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Temporal Aggregation and SVAR Identification, with an Application to Fiscal Policy*

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Abstract

We show how to assess the plausibility of identifying assumptions for a low-frequency structural vector autoregression (SVAR) using estimates from a higher-frequency model. We apply this method to an SVAR in government spending and output. Results from quarterly data show that it seems reasonable here to identify the annual fiscal SVAR by imposing a zero within-year impact of output on government spending. Our method can also be applied to other areas, for instance to examine the impact of monetary policy.

Keywords: SVAR, identification, high-frequency, low-frequency.

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1 Introduction

Structural vector autoregressions (SVARs) are often used to estimate the effects of economic impulses, such as fiscal and monetary policy shocks. High-frequency data can help identify the shocks. For instance, to study the impact of fiscal shocks many contributions (e.g., Ravn et al., 2007) use a quarterly analysis, where implementation lags and other restrictions motivated by institutional features provide natural identifying assumptions. However, data of a sufficiently high frequency are sometimes only available for a small number of countries and often over a shorter period. Thus, to examine a large cross-section of countries over a sufficiently long period, one needs to have low-frequency data (for fiscal SVARs see e.g., Beetsma et al., 2006, and Ardagna et al., 2007). However, temporal aggregation can invalidate the absence of a contemporaneous impact used to identify the high-frequency model (see Granger, 1988, among others), hampering identification in the low-frequency model.

This letter describes a way to examine how serious this is. More specifically, we exploit high-frequency identification restrictions and transform the estimates of a high-frequency model into (approximate) estimates for a low-frequency SVAR specification. This allows one to assess the plausibility of the identifying assumptions in the low-frequency specification. Hence, this way one can combine the advantages of using low- and high-frequency observations. We believe that this approach may be useful not only for fiscal but also for monetary policy, where researchers often use SVARs at different frequencies.

The next section lays out the identification problem and is followed by, respectively, a section that describes the temporal aggregation, an implementation section and a section that applies our method to the estimation of the within-year impact of output on government purchases from quarterly data. A brief conclusion ends this letter.

2 The identification problem

Let x_t be a K-vector of flow variables in the *low* (for instance, annual) frequency period t. We assume that x_t follows an SVAR (though the procedure also works for SVARMA):

$$A_0 x_t = A(L)\, x_{t-1} + \varepsilon_t,$$

where A_0 is a $K \times K$ matrix with 1's on the diagonal. The problem is how to obtain an estimate of the $(K-1)\,K$ unknown parameters in A_0. Without additional restrictions these parameters are not identified. The idea of our approach is to exploit high-frequency restrictions (e.g. a triangular ordering) to identify and estimate a high-frequency model and then derive estimates for A_0 of the low-frequency SVAR.

3 The Temporal Aggregation Procedure

From the SVAR specification we know that feeding a shock of size ε_t instead of 0 into the system leads to a change dx_t in x_t that fulfils

$$A_0 \cdot dx_t = \varepsilon_t.$$

Note that dx_t is the *within-period* change $x_t - x_t^0$, where x_t^0 is defined as the value of x_t when $\varepsilon_t = 0$. Suppose that we know the shock comes from variable k only ($k = 1, ..., K$). Let ε_t^k denote the K-vector with the shock as k-th element (and $K-1$ zeros) and denote the resulting change in x_t by dx_t^k. Then subtracting dx_t^k and stacking for all k gives

$$\begin{bmatrix} (A_0 - I_K)\, dx_t^1 \\ \vdots \\ (A_0 - I_K)\, dx_t^K \end{bmatrix} = \begin{bmatrix} \varepsilon_t^1 - dx_t^1 \\ \vdots \\ \varepsilon_t^K - dx_t^K \end{bmatrix},$$

where I_K is the K-dimensional identity matrix, so that the diagonal of $(A_0 - I_K)$ is zero. Let vec denote the columnwise vectorization of a matrix. Then, we can rewrite the system in block-diagonal format (see Section 5 for a concrete example)

$$\left(I_K \otimes \begin{bmatrix} dx_t^{1\prime} \\ \vdots \\ dx_t^{K\prime} \end{bmatrix} \right) vec\,(A_0' - I_K) = vec \left(\begin{bmatrix} \varepsilon_t^{1\prime} - dx_t^{1\prime} \\ \vdots \\ \varepsilon_t^{K\prime} - dx_t^{K\prime} \end{bmatrix} \right). \tag{1}$$

Suppose we have an observation of dx_t^k. We know that it originates from a shock to variable k only, but we do not observe the size of that shock. Consider the k-th block of equations in (1). The k-th equation of that block contains the same set of A_0 parameters as the other $K-1$ equations, but in contrast to those equations the k-th one also includes the unknown k-th element of ε_t^k. Hence, the k-th equation embodies no information on A_0 and we remove it. We do this for all K blocks (so all ε_t^k disappear from (1), because precisely the nonzero elements of all ε_t^k are deleted). Next, we remove the K columns in the Kronecker matrix and K zero elements in $vec\,(A_0' - I_K)$ that correspond to the diagonal of $(A_0' - I_K)$. This gives a square block-diagonal system of $(K-1)\,K$ equations in the $(K-1)\,K$ unknown elements of A_0; we can solve this for A_0, provided there are no singularities.

To operationalize our method we need values of dx_t^k that come from shocks to variable k in period t. We generate such shocks from an identified model (not necessarily an SVAR) for the vector $x_{t,q}$ of high-frequency values corresponding to the low-frequency $x_{t,q}$, where $q = 1, ..., Q$. Then, we compute the low-frequency changes dx_t^k by combining the Q high-frequency changes.

More precisely, suppose $x_{t,q}$ has an infinite moving average representation, so that for the $x_{t,q}$ in the low-frequency period t we have

$$\begin{cases} x_{t,1} = & \Psi_0\varepsilon_{t,1} + \Psi_1\varepsilon_{t-1,Q} + ... \\ x_{t,2} = & \Psi_0\varepsilon_{t,2} + \Psi_1\varepsilon_{t,1} + \Psi_2\varepsilon_{t-1,Q} + ... \\ \vdots & \\ x_{t,Q} = & \Psi_0\varepsilon_{t,Q} + \ldots + \Psi_{Q-1}\varepsilon_{t,1} + \Psi_Q\varepsilon_{t-1,Q} + ... \end{cases},$$

where the Ψ are $K \times K$ matrices and the $\varepsilon_{t,q}$ are K-vectors of structural disturbances of the high-frequency model. Assuming x_t and $x_{t,q}$ are in logarithms, one cannot simply add $x_{t,1}, ..., x_{t,Q}$ to obtain x_t. In fact,

$$x_t = \log\left[\exp\left(x_{t,1}\right) + \ldots + \exp\left(x_{t,Q}\right)\right],$$

where the log and exp functions of vectors are defined element by element. We now linearize around the value of $x_{t,q}$ that results from $\varepsilon_{t,1} = \ldots = \varepsilon_{t,Q} = 0$, denoted by $x^0_{t,q}$, so that

$$dx_t \approx diag\left(\sum_{q=1}^{Q}\exp\left(x^0_{t,q}\right)\right)^{-1}\left[diag\left(\exp\left(x^0_{t,1}\right)\right)dx_{t,1} + ... + diag\left(\exp\left(x^0_{t,Q}\right)\right)dx_{t,Q}\right],$$

where, for some generic K-vector z, $diag\left(z\right)$ is the diagonal matrix with the elements of z on its diagonal, and $dx_{t,q} \equiv x_{t,q} - x^0_{t,q}$. We approximate

$$diag\left(\sum_{q=1}^{Q}\exp\left(x^0_{t,q}\right)\right)^{-1} diag\left(\exp\left(x^0_{t,q}\right)\right) \approx \frac{1}{Q}I_K,$$

for all q. As we want to know the effect of shocks in period t only, we have

$$\begin{cases} dx_{t,1} = & \Psi_0\varepsilon_{t,1} \\ dx_{t,2} = & \Psi_0\varepsilon_{t,2} + \Psi_1\varepsilon_{t,1} \\ \vdots & \\ dx_{t,Q} = & \Psi_0\varepsilon_{t,Q} + \ldots + \Psi_{Q-1}\varepsilon_{t,1} \end{cases}.$$

This yields

$$dx_t \approx \frac{1}{Q}\left[\Psi_0\varepsilon_{t,Q} + \left(\Psi_0 + \Psi_1\right)\varepsilon_{t,Q-1} + \ldots + \left(\Psi_0 + \ldots + \Psi_{Q-1}\right)\varepsilon_{t,1}\right]. \quad (2)$$

Hence, for each k, if we have high-frequency shock vectors $\varepsilon^k_{t,1}, \ldots, \varepsilon^k_{t,Q}$, we can use (2) to compute (an approximate value of) the low-frequency change dx^k_t, which can then be used in (1) to derive the corresponding low-frequency contemporaneous correlations in A_0.

4 Implementation

To obtain the actual estimates of the low-frequency contemporaneous correlations, we first estimate the high-frequency model; this yields an estimate of the $K \times K$ diagonal covariance matrix Σ of the structural innovations $\varepsilon_{t,q}$, while a standard Monte Carlo method generates S sets of relevant impulse response matrices $(\Psi_0, ..., \Psi_{Q-1})$. Second, for each k we draw R sets of uncorrelated shock vectors $\left(\varepsilon_{t,1}^k, \ldots, \varepsilon_{t,Q}^k\right)$, where the k-th element of each vector is a draw from the estimated error distribution $N\left(0, \widehat{\Sigma}_{kk}\right)$. This gives R sets of shock vectors $\left\{\left(\varepsilon_{t,1}^1, \ldots, \varepsilon_{t,Q}^1\right), \ldots, \left(\varepsilon_{t,1}^K, \ldots, \varepsilon_{t,Q}^K\right)\right\}$. Third, we combine one set $(\Psi_0, ..., \Psi_{Q-1})$ with one collection $\left\{\left(\varepsilon_{t,1}^1, \ldots, \varepsilon_{t,Q}^1\right), \ldots, \left(\varepsilon_{t,1}^K, \ldots, \varepsilon_{t,Q}^K\right)\right\}$ to compute dx_t^k for all k using (2) and then derive the unknown elements in A_0 from (1); repeating this for all combinations yields SR draws for A_0 (the variation in the Ψ sets ensures that these A_0 draws capture the uncertainty of the high-frequency estimation step). We use the 5% and 95% quantiles to summarize the range of the A_0 draws.

5 Application

We apply our method to a 7-country panel SVAR in public spending g and output y (both in logarithms of their real values and seasonally adjusted) over the period 1965-2004. The country sample is Finland, France, Germany, Italy, the Netherlands, Sweden and the UK, for which quarterly public spending data are available (source: OECD Economic Outlook). Moreover, these are all EU countries, which guarantees a certain degree of homogeneity. Nevertheless, we control for country and time fixed effects, as well as country-specific time trends. After these corrections, the *annual-frequency* panel SVAR reads:

$$\begin{bmatrix} 1 & -\alpha_{12} \\ -\alpha_{21} & 1 \end{bmatrix} \begin{bmatrix} g_{it} \\ y_{it} \end{bmatrix} = A\left(L\right) \begin{bmatrix} g_{i,t-1} \\ y_{i,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{it}^g \\ \varepsilon_{it}^y \end{bmatrix},$$

where i indexes the country and t the year. We assume that the shock distribution is the same for all countries. System (1) becomes:

$$\begin{bmatrix} dg_{it}^1 & dy_{it}^1 & 0 & 0 \\ dg_{it}^2 & dy_{it}^2 & 0 & 0 \\ 0 & 0 & dg_{it}^1 & dy_{it}^1 \\ 0 & 0 & dg_{it}^2 & dy_{it}^2 \end{bmatrix} \begin{bmatrix} 0 \\ -\alpha_{12} \\ -\alpha_{21} \\ 0 \end{bmatrix} = \begin{bmatrix} \varepsilon_{it}^{g,1} - dg_{it}^1 \\ 0 - dg_{it}^2 \\ 0 - dy_{it}^1 \\ \varepsilon_{it}^{y,2} - dy_{it}^2 \end{bmatrix}.$$

Removing the redundant rows and columns yields

$$\begin{bmatrix} dy_{it}^2 & 0 \\ 0 & dg_{it}^1 \end{bmatrix} \begin{bmatrix} \alpha_{12} \\ \alpha_{21} \end{bmatrix} = \begin{bmatrix} dg_{it}^2 \\ dy_{it}^1 \end{bmatrix} \Rightarrow \alpha_{12} = \frac{dg_{it}^2}{dy_{it}^2} \text{ and } \alpha_{21} = \frac{dy_{it}^1}{dg_{it}^1},$$

provided $dy_{it}^2 \neq 0$ and $dg_{it}^1 \neq 0$. With $Q = 4$, the changes follow from (2) as:

$$\left\{ \begin{array}{l} \begin{bmatrix} dg_{it}^1 \\ dy_{it}^1 \end{bmatrix} = \frac{1}{4}\left[\Psi_0 \varepsilon_{t,4}^1 + \left(\Psi_0 + \Psi_1\right) \varepsilon_{t,3}^1 + \ldots + \left(\Psi_0 + \ldots + \Psi_3\right) \varepsilon_{t,1}^1\right] \\ \begin{bmatrix} dg_{it}^2 \\ dy_{it}^2 \end{bmatrix} = \frac{1}{4}\left[\Psi_0 \varepsilon_{t,4}^2 + \left(\Psi_0 + \Psi_1\right) \varepsilon_{t,3}^2 + \ldots + \left(\Psi_0 + \ldots + \Psi_3\right) \varepsilon_{t,1}^2\right] \end{array} \right. .$$

To obtain the Ψ and ε we estimate a *quarterly* panel SVAR in g and y (we thus consider the annual model as an approximation to the SVARMA implied by the temporally aggregated quarterly model). We impose the commonly-used identifying assumption that g within the quarter does not react to y (i.e. the coefficient that corresponds to the annual α_{12} is imposed to be zero). Application of the above procedure (with $S = R = 1000$) yields a 90% confidence interval (–0.082, 0.168) on the annual, within-year response coefficient α_{12} of g to y. This suggests that imposing a zero within-year response of g to y to identify an annual SVAR is a reasonable identifying restriction for the data set at hand. Moreover, the confidence interval can be used to examine the sensitivity of the annual SVAR results to realistic non-zero impacts.

6 Conclusion

We have shown how one may assess the sensibility of identifying assumptions in a low-frequency SVAR using estimates of a high-frequency model. We have applied our procedure to an annual SVAR in government purchases and output for a sample of EU countries using quarterly estimates. Imposing a zero effect of output on government spending at annual frequency appeared quite reasonable for our application. This finding is in line with fact that the government budget is usually set once a year, while within-year revisions tend to be relatively small and implemented with substantial delay.

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