The physics of line-driven winds of hot massive stars
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4 Wind models for very massive stars up to 300 solar masses

J.S. Vink, L.E. Muijres, A. de Koter, B. Anthonisse, G. Gräfener, N. Langer

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Abstract

The upper limit of massive stars is highly uncertain. Some studies have claimed the existence of a universal stellar mass upper limit of \( \sim 150 M_\odot \), but the true initial masses of these objects may have been significantly higher, possibly superseding 200 \( M_\odot \). We present mass-loss predictions from Monte Carlo radiative transfer models for very massive stars in the mass range 40-300 \( M_\odot \), and with luminosities 6.0 \( \leq \log(L_*/L_\odot) \leq 7.03 \). Such objects have a high Eddington factor \( \Gamma \).

Using a new dynamical approach, we find an upturn (or “kink”) in the mass-loss versus \( \Gamma \) dependence, where the model winds become optically thick. This is also the point where our wind efficiency numbers surpass the single-scattering limit (of \( \eta = 1 \)), reaching values up to \( \eta \approx 2.5 \). Our modelling thus suggests a natural transition from common O-type stars to Wolf-Rayet characteristics when the wind becomes optically thick. This “transitional” behaviour is also found in terms of the wind acceleration parameter \( \beta \), which naturally reaches values as high as 1.5-2, as well as in the spectral morphology of the Of and WN characteristic He \( \Pi \) line at 4686Å.

When we express our mass-loss predictions as a function of the electron scattering Eddington factor \( \Gamma_e \sim L_*/M_* \) only, we obtain an \( \dot{M} \) vs. \( \Gamma_e \) dependence that is consistent with a previously reported power-law \( \dot{M} \propto \Gamma_e^5 \) (Vink 2006). However, when we express \( \dot{M} \) in terms of both \( \Gamma_e \) and stellar mass, we find optically thin winds and \( \dot{M} \propto M_*^{0.68} \Gamma_e^{-2.2} \) for the \( \Gamma_e \) range 0.4 \( \lesssim \Gamma_e \lesssim 0.7 \), and mass-loss rates that are in agreement with the standard Vink et al. (2000) recipe for normal O stars. For higher \( \Gamma_e \) values the winds are optically thick and, as pointed out, the dependence is much steeper, \( \dot{M} \propto M_*^{0.78} \Gamma_e^{-4.77} \).
4. Wind models for very massive stars up to 300 solar masses

4.1 Introduction

The prime aim of this paper is to investigate the mass-loss behavior of stars with masses up to 300 $M_\odot$ that are approaching the Eddington limit.

Mass loss from massive stars is driven by radiative forces on spectral lines (Lucy & Solomon 1970; Castor, Abbott & Klein 1975; CAK). CAK developed the so-called ‘force multiplier formalism’ to treat all relevant ionic transitions. This enabled them to simultaneously predict the wind mass-loss rate, $\dot{M}$, and terminal velocity, $v_\infty$, of O-type stars. Although these predictions provided reasonable agreement with observations, they could not account for the large wind efficiencies $\eta = \dot{M} v_\infty / L/c$ of the denser O-type stars, with their strong He II 4686Å lines, nor that of the even more extreme Wolf-Rayet (WR) stars. This discrepancy had been proposed to be due to the neglect of multi-line scattering (Lamers & Leitherer 1993, Puls et al. 1996). Using a “global energy” Monte Carlo approach (Abbott & Lucy 1985, de Koter et al. 1997) in which the velocity law was adopted aided by empirical constraints, Abbott & Lucy (1985) and Vink et al. (2000) provided mass-loss predictions for galactic O stars including multi-line scattering. This appeared to solve the wind momentum problem for the denser O-star winds. Mass-loss rates were obtained that were a factor $\sim 3$ higher than for cases in which single scattering was strictly enforced.

Historically, the situation for the WR stars was even more extreme. Here $\eta$ values of $\sim 10$ had been reported (e.g. Barlow et al. 1981). With the identification of major wind-clumping effects on the empirical mass-loss rates (Hillier 1991; Moffat et al. 1994; Hamann & Koesterke 1998) these numbers should probably be down-revised to values of $\eta \approx 3$. Although it was argued that WR winds were also driven by radiation pressure (Lucy & Abbott 1993, Springmann 1994, Gayley et al. 1995, Nugis & Lamers 2002, Gräfener & Hamann 2005) the prevailing notion is still that these optically thick outflows of WR stars, where the sonic point of the accelerating flow lies within the (pseudo or false) photosphere, are fundamentally different from the transparent CAK-type O-star winds (e.g. Gräfener & Hamann 2008).

Müller & Vink (2008) have recently suggested a new parametrization of the line acceleration, expressing the line acceleration as a function of radius rather than of the velocity gradient (as in CAK theory). The implementation of this new formalism improves the local dynamical consistency of our models that initially adapted a velocity law. Not only do we find fairly good agreement with observed terminal velocities (see alsoMuijres et al. 2010b), but as our method naturally accounts for multi-line scattering it is also applicable to denser winds, such as those of WR stars.

Still adopting a velocity stratification, Vink & de Koter (2002) and Smith et al. (2004) predicted mass-loss rates for Luminous Blue Variables (LBVs), and showed that for these objects $\dot{M}$ is a strong function of the Eddington factor $\Gamma_e$. They also showed that despite their extremely large radii, even LBV winds may develop pseudo-
photospheres under special circumstances: when they find themselves in close proximity to both the bi-stability and Eddington limit – at a transition value for $\Gamma_c$ of approximately 0.7.

In this paper, our aim is to study the mass-loss behaviour of stars as they approach the Eddington limit, in a systematic way targeting very massive stars in the range 40-300 $M_\odot$. A pilot study was performed by Vink (2006) who found a steep dependence of $\dot{M}$ on $\Gamma_c$, finding $\dot{M} \propto \Gamma_c^5$, but this was obtained using the earlier Vink et al. (2000) global energy approach (in which the velocity stratification was adopted) rather than the improved dynamically consistent approach applied here.

The upper limit of massive stars is highly controversial. On purely statistical grounds some studies have claimed the existence of a universal stellar mass upper limit of $\sim 150$ $M_\odot$ (e.g. Weidner & Kroupa 2004, Oey & Clarke 2005, Figer 2005). However as a result of strong mass loss, the true initial masses of these objects may have been significantly higher, likely super-seeding 150-200 $M_\odot$ (e.g. Figer et al. 1998, Crowther et al. 2010). This illustrates that the issue of the highest mass star is highly uncertain because of the limited quantitative knowledge of mass-loss rates of stars close to their Eddington limit. Our aim is thus to model the mass-loss rates of stars with masses up to 300 $M_\odot$. Very massive stars have been proposed to lead to the production of intermediate mass (of order 100$M_\odot$) black holes that have been suggested to be at the heart of ultra-luminous X-ray sources (Belkus et al. 2007 and Yungelson et al. 2008). Clearly, the success of such theories depends critically on the applied mass-loss rates. The present study may help advance these theories.

Our paper is organized as follows. In Sect. 4.2 we briefly describe the Monte Carlo mass-loss models, before presenting the parameter space considered in this study (Sect. 4.3). The mass-loss predictions (Sect. 4.4) are followed by a description of the spectral morphology of the Of-WN transition in terms of the characteristic He II 4686Å line in Sect. 4.5. Subsequently, we compare our new mass-loss predictions against empirical values determined by Martins et al. (2008) for the most massive stars in the Arches cluster in Sect. 4.6, before ending with a discussion and summary in Sects. 4.7 and 4.8.

### 4.2 Monte Carlo models

Mass-loss rates are calculated with a Monte Carlo method that follows the fate of a large number of photon packets from below the stellar photosphere throughout the wind. The core of our approach is related to the total loss of radiative energy that is coupled to the momentum gain of the outflowing material. Since the absorptions and scatterings of photons in the wind depend on the density in the wind and hence on the mass-loss rate, one can find a consistent model where the momentum of the wind material equals the transferred radiative momentum. We have recently improved our
4. Wind models for very massive stars up to 300 solar masses

dynamical approach (Müller & Vink 2008, Muijres et al. 2010b) and we are now able to predict $\dot{M}$ simultaneously with $v_{\infty}$ and the wind structure parameter $\beta$. The essential ingredients and assumptions of our approach have more extensively been discussed in Abbott & Lucy (1985), de Koter et al. (1997) and Vink et al. (1999). Here we provide a brief summary.

The Monte Carlo code mc-wind uses the density and temperature stratification from a prior model atmosphere calculation performed with isa-wind (de Koter et al. 1993, 1997). These model atmospheres account for a continuity between the photosphere and the stellar wind, and describes the radiative transfer in spectral lines adopting an improved Sobolev treatment. The chemical species that are explicitly calculated (in non-LTE) are H, He, C, N, O, S, and Si. The iron-group elements, which are crucial for the radiative driving and the $\dot{M}$ calculations, are treated in a generalized version of the “modified nebular approximation” (e.g. Schmutz 1991). However, we performed a number of test calculations in which we treated Fe explicitly in non-LTE. These tests showed that differences with respect to the assumption of the modified nebular approximation for Fe were small. Therefore, we decided to treat Fe in the approximate way, as was done in our previous studies.

The line list used for the MC calculations consists of over $10^5$ of the strongest transitions of the elements H - Zn extracted from the line list constructed by Kurucz & Bell (1995). The wind was divided into 90 concentric shells, with many narrow shells in the subsonic region, and wider shells in supersonic layers. For each set of model parameters a certain number of photon packets was followed, typically $2 \times 10^6$.

Other assumptions in our modelling involve wind stationarity and spherical geometry. The latter seems to be a good approximation, as the vast majority of O-type stars show little evidence of significant amounts of linear polarization (Harries et al. 2002, Vink et al. 2009). Nevertheless, asphericity has been found in roughly half the population of Luminous Blue Variables (Davies et al. 2005, 2007), although those polarimetry results have been interpreted as the result of small-scale structure or ”clumping” of the wind, rather than of significant wind asymmetry.

With respect to wind clumping, it has been well-established that small-scale clumping of the outflowing gas has a pronounced effect on the ionization structure of both O-star and Wolf-Rayet atmospheres (e.g. Hillier 1991). This has lead to a downward adjustment of empirical mass-loss rates, by factors of up to three (e.g. Moffat et al. 1994, Hamann & Koesterke 1998, Mokiem et al. 2007, Puls et al. 2008), and possibly even more (Bouret et al. 2003; Fullerton et al. 2006). In addition, clumping may have a direct effect on the radiative driving, therefore on predicted mass-loss rates. This was recently investigated by Muijres et al. (2010a) for O dwarfs and supergiants. Whilst it was found that the impact on $\dot{M}$ can be large for certain clumping prescriptions, the overall conclusion was that for moderate clumping factors and porosity, clumping does not affect the wind properties dramatically. Stars approaching the Ed-
dington limit, however, may be much more susceptible to – even modest – clumping (Shaviv 1998, 2000; van Marle et al. 2008). In the present set of computations we do not account for the effects of clumping.

4.3 Parameter space and model applicability

Stars approach the Eddington limit when gravity is counterbalanced by the radiative forces, i.e. $\Gamma = g_{\text{rad}}/g_{\text{Newton}} = 1$. Photons can exert radiative pressure through bound-free, free-free, electron scattering and bound-bound interactions. In early type stars hydrogen, the dominant supplier of free electrons, is fully ionized. Therefore $\Gamma_e = g_e/g_{\text{Newton}}$ is essentially independent of distance and a fixed number for each model. Because of this useful property, that provides a well-defined and simple quantitative handle, we opt to discuss our results in terms of $\Gamma_e$. We discuss this choice in more detail in Sect. 4.3.1

The dependence of the mass-loss rate $\dot{M}$ on $\Gamma_e$ represents a non-trivial matter as $\dot{M}$ depends on both the mass $M$ and the stellar luminosity $L$. In order to properly investigate the effect of high $\Gamma_e$ on mass-loss predictions, we first need to establish the relevant part of parameter space in terms of $M$, $L$, and $\Gamma_e$. We express $\Gamma_e$ as:

$$\Gamma_e \equiv \frac{g_e}{g_{\text{Newton}}} = \frac{L_\star \sigma_e}{4\pi c G M_\star} = 7.66 \times 10^{-5} \sigma_e \left( \frac{L_\star}{L_\odot} \right) \left( \frac{M_\star}{M_\odot} \right)^{-1}. \quad (4.1)$$

In order to determine the electron scattering coefficient $\sigma_e$ the prescription from Lamers & Leitherer (1993) is used, which includes a dependence on the helium abundance. The luminosities are chosen in such a way that in combination with the stellar mass $M$, the desired $\Gamma_e$ value is obtained. The effective temperature sets the ionization stratification in the atmosphere and thus determines which lines are most active in driving the wind. As a result, $T_{\text{eff}}$ affects the predicted mass-loss rate. For most parts of this paper, we investigate the influence of $\Gamma_e$ for a fixed stellar temperature of 50 000 K. The $T_{\text{eff}}$ dependence is studied separately in Sect. 4.4.4. All models are for a solar chemical composition, as derived by Anders & Grevesse (1989).

We divide our model stars in three different groups according to their characteristics. The first group comprises objects that have relatively common O-star masses in the range 40-70 $M_\odot$. They are approaching their Eddington limit as a result of prior mass loss. The second group of objects are rather high-mass stars within the “observable range” of 70-120 $M_\odot$. They might be close to the Eddington limit already early-on on the main sequence because of their intrinsically high luminosity. The third group involves very massive stars in the mass range 120-300 $M_\odot$. They are near the Eddington limit for the same reason as the second group. So far, there is a lack of compelling observational evidence for the actual existence of such stars in the
present-day universe. We note, however, that Crowther et al. (2010) have suggested a revision of the upper mass limit to \(\sim 300 \, M_\odot\).

The bulk of the models in our grid have been chosen such that the behavior of mass loss as a function of \(M\) and \(L\) can be studied separately. The grid is presented in Table 4.1. We note that the \((M, L)\) combinations are intentionally rather extreme to assure high \(\Gamma_e\) values. The reason is to specifically map that part of parameter space where physically the most extreme winds are expected to appear.

### 4.3.1 Model applicability regime

With respect to the potential limitations of our modelling approach, we make one rather stringent assumption in the manner the (sub-) photospheric density structure is set-up. In the deepest layers of the model atmosphere (with \(v \ll 1 \text{km/s}\)) we assume that the run of density is provided by the equation of motion using \(g_{\text{rad}} = g_e\), hence we apply \(\Gamma = \Gamma_e\). In reality \(\Gamma > \Gamma_e\), as well as being depth-dependent, as a result of bound-bound, bound-free, and free-free processes. Notably, the opacities from millions of weak iron lines may contribute significantly, but they are largely neglected in the deep layers of our models.

Nugis & Lamers (2002) highlighted the importance of the iron peak opacities in deep photospheric layers for the initiation of Wolf-Rayet winds (see also Heger & Langer 1996). This approach was subsequently included in models by Gräfener & Hamann (2005, 2008) for WC and WNL stars. They found that the presence of these opacity bumps may locally cause \(\Gamma\) to approach unity, leading to the formation of optically thick winds. In our Monte Carlo approach we trace the radiative driving of the entire wind, and as most of the energy is transferred in the supersonic part of the outflow, we are less susceptible to the details of the (sub)photospheric region. However, this also means that we do not treat these deep regions self-consistently. This implies that we can (and we will) compute model atmospheres with values of \(\Gamma_e\) very close to one. In reality this may not be achieved for solar-metallicity stars, as this would result in super-critical \(\Gamma\)-values below the sonic point. However, our strategy has the advantage to allow us to explore the transition from transparent to dense stellar winds. As our models do capture the full physics in the layers around and above the sonic point, we argue that they correctly predict the qualitative behaviour of dense winds, but that \(\Gamma_e\) for one of our optically thick wind models would correspond to a model with smaller \(\Gamma_e\) if the ionic contributions were included in the deepest parts of the atmosphere. This "shift" in \(\Gamma_e\) is not fixed but would depend on the sonic point temperature and density. From the behaviour of the Rosseland mean opacity, we would expect the size of the shift to increase at higher \(\Gamma_e\) and higher temperatures.

If \(\Gamma\) exceeds unity at some depth in the sub-photospheric part of the atmosphere, a density inversion is expected to occur for the static case, i.e. for increasing radial distance from the center the density very near the domain where \(\Gamma > 1\) is anticipated
to increase. This is encountered in studies of stellar structure and evolution, but it is unclear what really happens in nature. The potential effects may involve strange-mode pulsations (e.g. Glatzel & Kiriakidis 1993), sub-surface convection (Cantiello et al. 2009), or an inflation of the outer stellar envelope (e.g. Ishii et al. 1999). These processes tend to occur only when \( \Gamma \) is very close to unity, or above (see e.g. Petrovic et al. 2006). In assessing the outcome of our computations, we find that at \( \Gamma_e > 0.95 \) the results behave rather suspect. Though we show the wind results over the entire \( \Gamma_e \) range, we only quantify the mass-loss rates up to this value of \( \Gamma_e \). This boundary is indicated with a vertical dashed line in all relevant figures.

### 4.4 Results

#### 4.4.1 Mass-loss predictions at high \( \Gamma_e \)

Table 4.1 lists our mass-loss predictions for all three considered mass ranges. Most columns are self-explanatory, but we note that the effective escape velocity \( \Gamma_{\rm esc}^{\rm eff} \) (6th column) is defined as \( \sqrt{2GM_*(1-\Gamma_e)/R} \). The predicted wind terminal velocities, mass-loss rates, wind efficiency numbers, and wind acceleration parameter \( \beta \) are given in columns (7), (8), (9), and (10) respectively. For comparison column (11) lists the mass-loss values from the standard mass-loss recipe of Vink et al. (2000) where \( \beta \) was held fixed at unity.

The predicted mass-loss rates (column 8) are shown in Fig. 4.1. Different symbols are used to identify the different mass ranges. We repeat that all models are computed at an effective temperature of \( T_{\rm eff} = 50,000 \) K (but see Sect. 4.4.4).

The figure shows that \( \dot{M} \) increases with \( \Gamma_e \). This is in qualitative agreement with the luminosity dependence of the standard mass-loss recipe of Vink et al. (2000), derived from a set of models with \( \Gamma_e \lesssim 0.4 \). Analogous to the results from the standard Vink et al. (2000) recipe, Fig. 4.1 suggests that there exists an additional mass-loss dependence on mass, as for fixed \( \Gamma_e \) the higher mass stars have larger mass-loss rates. This finding confirms that mass-loss rates cannot solely be described by a dependence on luminosity or Eddington factor. This will be discussed in Sect. 4.4.2.

When comparing columns (8) and (11) from Table 4.1, it can be noted that our new high \( \Gamma_e \) mass-loss predictions tend to be larger than those determined using the standard Vink et al. (2000) recipe. In order to quantify these differences, we divide the new mass-loss rates over those determined using the Vink et al. (2000) recipe (with the derived terminal wind velocities as input), and show the results in Fig. 4.2. For the range \( \Gamma_e \lesssim 0.7 \), the differences are small. However, for values of \( \Gamma_e \) exceeding \( \sim 0.7 \), the new and the old results diverge sharply. The maximum difference reaches a factor of five, which is similar in magnitude as reported previously for LBVs (Vink & de Koter 2002) and WR stars (Vink & de Koter 2005). We note that although these
4. Wind models for very massive stars up to 300 solar masses

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Note: The wind efficiency number $\eta$ is kept constant at 50,000 K. The wind velocity $V_\infty$ is used to compute the mass-loss rate $\dot{M}$ using the formula $\dot{M} = \eta \beta V_\infty^2 \rho_{\text{gas}}$. The last column provides the mass-loss rates as computed using the formula by Vink et al. (2000)
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<td>(1752)</td>
</tr>
<tr>
<td>22</td>
<td>85</td>
<td>6.50</td>
<td>0.987</td>
<td>23.7</td>
<td>1169</td>
<td>133</td>
<td>(1808)</td>
<td>(-3.89)</td>
<td>(3.11)</td>
<td>(2.09)</td>
<td>(1808)</td>
</tr>
</tbody>
</table>

Table 4.1: Continued...
4. Wind models for very massive stars up to 300 solar masses

<table>
<thead>
<tr>
<th>Mass Range III:</th>
<th>[1 - \log_{10} M]</th>
<th>[1 - \log_{10} \dot{M}]</th>
<th>[1 - \log_{10} L]</th>
<th>[1 - \log_{10} R]</th>
<th>[1 - \log_{10} \dot{M}_{\infty}]</th>
<th>[1 - \log_{10} \dot{M}_{\infty}]</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>1.18</td>
<td>0.43</td>
<td>0.02</td>
<td>0.69</td>
<td>0.34</td>
<td>0.75</td>
<td>4.15</td>
</tr>
<tr>
<td>0.94</td>
<td>1.01</td>
<td>0.83</td>
<td>0.21</td>
<td>0.67</td>
<td>0.70</td>
<td>0.97</td>
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<td>0.69</td>
<td>0.86</td>
<td>0.87</td>
<td>0.36</td>
<td>0.61</td>
<td>0.85</td>
<td>0.93</td>
<td>4.23</td>
</tr>
<tr>
<td>0.90</td>
<td>0.82</td>
<td>0.71</td>
<td>0.45</td>
<td>0.60</td>
<td>0.82</td>
<td>0.91</td>
<td>4.24</td>
</tr>
<tr>
<td>0.69</td>
<td>0.67</td>
<td>0.65</td>
<td>0.50</td>
<td>0.59</td>
<td>0.66</td>
<td>0.88</td>
<td>4.30</td>
</tr>
</tbody>
</table>

Continued...
4.4 Results

Figure 4.1: The predicted mass-loss rates versus $\Gamma_e$ for models approaching the Eddington limit. Asterisks, diamonds, and triangles correspond to models of the respective mass ranges I, II, and III. Our model assumptions likely break down to the right of the vertical dashed line.

Figure 4.2: The logarithmic difference between the new $\Gamma_e$ mass-loss predictions and the standard Vink et al. (2000) recipe for models approaching the Eddington limit. Symbols are the same as in Fig.4.1.
prior results were based on global energy consistency, where the velocity stratification was adopted, the reason for the differences revealed in Fig. 4.2 is that we probe an entirely different part of parameter space.

We now turn our attention to the wind velocity structure. We first inspect the associated terminal wind velocity predictions. Figure 4.3 shows the behaviour of terminal wind velocity versus $\Gamma_e$. The highest values are reached for the highest mass stars and exceed 5000 km/s. As expected: $v_\infty$ drops with $\Gamma_e$. In the $\Gamma_e$ range 0.4-0.95, the terminal wind velocity divided over the escape velocity is of the order 3-4, which is similar to the values for common O-type stars (Muijres et al. 2010b), where $T_{\text{eff}}$ is in the range 30-40 kK, and the wind velocities are closer to 3000 km/s (see Sect. 4.4.4).

We next turn our attention to the other wind velocity structure parameter, $\beta$, which describes how rapidly the wind accelerates. The predicted values of $\beta$ are depicted in Fig. 4.4. $\beta$ does not show a significant dependence on stellar mass. For $\Gamma_e$ up to 0.7, $\beta$ values are of order unity, in accordance with the dynamical consistent models of Pauldrach et al. (1986), Müller & Vink (2008) and Muijres et al. (2010b). However, when $\Gamma_e$ exceeds 0.7 and approaches unity, $\beta$ steadily rises to values of about 1.7. These larger $\beta$ values have been suggested to be more commensurate in Wolf-Rayet stars (see e.g. Ignace et al. 2003) and it is reassuring to find that our models naturally predict this transition, without the use of any free parameters. In fact, our overall results suggest a natural extension from O-type mass loss to more extreme WR behaviour for increasing $\Gamma_e$. An upturn in the $\dot{M}$ behaviour is found at $\Gamma_e \sim 0.7$. 

Figure 4.3: The predicted terminal wind velocities versus $\Gamma_e$ for models approaching the Eddington limit. Symbols are the same as in Fig.4.1.
4.4 Results

**Figure 4.4:** The predicted wind velocity structure parameter $\beta$ versus $\Gamma_e$ for models approaching the Eddington limit. Symbols are the same as in Fig.4.1.

### 4.4.2 $\Gamma_e$ dependence of mass loss

In order to determine the dependence of the mass-loss rate on $\Gamma_e$, we could simply fit the data-points to a power law:

$$\dot{M} \propto \Gamma_e^p.$$  \hfill (4.2)

Vink (2006) found $p$ to be equal to \sim 5, and our dynamically consistent results agree. However, in order to take the mass dependence into account, we divide the mass-loss rates by $M^q$. We show the result in Fig. 4.5 and fit the data with the following power-law:

$$\dot{M} \propto M^q \Gamma_e^p.$$ \hfill (4.3)

Figure 4.5 showcases two mass-loss regimes for $\Gamma_e \lesssim 0.95$, and we identify this boundary at $\Gamma_e \sim 0.7$. This is not only the point where the slope of the mass-loss versus $\Gamma$ relation changes, but also where $\beta$ increases and the wind efficiency parameter $\eta$ surpasses the single scattering limit (see below). Upon further inspection of our models we find that as long as $\Gamma_e \lesssim 0.7$, the winds are optically thin, meaning that the sonic point of the outflowing material lies outside the photosphere, whilst the winds become optically thick – with the photosphere moving outside of the sonic point – for $\Gamma_e$ values exceeding $\gtrsim 0.7$.

We determine two mass-loss recipes for the two separate $\Gamma_e$ regimes.
4. Wind models for very massive stars up to 300 solar masses

Figure 4.5: The predicted mass-loss rates divided by $M^{0.7}$ versus $\Gamma_e$ for models approaching the Eddington limit. The dashed-dotted line represents the best linear fit for the range $0.4 < \Gamma_e < 0.7$. The dashed line represents the higher $0.7 < \Gamma_e < 0.95$ range. Symbols are the same as in Fig.4.1.

For $0.4 < \Gamma_e < 0.7$ we find:

$$\log \dot{M} = -5.87(\pm0.08)$$
$$+2.2(\pm0.3)\log(\Gamma_e)$$
$$+0.68(\pm0.11)\log(M*/M_\odot).$$  \hspace{1cm} (4.4)

For $0.7 < \Gamma_e < 0.95$ we determine that:

$$\log \dot{M} = -5.71(\pm0.10)$$
$$+4.77(\pm0.46)\log(\Gamma_e)$$
$$+0.78(\pm0.04)\log(M*/M_\odot).$$  \hspace{1cm} (4.5)

These relationships can easily be transformed using Eq. (4.1). Eq. (4.4) is entirely analogous to $\dot{M} \propto L^{0.68}\Gamma_e^{1.52}$ and Eq. (4.5) to $\dot{M} \propto L^{0.78}\Gamma_e^{3.99}$. Interestingly, if one subsequently applies a mass-luminosity relationship for classical (He-rich) WR stars of Maeder & Meynet (1987) or for very massive H-rich stars such as that of Yungelson et al. (2008), with $L \propto M^{1.34}$ for both cases, it follows that $\dot{M} \propto M^{2.4}$.  

92
This appears to be in good accord with the radio mass-loss rate relation $\dot{M} \propto M^{2.3}$ for classical WR stars with measured masses from binaries by Abbott et al. (1986). It is also in agreement with the $\dot{M}$ versus stellar mass relationship of $\dot{M} \propto M^{2.5}$ that has been applied in WR evolution models by Langer (1989).

### 4.4.3 Increased wind efficiency close to the Eddington limit?

In order to learn whether radiation-driven mass-loss rates continue to increase with increasing $\Gamma_e$ or reach a maximum in $\dot{M}$ instead, it is insightful to consider the wind efficiency parameter $\eta = \dot{M}v_{\infty}/(L/c)$. We show the predicted values of $\eta$ in Fig. 4.6. As the symbols denote different mass ranges, the small scatter on the data-points shows that $\eta$ is not very sensitive to stellar mass. At values of $\Gamma_e \sim 0.5$ we find wind efficiency numbers $\eta$ of order 1, in accordance with standard Vink et al. (2000) models. However, when $\Gamma_e$ approaches unity, $\eta$ rises in a curved manner to values as high as $\eta \approx 2.5$. Such large $\eta$ values are more commensurate with Wolf-Rayet winds than with common O star winds, and these results thus confirm a natural extension from common O-type mass loss to more extreme WR behaviour.

The maximum mass loss in our models up to $\Gamma_e = 0.95$ is $\log \dot{M}_{\text{max}} = -3.8$. This is the mass-loss rate retrieved for the most extreme models in our grid. Owocki et al. (2004) investigated the mass loss of stars that formally exceed their Eddington limit and showed – as is implied by our results – that the expected mass loss falls well below the values required to account for the mass that is lost during LBV giant eruptions, such as that of $\eta$ Carinae in the 1840s. Interestingly, they introduce...
4. Wind models for very massive stars up to 300 solar masses

Figure 4.7: The predicted mass-loss rates versus effective temperatures for several values of $\Gamma_e$, with from top to bottom $\Gamma_e$ equal to 0.90 (model 29; open square), 0.87 (model 28; open triangle), 0.83 (model 25; open diamond), and 0.58 (model 23; asterisk) respectively.

a porosity-moderated *continuum* driven mass loss that might account for the huge mass-loss rates associated with LBV eruptions (which may be of order $1M_\odot$/yr).

### 4.4.4 Effect of $T_{\text{eff}}$ on high $\Gamma_e$ models

In order to establish whether there exists an additional temperature dependence on $\dot{M}$, we varied $T_{\text{eff}}$ over the range 50-30 kK for selected $\Gamma_e$ models, with mass-loss predictions presented in Fig. 4.7 and terminal wind velocities shown in Fig. 4.8. As we wish to stay above the temperature of the bi-stability jump (which starts at $T_{\text{eff}}$ values below $\sim 27.5$ kK; see Vink et al. 2000), we restrict our $T_{\text{eff}}$ range to a minimum value of 30 kK. We find that for all $\Gamma_e$ values $\dot{M}$ is not a strong function of temperature. In other words, for high $\Gamma_e$ models, the influence of effective temperature on the mass-loss rate does not seem to be significant, as long as we refrain from approaching the bi-stability jump. In terms of the terminal velocity dependence, Fig. 4.8 shows a rather steep dependence on temperature, with $v_\infty$ dropping by a factor of two. This is merely a reflection of the fact that the escape velocity drops by a similar factor of two over the temperature range under consideration.

### 4.4.5 Effect of the helium abundance on high $\Gamma_e$ models

To establish the existence of a potential helium dependence on $\dot{M}$, we computed additional models across the entire $\Gamma_e$ region, setting the hydrogen abundance to zero.
4.4 Results

Figure 4.8: The predicted terminal velocities versus effective temperatures for several values of $\Gamma_e$, with $\Gamma_e$ equal to 0.90 (model 29; open square), 0.87 (model 28; open triangle), 0.83 (model 25; open diamond), and 0.58 (model 23; asterisk) respectively.

Table 4.2: Helium enriched mass-loss predictions. All parameters that are not listed are the same as in Table 4.1. The masses have been lowered to keep $\Gamma_e$ the same.

<table>
<thead>
<tr>
<th>model number</th>
<th>$M_{\star,\text{old}}$ [$M_\odot$]</th>
<th>$M_{\star,\text{new}}$ [$M_\odot$]</th>
<th>$\log L$</th>
<th>$\Gamma_e$</th>
<th>$v_\infty$ [km s$^{-1}$]</th>
<th>$\log \dot{M}$ [M$_\odot$ yr$^{-1}$]</th>
<th>$\beta$</th>
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<tbody>
<tr>
<td>2He</td>
<td>60</td>
<td>35.8</td>
<td>6.0</td>
<td>0.43</td>
<td>4552</td>
<td>-5.36</td>
<td>1.12</td>
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<tr>
<td>5He</td>
<td>40</td>
<td>23.1</td>
<td>6.08</td>
<td>0.80</td>
<td>3141</td>
<td>-4.86</td>
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<tr>
<td>10He</td>
<td>85</td>
<td>49.0</td>
<td>6.25</td>
<td>0.55</td>
<td>4567</td>
<td>-5.04</td>
<td>1.21</td>
</tr>
<tr>
<td>14He</td>
<td>85</td>
<td>49.0</td>
<td>6.39</td>
<td>0.77</td>
<td>4144</td>
<td>-4.69</td>
<td>1.40</td>
</tr>
<tr>
<td>24He</td>
<td>120</td>
<td>69.5</td>
<td>6.50</td>
<td>0.70</td>
<td>4800</td>
<td>-4.70</td>
<td>1.32</td>
</tr>
<tr>
<td>26He</td>
<td>180</td>
<td>104.1</td>
<td>6.76</td>
<td>0.85</td>
<td>4759</td>
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</tr>
<tr>
<td>29He</td>
<td>275</td>
<td>159.5</td>
<td>6.97</td>
<td>0.90</td>
<td>4958</td>
<td>-3.99</td>
<td>1.70</td>
</tr>
<tr>
<td>30He</td>
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<td>175.0</td>
<td>7.03</td>
<td>0.95</td>
<td>4934</td>
<td>-3.88</td>
<td>1.75</td>
</tr>
</tbody>
</table>

95
and increasing the helium abundance. The results are listed in Table 4.2 and shown in Fig. 4.9. The mass-loss rates are similar to those of H-rich models for objects with the same $\Gamma_e$ (see Table 4.1). This is not too surprising given that the indirect effects of different continuum energy distributions for H-rich versus H-poor are rather subtle (Vink & de Koter 2002). However, when the He-rich results are plotted in Fig. 4.9 they lie above the H-rich models. For equal luminosity and $\Gamma_e$ the masses of the H-rich models are lower since $\Gamma_e$ is a function of the chemical composition through $\sigma_e$ (see Eq. 4.1); $\sigma_e$ is lower for He-rich models therefore the mass must be lowered to keep $\Gamma_e$ constant. Similar to the H-rich models, there appears to be an upturn in the mass-loss vs. $\Gamma_e$ dependence for models at about $\Gamma_e \sim 0.7$.

With respect to the terminal velocity and $\beta$-dependence, we do not find any significant differences between H-rich and He-rich models (see Table 4.1 versus Table 4.2.).

4.5 Spectral morphology: the characteristic He 4686 Ångström line

In the previous section, we provided evidence for a natural transition in the mass-loss-$\Gamma_e$ exponent, as well as in the velocity parameter $\beta$ and wind-efficiency $\eta$ from moderate $\Gamma_e$ “optically thin wind” cases to “optically thick wind” cases for objects that find themselves above $\Gamma_e \gtrsim 0.7$. We have inspected our models and confirmed that for $\Gamma_e \lesssim 0.7$, the sonic velocity is reached outside the photosphere, whilst for $\gtrsim 0.7$ the stars form a pseudo-photosphere.
4.5 Spectral morphology: the characteristic He 4686 Ångström line

Figure 4.10: The predicted normalized He II λ4686 flux versus wavelength for three values of \( \Gamma_e \), with from top to bottom \( \Gamma_e \) equal to 0.93 (model 20; 90\( M_\odot \)), 0.84 (model 15; 100\( M_\odot \)), 0.70 (model 24, 120\( M_\odot \)) respectively. Note that wind clumping has not been taken into account.

We expect that the occurrence of a pseudo-photosphere has a consequence for the spectral morphology of the stars in question. We might suspect that the transition \( \Gamma_e = 0.7 \) is the point where the spectral morphology of normal O stars changes from the common O and Of-types into a WN-type spectrum. The spectral sequence involving the Of/WN stars has a long history (e.g. Conti 1976, Walborn et al. 1992, de Koter et al. 1997, Crowther & Dessart 1998) but it has yet to be placed into theoretical context. Figure 4.10 shows a sequence for the predicted He II 4686Å lines for three gradually increasing values of \( \Gamma_e \): 0.70 (model 24), 0.84 (model 15), and 0.93 (model 20) respectively. These models have been selected to be objects with a constant luminosity of \( \log(L/L_\odot) = 6.5 \) and we have simply lowered the mass from 120\( M_\odot \) to 100\( M_\odot \) to 90\( M_\odot \). It is insightful to note that although the first spectrum below the transition \( \Gamma_e \) already shows filled-in emission – characteristic for Of stars – the line-flux is rather modest in comparison to that found for the next two cases with \( \Gamma_e \) values exceeding the critical value of 0.7. These objects show very strong and broad He II 4686Å emission lines that are characteristic for full-blown Wolf-Rayet stars of the nitrogen sequence (WN).

These models thus indicate that the observed spectral transition from Of to WN corresponds to a transition from relatively low \( \Gamma_e \) to high \( \Gamma_e \) values (and larger \( \beta \)) for WN stars. This assertion is not only based on the larger predicted mass-loss rates themselves, but also on the finding that at \( \Gamma_e = 0.7 \) the mass-loss behaviour (as a function of \( \Gamma_e \) ) changes.
4. Wind models for very massive stars up to 300 solar masses

4.6 Comparison with empirical mass-loss rates and wind velocities

Comparing our new mass-loss predictions against observed mass-loss rates is a non-trivial undertaking as high $\Gamma_e$ objects are scarce. The largest sample of such potentially high $\Gamma_e$ objects that involves state-of-the-art modelling analysis is probably that of the Arches cluster by Martins et al. (2008). They provided stellar and wind properties (accounting for wind clumping) of 28 of its brightest members from $K$-band spectroscopy. Roughly half of their sample comprises O4-O6 supergiants whilst the other half includes H-rich WN7-9 stars.

It is not possible to quote direct mass-loss predictions, as the Martins et al. analysis did not yield object masses. However, on the basis of the high stellar luminosities, with $\log(L/L_\odot)$ up to 6.3, these objects were suggested to be consistent with initial masses of up to $\sim 120 M_\odot$. For the O4-6 supergiant population, luminosity values are in the range $\log(L/L_\odot) = 5.75-6.05$, consistent with initial masses $M \simeq 55 - 95 M_\odot$. For this mass and luminosity range, $\Gamma_e \simeq 0.2$ – comfortably within our low-$\Gamma_e$ regime. Assuming the current mass of these objects is about the same as their initial mass, our mass-loss formula yields values of $\log \dot{M} \simeq -6.1$, which is in good agreement with the lower end of the Martins et al. mass-loss rates for their O4-O6 I objects.

The second group of Martins et al. objects comprise the WN7-9 objects. If we again assume their current masses can directly be inferred from the observed luminosities, we find $\Gamma_e \simeq 0.4$ and mass-loss rates $\log \dot{M} \simeq -5.3$. Even if the helium abundances of these objects are increased, these properties would not result in a pronounced emission profile of the He II $\lambda4686$ line, i.e. a profile shape that is typical for late-WN stars. For this to happen the mass-loss rates need to be higher by at least a factor of a few. This seems to require a high $\Gamma$. In the framework of our models this could be achieved by lowering the mass. However as the non-electron contribution in our models is not self-consistently treated in the high $\Gamma_e$ regime, and because Crowther et al. (2010) have challenged the stellar parameters of Martins et al., we refrain from providing quantitative assessments.

As the O4-O6 supergiants from the Martins et al. (2008) analysis have effective temperatures in the range 32-40 kK, we expect $v_\infty$ to fall in the range 2500-3500 km s$^{-1}$ (see Fig. 4.8), which is in reasonable agreement with the upper end of the Arches O4-O6 supergiant stars. However, for the late WN stars, the $v_\infty$ values presented by Martins et al. drop to values as low as 800 km s$^{-1}$ which is significantly lower than we predict. We identify two possible reasons for this discrepancy. The first option could be that as a result of our model assumptions (1D, smooth winds, etc.) we over-predict the wind terminal velocity. A second possibility is that the K-band spectral fits of Martins et al. (2008) yield terminal velocities that are too low (note that no ultra violet P Cygni blue edges are available for these obscured objects).
4.7 Discussion

In this section we compare our results with alternative model predictions. In the above we have investigated the mass-loss behavior at high $\Gamma_e$ for an extensive grid of models, and we obtained two mass-loss regimes. Moreover, we found that the mass-loss rate is dependent on $\Gamma_e$ and stellar mass, and that the shape of the dependence is well-described by a power law.

4.7.1 Comparison to CAK and other O-type star mass-loss models

In classical CAK theory, the mass-loss rate is proportional to:

$$\dot{M} \propto L \left( \frac{\Gamma_e}{1 - \Gamma_e} \right)^{1 - \frac{a}{\alpha}},$$

(4.6)

where $\alpha$ is a force multiplier parameter expressing the importance of optically thin lines to the total ensemble of lines. It is generally assumed that $\alpha$ is $\sim 2/3$ for galactic O-type stars (Puls et al. 2008) and constant throughout the atmosphere. In reality $\alpha$ is however depth-dependent (Vink 2000, Kudritzki 2002,Muijres et al. 2010b) which is better captured with an alternative representation of the line acceleration (Müller & Vink 2008). Nevertheless, the classical CAK formalism as described by Eq. 4.6 already shows a dependence on both $L$ and $\Gamma_e$, and one could rewrite this mass-loss dependence as a function of $M$ and $\Gamma_e$ using a mass-luminosity relation.

In the standard Vink et al. (2000) mass-loss parametrization $\dot{M} \propto L^{2.2} M^{-1.3} (v_{\infty}/v_{\text{esc}})^{-1.2}$, which can be re-organized to $\dot{M} \propto \Gamma_e^{1.9} M^{1.2}$. This is the type of mass-loss parametrization that is currently employed in modern evolutionary computations (see e.g Meynet & Maeder 2003, Palacios et al 2005, Limongi & Chieffi 2006, Eldridge & Vink 2006, Brott et al. 2009, and Vink et al. 2010). In other words, stellar models do include the important effect of positive mass-loss feedback (contrary to recent claims by Smith & Conti 2008), which describes how the mass-loss rate gradually increases whilst – as a result of mass loss – the stellar mass decreases. Having noted this, as we here find a much steeper $\dot{M}$ vs. mass dependence, in agreement with our earlier steeper $\dot{M} \propto M^{-1.8}$ for constant-luminosity LBVs (Vink & de Koter 2002, Smith et al. 2004), it is likely that the mass-loss feedback effect currently employed in the stellar evolution models is not strong enough over all areas of the Hertzsprung-Russell diagram. We therefore concur with the notion of Smith & Conti (2008) that new stellar evolution computations that take this effect properly into account are needed.

For the “low” $\Gamma_e$ range considered here, we found $\dot{M} \propto \Gamma_e^{2.2} M^{0.68}$, which is given the somewhat different $\Gamma_e$ range and difference in underlying approach (global energy versus local dynamical consistency) quite similar. However for the “high” $\Gamma_e$
regime we considered here (0.7 < Γ_e < 0.95) we found \( \dot{M} \propto \Gamma_e^{4.77} M^{0.78} \) which is a much steeper dependence on \( \Gamma_e \) than any previous radiation-driven wind model has delivered (see also Vink 2006).

### 4.7.2 Comparison to alternative Wolf-Rayet mass-loss models

We now compare our models to the optically thick wind models for Wolf-Rayet stars by Nugis & Lamers (2002) and Gräfener & Hamann (2008). As there is a significant qualitative difference between our Monte Carlo approach and these optically thick wind approaches, a meaningful quantitative comparison is a non-trivial undertaking (see section 4.3.1).

First we quantitatively compare our \( \dot{M} \) versus \( \Gamma_e \) dependence to the WNL star mass-loss dependence recently suggested by Gräfener & Hamann (2008). For the models in our grid at \( T_{\text{eff}} = 50 \, \text{kK} \), we find very good agreement with the Gräfener & Hamann (2008) mass-loss rates and also find that the power-law slope of our dependence is very similar. However, the onset of WR-type behaviour occurs earlier, i.e. for lower \( \Gamma_e \) in the models by Gräfener & Hamann.

In Sect. 4.3, we discussed the possibility of such a shift in \( \Gamma_e \), because the actual Eddington parameter \( \Gamma \) is expected to be affected by free-free and bound-free contributions and peaks in the iron opacity. By comparison with OPAL opacity tables (Iglesias & Rogers 1996), we estimate an increase of \( \Gamma \) by \( \sim 20\% \) in the region of the sonic point, assuming the location of the sonic point remains unaffected. This value corresponds roughly to a \( \sim 25\% \) shift in \( \Gamma_e \) between our relation and that by Gräfener & Hamann for typical parameters of Galactic WNL stars (\( T_{\text{eff}} = 45 \, \text{kK}, \log(L/L_\odot) = 6.3 \)). Note that this could be considered a maximum shift since we may overestimate the line force near the sonic point by applying the Sobolev approximation (Pauldrach et al. 1986). However, a change in \( \Gamma \) affects the atmospheric structure and therefore the location of sonic point, consequently the effect on \( \dot{M} \) is hard to establish.

The Gräfener & Hamann mass-loss rates also display a strong temperature dependence, with \( \dot{M} \propto T_{\text{eff}}^{-3.5} \) (and an additional strong dependence on the clump filling factor). The actual size of the shift in \( \Gamma_e \) is thus strongly dependent on the specific stellar parameters. Our Monte Carlo models suggest a much smoother dependence on \( T_{\text{eff}} \) (see Fig. 4.7) as long as we stay above the location of the predicted bi-stability jump, where the mass-loss properties jump drastically (Vink et al. 1999, Pauldrach & Puls 1990). What we wish to stress is that both modelling approaches show a \( \dot{M} \) versus \( \Gamma_e \) dependence that is much stronger than any additional mass or luminosity dependence. Where the two distinct mass-loss prescriptions differ is in the \( \Gamma_e \) value for the onset of WR-type mass loss behaviour. The exact location of this transition is of paramount importance for the evolution of the most massive stars. Ultimately, this should be testable with comparisons to observational data when sufficient objects are
available in the appropriate $\Gamma_e$ range. This will be a crucial aim of the Very Large Telescope (VLT) Tarantula survey (Evans et al. 2010).

4.8 Summary

We presented mass-loss predictions from Monte Carlo radiative transfer models for very massive stars in the mass range $40-300M_\odot$ and with Eddington factors $\Gamma_e$ in the range 0.4–1.0. An important outcome is that when winds become optically thick their (physical) properties change. This transitional behaviour can be summarized as follows:

(I) Our modelling suggests a natural transition from common O-type stars to more extreme Wolf-Rayet behaviour when $\Gamma_e$ exceeds a critical value, which is found to be $\approx 0.7$.

(II) The way in which the mass-loss rate depends on $\Gamma_e$ in the range $0.4 \lesssim \Gamma_e \lesssim 0.7$ is $\dot{M} \propto M_*^{0.68} \Gamma_e^{2.2}$, where rates are found to be consistent with the standard Vink et al. (2000) mass-loss rates.

(III) At $\Gamma_e \approx 0.7$ the $\dot{M}$ dependence shows a ”kink”, i.e. the slope is steeper for objects closer to the Eddington limit. Here the slope becomes $\dot{M} \propto M_*^{0.78} \Gamma_e^{4.77}$. This slope is in agreement with WNL models by Gräfener & Hamann (2008).

(IV) When $\Gamma_e$ approaches unity, the wind efficiency number $\eta$ rises in a curved manner to values as high as $\eta \approx 2.5$. Such large $\eta$ values are more commensurate with Wolf-Rayet winds than with common O stars winds, and these results thus confirm a natural extension from common O-type mass loss to more extreme WR behaviour.

(V) This transitional behaviour is also found in terms of the wind acceleration parameter $\beta$, which naturally reaches values as high as 1.5

(VI) The spectral morphology of the He $\pi$ line at 4686Å changes gradually as a function of $\Gamma_e$. This links the spectral sequence O-Of-Of/WN-WN to a transition of optically thin to optically thick winds.

(VII) The mass-loss rate is found to be only modestly dependent on the effective temperature for the range of 30 to 50 kK.