu} = \frac{\tau_{lu}}{L_{\text{Sob}}}. 
\]  

(2.11)

This implies that we assume the line profile function to be a box function. The situation of a clumpy medium is depicted in Fig. 2.3 using a dashed line. If the dashed line runs flat, the photon is not encountering any material. If the randomly selected optical depth the photon will travel is within the Sobolev region of one (or more) lines, a random selection, using the opacities of the contributing extinction processes at the point of interaction as a weighing factor, will determine the type of interaction.
The outcome of this random process can be a free-free or bound-free interaction, an electron scattering, or a line interaction.

The clumps themselves are assumed to be cubes, of which the length of the edge is \( \ell = L/C_c^{1/3} \). As explained, each volume \( L^3 \) contains a clump. The probability that a photon traveling this volume encounters a clump is given by the cross section of the clump relative to the cross section of the volume, i.e. \( \ell^2/L^2 = C_c^{-2/3} \). The clumps are randomly placed along the path of the photon using this probability.

On average, due to the effect of porosity and because part of the lines become ineffective (i.e. those lines that have their line interaction region completely or partially in the inter-clump medium), photons need to travel a larger geometrical distance in a clumpy medium before being absorbed. Therefore, the dashed line in Fig. 2.3 is drawn such that it falls below the solid line. If a packet of photons interacts with material in a clump it will be re-emitted in a random direction. For this new direction we account for the fact that the photon packet starts in a clump.

### 2.2.4 Model grid

In order to study the effects of clumping and porosity we have set up a small grid of main sequence stars and supergiants. The input parameters are listed in Table 2.1. To facilitate a comparison with the results of Vink et al. (2000) we have adopted their solar abundance pattern, which follows Anders & Grevesse (1989). We note that applying the solar abundances by Asplund et al. (2005) would result in a typical reduction of the mass loss rate by 0.1 dex (see Krtička & Kubát 2007). Though the mass-loss rate is calculated, the velocity stratification is prescribed (see Sect. 2.2.1). We adopt a terminal velocity \( v_\infty \) that is 2.6 times the effective surface escape velocity, where 'effective' implies that the surface gravity is corrected for radiation pressure on free electrons. The parameter \( \beta \) describing the rate of acceleration of the flow is set to unity. Again this is similar to Vink et al. (2000). Theoretical support for this choice is given by Müller & Vink (2008).

For our five different clumping stratifications the predicted mass-loss rates are given, each time for five different clumping factors. These results will be discussed in the next section.

### 2.3 Results

#### 2.3.1 The effect of clumping on \( \dot{M} \) through its impact on the photospheric radiation field and ionization of the gas

In Fig. 2.4 we show predicted wind energies \( E_{\text{kin}} = 1/2 \dot{M} v_\infty^2 \) as a function of clumping prescription and clumping factor for two typical main sequence stars (top panels) and two typical supergiants (bottom panels). \( E_{\text{kin}} \) therefore has a dimension of energy
2. Predictions of the effect of clumping on the wind properties of O-type stars

Table 2.1: Adopted model parameters together with predicted mass loss rates for all clumping assumptions. Possibly is not included in these predictions. i.e. the clumps are assumed to be optically thin. We stress that the mass-loss rates given here have been calculated assuming that all of the change in wind energy benefits $\dot{M}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>C_4</th>
<th>C_5</th>
<th>C_6</th>
<th>C_7</th>
<th>C_8</th>
<th>C_9</th>
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<tbody>
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<td>( C_{\infty} )</td>
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</tr>
<tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<tr>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Note: Except for instance for the case in which clumping sets in at 0.4σ this is unlikely and most of the effect will benefit $\dot{M}$. However, for instance for the case in which clumping sets in at 0.4σ this is .
### Table 2.1: Continued...

<table>
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<tr>
<th>$T_{\text{eff}}$ [K]</th>
<th>$R$ [R$_\odot$]</th>
<th>$\log L$</th>
<th>$M$ [M$_\odot$]</th>
<th>$v_{\infty}$ [km/sec]</th>
<th>$\log C_c$ constant</th>
<th>$C_c$ from $v_s$</th>
<th>$C_c$ from $0.4v_{\infty}$</th>
<th>$C_c$ up to $v_s$</th>
<th>$C_c$ up to $0.4v_{\infty}$</th>
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</table>
2. Predictions of the effect of clumping on the wind properties of O-type stars

per unit time. The reason why we discuss the results in terms of $E_{\text{kin}}$ and not in terms of $\dot{M}$ is that our Monte Carlo calculation essentially predicts the change in kinetic energy (see Sect. 2.2.1), but does not predict the effect on the velocity structure. If in presenting the results we assume that the terminal velocity is not affected$^1$, the effect of clumping can be expressed in a (change in) mass loss rate. This most certainly is not appropriate for the case in which clumping starts at $0.4v_\infty$. It is to be expected that only for those cases where clumping has developed near the sonic point the above assumption has merit. So; though we cannot disentangle the effects on $\dot{M}$ and $v_\infty$, we still opt to present changes in $\dot{M}$ only in all clumping prescription in Table 2.1 (and also in Table 2.2). However, in the discussion Sect. 2.4 we will concentrate on the physically most relevant cases.

As explained in section 2.2.4, the first four columns of Table 2.1 list stellar parameters and the fifth the adopted terminal velocity. In column six the clumping factor is given that is used in five different clumping prescriptions: constant clumping (column 7; filled red circles in the plot); clumping starting at the sonic point (column 8; green crosses) and at $0.4v_\infty$ (column 9; blue plusses), and clumping in the photosphere and lower part of the wind up to the sonic point (column 10; purple crosses) and up to $0.4v_\infty$ (column 11; black triangles). The fact that in these predictions porosity is not included implies that we assume the clumps to be optically thin.

Clumping introduces two effects that impact the mass loss of the star. First, if clumping occurs in the stellar photosphere the increased continuum opacity will shift the layer of continuum formation to lower temperatures, i.e. softening the radiation field. As a result, the Lyman and He $\iota$ continuum flux decrease, while the Balmer continuum flux increases. For a 30 000 K star the wind driving relies strongly on the contribution of Fe $\text{iv}$, with Fe $\text{iii}$ supplying a non-negligible part. As the lines of these ions tend to cluster in the Balmer continuum, clumping in the photosphere will result in an increase of the mass-loss rate. For a 40 000 K star the wind driving relies on Fe $\text{v}$ and Fe $\text{iv}$ lines (as well as on lines of carbon, nitrogen and oxygen), preferentially located in the Lyman and He $\iota$ continuum. Therefore in this case clumping in the photosphere will slightly lower the mass-loss rate. Second, clumping in the stellar wind will push the ionization balance of the wind driving ions towards a lower ionization stage. As has been shown by Vink et al. (1999) a dramatic increase in the mass loss rate is to be expected when iron recombines from Fe $\text{iv}$ to Fe $\text{iii}$ near the sonic point. This occurs at $\sim 25 000$ K in a smooth wind and is referred to as the bi-stability jump. Though Fe $\text{iii}$ does not become dominant in even the most clumped winds (i.e. $C_c = 100$) of our 30 000 K star, the Fe $\text{iii}$ contribution does incease very substantially for large clumping factors causing a higher mass-loss. For the 40 000 K

$^1$Notice that this neglects the possibility of feedback, i.e. a fully consistent hydrodynamical treatment of clumping might in principle result in a complex reaction that does not obey $\Delta \dot{M} \propto \Delta u^2$ for a given $\Delta L$. 

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star the increased importance of Fe IV relative to Fe V (and e.g. C III and C IV relative to C V) in a clumped outflow also has a positive effect on $\dot{M}$, though not as pronounced as in the 30 000 K star.

These considerations allow to interpret the results in Fig. 2.4. In the 30 000 K stars both the effect of clumping on the photospheric radiation field and the ionization balance work in the direction of an increase in the mass-loss rate (see also Gräfener & Hamann 2008). The small effect on $\dot{M}$ in clumping prescriptions 4 and 5, which both have clumping in the photosphere but not in the outer wind, shows that for this model the impact of clumping on the photospheric radiation field is negligible. The other three clumping prescriptions, 1, 2 and 3, show the importance of clumping in the outer wind. The situation of omnipresent clumping (prescription 1) is most extreme. Here the mass loss may increase by up to a factor 3 for $C_c = 10$ and 7 for $C_c = 100$. Notice that if clumping is only modest ($C_c \lesssim 3$) only small changes in the mass-loss are expected.

The situation for the 40 000 K stars is slightly different. Here the two effects of clumping – the softening of the radiation field and the recombination of the gas – work in opposite directions. In clumping prescriptions 4 and 5 the impact of clumping on the photospheric spectrum causes a modest decrease in the mass loss (less than a factor of two for even the most extreme clumping). Clumping in the outer wind has the reverse effect, which is now most prominent in clumping prescriptions 2 and 3 that have no clumping in the photosphere. As prescription 1 has clumping everywhere this model now falls in between 2 and 3 and 4 and 5. For the strongest clumping the most extreme effect is found for prescription 2 though here the increase in mass loss is only about a factor of two. Notice that modest clumping ($C_c \lesssim 3$) has about a 30% effect on $\dot{M}$.

### 2.3.2 The effect of porosity on $\dot{M}$

So far, the results that we have presented assume the clumps are optically thin. Now, we will account for the actual optical depth of the clumps. This may cause the medium to become porous. To do so, we have to introduce the physical scale of the separation of the clumps $L$, as introduced in Sect. 2.2.2. The results for this case are given in Table 2.2. In Fig. 2.5 we show the effect of porosity for the case that clumping sets in at the sonic point (i.e. model 2 in Fig. 2.4) and at 0.4 $v_\infty$ (model 3). The separation between the clumps is either 1/10, 1/100 or 1/1000 of the local scale-height, i.e. $D = 10, 100$ or 1000. Though it seems reasonable to assume that such separations may develop in the wind, these rather large separations (small values of $D$) are very unlikely to occur in the stellar photosphere. For this reason we focus on models 2 and 3.

In interpreting the results it is important to realize that one should compare these to the predicted mass-loss rates shown in Fig. 2.4 for the relevant clumping factor, and
2. Predictions of the effect of clumping on the wind properties of O-type stars

Figure 2.4: Wind energy predictions for different clumping stratifications of selected O-type stars. The smooth wind models have $C_c = 1$. The numbers refer to those used in Fig. 2.1 to identify the clumping behavior. Clumping in the outer winds (stratifications 1 through 3) results in an increase of $\dot{E}_{\text{kin}}$ because of an increased number of effective driving lines. The effect of clumping on the photospheric spectrum, which occurs in stratifications 4 and 5, is temperature dependent for the 30000 K (40000 K) model. It leads to an increase (decrease) of $\dot{M}$. See Sect. 2.3.1 for a discussion.

See Sect. 2.3.1 for a discussion.
Figure 2.5: Wind energy predictions accounting for both clumping and porosity for the clumping stratification in which clumping sets in at the sonic velocity (models labeled 2) and at $0.4v_{\infty}$ (models labeled 3). The scale of the clumps is given by $H_p/D$, where $D = 10, 100, 1000$. These results should be compared to the corresponding models in Fig. 2.4 that account for clumping but not for porosity. The addition of porosity always causes a decrease in $E_{\text{kin}}$. See Sect. 2.3.2 for a discussion.
not to the model that has a homogeneous outflow. Only through such a comparison one will single out the effect of porosity on $\dot{M}$. It is clear that if the separation between the clumps is large ($D$ is small) the effect of porosity will be strongest. Photons may travel relatively undisturbed through the interclump medium, avoiding interactions with the gas. For the extreme case $D = 10$ the drop in $\dot{M}$ due to the geometrical effect of porosity may be as large as one to two orders of magnitude. For $D = 1000$ the drop is at most a factor of three. The impact of porosity is less severe if clumping sets in farther out in the wind, as expected.

As an example of the quantitative behavior let us concentrate on the model where clumping sets in at the sonic point and $C_c = 100$. The drop in mass-loss (relative to the results discussed in the previous section) is 0.9 dex for the 30 000 K models and 0.7 dex for the 40 000 K models. For the 30 000 K supergiant model the increase in mass-loss due to the clumping effects discussed in Sect. 2.3.1 is essentially cancelled when porosity is also accounted for. In all other cases of this particular clumping stratification and clumping factor, the inclusion of porosity overcompensates for the effects of clumping on the photospheric radiation field and ionization of the gas and causes a decrease in the mass loss relative to a smooth outflow. Notice that for the smallest clump separation ($D = 1000$) the increase in $\dot{M}$ due to clumping effects alone is typically not fully compensated by the porous nature of the medium. The mass-loss rate may go up, though not more than a factor of three.

Notice that for modest clumping ($C_c \leq 3$) the combined effects of clumping and porosity has a negligible effect for the case $D = 100$; leads to about a 20% decrease in $\dot{M}$ for $D = 100$, while the mass-loss may drop by about a factor of two in case $D = 10$.

### 2.4 Discussion

In order to facilitate a quantitative discussion of the effects of clumping and porosity on predicted values of the mass-loss rates of O-type stars and to be able to assess what the actual clumping factors should be in order to bring agreement between empirical and predicted mass-loss rates, we approximate the clumping and porosity effects on the predicted total kinetic energy $E_{\text{kin}}^{\text{pred}}$ by a power-law. We thus assume that

$$E_{\text{kin}}^{\text{pred}}(C_c) = C_c^{\alpha} \times E_{\text{kin}}^{\text{pred}}(C_c = 1),$$

(2.12)

where the superscript “pred” stands for prediction and $E_{\text{kin}}(C_c = 1)$ implies a homogeneous outflow. The typical uncertainty in this relation is 10 to 20 percent for the 30 000 K stars and less than 10 percent for the 40 000 K stars. Values for the power-law index $\alpha$ are given in Table 2.3 in case clumping starts at the sonic point (model 2) and at $0.4v_{\infty}$ (model 3) for different values of the porosity length, prescribed by the parameter $D$ (see Eq. 2.7).
Table 2.2: Adopted model parameters together with predicted mass loss rates for two clumping stratifications and porosity descriptions. The scale of the clumps is given by $H_p/D$. The clumping laws are: clumping starts at the sonic point (i.e. $C_{cc}$ from $v_s$) and clumping starts at 0.4$v_{\infty}$ (i.e. $C_{cc}$ from $v_{\infty}$).

We again stress that the mass-loss rates given here have been calculated assuming that all of the change in wind energy benefits $\dot{M}$. However, for instance for the case in which clumping and porosity set in at 0.4$v_{\infty}$ this is highly unlikely; most of the effect will benefit $v_{\infty}$.

<table>
<thead>
<tr>
<th>T$_{\text{eff}}$</th>
<th>R</th>
<th>log L</th>
<th>M</th>
<th>log C$_{cc}$</th>
<th>$v_{\text{cc}}$</th>
<th>log $\dot{M}$ [$M_\odot$ yr$^{-1}$]</th>
<th>C from $v_{\text{cc}}$</th>
<th>C from 0.4$v_{\infty}$</th>
</tr>
</thead>
<tbody>
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<td>2.0</td>
<td>-6.17</td>
</tr>
</tbody>
</table>

Note: The values are given in solar units ($M_\odot$, $L_\odot$, $R_\odot$, $L_\odot$, $M_\odot$, $k$m/sec) for clarity.
## 2. Predictions of the effect of clumping on the wind properties of O-type stars

| D = 100 | D = 1000 | D = 10000 | 3.13 | 5.18 | 8.13 | 7.2 | 11.5 | 7.2 | 11.5 | 7.2 | 11.5 | 7.2 | 11.5 |
|----|----|----|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.39 | 0.70 | 0.39 | 0.39 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 | 0.70 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |

**Table 2.2 Continued...**
Empirical mass-loss rates derived using the Hα line or the radio continuum may suffer from the presence of clumping as these diagnostics scale with the square of the density. Fits to these data assuming a homogeneous outflow should thus be corrected for clumping according to the relation

\[
\dot{M}^{\text{emp}}(C_c) = C_c^{-1/2} \times \dot{M}^{\text{emp}}(C_c = 1),
\]  

(2.13)

where the superscript “emp” stands for empirical. To facilitate a further comparison we assume that for the models where clumping develops near the sonic point the effect of clumping dominantly impacts the mass-loss rate. If clumping sets in further out in the wind (at, say, 0.4v∞), the dominant impact is on the terminal flow velocity and not on \( \dot{M} \) (see Kršička et al. 2008). We therefore focus our discussion on the results that have been obtained for models in which clumping sets in at the sonic point. In doing so, we replace the term \( E_{\text{kin}}^{\text{pred}} \) in Eq. 2.12 by \( \dot{M}^{\text{pred}} \). In order to match empirical and predicted mass-loss rates for a wind that suffers from clumping and porosity it should thus hold that

\[
\log \left( \frac{\dot{M}^{\text{emp}}(C_c = 1)}{\dot{M}^{\text{pred}}(C_c = 1)} \right) = (\alpha + 0.5) \log C_c.
\]  

(2.14)

### 2.4.1 Accounting for clumping in both empirical estimates and predictions of mass-loss rates

In a comparison of empirical and predicted mass-loss rates of O-type stars brighter than 175 000 \( L_\odot \), having strong winds (\( \dot{M} \gtrsim 1 - 2 \times 10^{-7} M_\odot \text{yr}^{-1} \)), Mokiem et al. (2007) found that the empirical rates are consistently higher than the predicted rates. This implies that in principle a clumping factor (for given porosity length) can be found such that \( \dot{M}^{\text{emp}}(C_c) \) and \( \dot{M}^{\text{pred}}(C_c) \) match if \( \alpha > -0.5 \). For smaller values of \( \alpha \) the drop in predicted mass-loss rate due to the effect of clumping and porosity is so severe that it can never be matched by the correction of the empirical mass-loss rate for the effect of clumping. Our predictions show that this situation will occur if clumping develops relatively close to the surface (near the sonic point) for large clump separations \( L(r) \gtrsim 0.1 H_\rho \) (or \( D \lesssim 10 \)).

The offset between empirical and predicted mass-loss rates assuming homogeneous outflows as determined by Mokiem et al. (2007) is +0.27 dex for Galactic stars (see the left panel of their figure 4). This value is derived by comparing the empirical and predicted modified wind momentum (MWM) relation at a luminosity \( \log(L/L_\odot) = 5.75 \), which is typical for the stars investigated by these authors. As the slopes of these MWM relations show tiny differences, the choice of luminosity may in principle have a small effect on the derived offset. The offset of +0.27 dex in the mass-loss rate implies that a clumping \( C_c \approx 3.5 \) is sufficient to bring agreement
between $M_{\text{emp}}$ and $M_{\text{pred}}$ assuming clumping has no effect on the predicted mass-loss rates. If one does account for clumping and porosity effects in the theoretical values, the clumping that is required may increase up to $C_\alpha \sim 10$ for the case $D \sim 100$, but may be slightly lower ($C_\alpha \sim 2.5 - 3.5$) if $D \sim 1000$ as, on average, the derived $\alpha$ values are positive. For porosity lengths corresponding to $D \lesssim 100$ the required clumping factor will increase steeply.

### 2.4.2 Observational constraints on the number of clumps

In section 2.2.2, we estimated that the number of clumps per wind flow-time that we assumed in our models is $\sim 200 D^3$. The question is whether there are any empirical constraints either in support of, or contradicting the assumed clump sizes and numbers in our models. Currently, only rather rough order-of-magnitude estimates can be made.

Lépine & Moffat (1999) monitored a number of Wolf-Rayet stars spectroscopically discovering line-profile variations (LPVs) which were interpreted as a large number (more than $10^4$) of randomly distributed, radially propagating, discrete wind emission elements, or DWEEs, in order to account for the LPVs. Another way to derive the number and spatial scales of clumps is via the use of linear polarimetry that provides information on the geometry of the innermost portions of the stellar wind. Davies et al. (2007) showed that in order to reproduce the observed level of polarization variability of the LBVs P Cygni and AG Car the winds should consist of $\sim 10^3$ clumps per wind flow-time.

Quantitative estimates of the typical number of clumps in O-type stars have not yet been made, though line-profile variations do point to the presence of structure in their winds as well (see e.g. Eversberg et al. 1998; Lépine & Moffat 2008). The origin of the clumps likely controls their number. Cantiello et al. (2009) recently suggested that wind clumping might be induced by sub-surface convection induced by the iron opacity peak in massive stars, where the density scale height in the iron opacity zone is approximately a factor $10^2$ larger in LBV than in O star models. If wind clumping in LBVs and O stars were indeed induced by this iron opacity peak, one would then expect a factor $10^2$ more clumps per wind flow-time in O star than in LBV winds, which would bring us in the range of $10^5$ clumps per wind flow-time for O stars. This appears to be consistent with values of $D$ on the lower end ($D \sim 10$) of the range studied in our paper. As pointed out in section 2.4.1, such a relatively modest number of clumps would lead to lower expected mass-loss rates making it hard to reconcile empirical and predicted mass-loss rates for stars with dense winds.
Table 2.3: Fitted behavior of the effects of clumping and porosity for the case that clumping sets in at the sonic point (model 2) and at 0.4v∞ (model 3). The total kinetic energy is fitted to the function \( E_{\text{kin}} = E_{\text{kin}}(C_c = 1) \times C_{c}^{\alpha} \), for each type of star and clump separation \( L(r) = H_p(r)/D \). For given values of \( D \) the table lists the values of \( \alpha \). \( D = \infty \) implies non-porous models.

<table>
<thead>
<tr>
<th>( D )</th>
<th>30 000 V</th>
<th>40 000 V</th>
<th>30 000 I</th>
<th>40 000 I</th>
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</thead>
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<td></td>
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<tr>
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<td>-0.02</td>
<td>+0.17</td>
<td>-0.11</td>
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<tr>
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<td>+0.19</td>
<td>+0.43</td>
<td>+0.10</td>
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</tr>
<tr>
<td>( \infty )</td>
<td>+0.24</td>
<td>+0.21</td>
<td>+0.40</td>
<td>+0.20</td>
</tr>
</tbody>
</table>

2.4.3 The weak wind problem

For luminosities below about 175 000 \( L_\odot \), a comparison between empirical and predicted mass-loss rates shows a large discrepancy referred to as the “weak wind problem”: empirical mass-loss rates appear to be up to two orders of magnitude lower than the predicted rates. At the moment, the nature of this weak wind problems eludes us; for a discussion see e.g. Martins et al. (2004, 2005b); de Koter (2006); Fullerton et al. (2006); Mokiem et al. (2007); Puls et al. (2008). In the context of the current study a possible explanation could be that the wind of relatively low luminosity O stars become extremely clumpy and porous. Estimating the magnitude of the weak wind problem for stars of 100 000 \( L_\odot \) at about a factor of 30 (see for instance the right panel of figure 1 in Mokiem et al. 2007), a clumping factor \( C_c \approx 500 \) and porosity length \( D \approx 10 \) could bring empirical and predicted estimates into agreement. It is however unclear why preferentially in low density stellar winds, typically at \( \dot{M} \lesssim 1 - 2 \times 10^{-7} M_\odot \text{yr}^{-1} \), such extreme inhomogeneities would develop.
2. Predictions of the effect of clumping on the wind properties of O-type stars

2.5 Conclusions

We have investigated effects of clumping and porosity on predictions of the wind energy of O-type dwarf and supergiant stars using a method that is based on Monte Carlo radiative transfer. These results can be viewed as an addition to prescriptions provided by Vink et al. (2000, 2001). For five heuristic clumping stratifications we investigate the effects of a clumpy medium on the wind energy through induced changes in the (photospheric) radiation field and the excitation and ionization state of the gas throughout the wind. Also, we investigate the effect of porosity by introducing a prescription in which the clump size is expressed as a fraction of the local density scale height. Clumps of size $H_p/D$ equal to 1/10th, 1/100th and 1/1000th of $H_p$ are considered. The main conclusions are:

(I) The presence of optically thin clumps favors the recombination of the gas, which for the temperature range investigated (between 30 000 and 40 000 K) causes an increase in the line force, therefore an increase in the mass-loss rate. The larger the clumping factor $C_c$ and the closer the star is to the bi-stability jump (at $\sim 25 000$ K) the stronger is the effect.

(II) Accounting for porosity effects in the clumped medium and the (wavelength dependent) optical depth of the clumps, the mass-loss rate is found to decrease simply because photons may travel in between the clumps, avoiding interactions with the gas. For small clumps ($D \gtrsim 1000$) this effect is not very important, but for larger clumps ($D \lesssim 100$) the overall effect is a net decrease in $\dot{M}$.

(III) For clump sizes corresponding to $D \sim 100$ or larger the net effect on $\dot{M}$ is small. Assuming that the velocity structure is not affected by clumps the mass loss rate decreases by less than a factor of two, even for clumping factors $C_c \sim 100$. For large clumps, corresponding to $D \lesssim 10$, the effect is more dramatic. Already for modest clumping we find that $\dot{M} \propto C_c^{\alpha}$ with $\alpha \sim -0.5$ to -0.6. For such a steep dependence empirical mass-loss rates based on Hα measurements, which are found to be about a factor of two higher than predictions assuming smooth outflows, can no longer be reconciled with theoretical $\dot{M}$ values by applying clumping corrections.

(IV) Though large clumps and very large clumping factors may dramatically reduce the mass-loss rate, the occurrence of this type of structure is likely not the explanation for the “weak wind problem” for stars with $L \lesssim 10^{5.2}L_\odot$, unless a
mechanism can be identified causing extreme structure to develop in winds of 
\( \dot{M} \lesssim 1 - 2 \times 10^{-7} \, M_\odot \text{yr}^{-1} \) that is not active in denser winds.

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