Factors that influence the development of students' regulating activities as they collaborate in mathematics
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Factors that Influence the Development of Students’
Regulating Activities as they Collaborate in
Mathematics

Sonia Kafoussi, Petros Chaviaris & Rijkje Dekker

Abstract
This paper is focused on the investigation of factors that influence students’ self-regulation as they try to develop regulating activities when they collaborate in pairs in mathematics. We investigated the issue of how 10-11 year old students regulated their behavior during their mathematical activity as they reflected on their small-group interaction by observing and discussing their video-recorded collaboration. We studied the collaboration and the metadiscursive reflection of different cases of pairs. The results showed that the students’ development of their self-regulation is a complex process, as it is influenced by their beliefs about the role of the others, their beliefs about the role of collaboration in mathematical learning, the occurrence and the treatment of errors and disagreements as well as the difference of students’ cognitive levels.

Keywords: collaborative learning; mathematics; metadiscursive reflection; primary education; regulating activities
INTRODUCTION

Recently, the role of students’ self-regulation as they are engaged in mathematical activities has begun to gain a lot of interest (cf. Marcou & Lerman, 2006), although self-regulation has a long history in educational psychology (Fox & Riconscente, 2008). Self-regulation has been connected with metacognition, but nowadays researchers are trying to define these constructs more precisely (Dinsmore et al., 2008). Metacognition is described as cognition about cognition, that is, it concerns the awareness of an individual about his/her knowledge and the regulation of his/her cognitive activities in learning processes (Veenman et al., 2006). On the other hand, self-regulation concerns a “systematic process of human behavior that involves setting personal goals and steering behavior towards the achievement of established goals” (Zeidner et al., 2000, p.749, cf. Dekker et al., 2006) and it does not necessarily require that individuals are aware of the processes involved, as it involves motivational and socio-emotional processes. Zimmerman (1995) has mentioned that the interpretation of students’ self-regulation has to be treated as a complex interactive process influenced by students’ self-beliefs-system and “SRL (Self-Regulated Learning) involves more than metacognitive knowledge and skills, it involves a sense of personal influence, such as emotional processes as well as behavioural and social-environmental sources of influence” (p. 218).

Researches in mathematics education concerning self-regulation have mainly been developed in social contexts where the students have the opportunity to engage collaboratively in mathematical tasks (collaborative learning, students’ social interaction in the class) and discourse, as these socio-cultural settings facilitate the developing of these activities. They have shown that students regulate their own collaborative learning in the classroom setting according to their commitments, conceptualizations and strategies and there is some evidence that students can regulate their own collaborative learning activities (cf. Dekker, Elshout & Wood, 2006). However, more research is needed in order to clarify the conditions that
allow the development of self-regulation in collaborative learning in a classroom setting.

The purpose of this research is to investigate factors that influence students’ regulating activities when they try to collaborate in pairs. More specifically, we investigated the issue of how 10-11 year old students regulated their behavior during their mathematical activity as they reflected on their small-group interaction by observing and discussing their video-recorded collaboration. Critical moments of students’ collaboration in mathematics are discussed and the opportunities that students’ meta-discursive reflection offered to their self-regulation are presented. The study is focused on different pairs of students representing different self-belief systems.

THEORETICAL BACKGROUND

As the mathematical activity is considered as a process that takes place in a macro- and a micro-community with concrete socio-cultural characteristics, students’ awareness of their actions in mathematics has to be related not only with the reflection on cognitive aspects of their activity but with social and cultural aspects of it, too. Nowadays, the reduction of the student to a “cognitive subject” seems to be replaced by the acknowledgement of the student as a “social subject” that is influenced by his/her history and culture (Lerman, 1998; De Abreu, 2000; Valero, 2004).

Dekker & Elshout-Mohr (1998) have described an effective process model for interaction and mathematical level raising, of students working in small groups. Moreover, it has been mentioned that if the students often follow this model, they attain more mathematical level raising (Dekker & Elshout-Mohr, 2004; Pijls et al., 2007). The main activities described in the model are key and regulating activities. The key activities are crucial for level raising. The regulating activities provoke the key activities and in that way they regulate the process of level raising. We can represent them in the following way (Table 1):
Factors that Influence the Development of Students’ Regulating Activities as they Collaborate in Mathematics

**Table 1. Regulating activities**

<table>
<thead>
<tr>
<th>Regulating activities like:</th>
<th>Provoke key activities like:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A asks B to show his work</td>
<td>B shows his own work</td>
</tr>
<tr>
<td>A asks B to explain his work</td>
<td>B explains his own work</td>
</tr>
<tr>
<td>A criticizes B’s work</td>
<td>B justifies his own work</td>
</tr>
<tr>
<td></td>
<td>B reconstructs his own work</td>
</tr>
</tbody>
</table>

All the above activities can help children to become aware of their work and enhance their self-regulation. However, a participant of an interaction in the mathematics classroom “monitors his or her action in accordance with what he assumes to be the other participants background, understandings, expectations… At the same time the other participants make sense of the action by adopting what they believe to be the actor’s background, understandings, and intentions.” (Voigt, 1995, p. 169). As a consequence, every member of a mathematics class tacitly participates in his/her own way in interacting with others according to his/her personality (interests, expectations, intentions, and beliefs).

Moreover, as Sfard (2001) has mentioned students’ initiation to mathematical discourse depends on the “meta-discursive rules that regulate the communicative effort” (p.28). These rules are considered as the implicit regulators of interpersonal and intra-personal communication, as they determine the choices of the participants when they act and they embed their values and beliefs. She has emphasized the role of the interlocutors’ intentions in a mathematical discussion using the term “meta-discursive intentions” in order to describe the “interlocutors’ concerns about the way the interaction is being managed and the issues of the relationship between interlocutors” (p. 39).

In mathematics education, many researches investigating social interaction in classroom have been focused on cooperative learning contexts concerning small groups of students. These researches have revealed a lot of factors that influence students’ mathematical learning like group
composition, students’ beliefs about their cooperation in mathematics, students’ achievement in mathematics, the quality of mathematical activities (e.g. Good et al., 1992; Edwards, 2002; Kieran, 2001; Webb, 1989).

In our research we assumed that students can give their own explanations about their behavior as they try to collaborate in small groups in a mathematics classroom. In most researches the teacher has played a significant role in establishing the social and socio-mathematical norms of students’ collaboration (McClain & Cobb, 2001; Dekker & Elshout-Mohr, 2004). On the other hand, the teacher can not really know what the children, as they work in pairs, may discuss. Furthermore, if the students do not collaborate systematically during their mathematical activities, can they develop their regulating activities and take responsibility for the quality of their shared activities? How do the children think on their own about their collaborative learning?

Towards this effort we used the term “metadiscursive reflection” in order to describe this kind of students’ reflection that is related with the consciousness of relationships among cognitive, social and emotional components of their mathematical discourse. Metadiscursive reflection concerns students’ reflection on their own and their interlocutors’ beliefs and intentions about their social interaction and it is revealed through their explanations and justifications about their behavior. Our questions in this research are:

a) What factors influence the occurrence of each regulating activity as students try to collaborate in mathematics?

b) What are the critical situations of students’ metadiscursive reflection that allowed the development of these regulating activities for both partners?

**METHOD**

The research program took place in a fifth grade of a typical public school of Athens in Greece, in 2003–2004 and it lasted six months. The
participants were 18 students (9 boys and 9 girls) that worked in pairs, 4
times per week during math class teaching. The mathematical topic, in
which the students were engaged during the research program, concerned
the concept of fractions (equivalence, comparison and the four operations).
The activities about fractions have been given by the researchers in order to
be meaningful for the students according to the related literature
(Kieren,1992; Streefland,1991) and the students’ initial knowledge of
fractions. The research program was developed in three phases.

Initially, we studied the students’ profiles in order to organize them in
pairs. All the students were interviewed by the researcher about their beliefs
of their own participation and the others’ participation during the classroom
mathematical activity as well as about the nature and the goals of
mathematical activity (e.g. When do you feel really pleased in mathematics?
How do you feel when you make an error in mathematics? Do your
classmates help you in mathematics?). Every interview lasted about one
hour. Moreover, we investigated the students’ informal knowledge on
fractions using a questionnaire with mathematical problems on this topic.
Finally, the teacher of the class was asked to assess her students in
mathematics based on her personal evaluation by using the criterion of the
student’s need for help in order to solve a mathematical problem (He/she
managed in mathematics – He/she managed in mathematics but sometimes
with help – He/she managed in mathematics only with help) as well as to
provide the students’ grades in mathematics of the previous school year.

We based our research on a patchwork case-studies method (Jensen &
Rodgers, 2001), studying our cases horizontally. According to this method a
set of multiple cases of the same research entity (in our research we define
as entity a pair of students) allows a deeper and more holistic view of the
research subject.

The criteria for the organization of the students in pairs are presented in
the following table:
Criteria for the organization of pairs

<table>
<thead>
<tr>
<th>Criteria for the organization of pairs</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative beliefs about collaboration in mathematics</td>
<td>N</td>
</tr>
<tr>
<td>Positive beliefs about collaboration in mathematics</td>
<td>P</td>
</tr>
<tr>
<td>He/she managed in mathematics</td>
<td>1</td>
</tr>
<tr>
<td>He/she managed in mathematics but sometimes with help</td>
<td>2</td>
</tr>
<tr>
<td>He/she managed in mathematics only with help</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2. Criteria for pairs

On table 2 the notion “negative beliefs” was used to describe the students’ responses like: “I would like to solve alone the problems in mathematics” or “I would like the teacher to help me in order to solve the problems”.

The different cases of pairs that arised from this class are presented in table 3. As there were similar pairs of students in some cases, in our research we studied one pair of each case. The choice of the pairs was accidental.

<table>
<thead>
<tr>
<th>Case</th>
<th>Pairs’ profile</th>
<th>Number of pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N1 - P2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>N1 - N2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>N2 - N2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>P1 - N3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>P2 - P2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3. Pairs’ profiles

In the second phase of the program, the students’ collaboration was videotaped once a week for every pair in the class by the researcher(R) and then the members of the group participated in a session with him. These meetings with each group took place in the school library, immediately after the lesson in their regular classroom and they lasted about 30 minutes every time. Each group realized six meetings with the researcher. During this
session, the students observed and discussed issues concerning their video-recorded collaboration. The researcher had a role of coordinator during the students’ discussions. He clarified the context of these discussions by reminding them the special issues that they had to discuss, like the assessment of their collaboration, the significant moments of their work or their desires for the improvement of their interaction. These discussions were tape-recorded. On this meta-discursive level, the tape-recorded students’ discussions about their own videotaped collaboration were analyzed according to: (1) the way that the students assessed their collaboration (self-assessment), (2) the moments of their interaction that they considered as critical and (3) the targets and their behavior in their next collaboration (self-regulation). We will base on the protocols of 11 years old students’ dialogues (as they observed their videotaped collaboration) as evidence for the development of the regulating activities.

In the third phase, the members of each pair were interviewed again about their beliefs of their own and the others’ participation in mathematics as well as the nature and the goals of mathematical activity.

RESULTS
We should note that, according to the data from the initial interviews that were conducted in this class, the students that expressed the wish to collaborate with their classmates, were usually average or low achievers, and the students that expressed the wish to work alone, were usually high achievers (according to the criteria described in the method). Furthermore, the students’ justifications about their preferences revealed the following beliefs that prevented the development of collaboration in mathematics: a) mathematical knowledge is acquired with personal effort, b) different ideas in mathematics cause confusion and create difficulties in understanding, c) classmates’ errors in mathematics negatively influence pupil’s thinking and prevent their learning and d) the exposition of a pupil’s thinking to his/her classmates does not protect his/her self-image (Chaviaris et al., 2007). We
must mention that the students had not worked in groups in mathematics in previous years and their teacher was following a traditional approach in this subject (cf. Cobb et al., 1992).

In describing our results we will try to separate the three regulating activities in order to clarify the factors that influence the occurrence of each one as well as the students’ comments about them. Towards this effort, we will present illustrative episodes from the collaboration and the metadiscursive reflection of different pairs.

The first regulating activity “A asks B to show his/her work to her/his partner”

In order to reveal the factors that influenced the occurrence of students’ regulating activity “A asks B to show his/her work to her/his partner” using questions like: What are you doing?, we present and analyze two illustrative episodes of different pairs of students as they made their first efforts to collaborate in mathematics and as they reflected on their collaboration.

**Episode 1:**

Paul(P) and Nikos(N) had expressed negative beliefs about collaboration in mathematics (N1-N2) and Paul seemed to manage better in mathematics than Nikos. During their second mathematical activity, they had to solve the following problem:

Put the fractions $\frac{1}{3}$, $\frac{2}{6}$, $1\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{6}$ on the following number line.

![Figure 1.](Image)

Their dialogue as they tried to collaborate was the following:
Factors that Influence the Development of Students’ Regulating Activities as they Collaborate in Mathematics  

<table>
<thead>
<tr>
<th>utterances</th>
<th>writings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N: 1/3 will be here? (He is indicating the interval 0-1)</td>
</tr>
<tr>
<td>2&lt;sup&gt;a&lt;/sup&gt;</td>
<td>P: 1,2,3. (He is dividing the interval 0-1 by his finger)</td>
</tr>
<tr>
<td>2&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Then, it is here. (He is indicating the first third of the interval 0-1).</td>
</tr>
<tr>
<td>3</td>
<td>P: The 2/6 are equal with the 1/3, then it is at the same point.</td>
</tr>
<tr>
<td>4</td>
<td>N: Where?</td>
</tr>
<tr>
<td>5</td>
<td>P: Here, with 1/3.</td>
</tr>
<tr>
<td>6</td>
<td>P: The 1 ½ … in the middle after the point 1.</td>
</tr>
<tr>
<td>7</td>
<td>P: And 3/2 will be in the same point because they are equal.</td>
</tr>
<tr>
<td>8</td>
<td>N: Let me look. (He is looking at Paul’s worksheet)</td>
</tr>
<tr>
<td>9</td>
<td>P: Now, 5/6…</td>
</tr>
<tr>
<td>10</td>
<td>N: 5/6 will be somewhere about here? (He is indicating the interval 1-2)</td>
</tr>
<tr>
<td>11</td>
<td>P: They don’t pass over the point 1. So will be 1,2,3,4,5, here.</td>
</tr>
</tbody>
</table>

In the above dialogue, Nikos was not sure about the position of the fractions on the number line and he was trying to challenge Paul in order to show him his work (cf. ut. 1, 4, 8, 10). Paul showed his work without any question if Nikos could understand his thoughts. When they observed their video-taped collaboration, the following discussion took place:

[1] R: We’ll watch the video with your second collaboration in mathematics and after we’ll discuss about it.
[2] R: How was your second collaboration in relation with the first one?
[3] N: We collaborated more. In the first collaboration each one of us solved the problem alone.
[4] P: Yes. It was more collaborative than private.
[5] R: How did you collaborate?
[6] N: We talked to each other and…
[7] P: I think that I spoke more because he had some difficulty and I explained to him.
[8] N: Yes. I asked him because I hadn’t understood the number line.
[9] R: What did each of you do in this collaboration?
[10] P: I solved the problem in order to be on time and after I showed the solution to him.
[11] N: Paul knows more mathematics than me and he knew that I would ask him, so he finished quickly and after he showed me.
[12] R: Nikos, how did you feel about Paul’s behavior?
[13] N: Good. If I ask for help, it’s good. If someone helps me without my request, I become angry.
[14] R: Did you understand the number line after Paul’s help?
[16] R: Did you do something for this?
[17] N: What could I have done?
[18] P: You did not ask me. I could explain to you more.
[19] R: Ok. What would you like to change about your collaboration in mathematics?
[20] P: To discuss the problem more from the beginning.
[21] N: Yes. It is better to ask each other some explanations when we don’t understand the problem.
[22] R: Nice. We will meet again in a few days.

During the discussion Nikos justified his behavior (that is to ask Paul to show him his work) according to his beliefs about his interlocutor’s mathematical abilities (cf. phrase 11). Moreover, he seemed to accept that Paul’s role was to solve the problem alone and then to show him the solution. On the other hand, Paul has also accepted that role (cf. phrases 7, 10, 18). These perceptions about their roles during the solution of a
Factors that Influence the Development of Students’ Regulating Activities as they Collaborate in Mathematics

mathematical problem influenced the occurrence of the first regulating activity from the part of Nikos at the beginning of their interaction.

Episode 2:
Stavroula (S) and Alexia (A) had expressed different beliefs about the role of collaboration (N1-P2). Stavroula considered the collaboration to be an obstacle in the understanding of mathematics, because she believed that “if someone doesn’t work on his own, he cannot understand mathematics”. On the contrary, Alexia believed that collaboration could help her to control her thoughts before she announced them in the classroom and so that she could “avoid mistakes”.

At the beginning of the program, the children had to solve the following problem:

In Alexandra’s Avenue, public works are being made by 3 different firms of constructors. The works are being made at three different points. The first firm of constructors makes works at a point corresponding to the 1/3 of the avenue, if we count from its beginning. In the ¾ of the avenue there are works of the second firm of constructors and in the 5/6 of the avenue there are works of the third firm of constructors. Note in the following schema where the works are being made. Use red color for the first point, green for the second one and blue for the third one.

Figure 2.

<table>
<thead>
<tr>
<th>Utterances</th>
<th>writings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A: (She is reading the problem)</td>
</tr>
<tr>
<td>2</td>
<td>S: So…</td>
</tr>
<tr>
<td>3</td>
<td>S: 1/3 of the avenue…Which is 1/3?</td>
</tr>
<tr>
<td>4</td>
<td>A: Calm yourself.</td>
</tr>
<tr>
<td>5</td>
<td>A: Do you want to discuss it?</td>
</tr>
</tbody>
</table>
S: We have to count with a ruler.
A: Just a moment, \( \frac{1}{2} \) is the half and we have to share it in the middle, like yesterday. \( \frac{1}{3} \)?

S: This line is common for all the constructors.
1,2,3…Here is for the first firm.
Now, about the second firm with green color, \( \frac{3}{4} \)…
S: Did you finish with the first one?
A: I am confused, what did you do?
S:  Do you want me to help you?
1 cm for the first, and three more cm for the second …
Now we will see the third one… Ok, 5 more cm for the third.

[8b]She notes with the ruler 1 point on the line (distance 1 cm).
[11b] She notes with the ruler 3 more points on the line (distance 1 cm).

Alexia asked her partner to collaborate with her (cf. ut. 5, 10). However, the priority of Stavroula was to solve the problem by her own way. Her reaction “Do you want me to help you?” (cf. 11a) showed that her intention was not to find an acceptable solution by collaborating with her partner.

During their discussion as they reflected on their collaboration they made the following comments:

[1] S: When you find the solution and it is right, you don’t have to discuss it with your partner, because she may have a different opinion and she will confuse you.
[2] A: It is better to discuss it, because if it is wrong, you will think: why didn’t I ask?
[4] S: I told her to put centimeters, but Alexia told me to divide the whole in pieces. We made it wrong.
[5] R: The solution on the board with whom solution did it match?
[7] S: Ok, it is good to collaborate, but if you don’t find the solution alone, you
As the students reflected on their actions, Alexia justified her behavior (that is, to ask Stavroula to discuss with her about the problem) as she believed that in this way they could prevent mathematical errors (cf. phrase 2). On the other hand, Stavroula explained her behavior according to her beliefs about the negative consequences of collaboration in mathematics (cf. phrase 1).

Discussing on the above episodes we could mention that the occurrence of the first regulating activity (A asks B to show his/her work to her/his partner) was found to be influenced by two factors: a) the established roles that the partners perceived at the beginning of their interaction (who was the helper and who needed help) according to their beliefs about their own and their partner’s ability in mathematics (who considered him/herself as a good student in mathematics and who considered the opposite) and b) the students’ beliefs about the role of collaboration in mathematics.

The first factor was connected with students’ beliefs about their own and their partners’ learning ability in mathematics which influenced their behavior during their effort to cooperate. It was easier for the student that had lower self-estimation to perform this regulating activity (c.f the case of Nikos in episode 1). Paul’s and Nikos’ perceptions about their concrete roles in their interaction in the mathematics classroom provoked relationships of power among them and defined the way of their communication. In the regulating activity A asks B to show his/her work the student A seemed to be the one who needed help and the student B the other who was the helper.
The second factor that seemed to influence the occurrence of the first regulating activity was connected with the students’ beliefs about the role of collaboration in mathematics. Alexia who wanted to collaborate performed this regulating activity more usually in contrast of her partner who had negative beliefs (c.f episode 2). In the case of school mathematics the willingness to collaborate is important, because of the social dominant belief that learning of mathematics is an individual process and that the social interaction does not play any significant role in it (Chaviaris, 2006). As it was been revealed in students’ metadiscursive reflection the conflict of beliefs about the role of collaboration in mathematics defined the way that the student regulated their interaction.

Furthermore, we should note that although the students in the above episodes posed new targets for their next collaboration, they sometimes presented the same behavior during it. That is, they experienced a lot of difficulty in order to regulate themselves and to change their actions. The critical situation that helped the development of the first regulating activity for both students for the different pairs occurred when they experienced the effectiveness of their partner’s suggestion, if the solution of the “good” student was incorrect. These moments were discussed during their metadiscursive reflection, like in the dialogue presented above between Stavroula and Alexia. These topics for reflection helped mainly the “good” students to appreciate the efforts of their partner to contribute to the dialogue.

The second regulating activity “A asks B to explain his/her work to his/her partner.”

The process of explanation is considered as a significant process for the development of student’s mathematical reasoning and has substantial contribution in the development of students’ collaboration (e.g. Cobb & Bauersfeld, 1995). In the following we present and analyze two episodes concerning the conditions of the occurrence of the students’ request for
Factors that Influence the Development of Students’ Regulating Activities as they Collaborate in Mathematics

Episode 3:

Paul and Nikos (N1-N2) tried to collaborate as they were engaged in the following activity. The details of the students’ profile has been presented in the episode 1.

![Diagram of groups with cake pieces]

In which group would you like to be if you wanted to eat more cake?

Why?

How much cake will each child eat in every group?

Figure 3.

<table>
<thead>
<tr>
<th>Utterances</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>N: (He is reading the problem)</td>
<td>N: (He is reading the problem)</td>
<td>In which group would you like to be if you wanted to eat more cake?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N: We will eat the same piece but bigger quantity of these.</td>
<td>P: Look here. (He is showing the 2nd group). Each child will eat 2 pieces of these. 1/6 plus 1/6...2/6, that is, the same with 1/3.</td>
<td>Why?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N: Yes.</td>
<td>P: In the third group each child will eat 1,2,3… 3/9 of the cake, that is, the same too.</td>
<td>How much cake will each child eat in every group?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N: With which it is the same?</td>
<td>P: Both.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P: We will write that all the groups will eat the same.</td>
<td>N: Yes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although the students tried to communicate their solutions, they didn’t manage to give explanations about their thoughts (cf ut. 2, 5-7). When they observed the video of their collaboration, they gave the following explanation as they tried to collaborate in mathematics.
justifications about their actions:

[2] N: I said that they will eat more pieces but the same.
[4] N: But that is what I would like to say.
[5] R: So, Paul what did you do?
[6] P: I showed him the right.
[7] R: Did Nikos have wrong?
[8] P: Now, as he explained it, no.
[9] R: So?
[10] P: I had to ask him again…Nikos has to say what he means so I will not think that he will say something wrong.
[11] N: When you don’t understand you have to ask me.

In the above metadiscursive reflection, the interlocutors seemed to become aware that there were misunderstandings during their effort to communicate their mathematical ideas (cf. phrases 1-4). It was this fact that it provoked their reaction to pose questions concerning the second regulating activity, like to ask and to give explanations about their mathematical proposals (cf. phrases 8,10,11).

Episode 4:

The following episode concerns two students with negative beliefs about collaboration and with no differences in their mathematical ability (N2-N2). At the beginning of the program, both students, Apostolos(A) and Elsa(E), expressed their desire to collaborate only with their teacher in mathematics. Elsa justified her view as follows: “I have to try alone and only the teacher can help me when I have difficulties”. She declared that she didn’t want to help her classmates in mathematics because: “If I do not know it right, I will say it to the other students in a wrong way”. On the other hand, Apostolos
justified his own view as follows: “I have the impression that my classmates will think that I do not do well in mathematics and I don’t like this”. Although, both students expressed negative beliefs about collaboration in mathematics, their intentions differed. Apostolos wanted to protect his self-image and Elsa had low self-confidence in mathematics. Moreover, these views were connected with the targets that the two students posed for their mathematical activity. Apostolos declared that he felt happy in mathematics when he could solve a problem that his classmates “don’t manage”. In contrary Elsa said: “I feel happy when I answer correctly to my teacher’s questions”. In relation to the students’ abilities in mathematics, their teacher commented that both students very often needed help to complete a mathematical activity. In one of their collaborations, they had to solve the following problem:

_During an excursion, Helen used 2/4 of her film and Nikos used 2/6 of his film. The films were same. The children discussed about who took more photos. A. Who used more film and how much more? B. How much film was left over for every child?_

<table>
<thead>
<tr>
<th>utterances</th>
<th>writings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E: We have to find how much more.</td>
</tr>
<tr>
<td>2</td>
<td>A: How can we find it?</td>
</tr>
<tr>
<td>3</td>
<td>E: We will subtract 2/4 from 2/6 and we will find it.</td>
</tr>
<tr>
<td>4</td>
<td>A: It is 2/2, but it looks weird.</td>
</tr>
<tr>
<td>5</td>
<td>E: I thought something, the denominators are not the same.</td>
</tr>
<tr>
<td>6</td>
<td>A: We can simplify them. 2/4 can be ½ and 2/6 can be 1/3.</td>
</tr>
<tr>
<td>7</td>
<td>E: If we subtract them it is 1/1.</td>
</tr>
<tr>
<td>8</td>
<td>A: So, did we find how much more?</td>
</tr>
<tr>
<td>9</td>
<td>E: I don’t know, maybe.</td>
</tr>
</tbody>
</table>

[7]They wrote: 1/3-1/2=1/1

During their discussion as they reflected on their collaboration they made the following comments:
[1] E: Apostolos made an error in the subtraction, he subtracted 2/4 from 2/6 and he said it was 2/2.
[3] E: I said that the denominators were not the same.
[4] A: Yes, but then we made the same subtraction again with a simplification and you accepted it.
[5] E: Yes, it seemed right to me.
[7] E: And finally we found 1/1 again.
[8] A: It looked weird to me from the beginning.
[9] E: I was not sure at the end either. We didn’t think why it could not be 1/1.

Through their metadiscursive reflection, Apostolos and Elsa tried to explain their thinking (cf. ut. 2-3, 5-6). Both students tried to shift the responsibility of the error to their partner. However, they discussed on the quality of their explanations for the solution of a mathematical problem (cf. phrases 2, 4), that is if the given mathematical explanation made sense for both partners.

In general, we could mention that the second regulating activity did not easily occur during the collaboration of the groups. The first occurrence of it as a goal in the work of the pairs was observed when the partners did not understand the proposed solutions and they explicitly mentioned this during their metadiscursive reflection (c.f. episode 3). That is, the existence and the acceptance of the difficulties that a member of the pair experienced to understand his/her partner’s solution provoked the necessity of an explanation in order to proceed their communication smoothly. Furthermore, the development of the second regulating activity for both partners was connected with the awareness that the existence of explanations during their discussion in mathematics helped them to find a correct solution (cf. episode 4).

However, we have to stress that in the case of the pair of the students
where there was a big difference in cognitive level between the partners (case P1-N3), the effort for the occurrence of an explanation, as a regulating activity, was cancelled during their collaboration in mathematics. In that pair, the effort of the “good” student to explain his thought to his partner was continuously ineffective, as there was not a “domain of mathematical communication” between them.

The third regulating activity “A criticizes B’s work”

The action to ask your partner to criticize your own proposal and in the same time to be receptive to his assessment consists a high level of communicational behavior. This activity is important in collaborative settings in mathematics classroom according to the process model for interaction (cf. table 1). In the following, we present and analyze concrete episodes of students’ collaboration in mathematics in order to study the conditions under which this regulating activity was occurred.

Episode 5:

Paul and Nikos tried to solve the following problem:

5/8
I am bigger than
3/4

3/4
I am bigger than
5/8

2/3
I am bigger than
3/5

3/5
I am bigger than
2/3

Figure 4.

Facilitate the fractions to find their value. Use the strategy of changing them into fractions with the same denominator.
The following dialogue took place, as they reflected on their collaboration:

| Utterances                                                                 | Writings
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1  N: (He is reading the problem)</td>
<td></td>
</tr>
<tr>
<td>2  P: It suggests changing them into fractions with the same denominator.</td>
<td></td>
</tr>
<tr>
<td>3  N: ( \frac{3}{4} ) will be done ( \frac{6}{8} ), then it is bigger than ( \frac{5}{8} ).</td>
<td></td>
</tr>
<tr>
<td>4  P: Yes, I agree.</td>
<td></td>
</tr>
<tr>
<td>5  N: How will we make the second pair?</td>
<td></td>
</tr>
<tr>
<td>6  P: I don’t find something that makes them with the same denominator.</td>
<td></td>
</tr>
<tr>
<td>7a N: I found it!</td>
<td></td>
</tr>
</tbody>
</table>
| 7b If we subtract 2 from \( \frac{3}{5} \), it will be \( \frac{1}{3} \). So these fractions will have the same denominator and \( \frac{2}{3} \) is bigger than \( \frac{3}{5} \). | \[
\frac{2}{5} - \frac{2}{5} = \frac{1}{3}
\]
| 7c …It isn’t right?                                                        |          |
| 8  P: We can not find fractions with the same denominator by subtraction. We usually use division or multiplication. |          |
| 9  N: I know it, but we have to do it in this way because we can make the same denominator. |          |
| 10a P: \( \frac{1}{3} \) is not equal to \( \frac{3}{5} \).                  |          |
| 10b If we multiply it by 2, \( \frac{2}{3} \) will be \( \frac{4}{6} \).     |          |
| 11 P: If we divide it by 2… It doesn’t work. …By 3, it doesn’t make the denominator 5. |          |
| 12 N: We can not by 3 or 2, so what will we do?                            |          |
| 13 P: Oh! I found it.                                                       |          |
| 14 N: Let me see!                                                          |          |
| 15 P: 15, 3 times 5… 15. If we multiply it by 3 …and this one by 5 …it will be here 15 and here 15. | \[
\frac{5}{5} \times 3 = 9
\]
| 16 N: Ah! Yes, the first one is bigger.                                    |          |

[1] R: This is your fourth collaboration. What targets had you put last time?

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[2] P: To discuss the solution of the problem and to ask if we don’t understand.
[4] R: Good, so how do you feel about this collaboration?
[5] P: Very good, we continuously discussed.
[7] R: Let’s watch the second part of your collaboration. Do you want to observe something here?
[8] N: At the beginning it was not so good, we confused.
[10] R: What kind of disagreement did you have?
[11] N: I thought that if we subtracted 2 from 3/5 it would be right, because we would have the same denominator. Paul disagreed and he told me that it wasn’t right.
[12] P: Yes, I explained to him that it wasn’t right and then I thought how we had to do it.
[13] R: Nikos, were you convinced that your idea was wrong?
[16] N: Paul was right, you can not subtract because you don’t take equivalent fractions in this way, then he explained to me how he found the correct and I agreed.
[17] N: If Nikos had a correct idea, we would discuss it.

Paul and Nikos had a disagreement about the way of finding fractions with the same denominator (cf. ut. 7-12). This disagreement provoked the occurrence of the third regulating activity, as Nikos asked from Paul to criticize his proposal (cf. ut. 7c) and Paul presented his arguments in order to support his strategy (cf. ut. 8,10a). During their metadiscursive reflection both students had the opportunity to describe how they treated their disagreement (cf. phrases 8-17).
Episode 6:
Towards the end of the program, Elsa and Apostolos (N2-N2) had to engage in the following mathematical problem:

2 children fairly share 5/6 of a pizza. How much pizza will each child get?
4 children fairly share 2/3 of a pizza. How much pizza will each child get?

<table>
<thead>
<tr>
<th>utterances</th>
<th>writings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1   E: (She is reading the problem)</td>
<td></td>
</tr>
<tr>
<td>2a  A: 2 children, 5/6.</td>
<td></td>
</tr>
<tr>
<td>2b  We can multiply 2 by 5/6.</td>
<td></td>
</tr>
<tr>
<td>3   E: Yes.</td>
<td></td>
</tr>
<tr>
<td>4   A: Shall we do it?</td>
<td></td>
</tr>
<tr>
<td>5   E: Just a moment, it don’t write each one 5/6, but they share 5/6.</td>
<td></td>
</tr>
<tr>
<td>6a  A: Oh! Yes.</td>
<td></td>
</tr>
<tr>
<td>6b  There are 5 pieces. We share them in two, everyone will get 2 whole pieces and it rests one. Right?</td>
<td>[10]They try to draw in a cyclic disk 5/6.</td>
</tr>
<tr>
<td>7   E: Yes.</td>
<td></td>
</tr>
<tr>
<td>8   A: We will not divide this one in half?</td>
<td></td>
</tr>
<tr>
<td>9   E: Yes.</td>
<td></td>
</tr>
<tr>
<td>10  A: Can we make a drawing?</td>
<td>[17]They wrote:</td>
</tr>
<tr>
<td>11  E: Yes.</td>
<td>5/12</td>
</tr>
<tr>
<td>…</td>
<td></td>
</tr>
<tr>
<td>12  A: I can not make 6 pieces.</td>
<td></td>
</tr>
<tr>
<td>13  E: I have difficulties too.</td>
<td></td>
</tr>
<tr>
<td>14  A: Can we do operations?</td>
<td></td>
</tr>
<tr>
<td>15  E: Yes.</td>
<td></td>
</tr>
<tr>
<td>16  A: 5/6 will be 10/12, because it holds with 2.</td>
<td></td>
</tr>
<tr>
<td>17  E: We will divide it.</td>
<td></td>
</tr>
<tr>
<td>18  A: Good, 5/12 each one.</td>
<td>10/12</td>
</tr>
<tr>
<td>19  E: The next one, 4 children share fairly 2/3 of a pizza.</td>
<td>5/12</td>
</tr>
</tbody>
</table>
As we can observe in the above episode, both students continuously expressed comments and assessments about their own and their partner’s proposals. Apostolos and Elsa successively criticized the expressed proposals and they had an equivalent participation in their collaboration. This was a major advance during their collaboration, because Apostolos and Elsa had both negative beliefs about collaboration in mathematics at the beginning of the program.

Discussing about the conditions that influenced the occurrence of the third regulating activity (A criticizes B’s work) we could notice that its spontaneous occurrence was connected with the existence of one student’s disagreement during the interaction with his partner (cf. episode 5). However, the critical situation that allowed the occurrence of this regulating activity for both partners was the evaluation of all the different proposals that each partner offered, as they mutually tried to construct a common solution to a mathematical problem (cf. episode 6).

**CONCLUSIONS**

From the presented results, the students’ development of their self-regulation, as they tried to collaborate in pairs in mathematics, using as a theoretical context for our analysis the process model of interaction, is complicated. It is influenced mainly by their beliefs about the role of the others, their beliefs about the role of collaboration in mathematical learning, the occurrence and the treatment of errors and disagreements as well as the difference of students’ cognitive level. In our research, we found that the occurrence of each regulating activity was influenced by different factors. More specifically, the students’ activity to ask their partner in order to show his/her own work is related to their belief about their own and their partner’s learning ability and to their belief about collaboration; the students’ activity
to ask their partner to explain his/her mathematical solution is related to the existence of misunderstandings; the students’ activity to ask their partner to criticizing a solution is related to the disagreement about an explanation.

However, in collaborative settings, these regulating activities have to be developed by both students in order to exist an effective collaboration and this effort needs suitable learning environments. According to our opinion, every mathematical activity can give opportunities to students in order to reflect about their interaction. The students’ observations and discussions on their videotaped collaboration allowed them to become aware of multiple aspects of their mathematical activity and to improve their collaboration through the appearance of regulating activities. They had the opportunity to focus on different issues that are connected with the realization of a mathematical activity, like the treatment of an error or the treatment of different solutions. This means that students’ self-regulation could be studied through students’ metadiscursive reflection as this kind of reflection allows the understanding of factors that influence it. Table 4 summarizes the conclusions of this study.

<table>
<thead>
<tr>
<th>Regulating activities</th>
<th>Factors</th>
<th>Metadiscursive reflection on:</th>
</tr>
</thead>
</table>
| A asks B to show his/her work | - Beliefs about their own and their partner’s learning ability  
- Beliefs about collaboration  | the effectiveness of their partner’s suggestion         |
| A asks B to explain his/her work | - Misunderstanding of a problem solution               | the quality of their mathematical explanations       |
| A criticizes B’s work | - Disagreement about a mathematical solution.          | the evaluation of the proposals offered by each partner |

*Table 4. Development of regulating activities*
The results of this research show the necessity of the development of students’ metadiscursive reflection in the mathematics classroom setting. Towards this effort mathematical educators should help teachers to design suitable didactical situations that enhance self-regulated learning. The construction of tools that include metadiscursive self-questions that the students could use as they collaborate in order to solve a mathematical problem is open for future research.

According to our results the organization of mathematics classroom in pairs of students according to their cognitive level is not enough for students’ mathematical level raising. Teachers should be aware of their students’ self-belief systems in order to appreciate their effort to collaborate in pairs. Maybe many difficulties that children still experience in mathematics, in spite of the progress in didactics of mathematics, are related to an incomplete picture that we have about the interaction between students as human beings and not as mere cognitive subjects.

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