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Optimization and approximation on systems of geometric objects

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Chapter 11

Conclusion

In this thesis, we studied the approximability of hard combinatorial optimization problems on (intersection graphs of) systems of geometric objects. We gave both positive results, in the form of approximation algorithms, and negative results, in the form of hardness of approximation statements. Here we look back at these results and provide directions for further research.

First, it is clear that having (the intersection graph of) a system of geometric objects as the underlying structure of an optimization problem helps to approximate it better than would be possible in the general case. We provided new, improved approximation schemes for Maximum Independent Set and Minimum Vertex Cover on intersection graphs of fat objects in any constant dimension and in the weighted case. We presented the first approximation algorithms for Minimum (Connected) Dominating Set on intersection graphs of objects of arbitrary size and improved approximation schemes on unit fat objects. We gave the first approximation scheme for Geometric Set Cover on unit squares and the first approximation algorithms for Geometric Unique Coverage on unit disks and on unit squares.

Secondly, we demonstrated that the type of the objects in the system of geometric objects has a significant impact on the approximability of optimization problems on such systems. Maximum Independent Set and Minimum Vertex Cover on intersection graphs of three-dimensional convex polygons of ply 1 and Minimum Dominating Set, Geometric Set Cover, Geometric Unique Coverage, and Geometric Membership Set Cover on planar fat objects are as hard on systems of these objects as on general set systems. Minimum Dominating Set and Geometric Set Cover are APX-hard on arbitrary rectangles. This is in sharp contrast to the positive results of the previous paragraph.

The influence of the object type is nowhere more visible than with Minimum Dominating Set on intersection graphs of objects of arbitrary size. The immediate open question there is whether Minimum Dominating Set admits a constant-factor approximation algorithm or even a ptas on disk graphs of arbitrary ply. The hardness results of Section 8.5 show that on objects whose boundaries can intersect an arbitrary number of times, Minimum Dominating Set is very hard to approximate. On the contrary, if object boundaries intersect at most twice (i.e. the objects are pseudo-disks), a linear-size ϵ-net exists and at least for cases such as r-polygons with constant r or rectangles

with bounded aspect-ratio, we get constant-factor approximation algorithms. An intriguing question is whether Minimum Dominating Set on disk graphs is harder to approximate than on other intersection graph classes such as intersection graphs of squares, or whether the algorithmic ideas of Chapter 8 can be extended to disks or maybe even to arbitrary pseudo-disks (the linear bound on the size of an ϵ-net starts to fail 'naturally' beyond pseudo-disks).

Similarly, in Section 8.4 we used the shifting technique in a $(3+\epsilon)$-approximation algorithm for Minimum Dominating Set on disk graphs of bounded ply. However, we do not know if the shifting technique can be used to give a constant-factor approximation algorithm (or even a ptas) for Minimum Dominating Set on disk graphs of arbitrary ply. We can point to two reasons for this. First, there is no upper bound on the number of 'large' disks intersecting a j-square that are in the dominating set. Secondly, we cannot track which j-square is 'responsible' for dominating a disk intersecting more than one j-square at its level. The algorithms in Section 8.4 got around the first problem by assuming that the ply is bounded and around the second problem by considering $\preceq_{\mathsf{Leb}}$-dominating sets (Theorem 8.4.19), or by disregarding the domination of disks intersecting a boundary on their level and combining three result sets (Theorem 8.4.29).

The weighted case of Minimum Dominating Set also poses interesting new questions. We presented the first ptas for this problem on intersection graphs of two-dimensional geometric objects, namely on unit square graphs. On unit disk graphs however, the best result so far is a $(5+\epsilon)$-approximation algorithm [74]. Can one improve to a ptas in this case?

Another problem where we do not yet have a clear picture of its approximability is Geometric Set Cover and its variants. For Geometric Set Cover on unit squares we discovered a ptas, the first approximation scheme for this problem. The scheme even extends to the weighted case. Recently, a ptas for Geometric Set Cover on arbitrary disks was announced [217], but it does not extend to the weighted case. Does the weighted case have a ptas? Hopefully the ideas behind the scheme on unit squares can be used for a ptas on systems of unit disks or other objects.

With respect to variants of Geometric Set Cover, we considered Geometric Unique Coverage and gave constant-factor approximation algorithms on unit disks and unit squares. There is as yet however no reason to suspect that the attained constants are optimal. In particular, one wonders whether the ptas for Geometric Set Cover on unit squares can be adapted to this problem. The geometric version of Minimum Membership Set Cover presents even bigger challenges. We proposed a constant-factor approximation algorithm that runs in polynomial time if the optimum is constant. Can this restriction be lifted?

In summary, this thesis answered many question surrounding the approximability of optimization problems on systems of geometric objects. These answers in turn lead to new questions, that will be challenging to answer...