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# Dynamic efficiency of Cournot and Bertrand competition with cooperative R&D\*

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## Abstract

We consider the efficiency of Cournot and Bertrand competition when firms cooperatively conduct cost-reducing R&D. We decompose the combined-profits externality into three components: a strategic component, a size component, and a spillover component. The latter bears an opposite sign across competition types. Hence, under Bertrand competition the minimum spillover above which cooperative R&D exceeds noncooperative R&D is higher than under Cournot competition. Also, the traditional difference in R&D investment incentives between Cournot and Bertrand competition is exemplified if firms conduct R&D cooperatively. The Cournot-Nash price can then be below the Bertrand-Nash price, especially if spillovers are strong.

**Key words:** Bertrand competition; Cournot competition; R&D; dynamic efficiency.

**JEL Classification:** L13.

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# 1 Introduction

It is well known that in a duopoly with an exogenous market structure price (Bertrand) competition yields lower prices than quantity (Cournot) competition (Singh and Vives, 1984; Cheng, 1985). With Bertrand competition residual demand is more sensitive to changes in price thereby yielding lower equilibrium prices (Martin, 2002). Zanchettin (2006) shows that this result extends to duopolies with exogenous cost differences while for symmetric cost structures the result also extends to an oligopoly (Vives, 1985). Häckner (2000) reveals however that in an oligopoly of complementary goods with exogenous quality differences the low-quality firms may charge higher prices under Bertrand competition than under Cournot competition. The switch from Cournot competition to Bertrand competition induces the high-quality firms to charge a lower price. And the resulting upward pressure on the demand for the low-quality complement then allows for a price increase. Hence (Häckner, 2000, p. 238), “it is not evident which type of competition is more efficient.”

If market structure is endogenous the traditional welfare comparison of Cournot and Bertrand competition may indeed be reversed. Cellini *et al.* (2004) and Mukherjee (2005) show that under free entry the number of firms entering under Cournot competition exceeds that under Bertrand competition. The resulting increase in the number of product varieties can more than compensate for the higher price that always obtains under Cournot competition.

Alternatively, the production function is endogenous in the sense that competition in the product market is preceded by a stage where firms conduct research and development (R&D). This research can be aimed at lowering production costs (Qiu, 1997), or at increasing product quality (Symeonidis, 2003). Again the welfare comparison may be reversed as under Cournot competition incentives to invest

in R&D are higher than under Bertrand competition.<sup>1</sup> With process R&D, post-innovation production costs under Cournot competition are then reduced more than under Bertrand competition. The difference in profits under Cournot and Bertrand competition is then enhanced further. As a result, total surplus under Cournot competition can exceed total surplus under Bertrand competition, despite the fact that prices under Bertrand competition are always lower than under Cournot competition (Qiu, 1997). For product R&D similar results apply although here the higher welfare under Cournot competition is due to higher product quality which directly enhances consumers' surplus (Symeonidis, 2003).<sup>2</sup>

In this paper we also compare the efficiency of Cournot and Bertrand competition when production costs are endogenous. In contrast to the existing literature we consider firms to cooperate in R&D. An important aspect of R&D is its public good character (Jacquemin, 1988). This is reflected in the free flow of knowledge that is generated by any firm conducting R&D, the so-called technological spillover. Because of these spillovers firms are unable to appropriate exclusively all the proceeds of their R&D efforts. R&D investment levels then fall short of what is socially desirable. To alleviate this problem many jurisdictions, including the EU, the US and Japan, allow firms to set up R&D cooperatives (Martin, 1995). These cooperatives internalize the technological spillover which is thought to enhance firms' incentives to invest in R&D.

According to Kamien *et al.* (1992) there are two externalities that influence firms' R&D investment decision. First there is the competitive-advantage externality whereby any firms' R&D activities strengthen rival's position in the product

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<sup>1</sup>If firms conduct both process R&D and product R&D the incentives comparison between competition types is ambiguous (Lin and Saggi, 2002).

<sup>2</sup>Alternatively, production costs depend on a wage bargaining stage that precedes production (López and Naylor, 2004). Profits under Bertrand competition may then exceed those under Cournot competition. But in this scenario welfare under Cournot competition always falls short of that under Bertrand competition.

market through the reduction in rival's production costs because of the technological spillover. This reduces the incentives to conduct R&D. Second, any firm's reduction in production costs affects joint profits. This combined-profits externality can be either positive or negative, depending on the extent of technological spillover. The stronger is the technological spillover, the more likely it is that the combined-profits externality is positive. Firms competing in R&D consider the competitive-advantage externality only while R&D cooperatives also take the combined-profits externality into account. As a result, R&D cooperatives invest more in R&D than what a competitive R&D market would do whenever the combined-profits externality outweighs the competitive-advantage externality. This is the case if the technological spillover is above some threshold level. As shown by Kamien *et al.* (1992), this threshold is higher under Bertrand competition than under Cournot competition.

We identify three separate components that jointly make up the combined-profits externality: a strategic component, a spillover component, and a size component. The strategic component is always negative: firms that cooperate in R&D realize that a larger market share comes at rival's expense. This diminishes the incentive to invest in R&D. The size component on the other hand is always positive, provided that there exists a strictly positive technological spillover. Any firm's cost reduction that spills over to its rival is more to this rival's benefit the larger is its output. This enhances the R&D investment incentive of an R&D cooperative. The sign of the spillover component, however, is negative with Bertrand competition and positive with Cournot competition. Obviously, R&D results that spill over are beneficial to joint profits if firms compete in quantities as these are strategic substitutes. But if competition is over price, any reduction in rival's production costs due to the technological spillover leads to a reduction in rivals' profits as both firms will lower their price in response to the cost reduction.

Separating out these three components allows for a better understanding as to why cooperative R&D exceeds competitive R&D in case of relatively strong spillovers only. In that case the size component of the combined-profits externality is large enough to outweigh the strategic component and, in case of Bertrand competition, the spillover component. Also, the opposite sign of the spillover component explains why the threshold value of the technological spillover above which cooperative R&D exceeds noncooperative R&D is larger with Bertrand competition than with Cournot competition.

Perhaps more importantly, the difference in composition of the combined-profits externality across competition types implies that R&D cooperatives have stronger incentives to invest in R&D when competition is over quantities rather than price. As a result, the difference in post-innovation costs between Cournot competition and Bertrand competition is exemplified if firms cooperate in R&D. We show that this difference can be so large that in equilibrium the Cournot-Nash price is *below* the Bertrand-Nash price. Because the magnitude of the spillover component is one-to-one related to the size of the technological spillover, it follows that a lower price under Cournot competition is more likely to emerge the stronger is the technological spillover. At the same time we show that under Bertrand competition producers' surplus always falls short of that under Cournot competition. As a result, whenever the Cournot-Nash price is below the Bertrand-Nash price, total surplus under Cournot competition is higher than under Bertrand competition.

Our results are not without policy implications. It could be argued that R&D cooperatives are less desirable if the intensity of product market competition is relatively low, in order to avoid the creation of additional market power. But we show that the higher R&D investments under Cournot competition ultimately can lead to lower prices. Moreover, it is quite likely that the technological spillover

increases due to cooperation in R&D (Kamien *et al.* 1992). That enhances both the spillover and size component of the combined-profits externality. If anything, this increases the likelihood that competition over quantities leads to a lower price than what price competition would yield. Discuss different types of cooperatives.

We proceed as follows. We introduce the model in the next section, and derive the equilibrium in Section 3 for both types of product market competition. The efficiency of Cournot and Bertrand competition is compared in Section 4 and Section 5 concludes.

## 2 The model

Consider a two-stage duopoly where firms invest in cost-reducing R&D and then compete on the product market. Indirect market demand is given by:<sup>3</sup>

$$p_i = a - (q_i + \theta q_j), \quad (1)$$

$i, j = 1, 2, i \neq j$ , where  $q_i$  and  $p_i$  are the respective quantity and price of product  $i$ . The parameter  $\theta$  captures the extent of product differentiation. If  $\theta = 1$  both firms produce the same, homogeneous product; if  $\theta = 0$  both firms hold a local monopoly (i.e. products are independent). For the remainder of the paper we focus on all intermediate cases:  $\theta \in (0, 1)$ . Unless stated otherwise,  $i, j = 1, 2, i \neq j$  holds throughout the rest of the paper. In direct form market demand equals:

$$q_i = \frac{1}{1 - \theta^2} [(1 - \theta)a - (p_i - \theta p_j)]. \quad (2)$$

Each firm produces one version of the differentiated product with marginal costs  $c$  and no fixed costs, where we assume that  $c < a$ . Investments in process R&D can lower these marginal costs whereby either firm can appropriate part of

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<sup>3</sup>A standard quadratic utility function leads to these inverse demands (Singh and Vives, 1984).

rival's efforts without having to pay for it. In particular, if firm  $i$  invests  $x_i$  in R&D, its *effective* R&D investments  $X_i$  are given by:

$$X_i = x_i + \beta x_j. \quad (3)$$

In (3)  $\beta \in [0, 1]$  is the technological spillover. An R&D production function  $f$  then describes how much firm  $i$ 's marginal costs are reduced due to its effective R&D investments. Following Kamien *et al.* (1992) we assume diminishing returns to scale in R&D:  $f' > 0$ ,  $f'' < 0$  and  $f(0) = 0$ . In particular we set (see also Amir, 2000):

$$f(X_i) = \sqrt{X_i/\gamma}, \quad (4)$$

whereby  $\gamma > 0$  is related to the efficiency of the R&D process. A lower value of  $\gamma$  corresponds to a more efficient process of R&D. As the technological spillover affects any firm's effective R&D investments, it is an input of the R&D process.<sup>4</sup> Profits of firm  $i$  then equal

$$\pi_i = p_i q_i - (c - y_i) q_i - x_i, \quad (5)$$

where  $y_i = \sqrt{(x_i + \beta x_j)/\gamma}$ .

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<sup>4</sup>Alternatively R&D outputs spill over (as in d'Aspremont and Jacquemin (1988) and Qiu, 1997). Output spillovers in combination with diminishing returns to scale in R&D can be problematic however. In case one firm conducts much R&D while the other does not, it can be in the interest of the R&D-intensive firm to donate its next euro of R&D investment to its rival and to appropriate the R&D returns through the spillover, instead of investing this euro in own R&D (Amir, 2000). Moreover, empirical studies typically find the spillover to occur during the R&D process (Kaiser, 2002).



### 3 Market equilibria

#### 3.1 Second-stage Bertrand competition

Maximizing (5) over price yields equilibrium prices conditional on effective R&D efforts:<sup>5</sup>

$$\hat{p}_i(X_i, X_j) - c = \frac{(a - c)(2 + \theta)(1 - \theta) - 2y_i - \theta y_j}{4 - \theta^2}. \quad (6)$$

Inserting (6) into (5) and maximizing the concomitant sum of firms' profits over R&D investments results in the following cost reductions:<sup>6,7</sup>

$$\tilde{y}^B = \frac{(a - c)(2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2(1 - \theta^2) - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}, \quad (7)$$

Equilibrium output then equals:

$$\tilde{Q}^B = \frac{2\gamma(a - c)(2 + \theta)(1 - \theta)(4 - \theta^2)}{\gamma(4 - \theta^2)^2(1 - \theta^2) - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}. \quad (8)$$

Single-firm profits are given by:

$$\tilde{\pi}^B = \frac{\gamma(1 - \theta^2)(4 - \theta^2)^2 - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2} (\tilde{q}^B)^2, \quad (9)$$

where  $\tilde{q}^B = \tilde{Q}^B/2$ . Consumers' surplus and total surplus respectively equal:

$$\tilde{CS}^B = (1 + \theta) (\tilde{q}^B)^2, \quad (10)$$

and

$$\tilde{TS}^B = \frac{\gamma(4 - \theta^2)^2(1 + \theta)(3 - 2\theta) - 2(2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2} (\tilde{q}^B)^2. \quad (11)$$

<sup>5</sup>A hat refers to a conditional equilibrium outcome.

<sup>6</sup>A tilde refers to an unconditional equilibrium expression; superscript  $B$  stands for second-stage Bertrand competition.

<sup>7</sup>The second-order and stability conditions are examined in Section 3.3.

### 3.2 Second-stage Cournot competition

Maximizing (5) over quantities leads to equilibrium quantities conditional on R&D investments:

$$\hat{q}_i(X_i, X_j) = \frac{(a - c)(2 - \theta) + 2y_i - \theta y_j}{4 - \theta^2}. \quad (12)$$

Maximizing the concomitant sum of first-stage profits with respect to R&D investments gives:<sup>8</sup>

$$\tilde{y}^C = \frac{(a - c)(2 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)}, \quad (13)$$

and

$$\tilde{Q}^C = \frac{2\gamma(a - c)(4 - \theta^2)(2 - \theta)}{\gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)}. \quad (14)$$

Single-firm profits then equal:

$$\tilde{\pi}^C = \frac{\gamma(4 - \theta^2)^2 - (1 + \beta)(2 - \theta)^2}{\gamma(4 - \theta^2)^2} (\tilde{q}^C)^2, \quad (15)$$

with  $\tilde{q}^C = \tilde{Q}^C/2$ . Consumers' surplus and total surplus under second-stage Cournot competition then respectively equal:

$$\tilde{CS}^C = (1 + \theta) (\tilde{q}^C)^2, \quad (16)$$

and

$$\tilde{TS}^C = \frac{\gamma(4 - \theta^2)^2(3 + \theta) - 2(1 + \beta)(2 - \theta)^2}{\gamma(4 - \theta^2)^2} (\tilde{q}^C)^2. \quad (17)$$

### 3.3 Regularity conditions

The admissible parameter space is bounded by four conditions that emerge from the R&D stage: post-innovation costs have to be positive and the equilibrium must

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<sup>8</sup>Superscript  $C$  stands for second-stage Cournot competition.

be an interior solution. Under Bertrand and Cournot competition, the second-order conditions respectively require that:

$$\gamma \geq \frac{(1 + \beta)[(2 - \theta^2 - \theta\beta)^2 + (2\beta - \theta^2\beta - \theta)^2]}{(4 - \theta^2)^2(1 - \theta^2)(1 + \beta^2)} = \gamma_{R1}, \quad (\text{R1})$$

and

$$\gamma \geq \frac{(1 + \beta)[(2 - \theta\beta)^2 + (2\beta - \theta)^2]}{(4 - \theta^2)^2(1 + \beta^2)} = \gamma_{R2}. \quad (\text{R2})$$

The requirement that post-innovation costs are positive under Bertrand and Cournot competition respectively implies that:

$$\gamma > \frac{a(1 - \theta)(1 + \beta)}{c(1 + \theta)(2 - \theta)^2} = \gamma_{R3}, \quad (\text{R3})$$

and

$$\gamma > \frac{a(1 + \beta)}{c(2 + \theta)^2} = \gamma_{R4}. \quad (\text{R4})$$

Condition R3 turns out to be redundant (proofs of all lemmata are in Appendix A, Section 6.1):

**Lemma 1** *The parameter space is bounded by regularity conditions R1, R2 and R4.*

## 4 Cournot versus Bertrand

### 4.1 R&D investments

Cooperative R&D efforts under second-stage Cournot and Bertrand competition compare as follows (Appendix A, Section 6.2 contains the proofs of all propositions):

**Proposition 1** *For any  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$  we have that  $\tilde{y}^C > \tilde{y}^B$  under R1, R2 and R4.*

According to Proposition 1 cooperative R&D investments are always higher under Cournot competition than under Bertrand competition. At face value this result replicates Proposition 1 of Qiu (1997), who considers noncooperative R&D. He splits the incentives to invest in R&D into four different effects: a strategic effect, a spillover effect, a size effect and a cost effect. The latter three bear the same sign under Cournot and Bertrand competition. The spillover effect and the cost effect are negative. The more expensive is R&D, the lower will be the R&D investment. And the free flow of knowledge to competitors obviously reduces the incentive to invest in R&D. The size effect is positive and refers to the quantities produced. The more a firm produces, the more profitable will be a reduction in production costs, the higher is the incentive to invest in R&D that creates such a cost reduction.

The strategic effect however carries an opposite sign. With Cournot competition it is positive as quantities are strategic substitutes. It thus pays to have lower marginal cost as that translates into a larger market share and higher profits. With Bertrand competition it is negative. Reduced marginal costs induces a firm to cut price. Because prices are strategic complements, rivals' reaction will be to lower its price as well. In the end this reduces both firms' profits.

In case of cooperative R&D the incentives' decomposition of R&D investments is more involved. Considering the effect of any firm's R&D on total profits gives under second-stage Cournot competition (see Appendix B, Section 7 for details):

$$\begin{aligned}
\frac{\partial \Pi^C}{\partial x_i} = & \overbrace{\frac{y_j}{2\gamma y_i y_j \Omega^C} \left( \overbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}^{+} + \overbrace{-\frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_j^2}}^{-} \right)}^{\text{strategic effect (-)}} \\
& + \overbrace{\frac{\beta y_i}{2\gamma y_i y_j \Omega^C} \left( \overbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_i}{\partial q_i^2}}^{-} + \overbrace{-\frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial q_j q_i}}^{+} \right)}^{\text{spillover effect (-)}} + \overbrace{\frac{q_i}{2\gamma y_i} + \frac{\beta q_j}{2\gamma y_j}}^{\text{size effect (+)}} + \overbrace{-1}^{\text{cost effect (-)}},
\end{aligned}$$

where  $\Omega^C = (\partial^2 \pi_i / \partial q_i^2) (\partial^2 \pi_j / \partial q_j^2) - (\partial^2 \pi_i / \partial q_j \partial q_i) (\partial^2 \pi_j / \partial q_i q_j) > 0$ . The strategic effect, the spillover effect and the size effect all include an additional component which arise from firm  $i$ 's concern about firm  $j$ 's profits whenever it sets its optimal R&D investment, that is, the combined-profits externality. Considering the strategic effect first, a negative component is added. If firm  $i$ 's R&D activities reduce its production costs, it is ultimately at the expense of firm  $j$ 's profits because quantities are strategic substitutes. As shown in Appendix A (Section 7), this second component outweighs the positive component, which makes the strategic effect also negative under Cournot competition. The spillover effect includes a second, positive component. This positive component is due to any firm's R&D activities leading to a reduction in rival's production costs through the spillover. This enhances firm  $i$ 's R&D investments incentives. However, the spillover effect as a whole remains negative (see Appendix B, Section 7). Finally, as any firm's R&D activities are more valuable the larger is the quantity produced to which the reduced production costs apply, the size effect now also includes rivals' production, provided that the R&D results carry over to the rival (that is, provided that the spillover is positive).

D'Aspremont and Jacquemin (1988) show that cooperative R&D efforts exceed competitive levels of R&D when the technological spillover is above some thresh-

old level. Kamien *et al.* (1992) observe that in that case the combined-profits externality is positive and that it outweighs the competitive-advantage externality. Our decomposition of investment incentives towards cooperative R&D yields further insights. Note that the three additional components jointly constitute the combined-profits externality. If there are no technological spillovers only the component that is added to the strategic effect remains. As this is negative, cooperation in R&D absent technological spillovers would lead to a reduction in R&D activity. In that case the only additional consideration both firms have is that their R&D is detrimental to rival's profits as it reduces rival's market share.

The other two components of the combined-profits externality are both positive but feature only in case of positive spillovers. If these are high enough, the two positive components outweigh the negative one and the combined-profits externality becomes positive. In addition to the strategic effect, cooperating firms now also realize that their R&D efforts are, in fact, beneficial to rivals' profits as it reduces rival's production costs as well. For some higher level of the technological spillover the combined-profits externality is then so large that it overrules the competitive-advantage externality. If that is the case, competitive R&D efforts fall short of the level of cooperative R&D.

Under second-stage Bertrand competition the decomposition of cooperative R&D investment incentives yields (see Appendix B, Section 7):

$$\begin{aligned}
\frac{\partial \Pi^B}{\partial x_i} = & \overbrace{\frac{y_j}{2\gamma y_i y_j \Omega^B} \left( \frac{\partial q_i}{\partial p_i} \frac{\partial \pi_i}{\partial p_j} \frac{\partial^2 \pi_j}{\partial p_i \partial p_j} + \frac{\partial q_i}{\partial p_i} \frac{\partial \pi_j}{\partial p_i} \frac{\partial^2 \pi_j}{\partial p_j^2} \right)}^{\text{strategic effect (-)}} \\
& + \overbrace{\frac{\beta y_i}{2\gamma y_i y_j \Omega^B} \left( -\frac{\partial q_j}{\partial p_j} \frac{\partial \pi_i}{\partial p_j} \frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial q_j}{\partial p_j} \frac{\partial \pi_j}{\partial p_i} \frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \right)}^{\text{spillover effect (-)}} + \overbrace{\frac{q_i}{2\gamma y_i} + \frac{\beta q_j}{2\gamma y_j}}^{\text{size effect (+)}} + \overbrace{-1}^{\text{cost effect (-)}},
\end{aligned}$$

There are again three components that together constitute the combined-profits externality. The negative strategic effect becomes larger in absolute size as a reduction in firm  $i$ 's production costs induces it to lower its price. Because prices are strategic complements the rival firm follows suit which in the end lowers both firms' profits. Hence, firms that cooperate in R&D also cut back on their R&D investments if competition is over price and there are no technological spillovers. The size effect on the other hand is again positive because rivals' output is now also taken into account, provided that the spillover is positive. The crucial difference with Cournot competition is that a second, negative component is added to the spillover effect. Any reduction in rival's production costs due to the technological spillover leads to lower profits as both firms will lower their price in response to the cost reduction. As a result, the difference in R&D investment incentives between Cournot and Bertrand competition as reported by Qiu (1997) increases if firms cooperate in R&D.

As shown by Kamien *et al.* (1992), under second-stage Bertrand competition there exists a threshold value of the technological spillover as well, above which the combined-profits externality outweighs the competitive-advantage externality. That threshold is higher though than under second-stage Cournot competition. We now know that this is due to the opposite sign of the component that is added

to the spillover effect. It thus requires a larger spillover for the combined-profits externality to outweigh the competitive-advantage externality.

## 4.2 Price

Before we proceed with the comparison of equilibrium prices we first introduce an assumption:

$$\beta > \gamma(4 - \theta^2) - 1 = \beta^*. \quad (\text{A1})$$

Note that  $\beta^* < 0$  whenever  $\gamma < 1/(4 - \theta^2)$ , while  $\beta^* > 1$  for  $\gamma > 2/(4 - \theta^2)$ . That is, assumption A1 holds always if the R&D process is relatively efficient while it cannot hold if the R&D process is relatively inefficient. For intermediate cases the size of the technological spillover determines whether the condition in A1 is met or not. In general, the larger is the technological spillover, the more likely it is that A1 holds, all else equal. The assumption itself does not rule out the existence of equilibria:

**Lemma 2** *The set where regularity conditions R1, R2, R4 and assumption A1 hold is not empty.*

Note from the proof of Lemma 2 that whenever assumption A1 holds, the admissible parameter space is bounded by conditions R1 and R4.

Equilibrium prices compare as follows:

**Proposition 2** *For any  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$  we have that  $\tilde{p}^C < \tilde{p}^B$  under R1, R4 and A1.*

According to Proposition 2, with cooperative R&D prices can be lower with Cournot competition than with Bertrand competition. Put differently, whenever assumption A1 holds, consumers' surplus is larger under second-stage Cournot



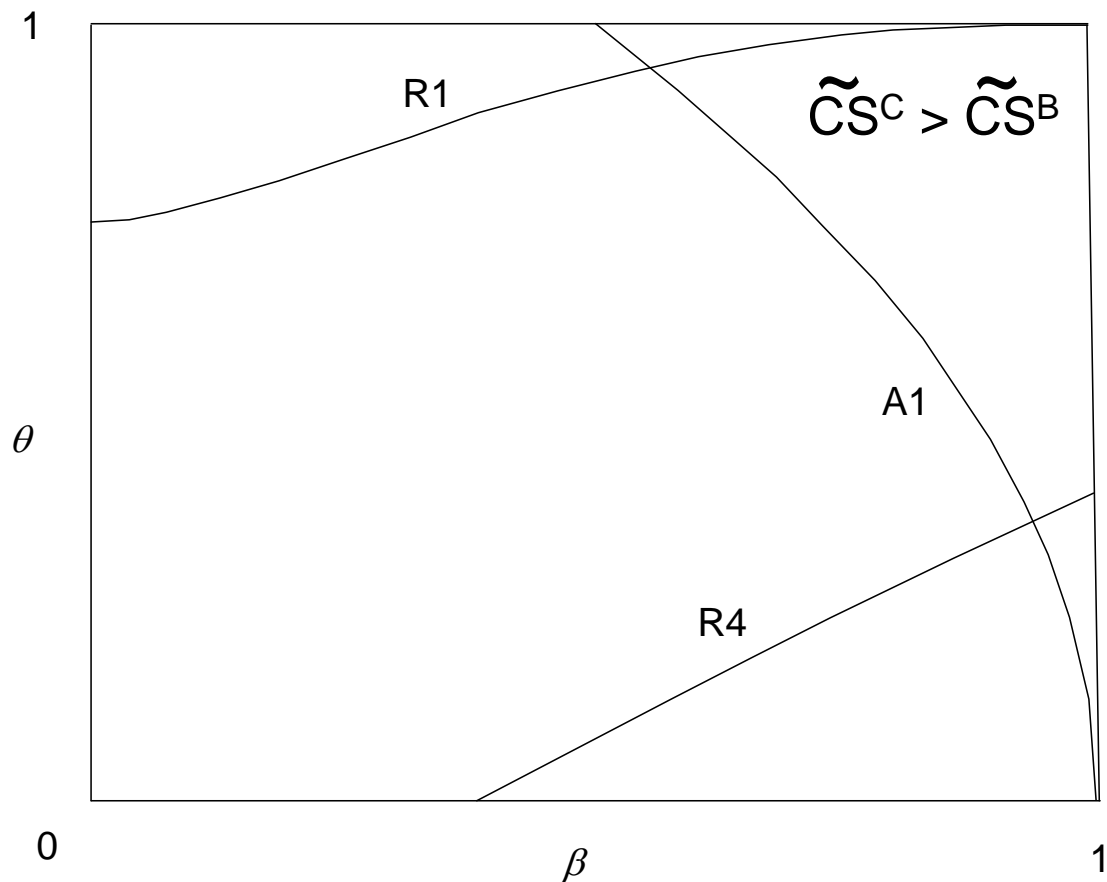


Figure 1: Comparing consumers' surplus under second-stage Cournot and Bertrand competition under assumption A1 and regularity conditions R1 and R4, where  $a = 100$ ,  $c = 70$ , and  $\gamma = 0.5$ ).

competition than with second-stage Bertrand competition. Post-innovation cost are then so much lower under second-stage Cournot competition that in equilibrium lower prices obtain. This, of course, is due to the different magnitude of the combined-profits externality for the two types of product market competition. Recall that this difference is more pronounced the larger is the technological spillover. In that case it is also more likely that assumption A1 holds. The conditions of Proposition 2 are illustrated in Figure 1.

### 4.3 Profits

Because firms enjoy more market power when they compete over quantities, it is not immediate what Propositions 1 and 2 imply for profits. The following proposition clarifies:

**Proposition 3** *For any  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$ , we have that  $\tilde{\pi}^C > \tilde{\pi}^B$  under R1, R2 and R4.*

Despite the fact that under Cournot competition firms always incur higher R&D costs and that for a subset of the parameter space the equilibrium price is below what obtains with Bertrand competition, producers' surplus is always higher when firms compete over quantities.

### 4.4 Welfare

Combining the message of Proposition 3 with that of Proposition 2 tells us that total surplus with Cournot competition exceeds that with Bertrand competition whenever Assumption A1 holds. Competition over quantities could also yield higher total surplus in combination with a higher equilibrium price if the concomitant difference in producers' surplus is large enough. The complete ranking of total surplus is as follows:

**Proposition 4** *For any  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$  the following holds under R1, R2, and R4:*

- (i)  $0 < \gamma < (1 + \beta)/(4 - \theta^2) : \quad \widetilde{TS}^C > \widetilde{TS}^B$
- (ii)  $(1 + \beta)/(4 - \theta^2) < \gamma$  there exists a unique  $\gamma^*(\theta)$  such that:
  - (a)  $\gamma > \gamma^*(\theta), \forall \beta \in [0, 1] : \quad \widetilde{TS}^B > \widetilde{TS}^C$
  - (b)  $(1 + \beta)/(4 - \theta^2) < \gamma < \gamma^*(\theta) \exists \beta^*(\gamma) \in [0, 1]$  such that:
    - $\beta < \beta^*(\gamma) : \quad \widetilde{TS}^C < \widetilde{TS}^B$

$$\begin{aligned} \beta = \beta^*(\gamma): & \quad \widetilde{TS}^C = \widetilde{TS}^B \\ \beta > \beta^*(\gamma): & \quad \widetilde{TS}^C > \widetilde{TS}^B. \end{aligned}$$

According to Proposition 4 there are two situations where Cournot competition leads to higher total surplus than Bertrand competition. First, as in part (i), where both consumers' surplus and producers' surplus are higher, and second, as in part (ii, b) with strong technological spillovers, where the lower consumers' surplus is offset by the higher producers' surplus.

## 5 Conclusions

We decompose the combined-profits externality into three components: a strategic component, a size component, and a spillover component. The latter two play a role only when technological spillovers are present. This explains why a threshold value of the technological spillover exists above which cooperative R&D exceeds noncooperative R&D. Because the spillover component bears an opposite sign under Bertrand and Cournot competition, our decomposition also explains why the threshold is larger under Bertrand competition than under Cournot competition.

The traditional difference in R&D investment incentives under the two types of product market competition is enhanced if firms cooperate in R&D. This is also due to the opposite sign of the spillover component. As a result, post-innovation cost under Cournot competition can be so much lower than under Bertrand competition that the Cournot-Nash price is below the Bertrand-Nash price. Total surplus under Cournot competition is then higher because producers' surplus with Cournot competition always dominates that under Bertrand competition. This situation is more likely to occur the stronger is the technological spillover because that increases the magnitude of the spillover component of the combined-profits externality.

An obvious policy implication is that sustaining R&D cooperatives can be particularly beneficial in markets where the intensity of competition in the product market is relatively low. Not only would this trigger higher R&D investments, it could also lead to a drop in consumer prices which are below the level that would obtain under more intense product market competition. This is all the more likely because the technological spillover is expected to increase whenever firms form an R&D cooperative.

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## 6 Appendix A Proofs

### 6.1 Proofs of lemmata

#### 6.1.1 Proof of Lemma 1

It is immediate that  $\gamma_{R4} > \gamma_{R3}$ .

*QED*

#### 6.1.2 Proof of Lemma 2

For A1 and R4 to hold jointly it must be that  $1 < a/c < (2+\theta)/(2-\theta)$ , or  $2(a-c) < \theta(a+c)$ . Indeed,  $a$  and  $c$  can always be chosen such that this inequality holds. For A1 and R1 to hold jointly it must be that  $1 > [(2-\theta^2-\theta\beta)^2 + (2\beta-\theta^2\beta-\theta)^2]/[(4-\theta^2)(1-\theta^2)(1+\beta^2)]$ , or  $\beta > \left(2 - \theta^2 - \sqrt{(4-\theta^2)(1-\theta^2)}\right) / \theta = f(\theta)$ . Note that  $f(\theta)$  is continuous and strictly decreasing in  $\theta \in (0, 1)$ , that  $\lim_{\theta \rightarrow 0} f(\theta) = 0$ , and that  $\lim_{\theta \rightarrow 1} f(\theta) = 1$ . For A1 and R2 to hold jointly it must be that  $1 > [(2-\theta\beta)^2 + (2\beta-\theta)^2]/[(4-\theta^2)(1+\beta^2)]$ , or  $\beta > \left(2 - \sqrt{(4-\theta^2)}\right) / \theta = g(\theta)$ . Note that  $g(\theta)$  is continuous and strictly increasing in  $\theta \in (0, 1)$ , that  $\lim_{\theta \rightarrow 0} g(\theta) = 0$ , and that  $\lim_{\theta \rightarrow 1} g(\theta) = 2 - \sqrt{3} > 0$ . Finally, note that  $f(\theta) - g(\theta) > 0 \forall \theta \in (0, 1)$ . That is, whenever A1 holds, the admissible parameter space is bounded by conditions R1 and R4.

*QED*

### 6.2 Proofs of propositions

#### 6.2.1 Proof of Proposition 1

$\tilde{y}^C > \tilde{y}^B \Leftrightarrow 2\gamma(a-c)(1+\beta)(4-\theta^2)^2(1-\theta)\theta^3 >$ , or  $\beta > -1$ .

*QED*

### 6.2.2 Proof of Proposition 2

Prices are lower under second-stage Cournot competition than under second-stage Bertrand competition if, and only if  $\tilde{Q}^C > \tilde{Q}^B \iff \gamma < (1 + \beta)/(4 - \theta^2)$ .

*QED*

### 6.2.3 Proof of Proposition 3

First note that

$$\tilde{\pi}^C = \frac{\gamma(a-c)^2(2-\theta)^2}{\Delta^C}$$

and

$$\tilde{\pi}^B = \frac{\gamma(a-c)^2(2+\theta)^2(1-\theta)^2}{\Delta^B},$$

where  $\Delta^C = \gamma(4 - \theta^2)^2 - (1 + \beta)(2 - \theta)^2$  and  $\Delta^B = \gamma(4 - \theta^2)^2(1 - \theta^2) - (1 + \beta)(1 - \theta)^2(2 + \theta)^2$ . The result then follows as:

$$\tilde{\pi}^C - \tilde{\pi}^B = \frac{2\gamma^2(a-c)^2(4-\theta^2)^2(1-\theta)\theta^3}{\Delta^C\Delta^B} > 0.$$

*QED*

### 6.2.4 Proof of Proposition 4

First note that  $\tilde{T}S^B - \tilde{T}S^C = \gamma(a-c)^2 F(\gamma, \theta, \beta) / [(\Delta^B)^2(\Delta^C)^2]$ , where  $\Delta^B = \gamma(1 - \theta^2)(4 - \theta^2)^2 - (2 + \theta)^2(1 - \theta)^2(1 + \beta)$ ,  $\Delta^C = \gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)$ , and  $F(\gamma, \theta, \beta) = [\gamma(2 + \theta)^2(1 - \theta)^2(4 - \theta^2)^2(3 - 2\theta)(1 + \theta) - 2(2 + \theta)^4(1 - \theta)^4(1 + \beta)] [\Delta^C]^2 - [\gamma(2 - \theta)^2(4 - \theta^2)^2(3 + \theta) - 2(2 - \theta)^4(1 + \beta)] [\Delta^B]^2$ .

Define  $G(\gamma, \theta, \beta) = F(\gamma, \theta, \beta) / (\gamma\theta^2(4 - \theta^2)^2)$ . Obviously,  $\text{sign}(\tilde{T}S^B - \tilde{T}S^C) = \text{sign}(G(\gamma, \theta, \beta))$ . Note that  $G(\gamma, \theta, \beta) = \gamma^2 g_1 + \gamma g_2 + g_3$ , where  $g_1 = (4 - \theta^2)^4(1 - \theta)^2(1 + \theta)(4 - 2\theta - \theta^2)$ ,  $g_2 = -2(4 - \theta^2)^2(1 + \beta)(16 - 32\theta + 8\theta^2 + 20\theta^3 - 15\theta^4 + 3\theta^5 - \theta^6 + \theta^7)$ , and  $g_3 = (1 - \theta)^2(2 + \theta)^2(2 - \theta)^2(1 + \beta)^2(4 - 2\theta + \theta^2 - \theta^3)$ . It follows that  $G(\gamma, \theta, \beta)$  is strictly convex in  $\gamma$  as  $\partial^2 G(\gamma, \theta, \beta) / \partial \gamma^2 = 2g_1 > 0$  (note:

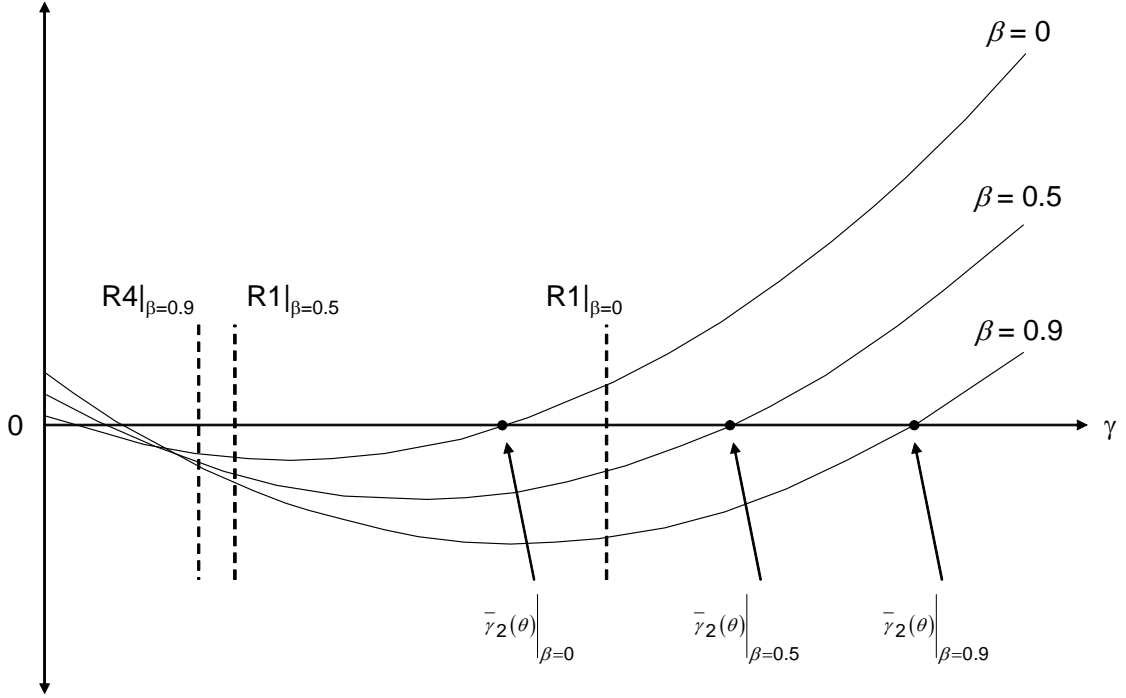


Figure 2:  $G(\gamma; \theta, \beta)$  for different sizes of the technological input spillover, whereby  $a = 100$ ,  $c = 70$ , and  $\theta = 0.9$ .

$\min_{\{\theta\}} g_1 = \lim_{\theta \rightarrow 1} g_1 = 0$ ). Moreover,  $g_2^2 - 4g_1g_3 > 0 \forall \theta \in (0, 1)$ . Hence, given any  $\theta \in (0, 1)$ , there are two real solutions to  $G(\gamma, \theta, \beta) = 0$ , in particular:

$$\bar{\gamma}_1(\theta) = \frac{-g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \text{ and } \bar{\gamma}_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$

The larger root is to be considered as  $\min_{\theta} \{\gamma^* - \bar{\gamma}_1(\theta)\} = \lim_{\theta \rightarrow 0} \{\gamma^* - \bar{\gamma}_1(\theta)\}|_{\beta=1} = 0$ , where  $\gamma^*$  is the threshold value induced by R2. Label the larger root  $\bar{\gamma}(\theta)$ . Then observe that  $\min_{\theta, \beta} \{\partial \bar{\gamma}(\theta) / \partial \beta\} = \lim_{\theta \rightarrow 0} \partial \bar{\gamma}(\theta) / \partial \beta|_{\beta=0.5} = 0.25$ . This gives rise to the different lines in Figure 2 for different values of  $\beta$ . Obviously, for any  $\gamma > \bar{\gamma}(\theta)$  we are in situation (i) while situation (ii) emerges for any  $\gamma > \bar{\gamma}(\theta)$ . The rest of the proof then follows.

*QED*



## 7 Appendix B Decomposition of R&D investment incentives

Second-stage profits of firm  $i$  equal  $\pi_i = p_i q_i - (c - y_i) q_i - x_i$ , with  $y_i = [(x_i + \beta x_j) / \gamma]^{\frac{1}{2}}$ .

The first-order condition for optimal outputs is:

$$\frac{\partial \pi_i}{\partial q_i} = p_i + q_i \frac{\partial p_i}{\partial q_i} - (c - y_i) \equiv 0. \quad (18)$$

Assume an interior solution to exist and that it is stable, i.e. the second-order condition and the stability condition hold:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = 2 \frac{\partial p_i}{\partial q_i} + q_i \frac{\partial^2 p_i}{\partial q_i^2} \leq 0, \quad \Omega^C = \frac{\partial^2 \pi_i}{\partial q_i^2} \frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_i}{\partial q_j \partial q_i} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} > 0.$$

Differentiating first-order conditions (18) with respect to  $x_i$  gives:

$$\begin{pmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} \\ \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} & \frac{\partial^2 \pi_j}{\partial q_j^2} \end{pmatrix} \begin{pmatrix} \frac{\partial q_i}{\partial x_i} \\ \frac{\partial q_j}{\partial x_i} \end{pmatrix} = -\frac{1}{2\gamma} \begin{pmatrix} \frac{1}{y_i} \\ \frac{\beta}{y_j} \end{pmatrix}.$$

From this we obtain:

$$\frac{\partial q_i}{\partial x_i} = \frac{1}{2\gamma y_i y_j \Omega^C} \left( \beta y_i \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} - y_j \frac{\partial^2 \pi_j}{\partial q_j^2} \right)$$

and

$$\frac{\partial q_j}{\partial x_i} = \frac{1}{2\gamma y_i y_j \Omega^C} \left( y_j \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \beta y_i \frac{\partial^2 \pi_i}{\partial q_i^2} \right).$$

Note further that  $\partial \pi_i / \partial q_j = q_i (\partial p_i / \partial q_j)$ , which is negative if products are demand substitutes, that  $\partial \pi_i / \partial x_i = q_i / (2\gamma y_i)$ , and that  $\partial \pi_j / \partial x_i = \beta q_j / (2\gamma y_j)$ . It then follows that

$$\begin{aligned} \frac{\partial \Pi^C}{\partial x_i} &= \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j}{\partial x_i} + \frac{\partial \pi_j}{\partial q_i} \frac{\partial q_i}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_i} - 1 \\ &= \frac{y_j}{2\gamma y_i y_j \Omega^C} \left( \frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_j^2} \right) \\ &\quad + \frac{\beta y_i}{2\gamma y_i y_j \Omega^C} \left( \frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial q_i^2} - \frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} \right) + \frac{q_i}{2\gamma y_i} + \frac{\beta q_j}{2\gamma y_j} - 1 \\ &= 0, \end{aligned}$$

from which the decomposition follows. Under second-stage Bertrand competition a similar reasoning applies which is omitted here but available upon request.

With Bertrand competition, the signs of the strategic and spillover effect are unambiguous. Under Cournot competition, the signs of the strategic and spillover effect respectively follow from the stability condition:

$$\begin{aligned} \overbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}^+ + \overbrace{-\frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_j^2}}^- &= \overbrace{\frac{\partial \pi_i}{\partial q_j}}^- \overbrace{\left( \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \frac{\partial^2 \pi_j}{\partial q_j^2} \right)}^+, \\ \overbrace{\frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_i}{\partial q_i^2}}^- + \overbrace{-\frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial q_j \partial q_i}}^+ &= \overbrace{\frac{\partial \pi_i}{\partial q_j}}^- \overbrace{\left( \frac{\partial^2 \pi_i}{\partial q_i^2} - \frac{\partial^2 \pi_i}{\partial q_j \partial q_i} \right)}^-. \end{aligned}$$