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**Optimization and approximation on systems of geometric objects**

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# Summary

Practical problems in for example wireless networks, computational biology, or cartography can often be modeled by defining an optimization problem on a system of geometric objects. Many optimization problems are NP-hard to solve and sometimes even provably hard to approximate. However, if the underlying structure of the optimization problem is a system of geometric objects, then the problem is usually easy to solve or can be approximated well. This PhD thesis investigates the approximability of hard optimization problems on systems of geometric objects. In particular it considers how the object type influences the approximability of these problems.

The most important structure on systems of geometric objects that we consider during this investigation is the intersection graph of the given objects, called a geometric intersection graph. Every vertex of this graph corresponds to an object and there is an edge between two vertices if and only if the corresponding objects intersect. A famous example of this is the disk graph, which is frequently used to model wireless networks. Several classic optimization problems, such as Maximum Independent Set, Minimum Vertex Cover, and Minimum (Connected) Dominating Set, are relevant in this context. With respect to the approximability of these problems, we conclude the following.

If a collection of  $n$  disks of equal size (unit disks) is given of which the density is  $d$ , then we can give a  $(1 + \epsilon)$ -approximation for each of the studied problems in  $d^{O(1/\epsilon)}n^{O(1)}$  time. This yields an eptas if  $d = d(n) = n^{o(1)}$  and a ptas in general. For Minimum Vertex Cover we can even strengthen the algorithm to an eptas in the general case. These schemes can be extended to intersection graphs of unit fat objects in constant dimension. Moreover we can show that, up to constants, there is no faster algorithm to find a  $(1 + \epsilon)$ -approximation for these problems, unless the exponential time hypothesis fails. Except for Minimum Vertex Cover, we also prove that there is no eptas for the studied problems if  $d = d(n) = n^\alpha$  for any constant  $\alpha > 0$ , unless  $\text{FPT} = \text{W}[1]$ . This gives a strong indication that the given schemes are optimal.

The approximation schemes for Maximum Independent Set and Minimum Vertex Cover can be extended to intersection graphs of arbitrarily sized disks. We obtain an eptas if the level density is  $d = d(n) = n^{o(1)}$ , a ptas in general, and for Minimum Vertex Cover we even obtain an eptas in the general case. The schemes also apply to intersection graphs of fat objects in constant dimension. As in the case of unit disk graphs, we can show that these schemes are optimal, up to constants.

If however we consider Minimum (Connected) Dominating Set, then it turns out that this problem becomes a lot harder on intersection graphs of ob-

jects of arbitrary size. If the objects are arbitrarily scaled and translated copies of a convex polygon, then there is a constant-factor approximation algorithm. This is the first approximation algorithm for this problem that beats the  $\ln n$ -approximation given by the greedy algorithm. Extending to disk graphs is not directly possible, because the constant in the approximation factor depends on the complexity of the polygon. If however each point of the plane is overlapped by a bounded number of disks, then a  $(3 + \epsilon)$ -approximation can be found in polynomial time (for fixed  $\epsilon > 0$ ). This also applies to intersection graphs of fat objects in constant dimension. If the number of objects overlapping some point is not bounded and the shape of the objects is only slightly different from the shape of a disk (but still is fat), then we can show that the  $\ln n$ -approximation algorithm gives the best possible approximation factor, unless  $\text{NP} \subset \text{DTIME}(n^{O(\log \log n)})$ .

Another important problem to study is the geometric version of the well-known minimum set cover problem and several of its variants. Given a set of geometric objects and a set of points in the plane, one needs to find a smallest subset of the objects that cover all points. If the objects are squares of equal size, then we give a ptas for this problem, one of the first approximation schemes for this problem in two dimensions and the first that also applies to the weighted case.

If we want to find a subset of the objects such that a maximum number of the given points is overlapped by precisely one object, then we give a  $1/18$ -approximation algorithm if the objects are unit disks and a  $1/2$ -approximation algorithm if the objects are unit squares. If the objects have arbitrary size and are fat, then we give an eptas if each point in the plane is overlapped by a bounded number of objects. If this number is not bounded however, then we can again prove that the  $\ln n$ -approximation algorithm gives the best possible approximation factor.

The above three algorithms can be extended to the case where each object has a cost, each point has a profit, and we aim to maximize the total profit of (uniquely) covered points such that the total cost is within a given budget.

The foremost conclusion of this thesis is that well-known optimization problems are often better approximable on systems of geometric objects than on general systems. We have shown however that the object type has strong influence on the approximability of these problems. In particular, problems such as Minimum Dominating Set, Minimum Set Cover, and several of its variants seem more difficult on a system of disks than on a system of (say) squares. Further investigation of this phenomenon poses a new challenge.