Title: Collaborative learning for mathematical level raising, what does it take?

Keywords: collaborative learning, interaction, mathematical level raising, learning materials, teacher interventions

At ICME-6 in Budapest in 1988, I gave a presentation about the learning of mathematics in heterogeneous small groups. I was a PhD-student and completely involved in classroom observations and the designing of good learning materials for small group learning. Freudenthal, whose ideas about the heterogeneous learning group had influenced me, was in my audience, giving me support with his presence. Now, 20 years later, I have been involved in many research projects on collaborative learning of mathematics. We know a lot more about the process of interaction which stimulates mathematical level raising. We also know more about the characteristics of the learning materials. For level raising isolated problem solving activities are not sufficient, we need at least a series of problems, with special problems in it to provoke level differences between the students. We know more about the favorable size of small groups, the pros and cons of couples: easily accessible for research, but less rich for a critical discussion between students. And we start to know more about the role of the teacher. Which interventions stimulate the interaction and the process of level raising? Which interventions can be disturbing? Which sort of whole class discussions supports the learning in small groups? Some say that whole class discussions are crucial to establish good social and sociomathematical norms and to consolidate level raising. Others think that they are mainly time-consuming and evoke all sorts of stereotypical behavior of the students, including off-task behavior. I will present some of our research findings over the last twenty years and I am sure we will have enough to discuss!