Track and vertex reconstruction in the ATLAS inner detector
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Appendix B

Converting between perigee and cartesian track representation

The seven cartesian track parameters at the point of closest approach to the z-axis can be calculated in terms of the five perigee-parameters, \( \vec{\eta} = (d_0, z_0, \phi_0, \theta, q_p) \), as shown below:

\[
\begin{align*}
p_x &= \frac{q_p}{p} \cos \phi \sin \theta \\
p_y &= \frac{q_p}{p} \sin \phi \sin \theta \\
p_z &= \frac{q_p}{p} \cos \theta \\
E &= \sqrt{\left(\frac{q_p}{p}\right)^2 + m^2} \\
x &= x_{ref} - d_0 \sin \phi \\
y &= y_{ref} + d_0 \cos \phi \\
z &= z_{ref} + z_0
\end{align*}
\]

The 7 x 7 covariance matrix for the cartesian parameters can be computed by taking the differentials of the above equations, with respect to the perigee parameters. For example:

\[
\text{cov}(p_x, p_x) \equiv (V_\alpha)_{11} = \frac{\delta p_x}{\delta \phi_0} \frac{\delta p_x}{\delta \phi_0} (V_\eta)_{33} + 2 \frac{\delta p_x}{\delta \phi_0} \frac{\delta p_x}{\delta \theta} (V_\eta)_{34} + 2 \frac{\delta p_x}{\delta \phi_0} \frac{\delta p_x}{\delta q_p} (V_\eta)_{35} \\
+ \frac{\delta p_x}{\delta \theta} \frac{\delta p_x}{\delta \theta} (V_\eta)_{44} + 2 \frac{\delta p_x}{\delta \theta} \frac{\delta p_x}{\delta q_p} (V_\eta)_{45} + \frac{\delta p_x}{\delta q_p} \frac{\delta p_x}{\delta q_p} (V_\eta)_{55}
\]

This follows from the definition \( \text{cov}(p_x, p_x) = \langle \delta p_x \delta p_x \rangle \). The set of differentials can be represented by the Jacobian matrix, \( \mathbf{J} \).
Converting between perigee and cartesian track representation

\[
J = \begin{pmatrix}
\frac{\partial H_1(\vec{\alpha}_A)}{\partial \alpha_1} & \cdots & \frac{\partial H_1(\vec{\alpha}_A)}{\partial \alpha_n} \\
\frac{\partial H_2(\vec{\alpha}_A)}{\partial \alpha_1} & \cdots & \frac{\partial H_2(\vec{\alpha}_A)}{\partial \alpha_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial H_n(\vec{\alpha}_A)}{\partial \alpha_1} & \cdots & \frac{\partial H_n(\vec{\alpha}_A)}{\partial \alpha_n}
\end{pmatrix}
\]

(B.9)

The Jacobian matrix can then be used to transform the covariance matrix in the perigee frame to the covariance matrix in cartesian frame:

\[
V_\alpha = J_{\eta\rightarrow\alpha} V_\eta J_{\eta\rightarrow\alpha}^{-1}
\]

(B.10)

The transformation-matrix \( J_{\eta\rightarrow\alpha} \), for transformation from the perigee to the cartesian representation, can be calculated to be equal to:

\[
J_{\eta\rightarrow\alpha} = \begin{pmatrix}
0 & 0 & -p_y & p_z \cos \phi & \frac{-qp_x}{q} & 0 & 0 \\
0 & 0 & p_x & p_z \sin \phi & \frac{-qp_y}{q} & 0 & 0 \\
0 & 0 & 0 & -\sin \theta & \frac{-qp_z}{q} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-q}{p} & 0 & 0 \\
-\sin \phi & -d_0 \cos \phi & 0 & 0 & 0 & 0 & 0 \\
cos \phi & -d_0 \cdot \sin \phi & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(B.11)

Similarly the transformation-matrix \( J_{\alpha\rightarrow\eta} \), for transformation from the cartesian to the perigee representation, can be calculated to be equal to:

\[
J_{\alpha\rightarrow\eta} = \begin{pmatrix}
0 & 0 & 0 & 0 & -\sin \phi & \cos \phi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{p_y}{p_x + p_y^2} \cos \phi & \frac{p_x}{p_x + p_y^2} & 0 & 0 & 0 & 0 & 0 \\
-\frac{p_z(1 + \tan \theta^2)}{q \cdot E} & \frac{p_z(1 + \tan \theta^2)}{q \cdot E} & -\tan \theta & 0 & 0 & 0 & 0 \\
-\frac{q \cdot E}{p^3} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(B.12)
It should be noted that the cartesian representation is used to represent the trajectory at a specific point along the track. This can be any point the track while the perigee parameters are by definition defined at the point of closest approach to the global z-axis. This means that before $J_{\alpha \rightarrow \eta}$ is used to obtain the perigee representation, the cartesian coordinates and their corresponding error-matrix should be transported to the perigee position.