A game for the Borel functions
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We have seen a number of games in this thesis. In Chapter 2, we saw the tree game and its characterization of the Borel functions. In the second part of the thesis, we saw more games for certain subclasses of Borel functions, and we saw that they can be used to prove decomposition theorems.

The question naturally arises: can we obtain a result for more general sub-classes of Borel functions? It is hoped that the game-theoretic tools we have developed in this thesis can be generalized to obtain a more elegant characterization theorem. In particular, all of the games we have looked at can be viewed as restricted tree games. The Wadge game can be viewed as the restricted tree game in which Player II is required to produce $\phi$ such that $\text{dom}(\phi)$ is linear; for the eraser game, we require that $\text{dom}(\phi)$ is finitely branching; for the backtrack game, we require that $\text{dom}(\phi)$ branches finitely at the root and is linear thereafter; for the game $G_{2,3}$, we require that $\text{dom}(\phi)$ may branch infinitely at the root but is finitely branching thereafter; and for the multitape game, we require that $\text{dom}(\phi)$ may branch infinitely at the root but is linear thereafter.

Thus, it would seem natural to come up with more general restrictions on $\text{dom}(\phi)$, and work with $m$'s and $n$'s or $\alpha$'s and $\beta$'s instead of numbers between 1 and 3. (The author refuses to prove any decomposition theorems with 4's in them.)

The tree game itself is simple and characterizes a class of functions widely considered in descriptive set theory. Going beyond the Borel functions, one might try to generalize the tree game to characterize classes of projective functions, possibly by allowing Player II to produce multiple infinite branches.