



## UvA-DARE (Digital Academic Repository)

### Dynamic efficiency of Cournot and Bertrand competition: Input versus output spillovers

Hinloopen, J.; Vandekerckhove, J.

**Publication date**  
2009

[Link to publication](#)

#### **Citation for published version (APA):**

Hinloopen, J., & Vandekerckhove, J. (2009). *Dynamic efficiency of Cournot and Bertrand competition: Input versus output spillovers*. Afdeling Algemene Economie.

#### **General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

#### **Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

# Dynamic efficiency of Cournot and Bertrand competition: input versus output spillovers\*

Jeroen Hinloopen,<sup>†</sup> Jan Vandekerckhove<sup>‡</sup>

March 18, 2009

## Abstract

We consider the efficiency of Cournot and Bertrand equilibria in a duopoly with substitutable goods where firms invest in process R&D that generates input spillovers. Under Cournot competition firms always invest more in R&D than under Bertrand competition. More importantly, Cournot competition yields *lower* prices than Bertrand competition when the R&D production process is efficient, when spillovers are substantial, and when goods are not too differentiated. The range of cases for which total surplus under Cournot competition exceeds that under Bertrand competition is even larger as competition over quantities always yields the largest producers' surplus.

**Key words:** Bertrand competition; Cournot competition; process R&D; efficiency.

**JEL Classification:** L13.

---

\*Thanks are due to two anonymous referees, to Thomas Rønne and to seminar participants at the Katholieke Universiteit Leuven, at the University of Amsterdam, at the joint KULeuven/ZEW Workshop on the Economics of Innovation, December 2007, and at the annual conference of the European Association for Research in Industrial Economics (EARIE) in Toulouse, September 2008. The usual disclaimer applies.

<sup>†</sup>Corresponding author. Universiteit van Amsterdam and Katholieke Universiteit Leuven. Correspondence: Universiteit van Amsterdam, FEB/ASE, Roetersstraat 11, 1018 WB, The Netherlands; J.Hinloopen@uva.nl; <http://www.fee.uva.nl/io/jhinloopen>.

<sup>‡</sup>Katholieke Universiteit Leuven, FEB/MSI, Naamsestraat 69, 3000 Leuven, Belgium; Jan.Vandekerckhove@econ.kuleuven.be.

# 1 Introduction

Competition over price (Bertrand competition) is known to yield lower prices than competition over quantities (Cournot competition). This result was first established by Singh and Vives (1984) for a symmetric duopoly supplying demand substitutes (see also Cheng, 1985). It is robust to various generalizations, including the extension to an oligopoly (Vives, 1985), to differences in costs under Cournot and Bertrand competition (Qiu, 1997; Lopez and Naylor, 2004; Zanchettin, 2006), to differences in product quality under the two types of competition (Linn and Saggi, 2002; Symeonidis, 2003), and to differences in market structure associated with price and quantity competition (Cellini *et al.*, 2004; Mukherjee, 2005).<sup>1</sup>

In this paper we qualify the celebrated result of Singh and Vives (1984) by showing that Cournot competition can yield *lower* prices than Bertrand competition in a duopoly with endogenous production costs that supplies demand substitutes. To the best of our knowledge we are the first to establish that lower prices can obtain under Cournot competition with an exogenous market structure. It occurs when products are relatively homogenous, when technological spillovers are strong, and when the R&D production process is sufficiently efficient. Indeed, under these circumstances the incentives to conduct R&D are much larger under Cournot competition than under Bertrand competition as in this case much more of the benefits of any cost reduction are given to consumers when competition is over price. As a result, post innovation costs are much lower under Cournot competition which translates into a lower equilibrium price. The range of cases for which total surplus under Cournot competition exceeds that under Bertrand competition is even larger because profits under Bertrand competition are always

---

<sup>1</sup>For an oligopoly supplying demand complements with quality differences an exception exists. Häckner (2000) shows that in this case the switch from Cournot competition to Bertrand competition induces the high-quality firm to charge a lower price. The resulting upward pressure on the demand for the low-quality complement then allows for a price increase of this complement.

below those under Cournot competition.

A motivating example for our analysis is the semiconductor industry. In this industry firms compete à la Cournot and technological spillovers are strong (de Bondt and Veugelers, 1989). It is precisely in this industry that prices have fallen at an astonishing rate of 36% per year in the 1990s. This pricing pattern can be attributed almost exclusively to quality increases that are associated with product innovations (Aizcorbe, 2006). Although the alternative pricing pattern under Bertrand competition is by definition not available, this example does suggest that low prices can also emerge under Cournot competition.

Our analysis is related to that of Qiu (1997). The main difference is that we consider technological spillovers to occur during the R&D process while Qiu (1997), following d'Aspremont and Jacquemin (1988), assumes that final R&D results spill over. That is, we consider input spillovers rather than output spillovers. There are three important reasons for doing so (see also Hinlopen, 2003). First, empirical studies indicate that spillovers indeed occur during the R&D process (Kaiser, 2002). This finding corresponds to the three channels that Geroski (1995) identifies through which a technological spillover can occur: (i) the exchange of ideas through publications, casual encounters and at seminars, (ii) the flow of knowledge when a knowledge worker changes employer, and (iii) the deduction of the line of reasoning of rivals by observing their behavior.

Second, Qiu (1997) assumes the R&D results of one firm to be *additive* (and possibly perfectly additive), to its rival's R&D results. There are at least three reasons to question this assumption. Note that the two firms operate in the same product market while initially using the same production technology. It is then most likely that there will be some overlap in their independently obtained research results that are aimed at reducing the costs of production. Also, the parts that do not overlap are expected not to be a perfect match to rivals' research

results. Finally, differences in corporate culture, research strategies, and internal organization hamper any firm's ability to appropriate fully rival's research results. In sum, high levels of technological output spillovers are not likely to be observed (Gerschbach and Schmutzler (2003) take an extreme position here by assuming that *all* of any firm's R&D results are perfectly additive to any of its rivals' R&D results).

Third, Qiu (1997) assumes diminishing returns to scale in R&D. In combination with additive output spillovers this has a counter-intuitive implication. If one firm has spent more on R&D than its rival, it could be in the interest of the former to donate its next R&D investment dollar to its rival and to appropriate the R&D results through the technological spillover. If these spillovers are substantial this could be a more effective additional cost reduction than spending this last R&D dollar on own R&D (Amir, 2000).

Recently, Amir *et al.* (2008) have formalized the latter flaw. They introduce the intuitive criterion that investing in one research laboratory for a firm should be at least as efficient as investing in several independent research laboratories that mutually benefit from spillovers. In particular (Amir *et al.*, 2008): "The R&D technology and the spillover process should be such that any total R&D investment level cannot generate more cost reduction if allocated to  $n$  labs, run independently but with spillovers at their natural rate, than if spent all in one lab." This means that any research entity absorbs better its own findings than those acquired through spillovers from other research entities. Indeed, the analysis of Qiu (1997) is at odds with this criterion.

Output spillovers are more beneficial to a firm than input spillovers if there are diminishing returns to scale in R&D. At the same time they reduce rivals' production cost more than what input spillovers would do. At forehand, the net effect of switching from output spillovers to input spillovers on the incentive

to invest in R&D is unclear. And because R&D investments rule the efficiency comparison between Cournot and Bertrand competition in Qiu (1997), it remains to be examined to what extent his analysis hinges on the assumption of output spillovers. For this reason we re-examine the dynamic efficiency of Cournot and Bertrand competition assuming input spillovers. In passing we reveal a technical error in Qiu (1997) related to the stability of equilibria when R&D is a strategic substitute.

The paper proceeds as follows. In the next section the model is introduced and the technical difference between input and output spillovers is discussed in detail. In Section 3 the equilibria are characterized under second-stage Cournot and Bertrand competition. The two competition types are compared next and Section 5 concludes.

## 2 The model

We consider a two-stage game. In the first stage firms invest in cost-reducing R&D. In the second stage they compete either over price or quantity. Market demand in indirect form is given by:<sup>2</sup>

$$p_i = a - (q_i + \theta q_j), \quad (1)$$

$i, j = 1, 2, i \neq j$ , where  $p_i$  and  $q_i$  are the respective price and quantity of product  $i$ , and where  $\theta$  captures the extent to which products are differentiated; in case  $\theta = 1$  products are homogeneous while  $\theta = 0$  corresponds to completely differentiated products (i.e. both firms have a local monopoly). These polar cases are further ignored, that is,  $\theta \in (0, 1)$ . Unless stated otherwise,  $i, j = 1, 2, i \neq j$  holds throughout the rest of the paper. Market demand in direct form is then given by:

---

<sup>2</sup>This follows from a standard quadratic utility function, see Singh and Vives (1984).

$$q_i = \frac{1}{1 - \theta^2} [(1 - \theta)a - (p_i - \theta p_j)]. \quad (2)$$

The industry consists of two firms each producing one version of the differentiated product. *Ex ante* marginal costs of production,  $c$ , are fixed. We assume that both firms are active, that is,  $c < a$ .

## 2.1 Input spillovers versus output spillovers

The fixed production costs can be reduced by investing in process-innovating R&D. Note that if one firm conducts R&D, the rival firm can absorb part of this effort without having to pay for it.<sup>3</sup> This process runs through the technological spillover. In modelling this externality we adhere to the criterion identified by Amir *et al.* (2008), which states that it should be more beneficial to a firm to invest in one research laboratory than to invest in several independent research laboratories that possibly benefit from mutual spillovers.

Qiu (1997) considers the final results of any firm's R&D efforts to spill over to its rival. There are diminishing returns to R&D in that any reduction in production costs  $x$  comes at a cost  $f(x)$ , with  $f' > 0$ ,  $f'' \geq 0$  and  $f(0) = 0$ . If  $\beta \in [0, 1]$  represents the technological spillover, firm  $i$  realizes a cost reduction of  $x_i + \beta x_j$  if it invests  $f(x_i)$  in R&D. The criterion of Amir *et al.* (2008) then requires:

$$f(x_i + \beta x_j) \leq f(x_i) + f(x_j). \quad (3)$$

For any  $f$  with  $f(0) = 0$  Amir *et al.* (2008, Proposition 2) show that condition (3) translates into  $\beta f'(x) \leq f'(0)$ . Note that this condition is violated for the formulation used by Qiu (1997), being  $f(x_i + \beta x_j) = \gamma x_i^2/2$ . Put differently, in Qiu (1997) it is beneficial to spread any R&D investment over a number of different

---

<sup>3</sup>It is understood that firms have to conduct at least some R&D themselves to share in rival's R&D activities (for an early recognition of this point see Cohen and Levinthal, 1989). We abstain from modelling this absorptive capacity as it would make the analysis intractable (cfr. Kamien and Zang, 2000).

independent research labs rather than investing it in one laboratory. In case of two firms a counter-intuitive result then obtains. Rather than investing its next R&D euro itself, it could be in the interest of a firm to give this euro to its rival for him to invest in R&D, and to appropriate the result through the output spillover.

In case of input spillovers the reduction in marginal cost brought about by an R&D investment is determined by an R&D production function  $g$ . Diminishing returns to R&D are present if  $g' > 0$ ,  $g'' < 0$ , and  $g(0) = 0$ . If firm  $i$  invests in R&D, its *effective* R&D investments  $X_i$  due to the input spillover are given by:

$$X_i = x_i + \beta x_j. \quad (4)$$

Investing  $x_i$  thus yields a reduction in cost of  $g(X_i)$ . The criterion of Amir *et al.* (2008) then requires:

$$g(X_i) \leq g(x_i) + g(x_j), \quad (5)$$

which holds for any  $g$  with diminishing returns. Following Amir (2000) we set:

$$g(X_i) = \sqrt{\frac{X_i}{\gamma}}, \quad (6)$$

whereby  $\gamma > 0$  determines the efficiency of the R&D phase. A higher value of  $\gamma$  corresponds to a less efficient production of R&D results. Note that R&D production function (6) fullfills criterion (5).

Firm  $i$ 's profits then equal

$$\pi_i = p_i q_i - (c - y_i) q_i - x_i, \quad (7)$$

with  $y_i = \sqrt{(x_i + \beta x_j)/\gamma}$ .



### 3 Market equilibria

#### 3.1 Second-stage Bertrand competition

Maximizing (7) over price yields equilibrium prices conditional on effective R&D efforts:<sup>4</sup>

$$\hat{p}_i(X_i, X_j) - c = \frac{(a - c)(2 + \theta)(1 - \theta) - 2y_i - \theta y_j}{4 - \theta^2}. \quad (8)$$

Inserting (8) into (7) and maximizing the resulting profits over R&D investments result in the following cost reduction:<sup>5,6</sup>

$$\tilde{y}^B = \frac{(a - c)(2 - \theta^2 - \theta\beta)}{\gamma(1 + \theta)(2 - \theta)(4 - \theta^2) - (2 - \theta^2 - \theta\beta)}, \quad (9)$$

and concomitant total output:

$$\tilde{Q}^B = \frac{2\gamma(a - c)(4 - \theta^2)}{\gamma(1 + \theta)(2 - \theta)(4 - \theta^2) - (2 - \theta^2 - \theta\beta)}. \quad (10)$$

Singel-firm equilibrium profits then equal:

$$\tilde{\pi}^B = \frac{\gamma(1 + \beta)(1 - \theta^2)(4 - \theta^2)^2 - (2 - \theta^2 - \theta\beta)^2}{\gamma(1 + \beta)(4 - \theta^2)^2} (\tilde{q}^B)^2, \quad (11)$$

where  $\tilde{Q}^B = 2\tilde{q}^B$ . Consumers' surplus and total surplus are then respectively given by:

$$\tilde{CS}^B = (1 + \theta) (\tilde{q}^B)^2, \quad (12)$$

and

$$\tilde{TS}^B = \frac{\gamma(1 + \beta)(1 + \theta)(4 - \theta^2)^2(3 - 2\theta) - 2(2 - \theta^2 - \theta\beta)^2}{\gamma(1 + \beta)(4 - \theta^2)^2} (\tilde{q}^B)^2. \quad (13)$$

<sup>4</sup>A hat refers to a conditional equilibrium outcome.

<sup>5</sup>A tilde refers to an unconditional equilibrium expression; superscript  $B$  stands for second-stage Bertrand competition.

<sup>6</sup>The concomitant second-order and stability conditions are dealt with below.

### 3.2 Second-stage Cournot competition

Maximizing (7) over quantities gives us:

$$\widehat{q}_i(X_i, X_j) = \frac{(a-c)(2-\theta) + 2y_i - \theta y_j}{4-\theta^2}. \quad (14)$$

Maximizing firm profits over R&D investments after inserting (14) into (7) yields as cost reduction and concomitant output level:<sup>7</sup>

$$\widehat{y}^C = \frac{(a-c)(2-\theta\beta)}{\gamma(2+\theta)(4-\theta^2) - (2-\theta\beta)}, \quad (15)$$

and:

$$\widetilde{Q}^C = \frac{2\gamma(a-c)(4-\theta^2)}{\gamma(2+\theta)(4-\theta^2) - (2-\theta\beta)}. \quad (16)$$

Singel-firm profits are given by:

$$\widetilde{\pi}^C = \frac{\gamma(1+\beta)(4-\theta^2)^2 - (2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2} (\widetilde{q}^C)^2, \quad (17)$$

with  $\widetilde{Q}^C = 2\widetilde{q}^C$ . Consumers' surplus and total welfare under second-stage Cournot competition then equal:

$$\widetilde{CS}^C = (1+\theta) (\widetilde{q}^C)^2, \quad (18)$$

and

$$\widetilde{TS}^C = \frac{\gamma(1+\beta)(3+\theta)(4-\theta^2)^2 - 2(2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2} (\widetilde{q}^C)^2. \quad (19)$$

### 3.3 Regularity conditions

The R&D stage gives rise to eight regularity conditions. In addition to the two second-order conditions, post-innovation costs have to be positive and the equilibrium has to be stable. The second-order conditions under Bertrand and Cournot competition require, respectively:

---

<sup>7</sup>Superscript  $C$  stands for second-stage Cournot competition.

$$\gamma \geq \frac{(2 - \theta^2 - \theta\beta)^3}{(1 - \theta^2)(4 - \theta^2)^2(2 - \theta^2 - \theta\beta^2)}, \quad (\text{R1})$$

and

$$\gamma \geq \frac{(2 - \theta\beta)^3}{(2 - \theta\beta^2)(4 - \theta^2)^2}. \quad (\text{R2})$$

Under Bertrand and Cournot competition positive post-innovation costs respectively imply:

$$\gamma > \frac{a(2 - \theta^2 - \theta\beta)}{c(2 - \theta)(1 + \theta)(4 - \theta^2)}, \quad (\text{R3})$$

and

$$\gamma > \frac{a(2 - \theta\beta)}{c(2 + \theta)(4 - \theta^2)}. \quad (\text{R4})$$

Finally, the Routh-Hurwitz stability condition is that:

$$\frac{\partial^2 \hat{\pi}_i(x_i, x_j)}{\partial x_i^2} \frac{\partial^2 \hat{\pi}_j(x_i, x_j)}{\partial x_j^2} - \frac{\partial^2 \hat{\pi}_i(x_i, x_j)}{\partial x_j \partial x_i} \frac{\partial^2 \hat{\pi}_j(x_i, x_j)}{\partial x_i \partial x_j} > 0. \quad (20)$$

This condition depends on the strategic nature of the R&D process. Following Bulow *et al.* (1985), label decision variable  $x$  a strategic substitute in case  $\partial^2 \hat{\pi}_i(x_i, x_j)/\partial x_i \partial x_j < 0$ , and a strategic complement if  $\partial^2 \hat{\pi}_i(x_i, x_j)/\partial x_i \partial x_j > 0$ . Accordingly, in a symmetric equilibrium condition (20) boils down to:

$$\frac{\partial^2 \pi_i(x_i, x_j)}{\partial x_i^2} < \frac{\partial^2 \pi_i(x_i, x_j)}{\partial x_i \partial x_j}, \quad (21)$$

for strategic substitutes. For strategic complements it reads as:

$$\frac{\partial^2 \pi_i(x_i, x_j)}{\partial x_i^2} < -\frac{\partial^2 \pi_i(x_i, x_j)}{\partial x_i \partial x_j}. \quad (22)$$

Under Bertrand competition these two stability conditions respectively translate into:

$$\gamma > \frac{(2 - \theta^2 - \theta\beta)^2}{(4 - \theta^2)(2 + \theta)(1 - \theta)(2 - \theta^2 + \theta\beta)}, \quad (\text{R5})$$

and

$$\gamma > \frac{(2 - \theta^2 - \theta\beta)}{(4 - \theta^2)(2 - \theta)(1 + \theta)}. \quad (\text{R6})$$

In case of Cournot competition the two stability conditions are:

$$\gamma > \frac{(2 - \theta\beta)^2}{(4 - \theta^2)(2 - \theta)(2 + \theta\beta)}, \quad (\text{R7})$$

and

$$\gamma > \frac{(2 - \theta\beta)}{(4 - \theta^2)(2 + \theta)}. \quad (\text{R8})$$

Five of these regularity conditions are redundant as the following lemma shows.

**Lemma 1** *The parameter space is bounded by regularity conditions R4, R5 and R7.*

**Proof.** It is immediate that R4 dominates R3, that R5 dominates R6, and that R7 dominates R8. Also, R5 dominates R1 and R7 dominates R2. ■

Note that Qiu (1997) considers stability conditions only in case where R&D is a strategic complement. In his model the stability conditions for R&D as a strategic substitute under Cournot and Bertrand competition are respectively given by (using the notation in Qiu, 1997):

$$v > \frac{2(2 - \theta\gamma)(1 - \theta)}{(2 - \gamma)(4 - \gamma^2)}, \quad (23)$$

and

$$v > \frac{2(1 - \theta)(2 - \theta\gamma - \gamma^2)}{(1 - \gamma)(2 + \gamma)(4 - \gamma^2)}, \quad (24)$$

where  $\theta \in [0, 1]$  is the output spillover, where  $v$  is the measure of the efficiency of the R&D process, and where  $\gamma \in [0, 1]$  indicates the extent of product differentiation. The analysis of Qiu (1997) applies only to R&D that is a strategic complement as it is straightforward to show that conditions (23) and (24) are more restrictive than the stability conditions when R&D is a strategic complement.

## 4 Cournot versus Bertrand

### 4.1 R&D investments

Comparing the effective R&D efforts of the different competition modes leads to the following proposition:

**Proposition 1** *For any given  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$ ,  $\tilde{y}^C > \tilde{y}^B$  under R4, R5 and R7.*

**Proof.**  $\tilde{y}^C > \tilde{y}^B \Leftrightarrow (1 + \theta)(2 - \theta)(2 - \theta\beta) > (2 + \theta)(2 - \theta^2 - \theta\beta)$ , or  $\beta > -1$ .

■

According to Proposition 1, R&D activity is higher under Cournot competition than under Bertrand competition. This result replicates Qiu (1997) who points out that there is a strategic effect at work when firms decide upon their R&D investments. In Cournot markets this strategic effect is positive. The firm with the lower production costs is the tougher competitor that has the largest market share. In Bertrand markets this strategic effect is negative. Any reduction in production costs induces rivals to cut price which is not in the interest of either firm. The switch from output spillovers to input spillovers does not affect this reasoning. The ranking in Proposition 1 is also found by Breton *et al.* (2004) who replicate the analysis of Qiu (1997) within an infinite horizon setting.

The actual difference in R&D activity that leads to the ranking in Proposition 1 is closely related to the efficiency of the R&D process. That is:

**Lemma 2** *Under R4, R5 and R7, the difference in R&D activity under Cournot and Bertrand competition is larger the more efficient is the R&D process.*

**Proof.** Note that

$$\tilde{y}^C - \tilde{y}^B = \frac{\gamma\theta^3(4 - \theta^2)(1 + \beta)(a - c)}{[\gamma(2 + \theta)(4 - \theta^2) - (2 - \theta\beta)][\gamma(1 + \theta)(2 - \theta)(4 - \theta^2) - (2 - \theta^2 - \theta\beta)]}.$$

Then observe that:

$$\frac{\partial (\tilde{y}^C - \tilde{y}^B)}{\partial \gamma} < 0 \Leftrightarrow \gamma^2 > \frac{(2 - \theta\beta)(2 - \theta^2 - \theta\beta)}{(1 + \theta)(4 - \theta^2)^3}.$$

This last conditions is less restrictive than condition R7 if, and only if  $(2 - \theta\beta)^3(1 + \theta)(4 - \theta^2) - (2 - \theta^2 - \theta\beta)(2 - \theta)^2(2 + \theta\beta)^2 > 0$ . Considering the left-hand side (LHS) of this last inequality, the result then follows as  $\min_{\{\theta, \beta\}} LHS = \lim_{\theta \rightarrow 0} LHS|_{\beta=1} = 0$ .

■

The larger is the reduction in production costs for any level of R&D investment, the more prominent is the strategic effect that affects any firms' incentive to conduct R&D. Hence, the more efficient is the R&D process, the larger is the difference in R&D investments under Cournot competition vis-à-vis Bertrand competition.

## 4.2 Profits

Under Cournot competition firms invest more in R&D than under Bertrand competition (Proposition 1). And larger R&D investments reduce profits, all else equal. The following proposition shows however that these higher R&D costs under Cournot competition are more than offset by the concomitant reduction in production cost:

**Proposition 2** *For any given  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$ ,  $\tilde{\pi}^C > \tilde{\pi}^B$  under R4, R5 and R7.*

**Proof.** First note that  $\tilde{\pi}^C - \tilde{\pi}^B = \gamma(a - c)^2(A - B)/(1 + \beta)$ , where

$$A = \frac{\gamma(1 + \beta)(4 - \theta^2)^2 - (2 - \theta\beta)^2}{[\gamma(2 + \theta)(4 - \theta^2) - (2 - \theta\beta)]^2},$$

and

$$B = \frac{\gamma(1 + \beta)(1 - \theta^2)(4 - \theta^2)^2 - (2 - \theta^2 - \theta\beta)^2}{[\gamma(1 + \theta)(2 - \theta)(4 - \theta^2) - (2 - \theta^2 - \theta\beta)]^2}.$$

Then observe that:

$$\tilde{\pi}^C - \tilde{\pi}^B > 0 \Leftrightarrow \gamma > \frac{2(4 - 3\theta^2) - \theta(1 - \beta)(\theta^2 - 2\theta - 4)}{2(1 + \theta)(4 - \theta^2)^2}.$$

This last condition is less restrictive than condition R7 if, and only if,  $(1 - \beta) [32 + 16\theta - 12\theta^2 - 16\theta\beta - 2\theta^3(1 + \beta) + 8\theta^3\beta] + \theta^2\beta [8\beta - \theta^2(1 + \beta)] > 0$ . Considering the LHS of this last inequality the result then follows as  $\min_{\{\theta, \beta\}} LHS = \lim_{\theta \rightarrow 0} LHS|_{\beta=1} = 0$ . ■

Proposition 2 states that producers' surplus under Cournot competition is always larger than under Bertrand competition. Because post-innovation production costs are lower under Cournot competition, this larger producers' surplus can exceed the lower consumers' surplus in Cournot markets compared to Bertrand markets. But before we analyze total surplus we first consider consumers' surplus.

### 4.3 Price

For comparing prices under Cournot and Bertrand competition we introduce the following assumption:

$$\gamma < \frac{1}{4 - \theta^2} \tag{A1}$$

If assumption A1 holds the R&D process is labelled 'efficient'. According to Lemma 2 this corresponds to situations where post-innovation cost under Cournot competition are particularly low compared to post-innovation costs under Bertrand competition. As will be shown below, this allows the equilibrium price under Cournot competition to be lower than under Bertrand competition. First note that assumption A1 does not rule out the existence of equilibria:

**Lemma 3** *The set where regularity conditions R4, R5, R7 and assumption A1 hold is not empty.*

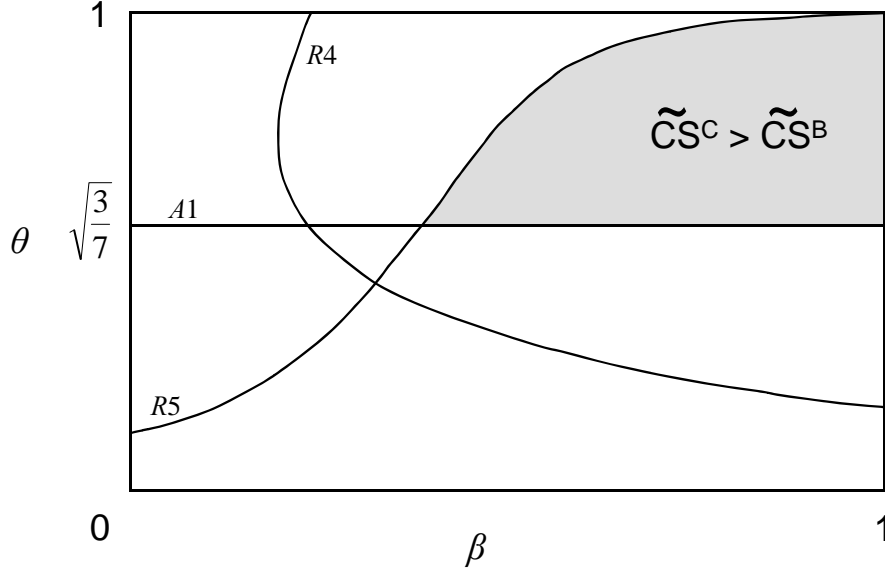


Figure 1: Comparing consumers' surplus with Cournot and Bertrand competition under assumption A1 and regularity conditions R4 and R5 ( $a = 100$ ,  $c = 70$ ,  $\gamma = \frac{7}{25}$ ).

**Proof.** For A1 and R4 to hold jointly it must be that  $1 < a/c < (2+\theta)/(2-\theta\beta)$ , or  $2(a-c) < \theta(a\beta+c)$ . Indeed,  $a$  and  $c$  can always be chosen such that this inequality holds. For A1 and R5 to hold jointly it must be that  $1 > (2-\theta^2-\theta\beta)^2/(2+\theta)(1-\theta)(2-\theta^2+\theta\beta)$ , or  $\beta > \left(6 - 3\theta^2 - \theta - \sqrt{(1-\theta)(36 + 16\theta - 19\theta^2 - 9\theta^3)}\right) / 2\theta = f(\theta)$ . Note that  $f(\theta)$  is continuous and strictly increasing in  $\theta \in (0, 1)$ , that  $\lim_{\theta \rightarrow 0} f(\theta) = \frac{1}{3}$ , and that  $\lim_{\theta \rightarrow 1} f(\theta) = 1$ . For A1 and R7 to hold jointly it must be that  $1 > (2-\theta\beta)^2/(2-\theta)(2+\theta\beta)$ , or  $\beta > \left(6 - \theta - \sqrt{(18-\theta)(2-\theta)}\right) / 2\theta = g(\theta)$ . Note that  $g(\theta)$  is continuous and strictly increasing in  $\theta \in (0, 1)$ , that  $\lim_{\theta \rightarrow 0} g(\theta) = \frac{1}{3}$ , and that  $\lim_{\theta \rightarrow 1} g(\theta) = (5 - \sqrt{17})/2 \approx 0.438$ . ■

Figure 1 displays the admissible parameter space and assumption A1 for particular values of  $a$ ,  $c$ , and  $\gamma$ . It follows that Note that from the proof of Lemma 3 follows that  $f(\theta) - g(\theta) > 0 \forall \theta \in (0, 1)$ . Hence, under assumption A1 the admissible parameter space is confined by conditions R4 and R5.



We can now state the main result of our analysis:

**Proposition 3** *For any given  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$ ,  $\tilde{p}^C < \tilde{p}^B$  under R4, R5, R7, and A1.*

**Proof.** Lower prices obtain under Cournot competition than under Bertrand competition if, and only if,  $\tilde{Q}^C > \tilde{Q}^B$ , or  $\gamma < 1/(4 - \theta^2)$ . ■

Proposition 3 conveys our new message. In a duopoly with substitutable products, prices can be *lower* under Cournot competition than under Bertrand competition. This happens when post-innovation costs under Cournot competition are sufficiently below post-innovation costs under Bertrand competition. Considering the admissible parameter space in Lemma 3, this occurs when the R&D process is efficient, when spillovers are substantial, and when products are not too differentiated. It is precisely under these circumstances that the benefits of any cost reduction are transferred much more to consumers under Bertrand competition than under Cournot competition. Hence, production costs under Cournot competition are much lower than under Bertrand competition which allows the equilibrium price to be lower as well.

#### 4.4 Welfare

As producers' surplus is always higher under Cournot competition than under Bertrand competition (Proposition 2), the result in Proposition 3 carries over to total surplus:

**Proposition 4** *For any given  $\theta \in (0, 1)$  and  $\beta \in [0, 1]$ ,  $\widetilde{TS}^C > \widetilde{TS}^B$  under R4, R5, R7, and A1.*

For a less efficient R&D production process it is still possible that total surplus under Cournot competition exceeds total surplus under Bertrand competition. In

that case consumers' surplus is lower when firms compete over quantities (Proposition 3). But this lower consumers' surplus is then more than compensated for by the higher producers' surplus under Cournot competition. To establish this result it is convenient to distinguish two cases: (i) no input spillovers, and (ii) positive input spillovers.

**Proposition 5** *For any given  $\theta \in (0, 1)$  and  $\beta = 0$ ,  $\widetilde{TS}^C < \widetilde{TS}^B$  under  $R4$ ,  $R5$ ,  $R7$ , and  $\neg A1$ .*

**Proof.** See Appendix. ■

Absent input spillovers the traditional welfare comparison emerges provided that the R&D production process is not too efficient. For positive input spillovers the difference in R&D investment incentives under Cournot and Bertrand competition becomes more pronounced. Indeed, a threshold value of the input spillover exists beyond which total surplus is larger if firms compete over quantity rather than over price:

**Proposition 6** *Suppose that  $\beta \in (0, 1]$ , and that  $R4$ ,  $R5$ ,  $R7$  and  $\neg A1$  hold. Then, given  $\theta \in (0, 1)$ ,  $\exists \bar{\gamma}(\theta)$  such that*

- (i) *if  $\gamma > \bar{\gamma}(\theta)$ , then  $\widetilde{TS}^B - \widetilde{TS}^C > 0 \forall \beta \in (0, 1]$ ; and*
- (ii) *if  $\gamma < \bar{\gamma}(\theta)$ , then  $\exists \bar{\beta}(\theta) \in (0, 1]$  such that*

$$\widetilde{TS}^B - \widetilde{TS}^C \begin{cases} > 0 & \forall \beta < \bar{\beta}(\theta) \\ = 0 & \text{if } \beta = \bar{\beta}(\theta) \\ < 0 & \forall \beta > \bar{\beta}(\theta). \end{cases}$$

**Proof.** See Appendix. ■

Technological spillovers carry a positive externality that raises total surplus. The combination of large R&D investments and strong technological spillovers contributes in particular to total surplus. Hence, as under Cournot competition R&D investments exceed those under Bertrand competition, total surplus can be larger under quantity competition when the input spillover is strong enough.

## 5 Conclusions

We have shown that in a duopoly with substitutable goods where firms invest in process R&D, price can be lower under Cournot competition than under Bertrand competition. This occurs when the R&D process is efficient, when spillovers are substantial, and when products are not too differentiated. Under these circumstances much more of the benefit of any cost reduction are given to consumers under Bertrand competition than under Cournot competition. As a result the post-innovation costs are much lower under Cournot competition than under Bertrand competition leading to lower prices when firms compete over quantities.

The robustness of our result should be checked along several dimensions. An obvious scenario would be to consider cooperative R&D prior to the production stage. Allowing firms to cooperate in R&D is an important policy tool to enhance incentives towards investment in R&D. As this policy is driven foremost by the resulting internalization of the technological spillover, it needs to be examined whether it affects the conclusion that price can be lower under Cournot competition than under Bertrand competition.

## References

- [1] Aizcorbe, A. M. (2002), “Why are semiconductor price indexes fall so fast in the 1990s? A decomposition”, *Economic Inquiry* 44: 485 - 496.
- [2] Amir, R. (2000), “Modelling imperfectly appropriable R&D via spillovers”, *International Journal of Industrial Organization* 18: 1013 – 1032.
- [3] Amir, R. Jim, J. Y. and Troege, M., 2008, “On additive spillovers and returns to scale in R&D”, *International Journal of Industrial Organization*, 26: 695 – 703.

- [4] Breton, M., Turki, A. and Zaccour, G. (2004), “Dynamic Model of R&D Spillovers, and Efficiency of Bertrand and Cournot Equilibria”, *Journal of Optimization Theory and Applications* 123: 1 – 25.
- [5] Bulow, J., Geanakoplos, J. and Klemperer, P. (1985), “Multimarket oligopoly: strategic substitutes and complements”, *Journal of Political Economy* 93: 488 – 511.
- [6] Cellini, R., Lambertini, L. and Ottaviano, G. I. P. (2004), “Welfare in a differentiated oligopoly with free entry: a cautionary note”, *Research in Economics* 58: 125 – 133.
- [7] Cheng (1985), “Comparing Bertrand and Cournot Equilibria: A Geometric Approach”, *RAND Journal of Economics* 16: 146 – 152.
- [8] Cohen, W.M. and Levinthal, D.A. (1989), “Innovation and learning: the two faces of R&D”, *Economic Journal* 99: 569 – 596.
- [9] d’Aspremont, C. and Jacquemin, A., 1988, “Cooperative and noncooperative R&D in duopoly with spillovers”, *American Economic Review* 78: 1133 – 1137.
- [10] de Bondt, R. and Veugelers, R. (1989), “Informatie en kennis in de economie”, in de Bondt, R, and Veugelers R (editors), *Referatenboek* (23ste VWEC), Leuven: Leuven Universitaire Pers, 19 – 33.
- [11] Geroski, P.A., 1995, “Do spillovers undermine the incentive to innovate?”, in Dowrick, S., editor, *Economic approaches to innovation*, Aldershot (UK): Edward Elgar, 76 – 97.
- [12] Gerschbach, H. and Schmutzler, A. (2003), “Endogenous spillovers and incentives to innovate”, *Economic Theory* 21: 59 – 79.

- [13] Häckner, J. (2000), “A note on price and quantity competition in differentiated oligopolies”, *Journal of Economic Theory* 93: 233 – 239.
- [14] Hinloopen, J. (2003), “R&D efficiency gains due to cooperation”, *Journal of Economics*, 80: 107 – 125.
- [15] Kaiser, U. (2002), “Measuring knowledge spillovers in manufacturing and services: an empirical assessment of alternative approaches”, *Research Policy* 31: 125 – 144.
- [16] Kamien, M. and Zang, I. (2000), “Meet me halfway: research joint ventures and absorptive capacity”, *International Journal of Industrial Organization* 18: 995 – 1013.
- [17] Lin, P. and Saggi, K. (2002), “Product Differentiation, Process R&D and the nature of market competition”, *European Economic Review* 46: 201 – 211.
- [18] López, M. C. and Naylor, R. A. (2004), “The Cournot–Bertrand profit differential: A reversal result in a differentiated duopoly with wage bargaining”, *European Economic Review* 48: 681 – 696.
- [19] Mukherjee, A. (2005), “Price and quantity competition under free entry”, *Research in Economics* 59: 335 – 344.
- [20] Qiu, L. D. (1997), “On the dynamic efficiency of Bertrand and Cournot equilibria”, *Journal of Economic Theory* 75: 213 - 229.
- [21] Singh, N. and Vives, X. (1984), “Price and quantity competition in a differentiated duopoly”, *RAND Journal of Economics* 15: 546 - 554.
- [22] Symeonidis, G. (2003), “Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R&D”, *International Journal of Industrial Organization* 21: 39 – 56.

[23] Vives, X. (1985), “On the efficiency of Bertrand and Cournot equilibria with product differentiation”, *Journal of Economic Theory* 36: 166 - 175.

[24] Zanchettin, P. (2006), “Differentiated duopoly with asymmetric costs”, *Journal of Economics & Management Strategy* 15: 999 - 1015.

## 6 Appendix

### 6.1 Proof of Proposition 5

First note that:

$$\widetilde{TS}^B - \widetilde{TS}^C = \frac{\gamma(a-c)^2}{\Delta_B^2 \Delta_C^2} F(\gamma; \theta),$$

where  $\Delta_B = \gamma(1+\theta)(2-\theta)(4-\theta^2) - (2-\theta^2)$ ,  $\Delta_C = \gamma(2+\theta)(4-\theta^2) - 2$ , and  $F(\gamma, \theta) = [\gamma(4-\theta^2)^2(1+\theta)(3-2\theta) - 2(2-\theta^2)^2] \Delta_C^2 - [\gamma(4-\theta^2)^2(3+\theta) - 8] \Delta_B^2$ . Define  $G(\gamma; \theta) = F(\gamma; \theta) / (\gamma\theta^2(4-\theta^2))$ . Obviously,  $\text{sign}(\widetilde{TS}^B - \widetilde{TS}^C) = \text{sign}(G(\gamma; \theta))$ . Note that  $G(\gamma; \theta) = \gamma^2 g_1 + \gamma g_2 + g_3$ , where  $g_1 = (4-\theta^2)^3(1+\theta)(4-2\theta-\theta^2)$ ,  $g_2 = -2(4-\theta^2)^2(1+\theta)(4-\theta-\theta^2) + 2\theta(4-\theta^2)(8+4\theta-4\theta^2-\theta^3)$ , and  $g_3 = (4-\theta^2)(4+4\theta-3\theta^2-\theta^3) - 8\theta(2-\theta^2)$ . It follows that  $G(\gamma; \theta)$  is strictly convex in  $\gamma$  as  $\partial^2 G(\gamma; \theta) / \partial \gamma^2 = 2g_1 > 0$  (indeed:  $\min_{\{\theta\}} g_1 = \lim_{\theta \rightarrow 1} g_1 = 54$ ). Moreover,  $g_2^2 - 4g_1g_3 > 0 \forall \theta \in (0, 1)$ . Hence, given any  $\theta \in (0, 1)$ , there are two real solutions to  $G(\gamma; \theta) = 0$ , in particular:

$$\bar{\gamma}_1(\theta) = \frac{-g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \text{ and } \bar{\gamma}_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$

When  $\beta = 0$ , regularity condition R5 is most binding. Label the resulting threshold value on the efficiency parameter  $\gamma^*$ . The result then follows as  $\min_{\theta} \{\gamma^* - \bar{\gamma}_2(\theta)\} = \lim_{\theta \rightarrow 0} \{\gamma^* - \bar{\gamma}_2(\theta)\} = 0$ . (see also Figure 2).

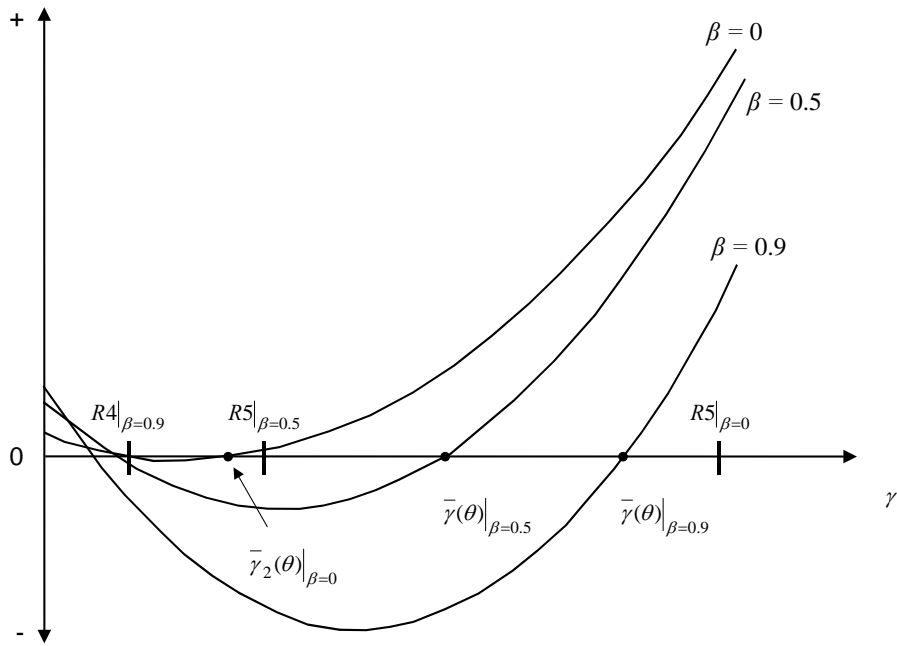


Figure 2:  $G(\gamma; \beta, \theta)$  for different levels of R&D input spillovers;  $a = 100$ ,  $c = 70$ ,  $\theta = 0.9$ .

## 6.2 Proof of Proposition 6

This proof is a general version of that in Section 6.1. Observe that:

$$\widetilde{TS}^B - \widetilde{TS}^C = \frac{\gamma(a-c)^2}{(1+\beta)\Delta_B^2\Delta_C^2} F(\gamma; \beta, \theta),$$

where  $\Delta_B = \gamma(1+\theta)(2-\theta)(4-\theta^2) - (2-\theta^2-\theta\beta)$ ,  $\Delta_C = \gamma(2+\theta)(4-\theta^2) - (2-\theta\beta)$ , and  $F(\gamma; \beta, \theta) = [\gamma(1+\beta)(4-\theta^2)^2(1+\theta)(3-2\theta) - 2(2-\theta^2-\theta\beta)^2] \Delta_C^2 - [\gamma(1+\beta)(4-\theta^2)^2(3+\theta) - 2(2-\theta\beta)^2] \Delta_B^2$ . Again we consider the related function  $G(\gamma; \beta, \theta) = F(\gamma; \beta, \theta) / (\gamma\theta^2(4-\theta^2))$ . It follows that  $\text{sign}(\widetilde{TS}^B - \widetilde{TS}^C) = \text{sign}(G(\gamma; \beta, \theta))$ . Note that  $G(\gamma; \beta, \theta) = \gamma^2 g_1 + \gamma g_2 + g_3$ , where  $g_1 = (1+\beta)(4-\theta^2)^3(1+\theta)(4-2\theta-\theta^2)$ ,  $g_2 = -2(1+\beta)(4-\theta^2)^2(1+\theta)(4-\theta^2-\theta(1-\beta)) - 2(4-\theta^2)[(4+2\theta-\theta^2)(2-\theta\beta)^2 - (2+\theta)^2(4-2\theta\beta-\theta^2)]$ , and  $g_3 = (1+\beta)(4-\theta^2)[2(2-\theta\beta)(1+\theta+\theta\beta) - (3+\theta)\theta^2] - 4\theta(1+\beta)(2-\theta\beta)(2-\theta^2-\theta\beta)$ . Then note that  $G(\gamma; \beta, \theta)$  is strictly convex in  $\gamma$  as  $\partial^2 G(\gamma; \beta, \theta) / \partial \gamma^2 = 2g_1 > 0$  (indeed:  $\min_{\{\theta, \beta\}} g_1 = \lim_{\theta \rightarrow 1, \beta \rightarrow 0} g_1 = 54$ ). Moreover,  $g_2^2 - 4g_1g_3 > 0 \forall \theta \in (0, 1)$ . Hence, given any  $\theta \in (0, 1)$ , there are two real solutions to  $G(\gamma; \beta, \theta) = 0$ , in particular:

$$\bar{\gamma}_1(\theta) = \frac{-g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \text{ and } \bar{\gamma}_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$

Only the larger root needs to be considered as  $\min_{\theta, \beta} \{\gamma^* - \bar{\gamma}_1(\theta)\} = \lim_{\theta \rightarrow 0} \{\gamma^* - \bar{\gamma}_2(\theta)\}|_{\beta=1} = 0$ , where  $\gamma^*$  is the threshold value induced by R7. Label the larger root  $\bar{\gamma}(\theta)$ . Then observe that  $\min_{\theta, \beta} \{\partial \bar{\gamma}(\theta) / \partial \beta\} = \lim_{\theta \rightarrow 0} \partial \bar{\gamma}(\theta) / \partial \beta|_{\beta=0.5} = 0$ . This gives rise to the different lines as drawn in Figure 2 for different values of  $\beta$ . Obviously, for any  $\gamma > \bar{\gamma}(\theta)$  we are in situation (i) while situation (ii) emerges for any  $\gamma < \bar{\gamma}(\theta)$ . The rest of the proof then follows.