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Monitoring process mean level using auxiliary information

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In this study, a Shewhart-type control chart is proposed for the improved monitoring of process mean level (targeting both moderate and large shifts which is the major concern of Shewhart-type control charts) of a quality characteristic of interest Y. The proposed control chart, namely the M_r chart, is based on the regression estimator of mean using a single auxiliary variable X. Assuming bivariate normality of (Y, X), the design structure of M_r chart is developed for phase I quality control. The comparison of the proposed chart is made with some existing control charts used for the same purpose. Using power curves as a performance measure, better performance of the proposed M_r chart is observed for detecting the shifts in mean level of the characteristic of interest.

Keywords and Phrases: control charts, power curves, simulations, $\bar{X}$ chart, M_r chart, regression-adjusted charts, cause-selecting charts, normality; auxiliary information.

1 Introduction

The monitoring of any process output demands an early detection of shifts in the process parameters. The shift may be in process variability, process mean level or both. The variability of any process is controlled first, followed by controlling of the mean level. In the 1920s, Walter A. Shewhart introduced the idea of control charts to monitor any process for variability or process mean level. The commonly used control charts for monitoring process variability are the R chart, the S chart and the S^2 chart and for process mean level are the $\bar{X}$ chart, the median chart, the trimmed mean chart and the mid-range chart.

For an improved monitoring of a quality characteristic of interest, the idea of exploiting correlation of the characteristic of interest with some other associated quality characteristic(s) had been used in the form of cause-selecting and regression-adjusted control charts, e.g. MANDEL (1969), ZHANG (1984, 1985), HAWKINS (1991, 1993), WADE and WOODALL (1993) and SHU, TSUNG and TSUI (2005). The cause-selecting and regression-adjusted control charts are constructed for a quality characteristic of interest after adjusting for the effect of some associated characteristic(s)

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(i.e. the residuals are obtained and used for monitoring the quality characteristic of interest).

By identifying and observing some auxiliary variables along with the variable of interest, the information on the relationship between auxiliary variables and variable of interest can be used to improve the precision with which parameters are estimated. Riaz (2008) introduced the idea of using information on an auxiliary characteristic in control charts for improved process monitoring. He considered the information on an auxiliary characteristic X for the improved monitoring of variance of a quality characteristic of interest Y. He proposed a regression-type estimator for variance of Y to capitalize on the correlation between Y and X and recommended its use in control charts for improved monitoring of variance of Y. He claimed the superiority of his proposed V_r chart to the well-known Shewhart control chart for variance (i.e. S^2 chart), conditioned on $|\rho_{yx}|$ [where ρ_{yx} is the correlation (linear relationship) between Y and X].

There are different classifications, in quality control literature, of control charts, e.g. according to the type of data, sample size, type of control, etc. Farnum (1994) classified two basic types of control: threshold control and deviation control. Threshold control is concerned with detecting the large shifts, whereas deviation control is concerned with detecting the small shifts in the process parameters. The Shewhart-type control charts are regarded as threshold control charts, while non- Shewhart control charts, (e.g. CUSUM and EWMA charts) are regarded as deviation control charts. In this study, the information about a single auxiliary characteristic X is introduced for the improved monitoring of the process mean level of a quality characteristic of interest Y following Riaz (2008). Assuming bivariate normality of (Y, X), a Shewhart-type process mean control chart, namely M_r chart (a threshold control chart), is proposed which is based on the regression estimator of process mean level. The focus of the proposed chart is on phase I quality control. The regression estimator for mean of Y, using a single auxiliary variable X, is defined for a bivariate random sample $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ of size n as:

$$M_r = \bar{y} + b(\mu_x - \bar{x}), \tag{1}$$

where $\bar{y}$ is the sample mean of Y, $\bar{x}$ is the sample mean of X, μ_x is the population mean of X (assumed to be known) and b is defined as:

$$b = r_{yx}(s_y/s_x), \tag{2}$$

where r_{yx} is the sample correlation coefficient between Y and X, s_x is the sample standard deviation of X and s_y is the sample standard deviation of Y.

In the following sections: (i) design structure of the proposed M_r chart is developed for improved monitoring of process mean level of Y, following the work of Pappanastos and Adams (1996) and Riaz (2006); (ii) power curves are constructed, as a performance measure, for the proposed M_r chart, following Nelson (1985) and Riaz (2008); (iii) performance of the proposed M_r chart is compared with those of some existing control charts (like Shewhart's well-known $\bar{X}$ chart, cause-selecting and regression-adjusted charts) used for the same purpose, following Tuprah and Ncube (1987), Acosta-Mejia, Pignatiello and Rao (1999) and Riaz (2008); and (iv) finally, an illustrative example to explain the working of the proposed M_r chart is also given.

2 The proposed chart

Assuming bivariate normality of (Y, X), a relationship between μ_y (the unknown process mean of quality characteristic of interest Y which is to be monitored) and M_r [the regression estimator of μ_y defined in (1)] is required to develop the structure of the M_r chart. Let $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ be a bivariate random sample of size n from a bivariate normal distribution, and let C be the random variable that defines a relationship between μ_y and M_r as:

$$C = \sqrt{n}(M_r - \mu_y)/\sigma_y, \tag{3}$$

where σ_y is the standard deviation of Y, which has already been controlled using some variability control chart. The relationship defined in (3) helps determining the parameters (i.e. centerline, lower and upper control limits) of the proposed M_r chart.

Now, if the distributional behavior of C is known then the sample statistic M_r can easily be used for the testing of hypotheses about shifts in μ_y. When (Y, X) follow bivariate normal distribution, the distributional behavior of C depends only on ρ_{yx} (the correlation between Y and X) and n. The distributional behavior of C, in terms of its mean, standard error and quantile points, is required for the development of the M_r chart, and is explored in the following paragraphs when (Y, X) follow a bivariate normal distribution.

First, for mean, applying expectations to (3) gives:

$$E(C) = E(\sqrt{n}(M_r - \mu_y)/\sigma_y) = \sqrt{n}E(M_r - \mu_y)/\sigma_y. \tag{4}$$

Here, $E(M_r)$ can safely be replaced by its estimate $\bar{M}_r$ (the mean of sample M_rs) using an appropriate number of random samples, as discussed in Hillier (1969) and Yang and Hillier (1970), from the process under study when the process is in the state of statistical control [just like $\bar{R}$ replaces $E(R)$ for the R-chart]. Thus, from (4), an estimate of μ_y, after rearranging and simplifying the terms, is given as:

$$\hat{\mu}_y = \bar{M}_r - \hat{\sigma}_y E(C)/\sqrt{n}. \tag{5}$$

The regression estimator M_r is generally a biased estimator of the population mean μ_y, but the bias vanishes when the relationship between Y and X is linear (see Sukhatme and Sukhatme, 1984, p. 238). So, for the case of bivariate normal (Y, X), M_r is unbiased for μ_y and hence $E(C) = 0$. Thus, (5) results into the following:

$$\hat{\mu}_y = \bar{M}_r. \tag{6}$$

Also, from (4), we have:

$$E(M_r) = \mu_y. \tag{7}$$

Replacing the estimate of μ_y [given in (6)] in (7) gives:

$$E\left(M_r\right) \simeq \bar{M}_r. \tag{8}$$

Second, for standard error, let the standard deviation of C (i.e. σ_C) be

$$\sigma_C = k_2. \tag{9}$$

It is not easy to get the analytical results for k_2 because $E(M_r^2)$ is difficult to obtain analytically. So, the simulation results are obtained for k_2 in this paper (in practice, simulation methods are often used to evaluate the expectation of a statistic, see Ross, 1990). The coefficient k_2 depends entirely on ρ_{yx} and n, in the case of bivariate normal distribution. Using 10,000 random samples generated from a standard bivariate normal distribution, without loss of generality, the results of k_2 have been obtained, for different combinations of ρ_{yx} and n, 1000 times each (for a detailed discussion regarding the number of simulations needed in control chart studies, see Schaffer and Kim, 2007). Based on these results, the mean values of k_2, along with their respective standard errors, are provided in Appendix Table A1 for $n = 5, 6, \ldots, 15, 20, 25, 30, 50, 100$ at some representative values of ρ_{yx}. The similar results can easily be obtained for any combination of ρ_{yx} and n.

In addition, taking variance of C and then simplifying finally gives the expression for σ_C as:

$$\sigma_C = \sqrt{n}\sigma_{M_r}/\sigma_y, \tag{10}$$

where σ_{M_r} represents the standard deviation of distribution of sample statistic M_r. Using (9) in (10) and rearranging yields the following result for σ_{M_r}:

$$\sigma_{M_r} = k_2\sigma_y/\sqrt{n}. \tag{11}$$

Substituting estimate for σ_y in (11), the estimate for σ_{M_r} is given as:

$$\hat{\sigma}_{M_r} = k_2\hat{\sigma}_y/\sqrt{n}, \tag{12}$$

where $\hat{\sigma}_y$ is controlled estimate of process standard deviation σ_y which can be obtained from some variability control chart.
An approximation for σ_{M_r}, when (Y, X) follows a bivariate normal distribution, is given as (see Sukhatme and Sukhatme, 1984, p. 267):

$$\sigma_{M_r} \simeq \sqrt{\sigma_y^2(1-\rho_{yx}^2)(1+1/(n-3))/n}. \tag{13}$$

Consequently,

$$k_2 \simeq \sqrt{(1-\rho_{yx}^2)(1+1/(n-3))}. \tag{14}$$

Asymptotically, and even for the smaller values of n, the approximation (14) works well, as can be seen from Appendix Table A1, e.g. for $n = 20$ and $\rho_{yx} = 0.70$, we have $k_2 = 0.7373$ (from Table A1), whereas relationship (14) gives $k_2 \simeq 0.7348$. The two results are very close to each other, which show the good approximation ability of result (14).

Lastly, for the quantile points of the distribution of C, let C_a represents the ath quantile point of the distribution of C (i.e. the point where C completes a% area). The analytical results for C_a are difficult to obtain; so, the simulation results are obtained for C_a. For a bivariate normal distribution of (Y, X), the quantile points of the distribution of C depends entirely on ρ_{yx} and n. Using the same 10,000 simulated random samples, the results of C_a have been obtained (like quantile points of $W = R/\sigma$ that determine the values of control limits of R chart and power of the chart) for different combinations of ρ_{yx} and n, 1000 times each. Based on these results, the mean values of some commonly used quantile points, along with their respective standard errors, are provided for $n = 5, 6, \ldots, 15, 20, 25, 30, 50, 100$ in Appendix Tables A2–A11 at some representative values of ρ_{yx}. The similar results can easily be obtained for any combination of ρ_{yx} and n. These quantile points help determining the control limits and the power of the proposed M_r chart to detect shifts in process mean level for the quality characteristic of interest Y. The distributional behavior of C is not symmetrical, at least for small values of n, as obvious from Appendix Tables A2–A11. Asymptotically, C is normally distributed, $N(0, (1 - \rho_{yx}^2)(1 + 1/(n - 3)))$.

Now, based on the results obtained in section 2, the parameters of the proposed M_r chart are discussed in section 3.

3 Parameters of proposed chart

The central line (CL), lower control limit (LCL) and upper control limit (UCL) are the three parameters of any Shewhart-type control chart. There are two approaches to express these parameters, namely probability limits approach and 3-sigma limits approach. In case of asymmetric distributional behavior of the relevant estimator, the probability limits approach is preferred. If the distributional behavior of the relevant estimator is nearly symmetric, then 3-sigma limits approach is a good alternative. The parameters of the proposed M_r chart using both the approaches are expressed in sections 3.1 and 3.2.

3.1 Probability limits approach

The value $\bar{M}_r$ corresponds to CL of the proposed M_r chart, just like $\bar{R}$ for R chart provided in Alwan (2000, p. 347) and $\bar{S}$ for S chart provided in Alwan (2000, p. 362). Assuming the probability of making a type I error to be less than a specified value say α, the control limits (which are actually the true probability limits) for the proposed M_r chart are defined as:

$$\left.\begin{array}{ll} \mathrm{LCL} = M_{r_1} & \text{with } P_n\left(M_r = M_{r_l}\right) \le \alpha_l \\ \mathrm{UCL} = M_{r\mathrm{u}} & \text{with } P_n\left(M_r = M_{r_u}\right) \ge 1 - \alpha_u \end{array}\right\}, \tag{15}$$

where $\alpha = \alpha_\mathrm{l} + \alpha_\mathrm{u}$ and P_n represents the cumulative distribution function for a given value of n.

Now using (3) and (6) in (15) and simplification finally gives the following:

$$\left.\begin{aligned} \text{LCL} = M_{r_\text{l}} = \bar{M}_r + C_l\hat{\sigma}_y/\sqrt{n} \quad \text{with } P_n\left(C = C_l\right) \le \alpha_l \\ \text{UCL} = M_{r_\text{u}} = \bar{M}_r + C_u\hat{\sigma}_y/\sqrt{n} \quad \text{with } P_n\left(C = C_u\right) \ge 1 - \alpha_u \end{aligned}\right\}. \tag{16}$$

Thus, the quantile points of the distribution of C, the average of sample M_rs (i.e. $\bar{M}_r$) and $\hat{\sigma}_y$ (a controlled estimate of process standard deviation σ_y) allow setting the true probability limits for the proposed M_r chart.

3.2 3-Sigma limits approach

If normal approximation to the distribution of C is used, then the parameters of M_r chart with the usual 3-sigma control limits are given as:

$$\left.\begin{aligned} \text{LCL} &= \bar{M}_r - 3\sigma_{M_r} \\ \text{CL} &= \bar{M}_r \\ \text{UCL} &= \bar{M}_r + 3\sigma_{M_r} \end{aligned}\right\}, \tag{17}$$

Using (12) in (17) gives the following result:

$$\left.\begin{aligned} \text{LCL} &= \bar{M}_r - 3k_2\hat{\sigma}_y/\sqrt{n} \\ \text{CL} &= \bar{M}_r \\ \text{UCL} &= \bar{M}_r + 3k_2\hat{\sigma}_y/\sqrt{n} \end{aligned}\right\}, \tag{18}$$

where the values of k_2 are provided in Appendix Table A1.
The validity of these 3-sigma limit-based parameters of the proposed M_r chart depends on how close the normal approximation is to the true distribution of C.

After deciding the control structure, for a given significance level, by either the probability limit approach or the 3-sigma limit approach, the sample statistic M_r is plotted against time order of the samples. If all the sample M_rs lie within control limits, there is reasonable evidence to conclude that there is no shift (particularly of larger amount) in the process mean level and the process is stable at $\bar{M}_r$. Otherwise, some assignable cause(s) is/are at work, causing a shift in the process mean level.

To address small and moderate shifts, using the developed structure of M_r chart, the runs rules (as discussed by Nelson, 1984, Wheeler, 1995 and Does and Schriever, 1992) may be supplemented to its basic structure. As a result, the risk of false alarms is increased.

4 Comparisons

In this section, some comparisons of the proposed M_r chart are made with well-known Shewhart $\bar{X}$ chart (as the characteristic of interest is denoted by Y in this study, the name $\bar{Y}$ chart will be used instead of the $\bar{X}$ chart later in this paper) and cause-selecting charts of Zhang (1984) and Wade and Woodall (1993). As the focus of the proposal is on phase I quality control, power curves have been used as

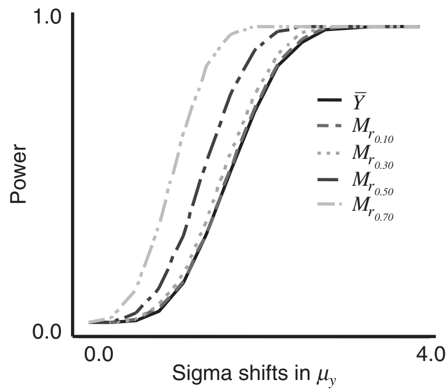

Fig. 1. Power curves of $\bar{Y}$ chart and M_r chart with $|\rho_{yx}|=0.10, 0.30, 0.50, 0.70$ using $\alpha=0.01$.

a performance measure (in contrast to phase II quality control where average run length (ARL) is used as a performance measure) of the control charts following Albers and Kallenberg (2006).

4.1 M_r chart vs. $\bar{Y}$ chart

Using the parameters for M_r and $\bar{Y}$ charts, as given in (16) and Alwan (2000, p. 394), respectively, the performance of the M_r chart has been compared with that of the $\bar{Y}$ chart in terms of discriminatory power. As the distributional behavior of C is not symmetrical, at least for smaller values of n, we have preferred to use the probability limits approach for the two charts to set control limits for a given significance level (α). Using the respective control structures of the two charts, the control limits of M_r and $\bar{Y}$ charts have been obtained for different combinations of ρ_{yx} and n, using different significance levels, and power curves for the two charts have been constructed. The power curves, for $n=15$, are produced here for some values of ρ_{yx} in Figure 1 (using $\alpha=0.01$).

In Figure 1, the curve referred to as $\bar{Y}$ represents the power curve of the $\bar{Y}$ chart, while the curves referred to as $M_{r_{0.10}}$, $M_{r_{0.30}}$, $M_{r_{0.50}}$ and $M_{r_{0.70}}$ represent the power curves of the M_r chart when $|\rho_{yx}|$ is 0.10, 0.30, 0.50 and 0.70, respectively. The similar behavior is observed for the other values of n. It has been observed that the discriminatory power of the M_r chart is higher than that of the $\bar{Y}$ chart and the gain in terms of discriminatory power for the M_r chart keeps increasing with an increase in $|\rho_{yx}|$, as can be seen in Figure 1.

4.2 M_r chart vs. cause-selecting and regression-adjusted charts

The cause-selecting and regression-adjusted control charts exploit the correlation between Y and X in the same fashion (i.e. both types of charts are constructed for a quality characteristic of interest Y after adjusting for the effect of some correlated characteristic X); so, the comparisons of the proposed M_r chart are made only with the cause-selecting charts. In this paper, the performance of the M_r chart is compared with those of cause-selecting approaches proposed by ZHANG (1984) and WADE and WOODALL (1993).

WADE and WOODALL (1993) proposed prediction limits by modifying ZHANG (1984) limits. They computed the powers of detecting shifts in μ_y for given x, using Zhang's limits and their proposed prediction limits. Particularly, they considered $n=50$, $\alpha=0.006, \sigma_y^2=\sigma_x^2=1$ and computed the powers for detecting different shifts in μ_y for given x, using Zhang's limits and their proposed prediction limits. They claimed slightly better performance of their proposed prediction limits than Zhang's limits, for detecting shifts in μ_y for given x. For the same environment (i.e. $n=50, \alpha=0.006$, $\sigma_y^2=\sigma_x^2=1$), the powers are computed using the control limits of the proposed M_r chart of this paper. The same shifts in μ_y for given x have been considered, as were considered by WADE and WOODALL (1993) for Zhang's limits and their proposed prediction limits. The powers are then plotted against different shifts in μ_y given x for ZHANG (1984) limits, WADE and WOODALL (1993) prediction limits and the limits based on the proposed M_r chart of this paper. The resulting power curves are shown in Figure 2 using $\rho_{yx}=0.90$ for comparison purposes. A similar pattern of power curves has been observed for $\rho_{yx}=0.50$ as well.

In the following figure, the curves referred to as '1', '2' and '3' are of Zhang's limits, Wade and Woodall's prediction limits and the M_r chart-based limits, respectively.

It has been observed that in terms of the discriminatory power, (i) the prediction limits of WADE and WOODALL (1993) are slightly better than ZHANG's (1984) (as claimed by WADE and WOODALL, 1993); (ii) the proposed M_r chart-based limits of this paper are slightly better than those of WADE and WOODALL (1993), as obvious from Figure 2. This improved performance of the proposed M_r chart-based limits may be due to the dual exploitation of the correlation between Y and X [once in estimating b of (1) and then in computing M_r defined in (1)]. These results may easily be claimed for the M_r chart relative to HAWKINS (1993) regression-adjusted

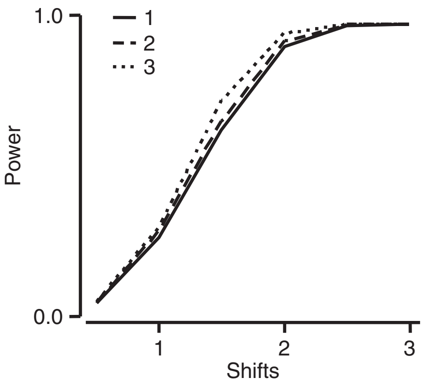

Fig. 2. Power curves for Zhang's limits, Wade and Woodall's limits and M_r chart-based limits using $\rho_{yx}=0.90$ and $\alpha=0.006$.

charts at least for the processes without cascade property (see HAWKINS, 1993) due to the similar structure of these charts.

5 An illustrative example

To illustrate the application of the proposed M_r chart, let us consider the data considered first by CONSTABLE *et al.* (1988) and then by WADE and WOODALL (1993) on X = ROLLWT and Y = BAKEWT. They considered the paired information on (Y, X) and used X to find the fitted values $\hat{Y}$ and then residuals $Y - \hat{Y}$ by fitting the least square regression line of Y on X when the process is assumed to be stable (in control). They used first 45 data points (which are from a stable state of the process) to find the fitted values $\hat{Y}$. After capitalizing on the relationship between Y and X this way, the residuals (i.e. $Y - \hat{Y}$) were computed for the improved process monitoring for Y. We consider here the same data set by giving the role of auxiliary variable to X and the role of characteristic of interest to Y. The same first 45 stable data points on (Y, X) are used and the mean values, standard deviations and correlation between Y and X are computed. These are mean$(Y)=201.18$, mean$(X)=210.24$, SD $(Y)=1.17$, SD $(X)=1.23$ and corr$(Y, X)=0.54$. As these results are obtained from stable points of the process, we assume here these estimates as the true parameter values just for illustration purposes (i.e. $\mu_y=201.18, \mu_x=210.24, \sigma_y=1.17, \sigma_x=1.23$ and $\rho_{yx}=0.54$). Assuming bivariate normality of (Y, X) and considering $\mu_x=210.24$ to be the known mean value of auxiliary variable X, 10 bivariate random samples (ideally these should be 20–30 initial random samples as recommended by Shewhart, but for convenience we have considered only 10 here) each of size 10 are simulated from the bivariate normal distribution, i.e. $N_2(\mu_y, \mu_x, \sigma_y, \sigma_x, \rho_{yx})$. The inspiration for this approach of simulation is taken from SINGH and MANGAT (1996, p. 221). The simulated data contains 90% values from $N_2(201.18, 210.24, 1.17, 1.23, 0.54)$ and 10% values from $N_2(203.18, 210.24, 1.17, 1.23, 0.54)$ (i.e. the data is contaminated for μ_y). The resulting data set of 10 bivariate random samples (Y, X), each of size 10, is given in Appendix Table A12. Now, the objective is to see whether the M_r chart is able to detect the contamination in the data

The sample statistics M_r and R_y for the data set given in appendix Table A12

Sample number	M_r	R_y
1	201.286	4.321
2	201.593	4.417
3	201.691	3.845
4	201.278	4.568
5	201.863	3.504
6	201.226	5.106
7	201.240	3.470
8	201.653	3.737
9	200.364	2.603
10	201.408	3.813

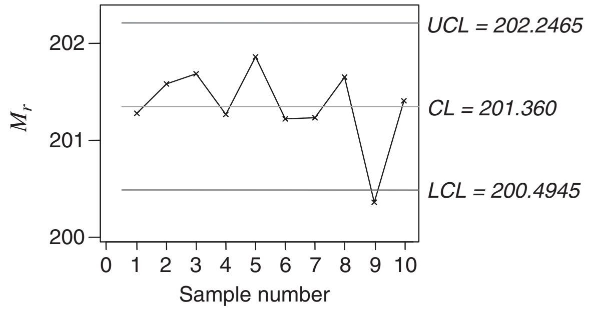

Fig. 3. M_r-Chart

or not? For this purpose, the sample statistics M_r and R_y (i.e. sample ranges for Y) are computed for these 10 samples and are given in the preceding table.
Based on the results of the above table, the control limits for the M_r chart using its control structure given in (16) are given as (using $\alpha = 0.02$):

$$\left.\begin{aligned} \mathrm{LCL} &= \bar{M}_r + C_{0.01}\hat{\sigma}_y/\sqrt{n} \\ \mathrm{CL} &= \bar{M}_r \\ \mathrm{UCL} &= \bar{M}_r + C_{0.99}\hat{\sigma}_y/\sqrt{n} \end{aligned}\right\}, \tag{19}$$

where $\hat{\sigma}_y = \bar{R}_y/d_2 = 3.938/3.078 = 1.2794$, $C_{.01} = -2.1391$.

The M_r chart using the control limits given in (19), for the simulated data set of Appendix Table A12, is given in Figure 3.

The sample statistic M_r, on its control chart, falling outside lower and upper control limits refers to some assignable cause(s) in the process at that time point. The M_r chart gives signal of some assignable cause(s) for sample at time point 9 in the simulated data set.

6 Conclusion

The proposal of this study is a Shewhart-type control chart for stage I quality control. The proposed M_r chart uses the information on a single auxiliary variable and capitalizes on its correlation with a quality characteristic of interest for the improved monitoring of the mean level of the quality characteristic of interest. It has been observed that the performance of the M_r chart, in terms of the discriminatory power, keeps improving with an increase in ρ_{yx}. In addition, comparisons with the conventional, cause-selecting and regression-adjusted control charts have proven a superior performance of the proposed M_r chart, in terms of the discriminatory power, for detecting shifts (especially of moderate and larger amount because Shewhart control charts target such shifts) in the process mean level for Y (i.e. μ_y).

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Appendix

In the following tables the results are reported up to four decimal places. The values reported in brackets are standard errors (reported up to five decimal places) for the results in each cell, reported to show how precise the results in each cell are.

Table A1. Values of k_2

n	ρ_{yx} 0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
5	1.2112	1.2029	1.1699	1.0944	1.0154	0.9717	0.8754	0.7282	0.5388	0.1715
	(0.00033)	(0.00012)	(0.00021)	(0.00049)	(0.00055)	(0.00037)	(0.00021)	(0.00012)	(0.00026)	(0.00019)
6	1.1493	1.1231	1.1090	1.0587	1.0021	0.9300	0.8206	0.7003	0.5124	0.1632
	(0.00121)	(0.00042)	(0.00052)	(0.00016)	(0.00032)	(0.00052)	(0.00111)	(0.00038)	(0.00014)	(0.00039)
7	1.1108	1.0901	1.0711	1.0213	0.9709	0.9056	0.8056	0.6765	0.5004	0.1596
	(0.00026)	(0.00037)	(0.00092)	(0.00037)	(0.00021)	(0.00028)	(0.00045)	(0.00019)	(0.00051)	(0.00033)
8	1.0884	1.0657	1.0407	1.0011	0.9449	0.8797	0.7765	0.6556	0.4910	0.1530
	(0.00008)	(0.00044)	(0.00032)	(0.00022)	(0.00019)	(0.00015)	(0.00011)	(0.00040)	(0.00029)	(0.00027)
9	1.0789	1.0630	1.0330	0.9911	0.9340	0.8654	0.7645	0.6467	0.4857	0.1521
	(0.00023)	(0.00039)	(0.00089)	(0.00039)	(0.00047)	(0.00048)	(0.00043)	(0.00084)	(0.00023)	(0.00021)
10	1.0678	1.0520	1.0210	0.9823	0.9219	0.8530	0.7587	0.6481	0.4728	0.1505
	(0.00041)	(0.00021)	(0.00065)	(0.00024)	(0.00060)	(0.00033)	(0.00065)	(0.00011)	(0.00067)	(0.00044)
11	1.0552	1.0421	1.0112	0.9732	0.9187	0.8496	0.7515	0.6390	0.4662	0.1497
	(0.00022)	(0.00077)	(0.00067)	(0.00055)	(0.00078)	(0.00070)	(0.00076)	(0.00066)	(0.0016)	(0.00040)
12	1.0509	1.0326	1.0088	0.9697	0.9166	0.8450	0.7490	0.6371	0.4608	0.1488
	(0.00030)	(0.00054)	(0.00029)	(0.00076)	(0.00067)	(0.00026)	(0.00025)	(0.00021)	(0.00072)	(0.00051)
13	1.0459	1.0275	1.0010	0.9649	0.9050	0.8404	0.7465	0.6303	0.4561	0.1479
	(0.00014)	(0.00047)	(0.00066)	(0.00008)	(0.00113)	(0.00047)	(0.00036)	(0.00074)	(0.00067)	(0.00033)
14	1.0447	1.0232	0.9978	0.9526	0.9035	0.8358	0.7437	0.6270	0.4530	0.1471
	(0.00022)	(0.00034)	(0.00030)	(0.00042)	(0.00080)	(0.00082)	(0.00022)	(0.00059)	(0.00050)	(0.00060)
15	1.0435	1.0201	0.9911	0.9471	0.9030	0.8288	0.7410	0.6245	0.4511	0.1463
	(0.00010)	(0.00033)	(0.00063)	(0.00040)	(0.00036)	(0.00091)	(0.00014)	(0.00026)	(0.00107)	(0.00016)
20	1.0310	1.0157	0.9833	0.9389	0.8988	0.8192	0.7373	0.6201	0.4491	0.1460
	(0.00029)	(0.00041)	(0.00021)	(0.00083)	(0.00033)	(0.00032)	(0.00071)	(0.00041)	(0.00014)	(0.00041)
25	1.0146	0.9959	0.9724	0.9357	0.8874	0.8162	0.7284	0.6125	0.4470	0.1445
	(0.00038)	(0.00020)	(0.00011)	(0.00053)	(0.00061)	(0.00021)	(0.00040)	(0.00063)	(0.00030)	(0.00022)
30	1.0122	0.9887	0.9647	0.9293	0.8834	0.8129	0.7263	0.6094	0.4389	0.1443
	(0.00061)	(0.00084)	(0.00033)	(0.00031)	(0.00055)	(0.00078)	(0.00024)	(0.00017)	(0.00028)	(0.00036)
50	1.0109	0.9799	0.9503	0.9230	0.8821	0.8088	0.7252	0.6085	0.4368	0.1429
	(0.00044)	(0.00028)	(0.00018)	(0.00048)	(0.00032)	(0.00033)	(0.00066)	(0.00051)	(0.00074)	(0.00017)
100	1.0029	0.9774	0.9487	0.9205	0.8718	0.8021	0.7230	0.6021	0.4351	0.1427
	(0.00012)	(0.00047)	(0.00081)	(0.00027)	(0.00050)	(0.00060)	(0.00019)	(0.00044)	(0.00041)	(0.00047)

Table A2. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.10$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.8952	−1.8710	−1.4488	−0.9400	−0.7538	0.7605	0.9510	1.4663	1.9397	2.9940
	(0.00033)	(0.00028)	(0.00021)	(0.00078)	(0.00042)	(0.00061)	(0.00022)	(0.00042)	(0.00018)	(0.00017)
6	−2.7204	−1.8508	−1.4198	−0.9258	−0.7429	0.7508	0.9497	1.4568	1.8823	2.7631
	(0.00010)	(0.00021)	(0.00041)	(0.00088)	(0.00031)	(0.00066)	(0.00074)	(0.00028)	(0.00049)	(0.00041)
7	−2.6103	−1.7997	−1.3965	−0.9094	−0.7324	0.7420	0.9372	1.4157	1.8292	2.6688
	(0.00032)	(0.00033)	(0.00091)	(0.00022)	(0.00041)	(0.00037)	(0.00061)	(0.00055)	(0.00019)	(0.00008)
8	−2.5648	−1.7790	−1.3926	−0.8987	−0.7224	0.7351	0.9239	1.4063	1.7824	2.5974
	(0.00028)	(0.00017)	(0.00036)	(0.00043)	(0.00026)	(0.00029)	(0.00041)	(0.00111)	(0.00056)	(0.00011)
9	−2.5126	−1.7529	−1.3703	−0.8828	−0.7203	0.7389	0.9107	1.3845	1.7696	2.5458
	(0.00026)	(0.00032)	(0.00035)	(0.00074)	(0.00011)	(0.00046)	(0.00034)	(0.00084)	(0.00030)	(0.00028)
10	−2.4720	−1.7482	−1.3516	−0.8780	−0.7188	0.7301	0.9010	1.3611	1.7513	2.4908
	(0.00014)	(0.00019)	(0.00062)	(0.00037)	(0.00066)	(0.00026)	(0.00078)	(0.00065)	(0.00052)	(0.00031)
11	−2.4578	−1.7350	−1.3424	−0.8767	−0.7123	0.7296	0.8993	1.3513	1.7383	2.4764
	(0.00020)	(0.00032)	(0.00028)	(0.00067)	(0.00037)	(0.00046)	(0.00056)	(0.00073)	(0.00040)	(0.00022)
12	−2.4308	−1.7284	−1.3384	−0.8654	−0.7061	0.7275	0.8830	1.3415	1.7218	2.4551
	(0.00032)	(0.00048)	(0.00031)	(0.00053)	(0.00077)	(0.00065)	(0.00041)	(0.00069)	(0.00038)	(0.00007)
13	−2.4216	−1.7109	−1.3283	−0.8601	−0.7009	0.7101	0.8711	1.3340	1.7185	2.4324
	(0.00021)	(0.00036)	(0.00047)	(0.00081)	(0.00013)	(0.00066)	(0.00043)	(0.00029)	(0.00042)	(0.00030)
14	−2.4118	−1.7015	−1.3159	−0.8562	−0.6947	0.6974	0.8674	1.3264	1.7057	2.4229
	(0.00040)	(0.00023)	(0.00078)	(0.00045)	(0.00034)	(0.00024)	(0.00068)	(0.00036)	(0.00055)	(0.00028)
15	−2.4048	−1.6942	−1.3060	−0.8590	−0.6841	0.6878	0.8657	1.3172	1.6972	2.4111
	(0.00022)	(0.00011)	(0.00090	(0.00031)	(0.00016)	(0.00063)	(0.00074)	(0.00047)	(0.00042)	(0.00037)
20	−2.3927	−1.6694	−1.3001	−0.8578	−0.6823	0.6841	0.8625	1.3088	1.6835	2.3789
	(0.00018)	(0.00045)	(0.00019)	(0.00087)	(0.00046)	(0.00024)	(0.00037)	(0.00065)	(0.00013)	(0.00026)
25	−2.3630	−1.6634	−1.2990	−0.8560	−0.6804	0.6824	0.8601	1.3015	1.6704	2.3621
	(0.00006)	(0.00071)	(0.00054)	(0.00036)	(0.00018)	(0.00047)	(0.00062)	(0.00071)	(0.00082)	(0.00043)
30	−2.3550	−1.6527	−1.2949	−0.8541	−0.6796	0.6811	0.8615	1.2971	1.6621	2.3585
	(0.00015)	(0.00029)	(0.00089)	(0.00041)	(0.00023)	(0.00092)	(0.00038)	(0.00049)	(0.00026)	(0.00033)
50	−2.3512	−1.6499	−1.2896	−0.8526	−0.6785	0.6796	0.8575	1.2899	1.6507	2.3511
	(0.00034)	(0.00039)	(0.00054)	(0.00017)	(0.00075)	(0.00048)	(0.00053)	(0.00036)	(0.00028)	(0.00026)
100	−2.3327	−1.6488	−1.2857	−0.8432	−0.6757	0.6766	0.8430	1.2859	1.6497	2.3336
	(0.00022)	(0.00051)	(0.00037)	(0.00046)	(0.00063)	(0.00078)	(0.00042)	(0.00026)	(0.00064)	(0.00015)

Table A3. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.20$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.8136	−1.8312	−1.4420	−0.9303	−0.7480	0.7542	0.9550	1.4708	1.9211	2.8641
	(0.00026)	(0.00042)	(0.00076)	(0.00089)	(0.00016)	(0.00039)	(0.00042)	(0.00028)	(0.00054)	(0.00036)
6	−2.6515	−1.8079	−1.4139	−0.9175	−0.7255	0.7382	0.9261	1.4266	1.8160	2.6985
	(0.00018)	(0.00043)	(0.00066)	(0.00052)	(0.00047)	(0.00025)	(0.00048)	(0.00034)	(0.00028)	(0.00019)
7	−2.5792	−1.7666	−1.3568	−0.8906	−0.7076	0.7187	0.9024	1.3813	1.7825	2.6234
	(0.00033)	(0.00084)	(0.00047)	(0.00035)	(0.00028)	(0.00076)	(0.00041)	(0.00055)	(0.00047)	(0.00025)
8	−2.5305	−1.7593	−1.3548	−0.8874	−0.7024	0.7084	0.8906	1.3622	1.7718	2.5722
	(0.00028)	(0.00046)	(0.00057)	(0.00074)	(0.00044)	(0.00093)	(0.00088)	(0.00036)	(0.00023)	(0.00042)
9	−2.4906	−1.7458	−1.3402	−0.8788	−0.7000	0.7089	0.8846	1.3586	1.758	2.5268
	(0.00009)	(0.00022)	(0.00038)	(0.00047)	(0.00054)	(0.00011)	(0.00017)	(0.00028)	(0.00049)	(0.00027)
10	−2.4339	−1.7296	−1.3305	−0.8748	−0.6984	0.7014	0.8815	1.3464	1.7370	2.4770
	(0.00032)	(0.00047)	(0.00028)	(0.00043)	(0.00019)	(0.00052)	(0.00036)	(0.00027)	(0.00056)	(0.00034)
11	−2.4094	−1.7063	−1.3275	−0.8637	−0.6851	0.6896	0.8780	1.3341	1.7137	2.4274
	(0.00013)	(0.00026)	(0.00095)	(0.00028)	(0.00062)	(0.00044)	(0.00038)	(0.00087)	(0.00032)	(0.00060)
12	−2.3962	−1.6927	−1.3201	−0.8583	−0.6805	0.6834	0.8707	1.3294	1.7097	2.4111
	(0.00025)	(0.00038)	(0.00043)	(0.00029)	(0.00070)	(0.00066)	(0.00045)	(0.00054)	(0.00035)	(0.00058)
13	−2.3873	−1.6873	−1.3123	−0.8500	−0.6765	0.6798	0.8610	1.3200	1.6978	2.3920
	(0.00042)	(0.00033)	(0.00010)	(0.00045)	(0.00087)	(0.00016)	(0.00065)	(0.00047)	(0.00039)	(0.00050)
14	−2.3714	−1.6717	−1.3065	−0.8442	−0.6760	0.6792	0.8560	1.3141	1.6877	2.3889
	(0.00051)	(0.00047)	(0.00062)	(0.00040)	(0.00039)	(0.00067)	(0.00028)	(0.00091)	(0.00043)	(0.00036)
15	−2.3645	−1.6582	−1.2968	−0.8391	−0.6701	0.6739	0.8406	1.2999	1.6652	2.3763
	(0.00044)	(0.00047)	(0.00048)	(0.00033)	(0.00028)	(0.00042)	(0.00057)	(0.00079)	(0.00064)	(0.00019)
20	−2.3592	−1.6436	−1.2889	−0.8364	−0.6697	0.6713	0.8398	1.2899	1.6487	2.3687
	(0.00022)	(0.00030)	(0.00029)	(0.00063)	(0.00076)	(0.00034)	(0.00017)	(0.00026)	(0.00048)	(0.00035)
25	−2.3311	−1.6319	−1.2767	−0.8308	−0.6675	0.6699	0.8337	1.2789	1.6366	2.3420
	(0.00011)	(0.00046)	(0.00071)	(0.00056)	(0.00037)	(0.00102)	(0.00044)	(0.00032)	(0.00018)	(0.00008)
30	−2.3056	−1.6224	−1.2601	−0.8252	−0.6630	0.6685	0.8282	1.2667	1.6261	2.3149
	(0.00024)	(0.00039)	(0.00061)	(0.00054)	(0.00047)	(0.00052)	(0.00046)	(0.00018)	(0.00082)	(0.00037)
50	−2.2804	−1.6115	−1.2558	−0.8243	−0.6602	0.6615	0.8251	1.2563	1.6126	2.2815
	(0.00048)	(0.00074)	(0.00057)	(0.00046)	(0.00028)	(0.00089)	(0.00049)	(0.00032)	(0.00026)	(0.00033)
100	−2.2742	−1.6073	−1.2519	−0.8222	−0.6597	0.6599	0.8232	1.2531	1.6079	2.2754
	(0.00025)	(0.00034)	(0.00043)	(0.00055)	(0.00047)	(0.00036)	(0.00074)	(0.00068)	(0.00017)	(0.00026)

Table A4. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.30$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.7649	−1.8119	−1.4278	−0.9187	−0.7260	0.7426	0.9211	1.4407	1.8676	2.8294
	(0.00012)	(0.00076)	(0.00042)	(0.00049)	(0.00068)	(0.00074)	(0.00045)	(0.00076)	(0.00022)	(0.00028)
6	−2.6101	−1.7705	−1.3780	−0.9035	−0.7130	0.7290	0.9177	1.3975	1.7955	2.6558
	(0.00019)	(0.00033)	(0.00028)	(0.00062)	(0.00011)	(0.00092)	(0.00073)	(0.00018)	(0.00053)	(0.00031)
7	−2.5254	−1.7530	−1.3480	−0.8885	−0.7039	0.7092	0.8961	1.3615	1.7767	2.5730
	(0.00013)	(0.00037)	(0.00074)	(0.00054)	(0.00033)	(0.00066)	(0.00042)	(0.00038)	(0.00027)	(0.00043)
8	−2.4748	−1.7404	−1.3413	−0.8833	−0.7011	0.7048	0.8900	1.3513	1.7698	2.5302
	(0.00050)	(0.00048)	(0.00084)	(0.00026)	(0.00032)	(0.00047)	(0.00028)	(0.00040)	(0.00011)	(0.00037)
9	−2.4697	−1.7376	−1.3383	−0.8707	−0.6920	0.6995	0.8886	1.3463	1.7480	2.4928
	(0.00027)	(0.00073)	(0.00062)	(0.00055)	(0.00048)	(0.00028)	(0.00029)	(0.00038)	(0.00035)	(0.00046)
10	−2.4001	−1.7077	−1.3281	−0.8676	−0.6861	0.6913	0.8706	1.3350	1.7213	2.4221
	(0.00036)	(0.00054)	(0.00081)	(0.00049)	(0.00020)	(0.00076)	(0.00042)	(0.00094)	(0.00042)	(0.00053)
11	−2.3809	−1.6929	−1.3155	−0.8603	−0.6815	0.6877	0.8689	1.3216	1.7010	2.4003
	(0.00063)	(0.00047)	(0.00054)	(0.00039)	(0.00058)	(0.00067)	(0.00080)	(0.00043)	(0.00064)	(0.00023)
12	−2.3649	−1.6856	−1.3082	−0.8519	−0.6789	0.6819	0.8604	1.3102	1.6893	2.3990
	(0.00071)	(0.00057)	(0.00026)	(0.00043)	(0.00042)	(0.00085)	(0.00029)	(0.00068)	(0.00037)	(0.00054)
13	−2.3446	−1.6728	−1.2988	−0.8498	−0.6724	0.6793	0.8521	1.3027	1.6799	2.3644
	(0.00054)	(0.00061)	(0.00078)	(0.00028)	(0.00062)	(0.00038)	(0.00073)	(0.00046)	(0.00078)	(0.00022)
14	−2.3329	−1.6516	−1.2850	−0.8369	−0.6626	0.6687	0.8414	1.2971	1.6608	2.3566
	(0.00018)	(0.00037)	(0.00054)	(0.00043)	(0.00048)	(0.00058)	(0.00088)	(0.00056)	(0.00029)	(0.00044)
15	−2.3286	−1.6422	−1.2753	−0.8300	−0.6605	0.6656	0.8377	1.2897	1.6492	2.3314
	(0.00063)	(0.00057)	(0.00072)	(0.00064)	(0.00016)	(0.00062)	(0.00045)	(0.00077)	(0.00086)	(0.00069)
20	−2.2958	−1.6260	−1.2505	−0.8272	−0.6596	0.6624	0.83027	1.2692	1.6393	2.3190
	(0.00048)	(0.00074)	(0.00069)	(0.00066)	(0.00084)	(0.00032)	(0.00029)	(0.00028)	(0.00042)	(0.00034)
25	−2.2547	−1.5971	−1.2488	−0.8102	−0.6589	0.6609	0.8265	1.2558	1.6016	2.2763
	(0.00036)	(0.00069)	(0.00028)	(0.00042)	(0.00058)	(0.00076)	(0.00063)	(0.00017)	(0.00055)	(0.00043)
30	−2.2395	−1.5775	−1.2301	−0.8096	−0.6477	0.6520	0.8178	1.2398	1.5845	2.2450
	(0.00025)	(0.00038)	(0.00045)	(0.00064)	(0.00071)	(0.00087)	(0.00054)	(0.00044)	(0.00026)	(0.00045)
50	−2.2140	−1.5678	−1.2181	−0.8047	−0.6390	0.6408	0.8021	1.2166	1.5689	2.2198
	(0.00060)	(0.00082)	(0.00067)	(0.00029)	(0.00042)	(0.00061)	(0.00057)	(0.00058)	(0.00066)	(0.00032)
100	−2.2074	−1.5602	−1.2151	−0.7980	−0.6392	0.6397	0.7987	1.2164	1.5607	2.2167
	(0.00028)	(0.00043)	(0.00047)	(0.00068)	(0.00059)	(0.00090)	(0.00029)	(0.00066)	(0.00022)	(0.00054)

Table A5. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.40$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.6943	−1.7723	−1.3833	−0.9024	−0.7162	0.7284	0.9173	1.4026	1.8195	2.8045
	(0.00042)	(0.00054)	(0.00077)	(0.00053)	(0.00060)	(0.00076)	(0.00044)	(0.00018)	(0.00023)	(0.00037)
6	−2.5290	−1.7400	−1.3590	−0.8872	−0.7103	0.7114	0.8988	1.3729	1.7665	2.5726
	(0.00034)	(0.00043)	(0.00073)	(0.00082)	(0.00079)	(0.00065)	(0.00036)	(0.00055)	(0.00069)	(0.00014)
7	−2.4467	−1.7104	−1.3356	−0.8790	−0.7004	0.7045	0.8886	1.3563	1.7311	2.4959
	(0.00026)	(0.00045)	(0.00028)	(0.00062)	(0.00028)	(0.00029)	(0.00073)	(0.00098)	(0.00045)	(0.00072)
8	−2.3880	−1.6826	−1.3255	−0.8719	−0.6971	0.7011	0.8812	1.3430	1.7110	2.4419
	(0.00047)	(0.00039)	(0.00028)	(0.00040)	(0.00089)	(0.00036)	(0.00056)	(0.00054)	(0.00011)	(0.00058)
9	−2.3376	−1.6683	−1.3108	−0.8658	−0.6900	0.6912	0.8714	1.3211	1.6841	2.4001
	(0.00018)	(0.00076)	(0.00032)	(0.00055)	(0.00057)	(0.00085)	(0.00022)	(0.00045)	(0.00016)	(0.00024)
10	−2.3029	−1.6369	−1.2876	−0.8560	−0.6802	0.6838	0.8632	1.2968	1.6690	2.3660
	(0.00035)	(0.00077)	(0.00028)	(0.00037)	(0.00064)	(0.00059)	(0.00073)	(0.00033)	(0.00047)	(0.00017)
11	−2.2742	−1.6186	−1.2747	−0.8536	−0.6722	0.6788	0.8598	1.2897	1.6328	2.3254
	(0.00017)	(0.00018)	(0.00068)	(0.00024)	(0.00082)	(0.00043)	(0.00027)	(0.00065)	(0.00060)	(0.00022)
12	−2.2552	−1.5902	−1.2615	−0.8429	−0.6670	0.6694	0.8529	1.2777	1.6196	2.3001
	(0.00008)	(0.00026)	(0.00042)	(0.00097)	(0.00074)	(0.00054)	(0.00076)	(0.00018)	(0.00038)	(0.00042)
13	−2.2271	−1.5875	−1.2544	−0.8328	−0.6619	0.6667	0.8479	1.2643	1.5948	2.2610
	(0.00012)	(0.00045)	(0.00066)	(0.00073)	(0.00062)	(0.00071)	(0.00042)	(0.00063)	(0.00068)	(0.00009)
14	−2.2172	−1.5713	−1.2471	−0.8258	−0.6577	0.6644	0.8332	1.2522	1.5835	2.2441
	(0.00065)	(0.00031)	(0.00063)	(0.00036)	(0.00088)	(0.00063)	(0.00032)	(0.00093)	(0.00024)	(0.00016)
15	−2.2004	−1.5572	−1.2312	−0.8187	−0.6521	0.6601	0.8248	1.2443	1.5725	2.2233
	(0.00046)	(0.00069)	(0.00056)	(0.00029)	(0.00048)	(0.00052)	(0.00034)	(0.00080)	(0.00035)	(0.00036)
20	−2.1805	−1.5419	−1.2285	−0.8002	−0.6474	0.6509	0.8146	1.2341	1.5603	2.1966
	(0.00034)	(0.00011)	(0.00056)	(0.00054)	(0.00054)	(0.00078)	(0.00026)	(0.00066)	(0.00052)	(0.00081)
25	−2.1606	−1.5322	−1.2100	−0.7938	−0.6381	0.6401	0.8018	1.2256	1.5508	2.1802
	(0.00043)	(0.00029)	(0.00042)	(0.00046)	(0.00037	(0.00047)	(0.00065)	(0.00037)	(0.00042)	(0.00072)
30	−2.1592	−1.5208	−1.1993	−0.7811	−0.6298	0.6307	0.7957	1.2071	1.5329	2.1649
	(0.00022)	(0.00038)	(0.00076)	(0.00032)	(0.00023)	(0.00049)	(0.00058)	(0.00066)	(0.00028)	(0.00032)
50	−2.1465	−1.5176	−1.1823	−0.7770	−0.6229	0.6233	0.7779	1.1831	1.5188	2.1477
	(0.00046)	(0.00045)	(0.00061)	(0.00073)	(0.00081)	(0.00036)	(0.00069)	(0.00024)	(0.00074)	(0.00028)
100	−2.1415	−1.5140	−1.1794	−0.7742	−0.6208	0.6211	0.7743	1.1797	1.5146	2.1419
	(0.00031)	(0.00024)	(0.00028)	(0.00039)	(0.00066)	(0.00072)	(0.00067)	(0.00051)	(0.00028)	(0.00047)

Table A6. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.50$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.6773	−1.7422	−1.3717	−0.8901	−0.7094	0.7183	0.9107	1.3910	1.7707	2.7893
	(0.00029)	(0.00018)	(0.00063)	(0.00061)	(0.00088)	(0.00074)	(0.00059)	(0.00034)	(0.00014)	(0.00011)
6	−2.5055	−1.7245	−1.3533	−0.8834	−0.7038	0.7105	0.8816	1.3632	1.7587	2.5475
	(0.00035)	(0.00061)	(0.00022)	(0.00076)	(0.00027)	(0.00065)	(0.00070)	(0.00054)	(0.00049)	(0.00041)
7	−2.4052	−1.6963	−1.3281	−0.8727	−0.6922	0.6905	0.8612	1.3302	1.7297	2.4703
	(0.00014)	(0.00043)	(0.00037)	(0.00113)	(0.00063)	(0.00083)	(0.00076)	(0.00042)	(0.00005)	(0.00015)
8	−2.3500	−1.6679	−1.3127	−0.8643	−0.6879	0.6870	0.8500	1.3004	1.6885	2.4230
	(0.00061)	(0.0005)	(0.00065)	(0.00100)	(0.00064)	(0.00025)	(0.00061)	(0.00087)	(0.00092)	(0.00016)
9	−2.3256	−1.6450	−1.3044	−0.8616	−0.6806	0.6801	0.8475	1.2954	1.6703	2.3846
	(0.00029)	(0.00027)	(0.00028)	(0.00071)	(0.00034)	(0.00059)	(0.00047)	(0.00018)	(0.00065)	(0.00010)
10	−2.2893	−1.6210	−1.2846	−0.8539	−0.6736	0.6772	0.8419	1.2803	1.6557	2.3505
	(0.00043)	(0.00050)	(0.00063)	(0.00084)	(0.00070)	(0.00061)	(0.00029)	(0.00075)	(0.00081)	(0.00064)
11	−2.2328	−1.5707	−1.2543	−0.8497	−0.6675	0.6729	0.8337	1.2640	1.6006	2.3174
	(0.00028)	(0.00032)	(0.00087)	(0.00026)	(0.00070)	(0.00088)	(0.00054)	(0.00034)	(0.00066)	(0.00035)
12	−2.1955	−1.5448	−1.2369	−0.8226	−0.6568	0.6663	0.8314	1.2535	1.5863	2.2724
	(0.00010)	(0.00039)	(0.00088)	(0.00028)	(0.00064)	(0.00079)	(0.00044)	(0.00045)	(0.00027)	(0.00037)
13	−2.1787	−1.5329	−1.2255	−0.8022	−0.6549	0.6658	0.8200	1.2315	1.5516	2.2478
	(0.00024)	(0.00066)	(0.00076)	(0.00011)	(0.00032)	(0.00016)	(0.00029)	(0.00036)	(0.00027)	(0.00024)
14	−2.1562	−1.5108	−1.2093	−0.7941	−0.6460	0.6560	0.8116	1.2158	1.5478	2.1949
	(0.00041)	(0.00059)	(0.00021)	(0.00022)	(0.00054)	(0.00034)	(0.00015)	(0.00073)	(0.00054)	(0.00014)
15	−2.1348	−1.5058	−1.1867	−0.7805	−0.6311	0.6511	0.8070	1.2028	1.5276	2.1629
	(0.00014)	(0.00092)	(0.00005)	(0.00029)	(0.00063)	(0.00097)	(0.00014)	(0.00028)	(0.00062)	(0.00044)
20	−2.1105	−1.4875	−1.1688	−0.7655	−0.6290	0.6352	0.7717	1.1688	1.4877	2.1375
	(0.00012)	(0.00042)	(0.00061)	(0.00070)	(0.00043)	(0.00052)	(0.00065)	(0.00087)	(0.00010)	(0.00028)
25	−2.0860	−1.4717	−1.1573	−0.7588	−0.6188	0.6224	0.7573	1.1478	1.4639	2.1092
	(0.00044)	(0.00036)	(0.00074)	(0.00027)	(0.00061)	(0.00027)	(0.00088)	(0.00066)	(0.00055)	(0.00031)
30	−2.0574	−1.4590	−1.1340	−0.7459	−0.6018	0.6011	0.7464	1.1361	1.4588	2.0600
	(0.00034)	(0.00018)	(0.00079)	(0.00045)	(0.00010)	(0.00041)	(0.00042)	(0.00070)	(0.00042)	(0.00042)
50	−2.0525	−1.4521	−1.1327	−0.7448	−0.6001	0.59899	0.7440	1.1320	1.4530	2.0527
	(0.00036)	(0.00054)	(0.00038)	(0.00066)	(0.00034)	(0.00046)	(0.00076)	(0.00010)	(0.00079)	(0.00025)
100	−2.0279	−1.4354	−1.1166	−0.7330	−0.5871	0.5875	0.7332	1.1163	1.4342	2.0284
	(0.00060)	(0.00014)	(0.00088)	(0.00017)	(0.00041)	(0.00074)	(0.00035)	(0.00059)	(0.00029)	(0.00038)

Table A7. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.60$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.3364	−1.5783	−1.1905	−0.7814	−0.6181	0.6545	0.8318	1.2919	1.6121	2.3754
	(0.00028)	(0.00061)	(0.00054)	(0.00027)	(0.00070)	(0.00038)	(0.00018)	(0.00088)	(0.00014)	(0.00027)
6	−2.2274	−1.5365	−1.1862	−0.7789	−0.6106	0.6421	0.8219	1.2756	1.5997	2.2943
	(0.00013)	(0.00022)	(0.00062)	(0.00057)	(0.00041)	(0.00042)	(0.00049)	(0.00054)	(0.00063)	(0.00030)
7	−2.1803	−1.5119	−1.1722	−0.7732	−0.6071	0.6341	0.8074	1.2476	1.5582	2.2388
	(0.00042)	(0.00041)	(0.00103)	(0.00076)	(0.00087)	(0.00079)	(0.00066)	(0.00086)	(0.00029)	(0.00016)
8	−2.1487	−1.4905	−1.1621	−0.7615	−0.6012	0.6247	0.7912	1.2139	1.5245	2.1852
	(0.00021)	(0.00011)	(0.00092)	(0.00074)	(0.00084)	(0.00024)	(0.00069)	(0.00059)	(0.00070)	(0.00017)
9	−2.0823	−1.4711	−1.1442	−0.7565	−0.5991	0.6107	0.7890	1.1846	1.4966	2.1133
	(0.00010)	(0.00029)	(0.00016)	(0.00034)	(0.00045)	(0.00028)	(0.00071)	(0.00064)	(0.00080)	(0.00026)
10	−2.0510	−1.4500	−1.1360	−0.7414	−0.5922	0.6084	0.7669	1.1584	1.4707	2.0879
	(0.00044)	(0.00029)	(0.00070)	(0.00059)	(0.00051)	(0.00061)	(0.00036)	(0.00063)	(0.00028)	(0.00035)
11	−2.0371	−1.4380	−1.1296	−0.7390	−0.5857	0.5973	0.7466	1.1339	1.4461	2.0416
	(0.00028)	(0.00042)	(0.00087)	(0.00028)	(0.00047)	(0.00041)	(0.00052)	(0.00112)	(0.00021)	(0.00032)
12	−2.0050	−1.4270	−1.1024	−0.7211	−0.5817	0.5911	0.7343	1.1188	1.4321	2.0213
	(0.00063)	(0.00028)	(0.00097)	(0.00046)	(0.00014)	(0.00024)	(0.00076)	(0.00029)	(0.00038)	(0.00041)
13	−1.9795	−1.4068	−1.0914	−0.7178	−0.5732	0.5878	0.7285	1.1085	1.4130	1.9958
	(0.00046)	(0.00064)	(0.00018)	(0.00041)	(0.00043)	(0.00027)	(0.00070)	(0.00028)	(0.00035)	(0.00045)
14	−1.9650	−1.3933	−1.0859	−0.7146	−0.5680	0.5762	0.7225	1.0958	1.4003	1.9809
	(0.00028)	(0.00045)	(0.00027)	(0.00088)	(0.00067)	(0.00041)	(0.00054)	(0.00087)	(0.00042)	(0.00039)
15	−1.9522	−1.3831	−1.0799	−0.7084	−0.5640	0.5712	0.7181	1.0858	1.3948	1.9688
	(0.00037)	(0.00059)	(0.00075)	(0.00021)	(0.00029)	(0.00049)	(0.00088)	(0.00023)	(0.00074)	(0.00034)
20	−1.9228	−1.3605	−1.0616	−0.6919	−0.5542	0.5605	0.7045	1.0711	1.3718	1.9366
	(0.00054)	(0.00027)	(0.00079)	(0.00038)	(0.00051)	(0.00010)	(0.00044)	(0.00092)	(0.00038)	(0.00027)
25	−1.9008	−1.3480	−1.0437	−0.6876	−0.5501	0.5555	0.6898	1.0501	1.3492	1.9057
	(0.00057)	(0.00061)	(0.00043)	(0.00066)	(0.00042)	(0.00120)	(0.00061)	(0.00064)	(0.00061)	(0.00048)
30	−1.8944	−1.3381	−1.0408	−0.6859	−0.5462	0.5498	0.6880	1.0416	1.3397	1.8917
	(0.00028)	(0.00028)	(0.00036)	(0.00076)	(0.00104)	(0.00063)	(0.00084)	(0.00010)	(0.00043)	(0.00019)
50	−1.8841	−1.3297	−1.0351	−0.6818	−0.5485	0.5494	0.6809	1.0344	1.3308	1.8820
	(0.00020)	(0.00014)	(0.00045)	(0.00086)	(0.00016)	(0.00047)	(0.00041)	(0.00018)	(0.00021)	(0.00043)
100	−1.8635	−1.3115	−1.0300	−0.6772	−0.5420	0.5418	0.6765	1.0308	1.3120	1.8611
	(0.00014)	(0.00070)	(0.00088)	(0.00010)	(0.00024)	(0.00059)	(0.00079)	(0.00066)	(0.00028)	(0.00033)

Table A8. Quantile points of the distribution of C(when $|\rho_{yx}|=0.70$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−2.1067	−1.4021	−1.0919	−0.6932	−0.5752	0.6092	0.7161	1.1307	1.4221	2.1497
	(0.00011)	(0.00076)	(0.00018)	(0.00061)	(0.00088)	(0.00014)	(0.00070)	(0.00045)	(0.00011)	(0.00027)
6	−1.9607	−1.3604	−1.0765	−0.6863	−0.5605	0.5800	0.7048	1.1081	1.3995	1.9995
	(0.00029)	(0.00007)	(0.00016)	(0.00073)	(0.00034)	(0.00074)	(0.00092)	(0.00063)	(0.00042)	(0.00014)
7	−1.9171	−1.3494	−1.0564	−0.6744	−0.5513	0.5799	0.6904	1.0939	1.3721	1.9451
	(0.00021)	(0.00027)	(0.00054)	(0.00042)	(0.00064)	(0.00079)	(0.00065)	(0.00014)	(0.00027)	(0.00029)
8	−1.8530	−1.3234	−1.0363	−0.6644	−0.5477	0.5530	0.6821	1.0729	1.3529	1.8838
	(0.00087)	(0.00066)	(0.00063)	(0.00046)	(0.00092)	(0.00076)	(0.00064)	(0.00061)	(0.00037)	(0.00041)
9	−1.8329	−1.3020	−1.0239	−0.6562	−0.5340	0.5422	0.6796	1.0514	1.3238	1.8679
	(0.00010)	(0.00059)	(0.00011)	(0.00057)	(0.00060)	(0.00041)	(0.00073)	(0.00086)	(0.00007)	(0.00022)
10	−1.8147	−1.2914	−1.0155	−0.6503	−0.5237	0.5308	0.6767	1.0342	1.3041	1.8344
	(0.00052)	(0.00070)	(0.00087)	(0.00043)	(0.00078)	(0.00037)	(0.00081)	(0.00024)	(0.00039)	(0.00047)
11	−1.7924	−1.2889	−0.9992	−0.6417	−0.5184	0.5289	0.6544	1.0102	1.2913	1.8085
	(0.00067)	(0.00041)	(0.00014)	(0.00053)	(0.00016)	(0.00059)	(0.00064)	(0.00121)	(0.00021)	(0.00066)
12	−1.7814	−1.2727	−0.9930	−0.6375	−0.5110	0.5211	0.6449	0.9992	1.2885	1.7964
	(0.00045)	(0.00086)	(0.00056)	(0.00027)	(0.00029)	(0.00018)	(0.00038)	(0.00047)	(0.00039)	(0.00026)
13	−1.7716	−1.2639	−0.9851	−0.6323	−0.5088	0.5198	0.6380	0.9904	1.2703	1.7815
	(0.00024)	(0.00057)	(0.00064)	(0.00065)	(0.00041)	(0.00054)	(0.00054)	(0.00028)	(0.00032)	(0.00029)
14	−1.7668	−1.2575	−0.9758	−0.6284	−0.5047	0.5180	0.6318	0.9852	1.2626	1.7759
	(0.00054)	(0.00079)	(0.00054)	(0.00028)	(0.00037)	(0.00084)	(0.00045)	(0.00070)	(0.00029)	(0.00021)
15	−1.7583	−1.2466	−0.9646	−0.6192	−0.5018	0.5163	0.6257	0.9741	1.2564	1.7602
	(0.00052)	(0.00073)	(0.00041)	(0.00010)	(0.00066)	(0.00075)	(0.00042)	(0.00080)	(0.00059)	(0.00033)
20	−1.7369	−1.2229	−0.9547	−0.6183	−0.4999	0.5063	0.6208	0.9683	1.2350	1.7452
	(0.00014)	(0.00021)	(0.00075)	(0.00047)	(0.00064)	(0.00097)	(0.00087)	(0.00050)	(0.00043)	(0.00063)
25	−1.7245	−1.2085	−0.9352	−0.6109	−0.4945	0.4998	0.6198	0.9462	1.2145	1.7395
	(0.00045)	(0.00028)	(0.00038)	(0.00066)	(0.00024)	(0.00076)	(0.00109)	(0.00086)	(0.00074)	(0.00037)
30	−1.6907	−1.1940	−0.9297	−0.6097	−0.4906	0.4915	0.6134	0.9315	1.1952	1.6917
	(0.00059)	(0.00043)	(0.00092)	(0.00034)	(0.00054)	(0.00088)	(0.00063)	(0.00086)	(0.00054)	(0.00020)
50	−1.6898	−1.2004	−0.9305	−0.6125	−0.4867	0.4870	0.6126	0.9311	1.1988	1.6888
	(0.00018)	(0.00061)	(0.00084)	(0.00045)	(0.00021)	(0.00079)	(0.00041)	(0.00028)	(0.00032)	(0.00043)
100	−1.6837	−1.1901	−0.9279	−0.6090	−0.4885	0.4893	0.6080	0.9288	1.1918	1.6826
	(0.00021)	(0.00088)	(0.00079)	(0.00029)	(0.00070)	(0.00063)	(0.00073)	(0.00014)	(0.00010)	(0.00052)

Table A9. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.80$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−1.7297	−1.1687	−0.9035	−0.6023	−0.4983	0.5133	0.6191	0.9223	1.1972	1.7607
	(0.00014)	(0.00061)	(0.00100)	(0.00073)	(0.00070)	(0.00018)	(0.00064)	(0.00066)	(0.00028)	(0.00021)
6	−1.6771	−1.1549	−0.8965	−0.5907	−0.4864	0.4977	0.6002	0.9104	1.1806	1.7148
	(0.00026)	(0.00074)	(0.00059)	(0.00043)	(0.00010)	(0.00086)	(0.00076)	(0.00032)	(0.00024)	(0.00029)
7	−1.6130	−1.1373	−0.8878	−0.5824	−0.4793	0.4934	0.5932	0.8972	1.1548	1.6448
	(0.00059)	(0.00064)	(0.00011)	(0.00056)	(0.00061)	(0.00069)	(0.00054)	(0.00079)	(0.00063)	(0.00044)
8	−1.5970	−1.1104	−0.8749	−0.5794	−0.4704	0.4833	0.5895	0.8865	1.1322	1.6107
	(0.00054)	(0.00084)	(0.00070)	(0.00045)	(0.00067)	(0.00042)	(0.00088)	(0.00029)	(0.00034)	(0.00018)
9	−1.5666	−1.0921	−0.8620	−0.5750	−0.4668	0.4788	0.5825	0.8758	1.1116	1.5893
	(0.00028)	(0.00063)	(0.00079)	(0.00016)	(0.00041)	(0.00005)	(0.00043)	(0.00092)	(0.00054)	(0.00025)
10	−1.5416	−1.0850	−0.8523	−0.5713	−0.4607	0.4720	0.5731	0.8649	1.0910	1.5599
	(0.00061)	(0.00028)	(0.00053)	(0.00014)	(0.00027)	(0.00061)	(0.00036)	(0.00011)	(0.00047)	(0.00053)
11	−1.5156	−1.0821	−0.8490	−0.5686	−0.4591	0.4689	0.5789	0.8595	1.0987	1.5372
	(0.00028)	(0.00042)	(0.00066)	(0.00037)	(0.00087)	(0.00049)	(0.00073)	(0.00060)	(0.00044)	(0.00072)
12	−1.5044	−1.0643	−0.8427	−0.5628	−0.4570	0.4614	0.5707	0.8508	1.0763	1.5162
	(0.00055)	(0.00064)	(0.00061)	(0.00059)	(0.00024)	(0.00033)	(0.00016)	(0.00109)	(0.00043)	(0.00047)
13	−1.4878	−1.0551	−0.8324	−0.5576	−0.4502	0.4599	0.5664	0.8444	1.0651	1.5097
	(0.00039)	(0.00042)	(0.00022)	(0.00038)	(0.00064)	(0.00027)	(0.00064)	(0.00075)	(0.00035)	(0.00068)
14	−1.4726	−1.0433	−0.8279	−0.5529	−0.4497	0.4539	0.5633	0.8346	1.0447	1.4926
	(0.00024)	(0.00054)	(0.00088)	(0.00041)	(0.00076)	(0.00064)	(0.00059)	(0.00027)	(0.00057)	(0.00081)
15	−1.4677	−1.0348	−0.8175	−0.5484	−0.4477	0.4493	0.5536	0.8299	1.0395	1.4752
	(0.00018)	(0.00075)	(0.00041)	(0.00016)	(0.00042)	(0.00021)	(0.00084)	(0.00074)	(0.00063)	(0.00014)
20	−1.4565	−1.0268	−0.8100	−0.5418	−0.4416	0.4444	0.5499	0.8160	1.0298	1.4607
	(0.00014)	(0.00032)	(0.00046)	(0.00093)	(0.00063)	(0.00061)	(0.00083)	(0.00021)	(0.00045)	(0.00054)
25	−1.4404	−1.0169	−0.8012	−0.5319	−0.4390	0.4379	0.5357	0.8077	1.0182	1.4458
	(0.00010)	(0.00010)	(0.00110)	(0.00038)	(0.00086)	(0.00029)	(0.00054)	(0.00051)	(0.00064)	(0.00060)
30	−1.4323	−1.0063	−0.7987	−0.5263	−0.4235	0.4267	0.5282	0.8002	1.0114	1.4378
	(0.00029)	(0.00043)	(0.00079)	(0.00086)	(0.00097)	(0.00086)	(0.00010)	(0.00039)	(0.00041)	(0.00023)
50	−1.4216	−0.9991	−0.7817	−0.5140	−0.4161	0.4174	0.5144	0.7889	0.9988	1.4238
	(0.00024)	(0.00064)	(0.000724)	(0.00076)	(0.00084)	(0.00070)	(0.00059)	(0.00018)	(0.00045)	(0.00054)
100	−1.3999	−0.9906	−0.7723	−0.5065	−0.4045	0.4064	0.5067	0.7724	0.9915	1.4006
	(0.00026)	(0.00066)	(0.00027)	(0.00041)	(0.00038)	(0.00043)	(0.00084)	(0.00088)	(0.00027)	(0.00034)

Table A10. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.90$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−1.2537	−0.8537	−0.6487	−0.4520	−0.3886	0.4160	0.4808	0.6974	0.8655	1.2980
	(0.00033)	(0.00061)	(0.00029)	(0.00059)	(0.00070)	(0.00027)	(0.00073)	(0.00028)	(0.00076)	(0.00024)
6	−1.2008	−0.8317	−0.6351	−0.4453	−0.3716	0.3908	0.4651	0.6704	0.8464	1.2407
	(0.00048)	(0.00054)	(0.00087)	(0.00053)	(0.00076)	(0.00024)	(0.00063)	(0.00042)	(0.00045)	(0.00014)
7	−1.1788	−0.8215	−0.6248	−0.4323	−0.3624	0.3818	0.4514	0.6623	0.8294	1.2097
	(0.00029)	(0.00038)	(0.00073)	(0.00084)	(0.00036)	(0.00061)	(0.00074)	(0.00054)	(0.00029)	(0.00047)
8	−1.1454	−0.8197	−0.6198	−0.4299	−0.3587	0.3756	0.4406	0.6546	0.8211	1.1720
	(0.00054)	(0.00059)	(0.00092)	(0.00011)	(0.00088)	(0.00010)	(0.00025)	(0.00059)	(0.00041)	(0.00018)
9	−1.1274	−0.8060	−0.6129	−0.4211	−0.3521	0.3689	0.4342	0.6414	0.8148	1.1454
	(0.00052)	(0.00066)	(0.00042)	(0.00016)	(0.00127)	(0.00036)	(0.00097)	(0.00070)	(0.00039)	(0.00055)
10	−1.1074	−0.7983	−0.6097	−0.4193	−0.3496	0.3626	0.4295	0.6312	0.8046	1.1187
	(0.00034)	(0.00064)	(0.00027)	(0.00043)	(0.00063)	(0.00086)	(0.00028)	(0.00087)	(0.00034)	(0.00026)
11	−1.0975	−0.7869	−0.6058	−0.4146	−0.3460	0.3533	0.4208	0.6280	0.7905	1.1057
	(0.00032)	(0.00028)	(0.00051)	(0.00014)	(0.00038)	(0.00075)	(0.00073)	(0.00064)	(0.00044)	(0.00063)
12	−1.0836	−0.7780	−0.5990	−0.4025	−0.3393	0.3416	0.4172	0.6104	0.7819	1.0930
	(0.00016)	(0.00034)	(0.00028)	(0.00043)	(0.00024)	(0.00024)	(0.00066)	(0.00054)	(0.00061)	(0.00035)
13	−1.0737	−0.7670	−0.5931	−0.3973	−0.3214	0.3378	0.4084	0.6037	0.7715	1.0896
	(0.00034)	(0.00041)	(0.00050)	(0.00092)	(0.00054)	(0.00076)	(0.00061)	(0.00064)	(0.00088)	(0.00077)
14	−1.0699	−0.7588	−0.5875	−0.3901	−0.3197	0.3305	0.4009	0.5981	0.7691	1.0739
	(0.00026)	(0.00010)	(0.00045)	(0.00035)	(0.00045)	(0.00092)	(0.00042)	(0.00079)	(0.00052)	(0.00048)
15	−1.0674	−0.7526	−0.5809	−0.3856	−0.3177	0.3282	0.3921	0.5930	0.7649	1.0731
	(0.00021)	(0.00034)	(0.00064)	(0.00105)	(0.00016)	(0.00072)	(0.00059)	(0.00084)	(0.00043)	(0.00078)
20	−1.0419	−0.7428	−0.5785	−0.3795	−0.3092	0.3161	0.3854	0.5861	0.7463	1.0509
	(0.00045)	(0.00086)	(0.00074)	(0.00097)	(0.00054)	(0.00079)	(0.00045)	(0.00018)	(0.00038)	(0.00041)
25	−1.0391	−0.7341	−0.5711	−0.3733	−0.3024	0.3039	0.3790	0.5787	0.7359	1.0416
	(0.00070)	(0.00088)	(0.00079)	(0.00005)	(0.00051)	(0.00064)	(0.00066)	(0.00026)	(0.00055)	(0.00032)
30	−1.0218	−0.7211	−0.5625	−0.3691	−0.2966	0.2969	0.3699	0.5644	0.7219	1.0232
	(0.00047)	(0.00061)	(0.00014)	(0.00059)	(0.00029)	(0.00038)	(0.00010)	(0.00041)	(0.00076)	(0.00028)
50	−1.0164	−0.7188	−0.5603	−0.3665	−0.2944	0.2955	0.3675	0.5612	0.7192	1.0170
	(0.00041)	(0.00024)	(0.00043)	(0.00070)	(0.00027)	(0.00038)	(0.00088)	(0.00034)	(0.00014)	(0.00039)
100	−1.0116	−0.7162	−0.5582	−0.3664	−0.2934	0.2937	0.3666	0.5580	0.7170	1.0123
	(0.00059)	(0.00018)	(0.00076)	(0.00066)	(0.00010)	(0.00084)	(0.00042)	(0.00021)	(0.00074)	(0.00026)

Table A11. Quantile points of the distribution of C (when $|\rho_{yx}| = 0.99$)

n	$C_{0.01}$	$C_{0.05}$	$C_{0.10}$	$C_{0.20}$	$C_{0.25}$	$C_{0.75}$	$C_{0.80}$	$C_{0.90}$	$C_{0.95}$	$C_{0.99}$
5	−0.4077	−0.2751	−0.2204	−0.1479	−0.1169	0.1201	0.1499	0.2244	0.2873	0.4100
	(0.00034)	(0.00053)	(0.00062)	(0.00054)	(0.00028)	(0.00076)	(0.00092)	(0.00034)	(0.00029)	(0.00025)
6	−0.3884	−0.2685	−0.2148	−0.1428	−0.1158	0.1197	0.1431	0.2166	0.2706	0.3967
	(0.00078)	(0.00048)	(0.00059)	(0.00059)	(0.00073)	(0.00054)	(0.00027)	(0.00079)	(0.00070)	(0.00041)
7	−0.3720	−0.2626	−0.2131	−0.1401	−0.1141	0.1180	0.1410	0.2149	0.2692	0.3866
	(0.00054)	(0.00088)	(0.00061)	(0.00016)	(0.00084)	(0.00105)	(0.00054)	(0.00059)	(0.00045)	(0.00029)
8	−0.3658	−0.2619	−0.2105	−0.1398	−0.1134	0.1164	0.1436	0.2127	0.2670	0.3788
	(0.00016)	(0.00014)	(0.00087)	(0.00029)	(0.00070)	(0.00024)	(0.00075)	(0.00043)	(0.00061)	(0.00018)
9	−0.3596	−0.2544	−0.2077	−0.1378	−0.1115	0.1133	0.1409	0.2142	0.2528	0.3642
	(0.00024)	(0.00027)	(0.00068)	(0.00025)	(0.00041)	(0.00042)	(0.00021)	(0.00010)	(0.00076)	(0.00034)
10	−0.3546	−0.2462	−0.2001	−0.1361	−0.1105	0.1129	0.1399	0.2115	0.2484	0.3560
	(0.00005)	(0.00073)	(0.00074)	(0.00097)	(0.00064)	(0.00060)	(0.00076)	(0.00041)	(0.00028)	(0.00054)
11	−0.3535	−0.2594	−0.1989	−0.1345	−0.1095	0.1197	0.1391	0.2110	0.2697	0.3618
	(0.00011)	(0.00026)	(0.00054)	(0.00034)	(0.00070)	(0.00088)	(0.00064)	(0.00045)	(0.00024)	(0.00051)
12	−0.3498	−0.2521	−0.1978	−0.1293	−0.1057	0.1186	0.1316	0.2034	0.2648	0.3588
	(0.00037)	(0.00046)	(0.00055)	(0.00054)	(0.00038)	(0.00043)	(0.00066)	(0.00027)	(0.00021)	(0.00068)
13	−0.3486	−0.2474	−0.1931	−0.1280	−0.1035	0.1171	0.1299	0.1953	0.2497	0.3543
	(0.00023)	(0.00054)	(0.00061)	(0.00033)	(0.00052)	(0.00079)	(0.00016)	(0.00048)	(0.00084)	(0.00052)
14	−0.3454	−0.2458	−0.1911	−0.1253	−0.1011	0.1090	0.1262	0.1930	0.2497	0.3492
	(0.00066)	(0.00059)	(0.00041)	(0.00064)	(0.00010)	(0.00059)	(0.00014)	(0.00054)	(0.00028)	(0.00014)
15	−0.3414	−0.2435	−0.1884	−0.1249	−0.0995	0.1032	0.1277	0.1895	0.2450	−0.3484
	(0.00041)	(0.00029)	(0.00045)	(0.00045)	(0.00079)	(0.00086)	(0.00073)	(0.00057)	(0.00014)	(0.00011)
20	−0.3399	−0.2410	−0.1874	−0.1232	−0.0987	0.1021	0.1265	0.1886	0.2416	0.3402
	(0.00023)	(0.00064)	(0.00024)	(0.00075)	(0.00177)	(0.00034)	(0.00005)	(0.00054)	(0.00037)	(0.00058)
25	−0.3365	−0.2386	−0.1856	−0.1220	−0.979	0.0984	0.1226	0.1861	0.2387	0.3368
	(0.00038)	(0.00026)	(0.00076)	(0.00084)	(0.00092)	(0.00059)	(0.00054)	(0.00063)	(0.00042)	(0.00039)
30	−0.3360	−0.2378	−0.1842	−0.1209	−0.970	0.0981	0.1222	0.1855	0.2381	0.3366
	(0.00018)	(0.00043)	(0.00064)	(0.00029)	(0.00090)	(0.00088)	(0.00028)	(0.00041)	(0.00086)	(0.00027)
50	−0.3327	−0.2352	−0.1834	−0.1205	−0.970	0.0968	0.1211	0.1833	0.2355	0.3330
	(0.00021)	(0.00010)	(0.00041)	(0.00074)	(0.00024)	(0.00097)	(0.00064)	(0.00045)	(0.00086)	(0.00005)
100	−0.3321	−0.2344	−0.1830	−0.1206	−0.0965	0.0969	0.1208	0.1838	0.2350	0.3327
	(0.00014)	(0.00028)	(0.00079)	(0.00088)	(0.00042)	(0.00059)	(0.00010)	(0.00087)	(0.00066)	(0.00016)

Table A12. Simulated data set

Sample number	Sample values in pairs (Y, X) of size 10										
1	Y	203.32	200.65	204.06	202.01	200.36	200.58	199.73	200.68	200.81	200.92
	X	212.04	210.38	209.50	210.66	210.12	212.05	209.91	209.14	210.30	212.90
2	Y	202.88	201.15	200.54	199.11	201.50	200.79	201.72	203.25	201.77	203.52
	X	211.23	209.11	210.18	210.92	209.62	209.86	213.26	211.31	211.36	210.90
3	Y	201.34	201.41	203.76	200.57	200.76	203.44	201.31	202.47	201.77	199.91
	X	209.86	207.93	209.26	208.50	210.21	211.61	210.99	210.08	210.94	209.50
4	Y	199.51	199.39	202.62	201.88	200.48	199.29	202.93	203.86	200.43	202.61
	X	208.94	210.75	210.71	211.01	212.09	207.76	210.77	211.87	210.40	211.52
5	Y	200.86	200.42	201.27	201.70	203.92	202.37	202.45	202.16	201.04	202.65
	X	211.20	208.89	210.53	209.16	211.59	211.06	211.72	209.34	211.77	212.02
6	Y	201.11	201.48	203.09	201.27	199.65	201.42	203.84	201.19	198.73	200.52
	X	209.99	212.08	209.67	211.64	208.98	212.15	211.39	209.78	207.87	209.84
7	Y	200.49	201.38	202.63	201.02	202.04	199.39	199.33	202.80	201.29	201.88
	X	209.35	211.11	209.27	210.23	209.74	210.75	208.71	210.73	209.88	210.55
8	Y	202.74	201.59	203.83	202.69	201.89	200.72	200.30	200.10	202.38	200.27
	X	211.43	208.45	211.29	212.20	208.75	208.55	211.41	212.91	208.98	207.56
9	Y	200.95	201.65	201.76	200.14	199.36	199.55	199.75	201.20	199.16	199.91
	X	212.17	210.69	209.30	209.60	206.72	209.72	209.94	210.41	208.13	208.53
10	Y	199.92	200.85	201.38	201.92	201.29	201.11	199.78	203.12	203.60	201.04
	X	208.93	210.28	210.57	210.31	209.22	210.51	209.29	210.02	212.12	210.14

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