Topics in market microstructure

Zovko, I.

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Chapter 5

Market imbalances and stock returns: heterogeneity of order sizes at the London Stock Exchange

5.1 Introduction

The connection of trade volume and stock returns has been well established (Andersen, 1996; Brock and LeBaron, 1996; Weber and Rosenow, 2004; Tauchen and Pitts, 1983; Chordia and Swaminathan, 2000). Large price moves are associated with large trade volume. On a finer level of detail, it has been shown that price moves are also driven by the properties of order flow (Plerou et al., 2002a; Berger et al., 2006; Carlson and Lo, 2006; Evans and Lyons, 2002; Payne, 2003; Farmer et al., 2005; Kaniel et al., 2008; Kumar and Lee, 2005; Gopikrishnan et al., 2000; Gabaix et al., 2003b; Maslov and Mills, 2001; Solomon and Richmond, 2001). For example, the number of buyer or seller initiated trades and volumes (signed order flow) has influence on price returns (Plerou et al., 2002a). In this paper we show that, in addition to the orderflow, the heterogeneity in order sizes plays a role in price formation. We investigate the pressures exhibited on the price by the trading of large orders either matched or unmatched by large orders on the other side of the market.¹

¹Throughout the paper, when referring to the two sides of the market, we mean the bid and ask sides of the market, not the sellers and buyers of financial services.
### Table 5.1: Basic facts and trading statistics for the analysed stocks.

From left to right the columns are: stock ticker symbol, company name, industry market capitalization in 2005, median daily number of trades in the analysed period for the on-book and off-book; median daily volume for the two markets and total; median daily number of trading firms.

<table>
<thead>
<tr>
<th>Stock ticker</th>
<th>Company name</th>
<th>Industry</th>
<th>Market cap (Bil.£)</th>
<th>Daily no. trades</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOD</td>
<td>Vodafone Group</td>
<td>Telecomm.</td>
<td>86.71</td>
<td>2,256</td>
</tr>
<tr>
<td>BPA</td>
<td>British Petroleum Amoco</td>
<td>Oil companies</td>
<td></td>
<td>1,395</td>
</tr>
<tr>
<td>HSBA</td>
<td>HSBC Holdings</td>
<td>Banking</td>
<td>99.19</td>
<td>1,199</td>
</tr>
<tr>
<td>SHEL</td>
<td>Shell Transport &amp; Trad.Co.</td>
<td>Oil companies</td>
<td></td>
<td>1,122</td>
</tr>
<tr>
<td>AZN</td>
<td>AstraZeneca</td>
<td>Pharmaceuticals</td>
<td>37.03</td>
<td>970</td>
</tr>
<tr>
<td>LLOY</td>
<td>Lloyds TSB Group</td>
<td>Banking</td>
<td>26.22</td>
<td>1,055</td>
</tr>
<tr>
<td>DGE</td>
<td>Diageo</td>
<td>Drinks</td>
<td>24.38</td>
<td>842</td>
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<td>PRU</td>
<td>Prudential</td>
<td>Insurance</td>
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</tr>
<tr>
<td>TSCO</td>
<td>Tesco</td>
<td>Mass distribution</td>
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<tr>
<td>CGNU</td>
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<td>Insurance</td>
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</tr>
<tr>
<td>RTR</td>
<td>Reuters Group</td>
<td>Media</td>
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<td>AAL</td>
<td>Anglo American</td>
<td>Mining</td>
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<td>74</td>
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</table>

<table>
<thead>
<tr>
<th>Stock ticker</th>
<th>Daily vol. (Mil.£)</th>
<th>Daily no. firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOD</td>
<td>97</td>
<td>510</td>
</tr>
<tr>
<td>BPA</td>
<td>138</td>
<td>930</td>
</tr>
<tr>
<td>HSBA</td>
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<td>927</td>
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<tr>
<td>SHEL</td>
<td>75</td>
<td>486</td>
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<tr>
<td>AZN</td>
<td>348</td>
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<tr>
<td>LLOY</td>
<td>117</td>
<td>449</td>
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<tr>
<td>DGE</td>
<td>57</td>
<td>330</td>
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<tr>
<td>PRU</td>
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<td>222</td>
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<td>CGNU</td>
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<tr>
<td>RTR</td>
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<td>226</td>
</tr>
<tr>
<td>AAL</td>
<td>59</td>
<td>285</td>
</tr>
</tbody>
</table>

From left to right the columns are: stock ticker symbol, company name, industry market capitalization in 2005, median daily number of trades in the analysed period for the on-book and off-book; median daily volume for the two markets and total; median daily number of trading firms.
On the other hand, the effect of volume moving the price has been interpreted in the context of informed and uninformed trading, as in for example Madhavan and Cheng (1997) and Smith et al. (2001). It is said that trades based on information (informed trades) move the price, while liquidity based trades (uninformed trades) do not. Our analysis shows evidence that it may be more appropriate to consider the lack of liquidity, not informed trading, that moves prices.

Using codes identifying member firms in a dataset from the London Stock Exchange (LSE), we disaggregate the daily and hourly trade volume into quantities bought and sold by individual member firms and call this the member firms’ size in the given interval. Unfortunately, codes in the LSE data identify only the aggregate order flow of a member firm, not individual orders. To obtain the actual orders from aggregate data, Vaglica et. al. (Vaglica et al., 2007) use an algorithm detecting “patches”. We take a different approach and analyse the data on two different timescales, hourly and daily. Given the indication that a firm’s trading is typically dominated by one large order at any given time (Vaglica et al., 2007), we assume that at one of the timescales the firms’ trades are corresponding to the firms’ orders and hope to detect the influence of order properties on at least one of the timescales. As we will show, the results are significant on both timescales and draw similar conclusions.

The order sizes are found to be extremely heterogenous (Vaglica et al., 2007), seemingly reflecting the heterogeneity of investor sizes (Zipf, 1949; Pushkin and Aref, 2004; Gabaix et al., 2006). Since large order sizes can incur the cost of significant market impact, different market mechanisms are designed to help limit this cost. As the LSE is a hybrid market with concurrent trading through an anonymous limit order book (on-book) and through a dealership market (off-book), by comparing them we show that indeed there are different impacts associated with different mechanisms.

The on-book market of the LSE largely uses the continuous double auction, while the off-book is an electronic quotation market where trading is ultimately done via phone. The member firms are institutions entitled to trade on the LSE. For example, large investment banks and hedge funds which directly send orders to the LSE are member firms. Investment banks can act either as a broker or can trade for their own account, while hedge funds normally trade for their own account.

We base the analysis on 12 stocks with ticker symbols AAL, RTR, CGNU, TSCO, PRU, DGE, LLOY, AZN, SHEL, HSBA, BPA, VOD. The period of the analysis is from 2002 to 2004 comprising 32 months of trading which corresponds to 674 trading days. Some basic facts and trading statistics for the analysed stocks is given in table 5.1. The markets are open from 8:00 to 16:30 but we discard data from
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HETEROGENEITY OF ORDER SIZES AT THE LSE

Figure 5.1: Distribution of daily order sizes for the on-book market (left) and off-book market (right) for all stocks pooled together. The order sizes are normalized by the respective stock mean. The insets show the collapse of the distributions for each stock separately onto each other, validating the pooling of data. The distributions for hourly disaggregation are remarkably similar. Lillo et al. (2005) have obtained results very similar to this.

Apart from being interesting in the context of market heterogeneity, the distribution of order sizes (the volume a firm has bought or sold in a day) is important in its own right. On the one hand it may be helpful to the literature on heterogenous agent based models Boswijk et al. (2007); Hommes (2006), on the other hand some rational expectation models Gabaix et al. (2006) use this distribution as an input of the agents optimization problem.

However, total trade volume has grown over the years and it is not a-priori clear that the order size distribution is a stable distribution. We find that on the LSE the increase in total trade volume was paralleled by the increase in the number of firms trading. For on-book trading, the number of firms trading in a low-activity stock has increased from about 30 to about 40. For a high-activity stock the
numbers increased from about 70 to about 90. For off-book trading, the numbers are approximately 50% higher. Therefore, the typical daily volume traded by a firm seems to have remained more or less constant. For high-activity stocks (e.g., BPA, VOD), the median daily volume traded by a firm on-book is about 5–7 million Pounds, while for low-activity stocks (e.g., AAL, CGNU, DGE) it is about 1–2 million Pounds throughout the sample. The numbers on the off-book market are lower, from 0.8–1 million for high activity stocks to 0.3–0.5 million for low activity stocks.

Therefore, it seems reasonable to decompose the total daily volume \( V_t \) on day \( t \) as

\[
V_t = \sum_{i=1}^{N_t} v_{t,i},
\]

where \( N_t \) is the number of firms trading on day \( t \) and \( v_{t,i} \) is the daily firm trade volume. Decompositions are done each day separately for the sell and buy side of the market.

As noted, we consider \( v_{t,i} \) to correspond the firm’s order size. At either the daily or hourly timescale, we believe this is a reasonable assumption. However, it is a caveat to keep in mind.

Apart from the different means from stock to stock, the distribution of order sizes \( v_{t,i} \) seems to be stable over time. When normalized by dividing with the mean size for each stock, the densities seem the same across stocks. In the insets of figure 5.1 we show the density functions of normalized order sizes for all stocks in the two markets. The curves collapse allowing us to pool the data to produce the density estimates in the main figures.

The main graphs of figure 5.1 represent the estimated density function\(^2\) of normalized order sizes pooled across all stocks. The left panel represents on-book trading, while the right represents off-book trading. Both densities show power-law behaviour in the tails with exponents around 3 for on-book and 3/2 for off-book trading, which were obtained by fitting a power-law to the tails of the distribution for values larger than 10. Looking at the Hill plots in figure 5.2 we see that the power law behavior in the on-book indeed seems to be valid for orders larger than 10 times the average size. For the off-book market the threshold is not as clear.

Disaggregating the volume on hourly intervals we obtain strikingly similar distributions and the same exponents. This remains to be investigated in the future. As a further speculation, the functional forms of the densities are very similar in shape to the Tsallis q-Gaussian (for the on-book market) and double q-Gaussian (for the off-book market) (Tsallis et al., 2003).

\(^2\)Actually shown is one minus the cumulative distribution function (CDF).
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HETEROGENEITY OF ORDER SIZES AT THE LSE

The model of Gabaix et al. (2006) predicts the tail exponent of optimal order size based on a rational agent minimizing impact to be equal to $3/2$. Their argument depends on using first order risk aversion. If one measures risk aversion using the variance, the exponent becomes 3 instead. In addition, their model rests on the assumption that investor sizes are distributed according to a power law, while new evidence suggests the distribution may be more close to a log-normal (Schwarzkopf and Farmer, 2008). In any case, the empirical exponents found in several studies are not in complete agreement. Vaglica et al. (2007) have found the exponent to be around 2 for the Spanish stock market. Gabaix et al. (2003b) and Gopikrishnan et al. (2000) quote for the transaction volumes of the Paris Bourse and the NYSE/NASDAQ exponents of $3/2$. Lillo et al. (2005), using transaction volumes from the same LSE dataset we use, found the exponents to be around 2.9 for the on-book market and 1.6 for the off-book market.

5.3 Order size heterogeneity and stock returns

Since the order size distribution is a fat tailed distribution, the likelihood that a disproportionally large order be present on the market on a given day is not negligible. In such a situation the composition

Figure 5.2: Maximum likelihood estimate of the Hill exponent for the on-book (left) and off-book (right) markets. Panels display Hill plots which show the value of the Hill exponent as a function of either the order statistics (the n-th largest value beyond which power-law behavior starts) or the threshold (the corresponding value of the variable).
of member firm order sizes is very heterogenous and extremely uneven. For example, it may happen that out of the total of 80 firms, the largest 2 firms account for 90% of the sell volume, while the remaining 78 account for only 10% of the sell volume. Similar size heterogeneity may or may not happen on the buy side of the market forcing the two large sellers to either transact with many counter parties to fill their order or to transact with a small number of large counter parties.

The terminology homogenous and heterogenous is inspired by the notion that a heterogenous partition of firm order sizes is a situation where there are great differences in the order sizes traded by member firms. A homogenous order partition is a situation when the order sizes traded by member firms are similar. An alternative choice of words would be to call a heterogenous composition of firms a concentrated composition, since most of the trade volume is concentrated in a few number of large firms. A homogenous composition would then be labeled as a diluted, or equal composition, where most firms are trading equal volumes.

The interplay of the two trading sides possibly determines demand and supply pressures and may ultimately influence the price move. However, it is not a-priori clear what kind of price pressure the above example situation may produce. In the setting of a financial market it may be plausible that a large seller will have to sell at a discount to a large number of buyers. In a traditional market for goods however, a seller which is the only source of a good usually is considered to have monopoly power over small buyers and may sell with a premium. We empirically address this question by quantifying the heterogeneity of firms’ order sizes and investigating its’ influence on price returns.

We use two similar approaches to quantify the size heterogeneity. One approach, rooted in statistical mechanics, uses the entropy, the other, rooted more in economics, uses the Gini index. Both approaches give identical results in terms of interpretation.

To calculate the entropy of a partition of the total volume for the sell (buy) side, we denote by \( w_i \) the fraction of the total volume sold (bought) by firm \( i \). We then calculate the entropy as

\[
E_s = \sum_{i=1}^{n} w_i \log(1/w_i), \quad i \in \text{firms},
\]

(5.2)

where by \( E_s \) we denote the sell side and analogously by \( E_b \) the buy side entropy. \( n \) is the number of firms on the side of the market in question. The imbalance between the two sides is naturally formed as the difference of entropies. We assign to the sell side the positive
sign and define the entropy imbalance as

$$\delta E = E_s - E_b.$$  \hspace{1cm} (5.3)

For a heterogenous volume partition the entropy is low\(^3\), therefore a positive value of the difference denotes that the buy side of the market is more heterogenous than the sell side. Alternatively, a large negative value of the difference means the sell side is more heterogenous and can be interpreted as the presence of a sell order on the market, unmatched by a large buy order.

To calculate the Gini index of a partition, we use the expression (Dixon et al., 1987; Sen, 1973)

$$G = \frac{1}{2n} \sum_{i=1}^{n} \sum_{j=1}^{n} |w_i - w_j|.$$  \hspace{1cm} (5.4)

The expression formulates the Gini index as the mean relative difference between all pairs of orders. If the partition is extremely uneven (one firm trading all the volume and the other firms trading infinitesimal amounts), the Gini index approaches one. If the partition is perfectly even (all firms trading equal amounts) the Gini index is zero.

In terms of the Gini indices we define the imbalance as

$$\delta G = \frac{G_b - G_s}{G_b + G_s},$$  \hspace{1cm} (5.5)

denoting the Gini index of the sell side of the market by \(G_s\) and the Gini index of the buy side of the market by \(G_b\). With this construction, similar to the entropy, the Gini imbalance is positive when buy orders are more heterogeneous, and is negative when the sell orders are more heterogeneous. The imbalance is zero when both sides of the market are similarly fragmented.

The difference between the two measures is that the entropy, in addition to the information about the partition of the total volume, contains also information about the number of firms. The larger the number of firms among which the volume is distributed, the larger the entropy.

Both the entropy and Gini measures are approximately normally distributed due to the sums in their definition and the central limit theorem. In terms of the time structure, both indices on a given side of the market are substantially serially correlated, however the imbalance variables are i.i.d.

We address the question of influence of the order size heterogeneity imbalance on the returns in terms of linear regressions. However, \(3^3\)In the terminology we adopt, a uniform partition of total trade volume among the member firms is a homogeneous partition.
Figure 5.3: Correlations, cross-dependencies and histograms of the four market imbalance variables and price returns. The diagonal shows the histogram. Values in the matrix below the diagonal are the absolute values of the correlation coefficients between the corresponding variables. The font size is proportional to the value of the coefficient. In the plots above the diagonal, we show scatter plots (tiny dots) as well as binned averages of the tiny dots (large points).

In addition to the size imbalance, we also include the imbalances in the signed order flow - it has been shown, for example in (Plerou et al., 2002a), that signed order flow has explanatory power for price returns. We include the imbalance in the signed number of trades $T$, signed transaction volume $V$ and numbers of buying and selling firms $N$. All imbalance variables are constructed as

$$\delta X = \frac{X_b - X_s}{X_b + X_s},$$  \hspace{1cm} (5.6)$$

where $X$ stands for any of the three variables. Finally, each imbalance variable is normalized by dividing with the standard deviation of the variable for the corresponding stock.
For the on-book market we have a full trading record (all limit orders, market orders and cancelations), and can therefore determine the sign of a trade precisely. However, for the off-book market this is not possible as the trades are negotiated via phone. To infer the trade sign we adopt a common algorithm where the trades are labeled as buyer or seller initiated depending on where the transaction price is in respect to the current bid and ask prices (Lee and Ready, 1991). The transactions close to the bid (i.e., below the mid-price) are labeled as seller initiated and the ones close to the ask (i.e., above the mid-price) as buyer initiated.

Our main result concerns the price returns and its relation to market imbalances. Other market variables such as the spread and volatility, though very important for markets, can not be analysed with a simple regression model as they are strongly correlated in time; this is left for future work.

The price returns are calculated as the log difference of the volume weighted average price (VWAP) of the last 10% and first 10% of trades of the day or hour, depending on the time scale of the analysis. The returns are normalized by the stock specific standard deviation. We also performed the analysis with market adjusted returns (subtracting from the non-normalized stock returns the FTSE100 returns) and obtained basically the same results.

As the results for the Gini imbalance and the entropy imbalance confirm each other, we show only the analysis and results for the entropy imbalance. We estimate the following regression of the price returns \( \delta P \)

\[
\delta P_t = \alpha \cdot \delta E_t + \beta \cdot \delta V_t + \gamma \cdot \delta T_t + \tau \delta N_t + \epsilon_t, \tag{5.7}
\]

where \( \delta E_t, \delta V_t, \delta T_t \) and \( \delta N_t \) are the previously described entropy, volume, number of trades and number of firms imbalance measures. To be able to compare the numerical values of the regression estimates between the hourly and daily regressions, we premultiply the hourly returns by the number of trading hours in a day (9 hours). This is to bring the scale of hourly returns to the scale of daily returns. The correlations, joint dependencies and histograms for the five variables are shown in figure 5.3. Some of the explanatory variables, such as signed trades and signed volume, are strongly correlated. This may lead to instabilities in coefficient estimates for those variables and we need to keep this in mind when interpreting results.

The results for the on- and off-book markets, as well as for the daily and hourly returns are collected in table II. Apart from the value of the coefficient, its error and p-value, we list also \( R^2_s \) and \( R^2_p \). \( R^2_s \) is the value of R-square of a regression with only the selected variable, and no others, included. It is equal to the square root of the absolute value of the correlation between the variable and the
Table 5.2: Regression results showing the significance of the market imbalance variables on price returns. Columns from left to right are estimated coefficient, its error and in the parenthesis the p-value of the test that the coefficient is zero assuming normal statistics; $R^2_s$ is the value of $R^2$ in a regression where only the selected variable is present in the regression. It expresses how much the variable on its own (solo) explains price returns. Final column $R^2_p$ is the partial $R^2$ of the selected variable. It expresses how much the variable explains price returns above the other three variables. We show separate results for the on- and off-book market, as well as for the daily and hourly returns. The constant in the regressions is not reported.
price returns. The column labeled $R^2_p$ is the partial R-square defined as $R^2_p = (R^2 - R^1)(1 - R^2_1)$ where $R^2$ is the R-square of a regression with all variables included, and $R^1$ is the R-square of a regression with all but the selected variable. $R^2_p$ is the partial contribution to the R-square due to the selected variable.

As we see from the table, the imbalances play a significant role in determining the price move on both markets and both time scales. All imbalance variables, except for the number of trades in the daily analysis, are significant beyond 1% level. However, signed trades are (statistically) insignificant only when analysed together with other orderflow variables due to the strong cross-correlations. In terms of economic significance, by looking at $R^2_s$ and $R^2_p$, the most important contribution seems to come from the signed volume imbalance, followed by the number of firms and entropy in the daily analysis, and the number of trades for the hourly analysis.

As expected, a large contribution to the price returns comes from the orderflow. Surprisingly however, one of the orderflow variables, the signed volume, changes sign between the two markets. We have performed reality checks by separating the data by stocks and by years (the data of each stock we separated in 3 one-year periods), and in all those subsamples found the same sign reversal. This suggests that an increase in the buyer initiated volume on the on-book market drives the price up, but a similar increase in buyer initiated volume on the off-book market drives the price down (analogously for the sell volume). The reason for this sign reversal seems to lie in cross market trading. The correlation of the volume imbalance between the two markets is significantly negative (about -0.15), implying that an increase in buyer initiated volume on the on-book volume is linked to an increase in the seller initiated volume on the off-book market (and vice versa). This effect is very suggestive of market making: for example, a firm buying on the on-book market and selling on the off-book market. Unfortunately, with our dataset we can not investigate further this behaviour as the member firm codes in the on- and off-book markets can not be linked.

However, an important point that is implied by this sign reversal is that it seems that the on-book market is responsible for the majority of price discovery on the LSE. The sign of the volume imbalance for the on-book market is positive, implying the intuitive behaviour that an increase in buyer initiated volume is linked with an increase in price. In contrast, an increase in buyer initiated volume on the off-book market leads to a decrease in price. We suspect that this is in fact due to the interaction of the two markets and that the increase of buyer-initiated volume on the off-book market is related to the increase of seller-initiated volume on the on-book market explaining the decrease in price. As we see here, one needs
to be very careful to include all sources of price adjustment forces when explaining price returns. This question is very interesting in its own right; unfortunately we leave it for further research.

In terms of the statistical specification of the model, residuals are i.i.d. and very close to normal. All the explanatory variables are exogenous to the model. Furthermore, we have also confirmed that the statistical errors under normality assumptions estimated in the model are correct by a bootstrap test. By shuffling the price returns and keeping all other variables intact, we obtain a realization of the null hypothesis where all the explanatory variables are correlated but are uncorrelated with the returns. Repeating the shuffling 1000 times and estimating the model on the bootstrapped data, we get a distribution of the coefficients under the null. The standard deviation of the estimates and the p-values obtained in this way coincide with the theoretical values shown in the table. Estimating recursive and sliding window regressions are all in line with the overall estimates. Looking at the regressions for each stock individually also leads to the same conclusions.

The overall $R^2$ on the on-book market is 32% for the daily analysis and 26% for hourly analysis. For the off-book market it is substantially lower at 7% for daily and 0% for hourly regressions. This suggests that the quotation market design (the off-book) substantially helps in limiting the price impact of order heterogeneity, but does not remove it completely. Finally, from the signs of the coefficient estimates we can summarize several facts.

1. The excess of buyer initiated over seller initiated volume drives prices up on the on-book market and down on the off-book market (and vice versa for the excess of seller initiated volume over buyer initiated volume).

2. Excess heterogeneity of buy order sizes seems to drive the price up; excess heterogeneity of sell orders seems to drive it down.

3. An excess in the number of buying firms pushes the price down, an excess of selling firms drives it up.

4. The number of transactions only plays a role at the hourly scale, not on the daily scale.

At this point it may be interesting to compare our regression results to the usual information based paradigm, such as for example in Keim and Madhavan (1996); Madhavan and Cheng (1997) and Smith et al. (2001). They also regress the price returns to signed volume (among other things) and compare the different effects on an on-book and off-book market, but differ in their interpretation of the price impact – or its lack of – by using concepts of informed and
uninformed trading. The regression results are nevertheless readily comparable.

The main difference in the regressions is their model specification which is parametrized for the comparison of the two markets. They estimate the effects using only one regression, where we use a separate regression for each market. By using only one regression equation, they are estimating the price effect of signed volume on both markets by two parameters. One is the overall effect of the signed volume (ignoring if the trade was coming from the on- or off-book market) and the other which is the correction to the first parameter for trades coming from the off-book market. They find a positive coefficient for the overall parameter, and a negative coefficient for the correction parameter, which they interpret as evidence that the off-book trades are less informed than the on-book trades. Indeed, we also find both a smaller coefficient and a substantially smaller $R^2$ for the off-book market trades. Our interpretation though is different, as we are saying that the off-book trades have a smaller price impact due to liquidity.

The issue of sign reversal of the volume effect is also not at odds with their results. Firstly, in the estimates of Smith et al. (2001) the magnitude of the correction parameter can easily be greater than the main effect parameter, effectively making the volume effect on the off-book market negative, and in one of their samples Madhavan and Cheng (1997) in fact report this. Another fact which may raise questions about the stability of their estimates is the disturbing asymmetry in the buyer and seller initiated trades that the studies find (they find the price effect for seller initiated trades, but no effect for buyer initiated trades). Asymmetry is always either very good, in which case it points to fundamental flaws in the current understanding of the problem, or is very bad, in which case it points to the problem of the analysis. The bottom line being, the reasons for accepting asymmetries where we do not expect them have to be very well grounded. Developed financial markets, such as all markets analysed in this discussion, to the authors’ knowledge do not show much evidence for the buyer/seller asymmetry (apart from trivial sign changes).

The information paradigm is also at odds with our following result. From “Fact 2” we can conclude that when a large order is transacted against multiple small orders it incurs a higher cost of price impact. In the information interpretation, this would suggest that large orders carry more information than small orders put together. This in itself can be interesting as it suggests that firms placing large orders are more ‘informed’ than the many small firms they transact with. The question that naturally arises in this context is what is the impact of a large order when it is transacted with
a comparably large order. To investigate this we create four dummy variables, each indexing one of the four market situations of interest. Variable D1 indexes instances when the sell side is heterogenous and the buy side homogenous, meaning there are a few large sell orders and many small buy orders. D2 indexes instances with the sell side homogenous and buy side heterogenous, the opposite of D1. D3 indexes instances with both sides being homogenous, i.e., many small orders on both sides. Finally D4 indexes situations with both sides being heterogenous, i.e., few large orders on both sides of the market. In order for a market side to be labeled as heterogenous the entropy needs to be smaller than the 25th quantile of the entropy density. A homogenous market side corresponds to the entropy being larger than the 75th quantile. In table 5.3 we display the results for a regression of daily price returns against signed orderflow and the four dummy variables. As expected, the order flow variables are significant and of the same sign as observed before. Of the four conditional means corresponding to the dummy variables, only D1 and D2 are significantly different from zero. This means that only situations with one side of the market being heterogenous leads to price impact. If both sides of the market are similarly fragmented, i.e., large sell orders trading with large buy orders, there is no large price impact. This suggests another fact:

5. Transacting of large buy orders against large sell orders does not substantially move the prices (and vice versa).

Using the informed vs. uninformed trading paradigm in this context, which of the oppositely trading large firms was correctly informed and which one was wrong? If both were transacting with small firms, they would incur a positive price impact, implying that their order was “informed”, however when transacting with a comparably large firm, there is no impact. Is it possible that somehow their different information sets cancel each other when confronted in the market? It is not obvious that such questions have an answer. To the authors this is suggestive that it may be liquidity and its absence that limits price impact of large trades, not the fact that large trades are in some ways more informed.

As a last corroboration of the significance of the entropy imbalance, we estimate partial regressions controlling for the influence of the orderflow. We first regress the returns $\delta P_t$ and the entropy imbalance $\delta E_t$ on the orderflow:

$$
\begin{align*}
\delta P_t &= \alpha_1 \cdot \delta V_t + \beta_1 \cdot \delta N_t + \gamma_1 \cdot \delta T_t + \epsilon_{1,t}, \\
\delta E_t &= \alpha_2 \cdot \delta V_t + \beta_2 \cdot \delta N_t + \gamma_2 \cdot \delta T_t + \epsilon_{2,t},
\end{align*}
$$

(5.8)

4To estimate the entropy density, we merge the sell and buy sides as they seem identical.
### Table 5.3: Regression results showing the effect of various market imbalances and its effect on the price returns. The dummy variables D1–D4 index market heterogeneity situations, for example the trading of few large sell orders with a large number of small buy orders. Details are in the text.
and then regress the residuals $\delta P_t'$ and $\delta E_t'$ obtained from the above regressions on each other

$$\delta P_t' = \alpha \cdot \delta E_t' + \epsilon_{3,t}. \quad (5.9)$$

In this way we remove the linear effect of the orderflow on both the price returns and the entropy imbalance. The entropy imbalance still remains significant: On the on-book market $\alpha = 0.20 \pm 0.01$, p-val = 0 and $R^2 = 3\%$. On the off-book market $\alpha = 0.039 \pm 0.008$ and p-val = 0. The on-book effect of the orderflow corrected entropy imbalance on the orderflow corrected price returns can also be seen in figure 5.4. The small points are the scatterplot of the variables while the larger points are the conditional means, binned on the x-axis.
5.4 Conclusions

We have shown that order size heterogeneity plays a role in price formation in addition to the signed order flow. Large orders which make up most of the trade volume in a given interval will incur the cost of market impact unless they transact with similarly large orders on the other side of the market. Alternatively, firms can limit the impact of large orders by trading in the off-book market. Trading in the off-book market however is not anonymous, which may be a concern in some situations.

The effect of size heterogeneity seems to be present on both daily and hourly time scales, though its economic importance on the hourly scale is very small. This large difference in order sizes can be attributed to the density of order sizes which is a fat tailed distribution with a tail exponent around 3/2 on the off-book market and 3 on the on-book market of the LSE and almost identical for the hourly and daily scale.

The fact that a large order moves the price when transacted with many small opposite orders, and does not move the price when transacted with a similarly large opposite order, is difficult to reconcile with the paradigm of information content of trades. For example, both the buy and sell large orders would have been informed if they transacted with multiple small orders (as they would move the price), but if they transact against each other they end up with no information content (as the price does not move). Did the information cancel out? It seems more likely that it is the limited liquidity that characterizes a situation of trading with multiple small orders which produces the price impact of a large order.

The interpretational differences between lack of liquidity and information content of trades is ultimately irrelevant. Neither interpretation really answers the question of why the price moves or not. In one interpretation, we sweep the question ab-initio under the rug by saying that the trades are informed or uninformed. As long as we do not have an explanation of which trades are informed beforehand, there is no way to know if a trade will or will not suffer price impact. On the other hand, in the interpretation using liquidity, we also do not provide a definitive answer as long as we have no explanation of liquidity fluctuations. However, many models, including the one tacitly implied here, have provided some insight into a possible mechanistic explanation of liquidity. Empirical behavioral finance studies may on the other hand provide insight for models of information content of trades.

In this paper, we have only touched on some aspects of the interaction of the member firms and market mechanisms in determining the price returns. Many other interesting things remain to be inves-
tigated, for example the cross correlations between the two markets. We have observed a curious fact that the entropy imbalance in the on-book market seems to be leading the imbalance in the off-book market: A shift in the entropy imbalance in the on-book market is substantially correlated with the shift in the imbalance in the off-book market up to three days later. This may be a signature of a firm first trying to transact its large order anonymously in the on-book market creating the imbalance there for two-three days, and then if it failed, resorting to the off-book market, where it then creates the imbalance there. This and other things seem interesting for future research.