Unknitting the black hole: black holes as effective geometries
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In this thesis, we will try to shed some light on the origin of black hole geometries in the context of the fuzzball proposal. The claim is that black holes are an effective description of an exponentially large number of smooth geometries. These geometries are hardly distinguishable from the black hole geometry outside the black hole horizon, and hence, can be seen as describing the same physics. In other words, these smooth geometries can be thought of as a manifestation of the black hole degrees of freedom whose number should reproduce the black hole entropy.

Though the idea described above is elegant, putting it to work is far from trivial. This is mainly due to the complicated nature of the gravitational equations. A way to overcome such an inconvenience is to appeal to as many symmetries as we need to gain control over our solutions. This is the strategy we will be using in this thesis. We will concentrate on two stringy systems which are tamed by their symmetries. The first one is the D1-D5 system which will be the study material of the second part. As it will be shown subsequently, this system turns out to be a successful testing ground for most of the fuzzball ideas. However, the D1-D5 system comes with a serious drawback; it is not quite a black hole as it has a vanishing horizon area. This motivates us to look for other black hole solutions with a large horizon but still under enough control. Such requirements are satisfied by a special class of black hole solutions of the $\mathcal{N} = 2$ four-dimensional supergravity. These black holes will be the subject of the third part of this thesis. Although, by requiring a large horizon we had to sacrifice simplicity, there are still enough symmetries to address some simple fuzzball questions regarding these black holes.

We close by a discussion about the successes, limitations, and some open questions of this program. Some technical details will be left to the appendices. Before embarking into this fascinating journey to visit our special classes of black holes, we will make a small detour in the first part of this thesis. The latter contains a review of the Hawking radiation of black holes, as well as a discussion of the general philosophy of the fuzzball scenario.
INTRODUCTION AND MOTIVATION

The beginning of the last century was marked by the emergence of two fascinating theories that led to a revolution in our way of thinking about nature: general relativity and quantum field theory. For a long time, physicists did not worry about the possible incompatibility of the two theories because they had different domains of applicability: general relativity was concerned with large distance physics (planets, galaxies, ...), while quantum field theory dealt with short distances (molecules, atoms, ...). The necessity of reconciling the two theories emerged with the study of quantum field theories in the presence of black holes following the seminal work of Hawking [1]. To the surprise of everybody, black holes seemed at the time to be in conflict with well established and cherished principles of modern physics.

Trying to understand black holes is the main subject of this thesis, but before going into technical details, we will first try to explain in simple words what is the problem with black holes, and what are the ideas explored in this thesis to understand them.

I-BLACK HOLES: CLASSICAL VS SEMI-CLASSICAL

One of the most puzzling objects that general relativity predicts are black holes. Classically, they are boring objects completely fixed once one is given the values of charges at infinity, this is the acclaimed no-hair theorem (See e.g. [2]). This picture changes drastically once quantum effects are taken into account. Black holes behave like thermodynamical objects [3, 1, 4]; they possess entropy, temperature ... etc. This naive “marriage” between classical general relativity and quantum field theory, the so-called “semi-classical quantum gravity”, leads to several paradoxes which are:
1-Entropy, Horizon Area and Holography

Entropy, as we know from statistical physics, is a quantity that measures the number of degrees of freedom of a system. According to the no-hair theorem, the entropy of a black hole should be zero. This is in clear conflict with what we have learned from semi-classical analysis: a black hole has an entropy proportional to its horizon area. The situation does not improve in the case where the uniqueness is violated, which happens in five dimensions for example [5, 6]. Even in this case, one still has far less degeneracy to account for the exponential number of black hole states. A possible way out would be to declare that these states are inherently quantum with no classical limit. However, this leaves the question of the entropy not being proportional to a volume, a logical guess based on extensivity, unanswered. To explain such an unexpected property, it was suggested that quantum gravity is holographic in nature [7, 8]: quantum gravity in $d$ dimensions should somehow be equivalent to a field theory without gravity in one dimension less. String theory gives a concrete realization of this idea that goes under the name of “AdS/CFT correspondence”, where, gravity (string theory) on AdS spacetime is equivalent to a conformal field theory that lives on the boundary of AdS [9].

2-Information Loss Paradox and Unitarity

Classically black holes are greedy objects; nothing escapes once the horizon is crossed. However, quantizing fields in a black hole background shows that the latter thermally radiates [4], and as a consequence, evaporates completely. It turns out that this radiation does not care about the state of the matter that collapsed to form the black hole, which is a potential source of information loss. However, as will be explained in the next chapter, this is due to the limitation of our derivation and is not of a fundamental origin. The other origin of information loss, which should be taken seriously, is the entangled nature of the radiation. In simple words, the radiated particles are correlated with “anti-particles” that fall behind the horizon. The information is lost when the black hole evaporates completely destroying part of the information. As a result, we are faced with the following dilemma:

- Information is lost [10], and one should drop the requirement of unitarity in the presence of gravitation.
- The semiclassical approximation breaks down, and quantum gravity becomes important at scales bigger than the natural Planck scale; see e.g. [11]. Essentially this is because, as we will see later (section 1.3.2), the Hawking radiation depends on the local physics around the black hole horizon. For large enough black holes, the curvature will be small everywhere near the horizon; and
hence, Hawking calculation will remain valid if the quantum effects are not
enhanced. As a result, in the absence of such large quantum effects informa-
tion will be lost after the complete evaporation of the black hole.

String theory, through AdS/CFT, suggests that gravity is unitary but does not shed
light on how information is restored yet.

3-What About the Black Hole Singularity?

Classically, every black hole has a singularity where the curvature blows up. As a
result, general relativity breaks down in a region surrounding this singularity. We
do not even know how to formulate physics laws in that region. Even worse, the
predictability of physics at a point in spacetime is destroyed whenever these singu-
larities are in the past lightcone of this point. To avoid such a wild behavior, the
“cosmic censorship” was conjectured [12]: every singularity should be shielded by a
horizon that causally disconnects us form it. Sadly, such a conjecture fails in some
cases, see e.g. [13, 14]. Another widely accepted way out, which remains to be
checked, is that the full theory of quantum gravity should resolve singularities.

II-Black Holes and String Theory

One of the highly controversial predictions of string theory is the existence of six
extra dimensions. Despite that, this turns out to be a key point in black hole physics
as we will explain in a moment. Before that, let us first understand how black holes
emerge in string theory. To do so, we will take a quick look at the objects described
by string theory.

String theory, as its name indicates, is a theory of vibrating strings, which means
that its fundamental objects are one-dimensional extended objects. But, strings are
not the only extended objects in string theory. A quick way to see that is to study
open strings. In principle, one can imagine attaching the ends of the open string
to an extended object called a “D-brane”. In such a situation, there will be a leak
of energy from the open string to the D-brane which already tells us that these D-
branes are heavy objects. What this does not tell us is that D-branes carry also a
charge like an electron which is true.

All in all, string theory is formulated on a ten-dimensional spacetime and describes
on top of strings, the dynamics of a host of extended, charged, and massive objects
called “D-branes”. But, what have these properties to do with black holes? To
understand that, let us discuss a toy model where we live in a two-dimensional
spacetime $\mathbb{R}^2$ (time plus a one-dimensional space $\mathbb{R}$) but string theory is living in one dimension higher [15]. We take the extra dimension to be a circle $S^1$, see picture 1. Suppose also that we have a one-dimensional D-brane which we will call a “D1-brane”. Such a D1-brane can be wrapped around the circle $S^1$. From our point of view, we cannot see the D1-brane because it lives in an extra dimension that we cannot access directly. Instead, we will observe a point particle in our spacetime. The metamorphosis of this point particle to a black hole happens as follows. Remember that our D1-brane is massive and charged. When we increase the intensity of the gravitational interaction, the D1-brane will try to decrease the size of the $S^1$ due to gravitational attraction. This process does not go on forever because there is a competing repulsive force due to the charge of the D1-brane. Such a force on the contrary will try to maximize the size of the $S^1$. The full spacetime $\mathbb{R}^2 \times S^1$ will stabilize once the two competing forces balance each other for a certain size of the $S^1$. Due to such process, the size of the $S^1$ will not be the same throughout the full spacetime $\mathbb{R}^2 \times S^1$. Such a non-uniform size will manifest itself in our spacetime as a curvature (see figure 1). In other words, we will feel a gravitational field as if there is a charged black hole sitting somewhere in our spacetime. This simple story generalizes to the full string theory, where now, the extra dimensions can be very complicated allowing for different kinds of charged black holes.

![Figure 1](image-url)  

**Figure 1:** (Left) A schematic depiction of $\mathbb{R} \times S^1$ the spatial part of spacetime where we live in $\mathbb{R}$ (the thick black line for example) while the extra dimension $S^1$ (dashed grey) is not visible to us. (Right) A D1-brane (thick grey) is wrapped around the $S^1$ making its size dependent on where we are on $\mathbb{R}$. As a result, our visible spacetime $\mathbb{R}^2$ is curved, which is felt as the presence of an attractive gravitational force. If we put, for example, a particle (small circle in grey) at point $p$ in $\mathbb{R}$ it will roll down as if it was subject to an attractive force. This does not happen in the original spacetime (left).

This is not the end of the story as string theory does more than just generating solutions describing black holes. String theory gives us a nice way to reproduce the entropy of a class of black holes by counting the underlying degrees of freedom [16]. Let us go back to our previous example to understand how this works in a simple set up. The story is more complicated but the idea is very simple; we just add vibrations to our D1-brane. Since these vibrations are living in the non-observed dimension of
spacetime we will not see them. Said differently, we will not be able to distinguish between different black holes associated to the D1-brane wrapping the extra $S^1$ with different vibration modes. In principle, counting the number of possible modes will give us the statistical explanation of the entropy of our black hole.

Although the idea described above is simple, the actual counting of microstates is more involved. It is true that one can reproduce the right entropy of some black holes this way, but, that does not really explain why is the entropy of a black hole proportional to its horizon area. Neither does this way of approaching black holes shed light on the other paradoxes of black holes. Things become a little bit better using AdS/CFT correspondence \[9\]. The essence of this correspondence is that “string theory (supergravity) on an AdS$_{d+1}$ spacetime is equivalent to a CFT$_d$ living on the boundary of AdS$_{d+1}$”. In this correspondence, black holes are described by a thermodynamical ensemble of CFT states \[17\], which is characterized by a set of potentials related to the conserved charges carried by the black hole. Since the CFT is extensive, such equivalence between gravity and a CFT living in one dimension lower gives a nice explanation to why the black hole entropy is proportional to a surface. It also suggests that quantum gravity is highly non-local. In principle, one can also argue, using such duality, that information is not lost since CFT is a unitary theory. However, we are still far from completely understanding black holes using AdS/CFT duality due to the limitation of our present understanding of this duality.

For more information about what string theory can and cannot do regarding black hole physics, the reader is urged to consult the literature. By now, there are a lot of good review papers on the subject, see \[18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31\] for a sample of them.

**III-The Fuzzball Program**

Although AdS/CFT taught us many key ideas about black hole physics, at present, its domain of applicability is very restricted due to our ignorance of the full map between bulk and boundary dynamics. To deal with such limitations, Mathur and collaborators suggested the following bright idea \[32, 33, 34, 35\]: instead of studying the black hole dual states in the CFT, why not study their manifestation in gravity. On general grounds, absence of entropy forces these dual geometries to be smooth. Since the CFT description of a black hole is a thermodynamical ensemble of states, one is tempted to declare that the black hole geometry should be an effective description of the same thermodynamical ensemble of these dual smooth geometries. But, is this the right way to think about black holes? After all, in a fundamental theory we expect to be able to describe a quantum system in terms of pure states.
This should apply to a black hole as well. At first glance, since the black hole carries an entropy, the thermodynamical ensemble description seems to be favorable. But, as we know from statistical physics, the thermodynamical ensemble can be regarded as a technique for approximating the physics of the generic microstate in the microcanonical ensemble with the same macroscopic charges. Thus, one should be able to speak of the black hole as a coarse grained effective description of a generic underlying microstate. Recall that, a typical or generic state in an ensemble is very hard to distinguish from the ensemble average without doing impossibly precise microscopic measurements. The entropy of the black hole is then, as usual in thermodynamics, a measure of the ignorance of macroscopic observers about the nature of the microstate underlying the black hole.

This ambitious program was first undertaken successfully in the case of the D1-D5 system [32, 33, 34, 35]. Then, it was extended to other supergravity solutions with a varied degree of success, for reviews see [36, 37, 11, 38, 39, 40]. We will leave the description of the fuzzball idea to the second chapter and content ourselves here by mentioning the following properties:

- The smooth geometries look indistinguishable form that of the associated black hole outside a compact region around the origin in space where the black hole sits.

- In the class of black holes we will study, the naive black hole geometry develops an infinite throat near its horizon in contrast to microstate geometries which have a finite deep throat, see picture 2. The incident quantum gets trapped inside the throat for a long time, but, eventually reflects off the tip of the throat and escapes to the outer region of the geometry in a process similar to the thermal radiation of a black hole. It is clear that in such a process information is not lost.

- Quantum effects are enhanced and seen at distances of the order of the horizon radius much larger than the natural Plank length. This allows for a better chance to understand the breakdown of semi-classical calculation as the horizon cannot be described by the naive vacuum state.

- Black hole singularity is an emergent phenomenon.

Despite these tantalizing properties, a good formulation of the fuzzball conjecture is still lacking. The idea that black holes are simply effective descriptions of underlying horizon-free objects is confusing because it runs counter to well-established intuitions in effective field theory. Most importantly, the idea that near the horizon of a large black hole the curvatures are small and hence so are the effects of quantum gravity is in clear conflict with the claim above that quantum effects become important at the horizon. A little thought reveals that such large quantum effects
are not that strange. Remember that quantum mechanics discretizes the phase space into $\hbar$-sized cells. It could happen that points belonging to the same cell describe states that differ from each other macroscopically. We will leave a thorough discussion of such subtleties and potential misconception to the conclusion at the end of this thesis.

IV-THE ORGANIZATION OF THE THESIS

This thesis is divided into three parts:

- The first part discusses background material. It starts by a reminder about four dimensional black holes building up to give a rough idea about Hawking radiation. In the second chapter, ideas about phase space quantization and coarse graining are explained.

- The second part deals with the D1-D5 system and its coarse graining. We start by a description of the system and its solution in the first chapter of this part. Then, coarse graining of simple thermodynamical ensembles of these geometries is discussed in the second chapter of this part of the thesis.

- The third part concerns the four dimensional $\mathcal{N} = 2$ multi-black hole solutions.
We start with a description of these solutions and discuss their most important properties in the first chapter of this part. Then, a phase space quantization of these solutions is described in the second chapter of this part of the thesis.

- We close by a conclusion discussing open issues about the fuzzball program and future directions of investigation. Some technicalities are left to the appendices.

We tried to be very basic in the first part of the thesis. The only required background materials to hopefully follow the discussion in this part are: general relativity, quantum field theory and statistical physics. Our apologies for the advanced reader. The last two parts on the other hand are meant for advanced readers. In those parts the reader is assumed to have elementary notions of string theory like D-branes, supergravity, and also differential geometry, though, we will spend sometime explaining some background material that is not widely known, such as phase space densities (fourth chapter) and geometric quantization (sixth chapter). We will try our best to specify at the beginning of each chapter what background is needed to understand it.