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Modeling options markets by focusing on active traders

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Abstract

In this work, we study the complex behavior of options markets characterized by the volatility smile phenomenon, through microsimulation (MS). We adopt two types of active traders in our MS model: speculators and arbitrageurs, and call and put options on one underlying asset. Speculators make decisions based on their expectations of the asset price at the option expiration time. Arbitrageurs trade at different arbitrage opportunities such as violation of put-call parity. Difference in liquidity among options is also included. Notwithstanding its simplicity, our model can generate implied volatility (IV) curves similar to empirical observations. Our results suggest that the volatility smile is related to the competing effect of heterogeneous trading behavior and the impact of differential liquidity.

Keywords:
options markets, volatility smile, microsimulation

1. Introduction

The Black-Scholes (BS) formula [1, 2] for pricing a European call/put option on a non-dividend-paying asset is

$$V_{BS}^{\phi,t}(S^t, K, r, T-t, \sigma) = \phi[S^t N(\phi d_1) - Ke^{-r(T-t)}N(\phi d_2)], \tag{1}$$

where S^t is the price of the underlying asset at time t, K is the strike price, r is the risk-free interest rate, T is the expiration time of the option and hence $T-t$ is the time to maturity, σ is the volatility of the asset, and $\phi = 1(-1)$ for a call (put) option. In the formula,

$$d_1 = \frac{\ln(S^t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \tag{3}$$

and $N(x)$ is the standard normal cumulative distribution function.

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All parameters in the BS model other than the volatility are observables. Moreover, according to the model, the theoretical value of an option is a monotonic increasing function of the volatility. A unique volatility is therefore implied by the market price of an option ($V^{\phi,t}$), namely, an implied volatility (IV):

$$\sigma_{imp}^{\phi,t}: \qquad V_{BS}^{\phi,t}(S^t, K, r, T-t, \sigma_{imp}^{\phi,t}) = V^{\phi,t}. \tag{4}$$

According to the BS model, the IV is independent of the strike for a fixed time to maturity. Hence, curves of IV against strike should be flat. In reality, however, it is well known that IVs exhibit a remarkable curvature [3, 4, 5, 6, 7, 8], which is commonly referred to as a volatility smile. In particular, equity index options tend to have a downward sloping IV curve, i.e. a volatility skew, and it has become much more pronounced only after the stock market crash of October 1987. Foreign currency options, however, typically show a symmetric smile, especially if the currencies are of equal strength. Contrary to equities, some commodity options often show an upward sloping skew.

Another important fact is that the volatility smile changes over time. In addition, it has been revealed that three principal components can sufficiently account for the observed deformation of the smile [7, 8]. The first component reflects the shift in its overall level; the second one conveys the change of its slope; and the third component accounts for the adjustment in its convexity.

Apparently, the volatility smile conflicts with the BS framework. To account for the deviations of option prices from the BS formula, models based on processes for the underlying asset other than geometric Brownian motion[2], such as stochastic volatility and jump diffusion processes, have been proposed [6]. They can include the smile effect on the valuation of options to some extent. However, market prices of options are ultimately determined by supply and demand, which result from traders' behavior. From this perspective, these alternative models do not explain the origin of the smile phenomenon.

In view of these facts, our motivation is to develop a microscopic simulation (MS) model with a simple structure that can reproduce the volatility smile and illustrate the mechanisms that govern the smile phenomenon. The general reason for the adoption of simple MS models to confirm stylized facts of financial markets has been discussed in Ref. [9].

In Sec. 2, we describe our MS model. We present it in the order of increasing level of sophistication, so that the cause(s) of certain stylized feature(s) can be clearly identified. The simulation results of the model are shown in Sec. 3. In the final section, we present our conclusions and plans for future work.

2. Modeling Options Markets by Focusing on Active Traders

In options markets, there are three main types of traders: hedgers, speculators, and arbitrageurs. Typically, hedgers use options as insurance to protect their financial interests, but do not attempt to gain profit from options markets; speculators trade options to bet on the future prices in the hope of making capital gains; and arbitrageurs involve making riskless profits by taking advantage of price disparities.

Traders are also different in their trading activity. Hedgers only need to trade once in a certain period. In contrast, speculators are much more active due to the benefits of option trading, i.e. its power of leverage, namely gaining exposure to larger amounts of assets for a much smaller investment, and its potential to profit whether the underlying asset moves up or down. Arbitrageurs are even more active. There are many traders of this type in options markets and their trading will quickly eliminate any detected arbitrage opportunity. Consequently, price disparities are usually small and transient, and arbitrageurs need to trade frequently in order to lock-in significant profits.

The smile dynamics exhibits apparent regularity even on short (e.g. daily) time scales, as shown in Refs. [7, 8]. We therefore consider that, whereas the smile is inevitably influenced by the less active traders, its basic dynamics should be accounted for by the behavior of the more active market participants. In modeling options markets, we hence focus on active traders, i.e. speculators and arbitrageurs.

In our model, agents trade in European call/put options on a single underlying asset with different strikes. All options have the same time to maturity. Speculators make decisions based on their expectations of the asset price at the option expiration time. In addition, their expected prices are influenced by news over time. Arbitrageurs trade at

[2]In the BS model a geometric Brownian motion is assumed for the price dynamics of the underlying asset.

different arbitrage opportunities, such as violation of put-call parity. The difference in liquidity between out-of-the-money (OTM) and in-the-money (ITM) options is also included. Changes of option prices are proportional to the excess demands.

2.1. Level I: The Market Consists of only Speculators

In our model, there are N_{tr} traders and N_{op} call/put options. At this level all traders are assumed to be speculators (SP). The strike price of the n-th call/put option is represented by K^n, and its market price at time t is denoted as $V^{n,\phi,t}$, and the i-th trader's expected asset price at the option expiration time is expressed as $S^{i,t}$.

We assume that the speculators' expected prices follow a log-normal distribution with mean m^t_{SP} and standard deviation σ^t_{SP}, based on the fact that prices can not be negative. Tests showed that adopting other distributions, e.g. a normal or Lorentzian distribution, does not influence the principal characteristics of the simulated IVs.

Based on their expected asset prices, the speculators estimate the profitability of trading each option. Without loss of generality, we assume that a speculator's transaction quantity $Q^{i,n,\phi,t}_{SP}$ is proportional to his estimated profit of the deal:

$$Q^{i,n,\phi,t+1}_{SP} = \lambda_{SP}[max(\phi(S^{i,t} - K^n), 0) - V^{n,\phi,t}], \tag{5}$$

where λ_{SP} is a positive parameter that controls the activity level. At this level $\lambda_{SP} = 1$. Here, $max(\phi(S^{i,t} - K^n), 0)$ is the expected payoff of the option. Notice that in this paper we assume that the interest rate is zero.

2.2. Level II: The Market Consists of both Speculators and Arbitrageurs

The arbitrageurs (AR) monitor the option prices in order to detect arbitrage opportunities. These agents will trade those options that are found to violate the arbitrage relations. Without loss of generality, we assume that an arbitrageur's transaction quantities are proportional to the extent to which the relations are violated. In the case of violation of put-call parity (PCP) [10], we therefore have

$$Q^{i,n,\phi,t+1}_{AR\_PCP} = -\phi\lambda_{AR\_PCP}A^{n,t}_{AR\_PCP} \tag{6}$$

with

$$A^{n,t}_{AR\_PCP} = (V^{n,1,t} - V^{n,-1,t}) - (S^t - K^n), \tag{7}$$

where $A^{n,t}_{AR\_PCP}$ indicates the extent to which put-call parity is violated and λ_{AR_PCP} is a positive parameter.

If the butterfly spread (BFS) relation [10] is violated, the following rule applies,

$$Q^{i,n,\phi,t+1}_{AR\_BFS} = -2\lambda_{AR\_BFS}A^{n,h,\phi,t}_{AR\_BFS}, \quad Q^{i,n-h,\phi,t+1}_{AR\_BFS} = Q^{i,n+h,\phi,t+1}_{AR\_BFS} = \lambda_{AR\_BFS}A^{n,h,\phi,t}_{AR\_BFS}$$

with

$$A^{n,h,\phi,t}_{AR\_BFS} = max(V^{n,\phi,t} - \frac{1}{2}(V^{n-h,\phi,t} + V^{n+h,\phi,t}) + \varepsilon^h, 0), \tag{8}$$

where $A^{n,h,\phi,t}_{AR\_BFS}$ indicates the extent to which the butterfly spread relation is violated and λ_{AR_BFS} is a positive parameter. Here the prices of the three options corresponding to the strikes K^n, K^{n-h}, and K^{n+h} are involved. The value of h varies, meaning that the arbitrageurs apply this rule to all the possible butterfly spreads of which K^n is the common middle strike. $\varepsilon^h \geq 0$ denotes the convexity of the curve considered by the arbitrageurs. For a typical convex IV curve, ε^h increases with increasing h. For simplicity, we adopt $\varepsilon^h = \alpha h$ where $\alpha \geq 0$.

2.3. Level III: Difference in Liquidity is Further Included

In real markets, the liquidity of options is not balanced across strikes [11]. In general, OTM options are more liquid than ITM options, implying that speculators trade in the former more actively than the latter. To reflect this fact, we modify the parameter λ_{SP} as follows,

$$\lambda_{SP}(K^n) = \eta_{SP}[\phi\, tanh(\gamma(K^n - S^t)) + 1], \tag{9}$$

where η_{SP} and γ are positive parameters. It is an increasing/decreasing function of the strike for call/put options.

In principle, arbitrage strategies are self-financing and risk-free, stipulating that all the options involved must be traded simultaneously and in the specified proportions. Liquidity unbalancing is therefore not applicable to the arbitrageurs in our model.

2.4. Option Price Updating

The prices of the options are updated according to the following rule, which can be explained as the effect of market makers' action to balance the supply and demand:

$$V^{n,\phi,t+1} = V^{n,\phi,t} + \frac{\beta Q^{n,\phi,t}}{N_{tr}}, \tag{10}$$

where $Q^{n,\phi,t}$ is the total transaction quantity or the excess demand of the n-th option at time t. Since the excess demand is proportional to N_{tr}, we rescale it with the latter. Here β is a positive parameter that indicates the sensitivity of the option price to the excess demand. Due to the fact that option prices cannot be negative, the lower bound of $V^{n,\phi,t}$ is 0.

3. Simulation Results

In the model we do not include a term structure of the smile. Therefore, we adopt a fixed value for $T - t$. For simplicity we also keep S^t and σ^t_{SP} constant. We change the value of m^t_{SP} in the sense that news alters the speculators' expected prices. For comparison, we display the simulated option prices together with the corresponding prices obtained by the BS model with the volatility σ_{BS}. The BS prices satisfy all the arbitrage relations.

The parameters λ_{SP}, λ_{AR_PCP} and λ_{AR_BFS} represent the traders' activity levels for employing the corresponding strategies (for the S and SA model, $\lambda_{SP} = \eta_{SP}$). We assume that their values are proportional to the traders' confidence levels about the profitability of the strategies. For example, speculative profits are much more uncertain than arbitrage gains, so the value chosen for η_{SP} is much smaller than that for λ_{AR_PCP} and λ_{AR_BFS}. In simulations, $\eta_{SP} = 0.1$, $\lambda_{AR\_PCP} = 1.0$, and $\lambda_{AR\_BFS} = 1.5$. The values for some other parameters are $N_{tr} = 5000$, $N_{op} = 11$, $\tau = 1$, $r = 0$, $\gamma = 1.5$, $\alpha = 0.1$ [3], $\beta = 0.1$, $m^t_{SP} = 19$, $\sigma^t_{SP} = 3.5$, $\sigma_{BS} = 0.2$, the fraction of speculators and that of arbitrageurs are both set to 0.5. We assume that the minimum IV accepted by the traders is 5%. The initial prices of all the options are equal to 1.

In the simulation based on the level I model, if m^t_{SP} coincides with S^t, the simulated option prices overlap with the corresponding BS prices and the resultant curve of IV against strike is flat (data not shown).

Next, we examine the IV smile in the case that $m^t_{SP} < S^t$. In this situation, the price of the underlying asset is considered by the speculators more likely to suddenly drop than to suddenly rise, in line with the nature of stock indexes. (Notice that we have assumed a zero interest rate and ignored dividends.) As shown in Figs. 1(a) and 1(b), the resultant prices of all call/put options are lower/higher than the corresponding BS prices. Consequently, the IVs of the call options are lower than the corresponding IVs for the put options. Obviously, the simulated option prices do not satisfy put-call parity.

As illustrated in Fig. 1(c), the simulated option prices based on the level II model satisfy both put-call parity and the butterfly spread relation. It can be observed that the speculators' trading and that of the arbitrageurs tend to move the option prices in opposite directions. Hence, the different strategies followed by the speculators and the arbitrageurs, respectively, lead to a competing effect on the simulated option prices. As shown in Fig. 1(d), at this level the IV curves still depart from empirical observations.

In the simulations based on the level III model, if the arbitrageurs only act on violation of put-call parity, we have obtained the results as shown in Figs. 2(a) and 2(b). Here we see that most of the simulated option prices satisfy put-call parity. In addition, the IVs of the options with strikes lower than S^t are higher than those IVs corresponding to the options with strikes higher than S^t. Here we can observe that the competition becomes unbalanced due to the difference in speculators' activity between OTM options and ITM options. However, the options do not satisfy the butterfly spread relation. Consequently, the IV curves are not convex as shown in empirical data. When the arbitrageurs also trade in response to violation of the butterfly spread relation, as displayed in Figs. 2(d), we finally have an IV skew similar to those observed in real markets for equity and index options.

[3] Solely for reference, the average alpha value of the corresponding BS prices where the common middle strike is 20 and the volatility is between 0.1 and 0.3 is around 0.08.

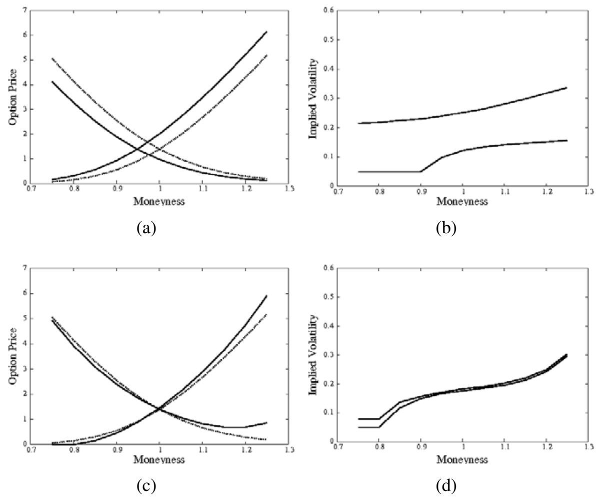

Figure 1: Simulation results of the level I model and those of the level II model plotted against moneyness (K^n/S^t). The simulated option prices (solid lines) are displayed together with the corresponding prices given by the BS model (dashed lines). The price curves of the call/put options are decreasing/increasing against moneyness and the IV curves of the call options are lower than or overlap the corresponding curves of put options. (a) Option prices of the level I model; (b) corresponding IVs. (c) Option prices of the level II model; (d) corresponding IVs.

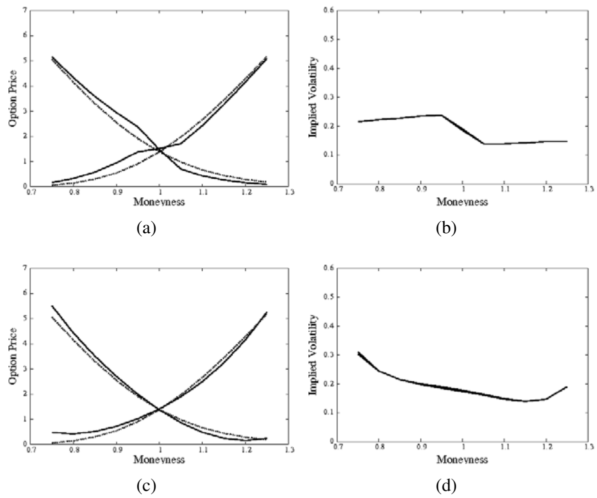

Figure 2: Simulation results of the level III model plotted against moneyness (K^n/S^t). The simulated option prices (solid lines) are displayed together with the corresponding prices given by the BS model (dashed lines). The price curves of the call/put options are decreasing/increasing against moneyness and the IV curves of the call options are lower than or overlap the corresponding curves of the put options. (a) Option prices when arbitrageurs only act on violation of put-call parity; (b) corresponding IVs. (c) Option prices when the arbitrageurs also act on violation of the butterfly spread relation; (d) corresponding IVs.

4. Conclusions

Notwithstanding its simplicity, our model can generate IV curves similar to empirical observations. Our results suggest that the volatility smile is related to the competing effect of the distinct behavior of option traders and the difference in speculators' trading activity for options with different strikes. Although not fully shown in this paper, it has been found that the shape of the smile is determined by the mean of the speculators' expected prices. In our future work we will investigate the origin of the difference in the shape of the empirically observed IV curves for options on various types of underlying assets. In addition, we will numerically compare our simulated IV surfaces with empirical observations in terms of principal components.

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