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Sub-Poissonian Atom-Number Fluctuations by Three-Body Loss in Mesoscopic Ensembles

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We show that three-body loss of trapped atoms leads to sub-Poissonian atom-number fluctuations. We prepare hundreds of dense ultracold ensembles in an array of magnetic microtraps which undergo rapid three-body decay. The shot-to-shot fluctuations of the number of atoms per trap are sub-Poissonian, for ensembles comprising 50–300 atoms. The measured relative variance or Fano factor \( F = 0.53 \pm 0.22 \) agrees very well with the prediction by an analytic theory \( (F = 3/5) \) and numerical calculations. These results will facilitate studies of quantum information science with mesoscopic ensembles.

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The study and control of particle number fluctuations in ultracold atomic systems has revealed a rich variety of intriguing quantum phenomena [1–6], and offers the potential to boost performance in cold atom technologies. Motivated by the prospects for quantum metrology [7], recent experiments have demonstrated the suppression of relative fluctuations between small atomic samples distributed over two or more traps or internal states, leading to number difference or spin squeezing and entanglement [8–11]. By contrast, work on suppressing absolute number fluctuations has been limited [12,13]. This is crucial, for example, in quantum information science using mesoscopic atomic ensembles [14–16], where recently observed collective excitations produced via Rydberg dipole blockade [17] could be exploited. Trapped ensembles would benefit from a \( \sqrt{N} \) collective enhancement of the Rabi frequency over single atoms, allowing fast quantum operations. However, intrinsic number fluctuations would adversely affect the fidelity. Suppressed fluctuations would yield robustness against such errors, especially if combined with composite pulse techniques [15,18].

In this Letter we show explicitly that three-body loss naturally reduces the shot-to-shot fluctuations of the absolute atom number in a trap to sub-Poissonian levels. Random particle loss is usually considered deleterious, and it is not generally recognized that random loss can suppress fluctuations, even below the Poisson level. This is the atomic analog to intensity squeezing in optics [19]. We use three-body loss to prepare small and well-defined numbers of atoms in each trap, ultimately enabling the study of collective excitations in mesoscopic ensembles. We trap a large number of dense mesoscopic ensembles in a lattice of microtraps which undergo rapid three-body decay. Through sensitive absorption imaging we measure the shot-to-shot distribution of atom numbers and find sub-Poissonian statistics for between 50 and 300 atoms per trap. The effects of residual imaging noise are greatly reduced through the application of spatial correlation analysis which exploits the lattice geometry and provides a way to isolate atom-number fluctuations. Our results are in very good agreement with a model for stochastic three-body loss which takes into account the fluctuations.

For ultracold gases in magnetic microtraps, inelastic density-dependent decay is the dominant loss process. In \(^{87}\text{Rb}\) this is typically due to three-body recombination [20], whereby all three atoms are lost from the trap. As this depends on the probability of finding three atoms together, three-body recombination is a sensitive probe of density fluctuations and correlations in degenerate Bose gases [21–23]. This previous work involved the macroscopic evolution of the mean number of remaining atoms, which decays proportional to the mean square density.

We are primarily interested in the fluctuations in the number of remaining atoms. We model this with the following master equation for the probability distribution \( P(N, t) \)

\[
\frac{dP(N, t)}{dt} = \sum_{\rho=1,2,3} \frac{k_{\rho}(\mathbb{E}^\rho - 1)}{\rho N_0^{\rho-1}} \frac{N!}{(N-\rho)!} P(N, t),
\]

which is valid for any birth-death process with multiple reactions involving \( \rho \) bodies [24]. Here \( N_0 \) is the initial mean atom number in a given trap, \( k_{\rho} \) are the scaled rate constants, and the step operator \( \mathbb{E}^\rho \) changes \( N \rightarrow N + \rho \). Equation (1) is a set of coupled differential equations, one for each possible value of \( N \). For small systems involving up to a few hundred atoms, these equations can be solved numerically to provide the full atom statistics (including fluctuations) as a function of time.

In our experiments \( k_2 = 0 \) and \( k_3 \gg k_1 \). For a nondegenerate gas at temperature \( T \) in a harmonic trap, \( k_3/N_0^3 = (2L_3/\sqrt{3})(\hbar \omega/2\pi k_B T)^3 \), where the mean trap frequency in our case is \( \hbar = 2\pi \times 10.0 \pm 0.5 \) kHz. The three-body rate constant is \( L_3 = 1.8(\pm 0.5) \times 10^{-29} \text{cm}^6/\text{s} \) for the \( J = m_F = 2 \) hyperfine state of \(^{87}\text{Rb} \) [22].

For the mean and variance of the distribution we can obtain approximate analytic expressions. Following [24] we perform a system size expansion for \( N_0 \gg 1 \) to obtain a linear Fokker-Planck equation and derive equations of motion for the moments. For combined one-body and...
three-body loss the evolution of the mean fraction of remaining atoms is

$$\eta = \frac{\langle N \rangle}{N_0} = \frac{\exp(-k_i t)}{\sqrt{1 + (k_3/k_1)[1 - \exp(-2k_i t)]}}. \quad (2)$$

We express the fluctuations in terms of the Fano factor, $F = \langle \langle N^2 \rangle - \langle N \rangle^2 \rangle / \langle N \rangle$, where the averages are taken over realizations ($F = 1$ for a Poisson distribution). The evolution of $F$ in time can be written as a function of $\eta$, leading to the differential equation:

$$\frac{dF}{d\eta} = \frac{k_3 \eta^2 [5F(\eta) - 3] + k_i [F(\eta) - 1]}{\eta(k_1 + k_3 \eta^2)}. \quad (3)$$

In the case where three-body loss dominates, we obtain the simple solution

$$F(\eta) = \frac{3}{5} + \eta^4 \left( F_0 - \frac{3}{5} \right). \quad (4)$$

where $F_0 = F(\eta = 1)$ is the initial Fano factor. As the atoms are lost from the trap the Fano factor asymptotes to a value of $F \rightarrow 3/5$, significantly below the Poissonian level $F = 1$. The memory of the initial Fano factor is lost very rapidly due to the fifth power of $\eta$, in contrast to one-body loss where $F = 1 + \eta(F_0 - 1)$. The result is easily generalized to an arbitrary $p$-body process yielding an asymptotic Fano factor $F \rightarrow r/(2p - 1)$. The results of this simple analytic model are in excellent agreement with the numerical solution to Eq. (1) for $\langle N \rangle \geq 10$.

Our experiment incorporates a two-dimensional lattice of optically resolvable magnetic microtraps produced by a magnetic film atom chip [25,26]. We load a few thousand atoms into each of about 250 traps, and evaporatively cool to the high trap depth, the role of heating-induced loss on the differential equation:

$$\frac{dF}{d\eta} = \frac{k_3 \eta^2 [5F(\eta) - 3] + k_i [F(\eta) - 1]}{\eta(k_1 + k_3 \eta^2)}.$$
atom number. In this way we determine a constant absorption
cross section of \((0.32 \pm 0.05)\sigma_0\) \((\sigma_0 = 3\lambda^2/2\pi)\), in
good agreement with the expected cross section of \(0.31\sigma_0\)
based on our imaging parameters. The maximum optical
depth for a trap containing 250 atoms is \(\sim 0.1\).

Figure 2 shows the measured atom-number statistics for
various hold times, corresponding to different mean atom
numbers in each trap. A histogram of the fitted number of
atoms in one specific trap for 19 repetitions of the experi-
ment at \(t = 25\) ms is shown in Fig. 2(a). The measured
\(\langle N \rangle = 280 \pm 3\) and the variance is \(\langle N^2 \rangle - \langle N \rangle^2 = 140 \pm 50\), indicated by the Gaussian distribution (solid line). The
distribution is significantly narrower than for a Poisson
distribution (dashed line), providing a direct observation
of sub-Poissonian number statistics. For longer hold times
[Fig. 2(b)] the mean number of atoms decreases due to
loss; however, the observed distribution does not become
significantly narrower. This is due to the added detection
noise contribution (dash-dotted line) which begins to
dominate the observed fluctuations for \(\langle N \rangle \leq 60\).

The same analysis is performed for each site and each
hold time independently to obtain the site-resolved relative
variance as a function of the mean number of atoms. Figure 2(c) shows the results of 245 \(\times\) 40 observations

\[
\frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2} = \frac{\sigma^2}{\langle N \rangle^2}
\]

where each point is derived from 19 measurements. The
observed fluctuations have two main contributions: atom
noise with a constant \(F\) (Poisson noise is indicated by a
dashed line) and a detection noise contribution correspond-
ing to a fixed variance of 64 atoms\(^2/\)trap/shot (dotted line).
We find for \(N \geq 100\) the vast majority of data points
fall well below the combined variance for Poisson fluctua-
tions (dash-dotted line) indicating \(F < 1\). Interestingly,
the deviation from Poisson statistics is most apparent for small
hold times (large \(\langle N \rangle\)), indicating three-body loss also has a
significant effect on the fluctuations before the end of the
evaporative cooling stage.

To account for detection noise and to investigate the sub-
Poissonian noise over the full range of atom numbers in our
experiment, we perform spatial correlation analysis of the
images. Here we benefit from the lattice geometry and
separate noise components based on their respective cor-
relation length scales to isolate the atom fluctuations in our
data. We compute, for each optical density image, the two-
dimensional fluctuation correlation function
\(\chi(x,\delta) = \int [n_i(x) - \langle n_i(x) \rangle] [n_i(x+\delta) - \langle n_i(x+\delta) \rangle] d^2x\),
which is then averaged over the realizations of the experiment
(indexed by \(i\)) for a given hold time (Fig. 3, inset). We
model the observed spatial distribution by
\(n_i(x) = c_i \sum_m N_{i,m} P_m(x) + d_i(x)\), where \(N_{i,m}\) and \(P_m(x)\)
are the number of atoms and local shape function, respectively,
for ensemble \(m\), \(c_i = 1\) accounts for correlated noise (due,
for example, to probe frequency noise), and \(d_i(x)\) accounts
for spatially uncorrelated imaging noise. The correlation
function \(\langle \chi(\delta) \rangle\) shows several distinct features (Fig. 3,
inset). A narrow spike at \(\delta = 0\) (central red pixel) repre-
sents the uncorrelated imaging noise. This sits on top of a

\[
F = \frac{\sigma^2}{\langle N \rangle^2}
\]

FIG. 2 (color online). Atom-number fluctuations measured for
each of 245 lattice sites during three-body decay. (a),(b) Number
distributions for one specific trap \((m = 138)\) at two hold times.
Histograms correspond to 19 measurements and each bin is 5
atoms wide. The lines indicate Gaussian fits to the data (solid
line), Poisson distributions (dashed line), and combined Poisson
and detection noise contributions (dash-dotted line). (c) The
relative variance versus \(\langle N_m \rangle\) for each lattice site and for each
hold time (points). Open circles indicate the measurements for
the selected trap, with a fit (including detection noise) for a
constant Fano factor \(F = 0.57\) (solid line). Arrows highlight the
two data points corresponding to the histograms (a),(b).

FIG. 3 (color online). Lattice-averaged Fano factor \(F\) as a
function of the mean number of atoms \(\langle N \rangle\). Horizontal lines
 correspond to \(F = 1\) (dashed line) and to \(F = 3/5\) (dotted line)
for strong three-body loss. The solid line is a model including
three-body and one-body loss terms. The shaded region indicates
systematic uncertainties described in the text. The inset shows an
example fluctuation correlation function \(\langle \chi(\delta) \rangle\) for \(t = 25\) ms.
broader peak (dark central feature) representing the fluctuations correlated over the length scale of approximately a single cloud which accounts for shot-to-shot fluctuations of the number of atoms within each trap. An array of neighboring peaks, spaced at the lattice period, represents the correlated noise across traps which we attribute to small fluctuations of the probe detuning.

In the analysis of \( \langle \chi(\delta) \rangle \), we subtract the calculated background-region correlation function and exclude the \( \delta = 0 \) pixel. We then fit two-dimensional Gaussian distributions to the central and neighboring correlation peaks. The lattice-averaged Fano factor (weighted by \( \langle N_n \rangle \)) is given by \( \tilde{F} = \left( \sum_n \langle N_n^2 \rangle - \sum_n \langle N_n \rangle^2 \right) / \sum_n \langle N_n \rangle \). Neglecting the small overlap between neighboring shape functions and noting that the fluctuations of \( N_{i,m} \) are uncorrelated between different traps, we obtain

\[ \tilde{F} = \frac{X_0 - X_A}{X_A + P_0} \langle \tilde{N} \rangle, \]

(5)

where \( X_0 \) and \( X_A \) are the fitted volumes of the central and neighboring peaks of \( \langle \chi(\delta) \rangle \), respectively, \( P_0 \) is the fitted volume of the preaveraged autocorrelation function peak \( \int (n_i(x) \langle n_i(x+\delta) \rangle) d^2x \), and \( \langle \tilde{N} \rangle \) is the weighted average atom number. Accounting for the overlap between neighboring shape functions yields a small correction factor, which for our lattice geometry is \( \approx 1.1 \).

Figure 3 shows the extracted Fano factor for two separately analyzed data sets as a function of \( \langle \tilde{N} \rangle \) during the hold time. Horizontal lines correspond to the Poissonian limit \( \tilde{F} = 1 \) (dashed line) and to the expected limit \( \tilde{F} = 3/5 \) (dotted line) for strong three-body decay. The data show sub-Poissonian atom-number fluctuations for \( \langle \tilde{N} \rangle \approx 50 \) up to 300 atoms per site. A fit over this range indicates \( \tilde{F} = 0.53 \) with a standard deviation of \( \pm 0.08 \). We independently estimate a systematic uncertainty of \( \pm 0.2 \) incorporating uncertainties in the absorption cross section, background noise contribution, and the overlap between neighboring traps. The measured fluctuations are clearly below the Poissonian level (dashed line) and in good agreement with the theoretical expectation of \( \tilde{F} = 3/5 \) (dotted line). For \( \langle \tilde{N} \rangle \approx 50 \) one-body loss dominates and we expect \( \tilde{F} \) to increase to 1. The solid line is the result of Eq. (3) including both three-body and one-body loss terms.

In conclusion, we have shown that normally undesirable density-dependent losses in small atomic ensembles naturally lead to suppressed fluctuations of the absolute atom number to below Poissonian noise levels. Three-body decay is a simple method to reliably prepare many well-defined ultracold ensembles comprising tens to a few hundreds of atoms. We expect this to be an ideal system for the study of collective excitations produced, for example, via laser-excited Rydberg states for quantum information processing with neutral atoms [14–17]. Such ensembles also have desirable properties for generation of many-particle entangled states [16] and states with a squeezed internal (spin) variable. Spin-squeezed samples currently receive great interest as a resource for cold atom-based metrology (interferometers, clocks), aiming to beat the standard quantum limit [10,11]. We thank N.J. van Druten and J.T.M. Walraven for fruitful discussions. We are grateful to FOM and NWO for financial support. S.W. acknowledges support from a Marie-Curie fellowship (PIIF-GA-2008-220794).

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