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ABSTRACT

Funded Pensions and Intergenerational and International Risk Sharing in General Equilibrium*

We explore intergenerational and international risk sharing in a general equilibrium multiple-country model with two-tier pensions systems. The exact design of the funded tier is key for the way in which risks are shared over the various generations. The laissez-faire market solution fails to provide an optimal allocation because the young cannot share in the risks. However, the existence of wage-indexed bonds combined with a pension system with a fully-funded second tier that pays defined wage-indexed benefits can reproduce the first best. If wage-indexed bonds are not available, mimicking the first best is not possible, except under special circumstances. We also explore whether national pension funds want to deviate from the first best by increasing domestic equity holdings. With wage-indexed bonds this incentive is absent, while there is indeed such an incentive when wage-indexed bonds do not exist.

JEL Classification: E2, F42, G23 and H55
Keywords: defined wage-indexed benefits, funded pensions, overlapping generations, risk sharing and wage-indexed bonds

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Submitted

1 Introduction

This paper explores intergenerational and international risk sharing in a general equilibrium multiple-country model with two-tier pensions systems. The first tier is a pay-as-you-go (PAYG) system that consists of a lump-sum component and a part that is linked to wages. The second tier involves a fully funded pension system that pays wage-indexed benefits. Our analysis is of interest from the perspectives of institutional design and of policy making. Given the ageing of their populations, many countries are reforming their pension systems by introducing a funded pension pillar.[1] Also the World Bank (see e.g. Holzmann and Hinz, 2005) has recommended pension funding to complement traditional public PAYG pension arrangements. The design of the funded component is of crucial importance for the way in which risks are shared over various generations.

Our model features a productivity shock and a depreciation shock (a financial shock) in each country. The laissez-faire market solution fails to provide an optimal allocation, because the young are born after the shocks have materialized and so cannot trade these risks. Introduction of a pension fund with benefits defined in wage-indexed terms allows for optimal intergenerational risk-sharing within each country.[2] Essential in this regard is that the pension fund features an (ex-post) mismatch between its assets and liabilities. For example, if pension benefits are defined in wage-indexed terms, any equity investment by the pension fund is effectively owned by the young generation and, in this way, the young share in the equity risks. Specific investment portfolios of the pension fund allow for the optimal allocation of the fundamental economic shocks between the young and old generations so that intergenerational risk sharing becomes optimal.

While intergenerational risk-sharing within each country may be optimal for given national resources, allocations can still be suboptimal when viewed from the perspective of an international planner. In the presence of multiple countries that are each characterized by their own shocks, replication of the international planner's optimum (the "first best") is possible only if pension funds have access to wage-indexed bonds of all countries. In particular, we show that in the presence of these wage-indexed bonds, a properly-designed pension system with a fully-funded second tier that pays a defined wage-indexed benefit reproduces the first best. If these wage-indexed bonds are not available, mimicking the first best is impossible, except

[1] For the European Union (EU), Economic Policy Committee and European Commission (2006, pp.28-31) provide an overview of recent and current reforms. Over the past decade, the volume of assets managed by pension funds in the OECD has grown much faster than GDP (see Boeri et al., 2006).

[2] Also a pension fund with benefits defined in real terms is able to produce optimal intergenerational risk-sharing within each country – see also Beetsma and Bovenberg (2008). However, with endogenous labor supply, the defined wage-indexed system becomes strictly preferable – see Beetsma et al. (2008).

under special circumstances. The analysis in this paper thus assigns an important role to the *combination* of a proper design of a two-tier pension system and the degree to which asset markets are complete.

Having identified the circumstances under which the first best can be replicated, we explore whether the national pension funds (or their designers) have an *incentive* to invest in accordance with the first-best allocation. The answer to this question depends on the incentive of pension funds to increase domestic equity investments and thereby again on the presence of wage-indexed bonds. With such bonds, the consequences for the factor returns of equity shifts in pension portfolios are spread evenly throughout the world, because countries' investment portfolios are sufficiently diversified in terms of both wages and equity. This is not the case, however, if wage-indexed bonds are absent. An increase in domestic equity holdings lowers the return on domestic equity but raises the domestic wage rate. This benefits the old generation to the extent that they have internationally diversified their equity investments but have a large stake in domestic wages through the pension system (via the wage-linked component of the first tier or the wage-indexed benefit they may receive via the second tier) that they cannot diversify by selling wage-indexed bonds to foreigners. The lower return on the domestic equity position of the pension fund causes only limited harm to the young as residual claimants of the defined-benefit pension fund, because the risk of a lower domestic return is diversified via the capital market. At the same time, they benefit substantially from the higher domestic wage (and more so if the wage-linked first- or second-tier pension payments to the old are smaller) if the lack of wage-indexed bonds prevents them from sharing domestic wage shifts internationally.

A growing literature explores intergenerational risk sharing through the pension system. A large part of this literature deals with risk sharing through PAYG systems – see, for example, Hassler and Lindbeck (1997), Thøgersen (1998), Krueger and Kubler (2002), Bohn (2003), Wagener (2004) and Campbell and Nosbusch (2008). Exceptions are De Menil and Sheshinski (2003) and Miles and Černý (2006). Matsen and Thøgersen (2004) investigate intergenerational risk sharing in two-tier pension systems but they do not include funded defined-benefit systems, while Oksanen (2006) and Teulings and De Vries (2006) explore funding in a partial equilibrium context. The current paper tries to extend the literature into various directions. In particular, we combine intergenerational and international risk-sharing within two-tier pension systems with a defined wage-indexed benefit pension fund and we explore the consequences of the presence of wage-indexed bonds in this context.

The remainder of the paper is structured as follows. Section 2 lays out the economy. In Section 3 we present the international planner. The market economy and the pension sector are discussed in Section 4. Section 5 investigates whether and how the market economy can mimic the international planner. In Section 6 we explore the consequences of an increase in domestic equity investment. Finally, Section 7 concludes the paper.

2 The economy

2.1 Individuals and preferences

There are two periods (0 and 1) and N countries. In period 0, a generation of mass 1 is born in each country. These generations live through periods 0 and 1. We call these the "old generations". Following Ball and Mankiw (2007), we assume that these generations invest their entire endowment in the first period and consume the principal and interest in the second period. The representative agent from the old generation in country i ($i = 1 \ldots N$) features the following utility function:

$$u_o^i = u_o\left(c_i^o\right), \tag{1}$$

where c_i^o is consumption in period 1 when this generation has become old. The utility function $u_o(\cdot)$ does not depend on i, hence preferences are homogeneous across countries. Function $u_o(\cdot)$ fulfills the usual Inada conditions.

In period 1, new generations of mass 1 are born in each country. We call these the "young generations". The lives of the two generations thus overlap in period 1. The young generations live for only one period and feature utility

$$u_y^i = u_y\left(c_i^y\right), \tag{2}$$

where c_i^y is consumption of a young generation person from country i. The young's utility function may differ from that of the old, but also fulfills the usual Inada conditions.

2.2 Production

There is only one, perfectly tradable good in the world. In each country the output level of the good in period 0 is exogenously given at per-capita level η_0. Production is endogenous only in period 1, when in country i it is given by

$$Y_i = A_i F\left(K_i, L_i\right), \tag{3}$$

where K_i represents the aggregate capital stock (the result of investment in period 0) and L_i aggregate employment. Stochastic parameter A_i captures total factor productivity (TFP). $F(\cdot)$ exhibits constant returns to scale and is identical across countries. The old generations are retired in period 1, while in that period each young individual exogenously supplies an amount of labour that we normalize to unity. Aggregate employment thus amounts to $\bar{L}_i \equiv 1$ in each country (the product of a unity mass of young workers and each worker's individual labour supply). For the sake of convenience we will drop the dependency on L_i in our notation and write $F(K_i)$ for $F(K_i, L_i)$ or $F^i \equiv F(K_i)$. The first- and second-order derivatives of the production function are denoted with subscripts, e.g. $F_K(K_i)$ or F^i_K, etcetera.

2.3 Uncertainty

Productivity and depreciation are uncertain in all countries. The vector of the stochastic shocks in the world is $\xi \equiv \{A_i, \phi_i\}_{i=1...N}$, where $0 \leq \phi_i \leq 1$ denotes the fraction of capital that survives the production process (i.e., one minus the depreciation rate). It is unknown in period 0, but becomes known before period 1 variables are determined. The probability density function of the shocks is denoted by $f(\xi)$. The shocks may be correlated within each country and across countries.

2.4 Assumption: countries ex ante identical

For the sake of simplicity we will assume throughout the paper that countries are ex ante equal. Formally, we make the following assumptions (some are already imposed)

Assumption 1. *We assume that (1) every country has the same initial endowment η_0, (2) utility functions are homogeneous among the same generation within different countries, (3) production functions are the same for each country, and (4) the probability density function $f(\xi)$ is symmetric on a country-by-country basis, i.e.*

$$f(A_1, \phi_1, A_2, \phi_2, \ldots, A_N, \phi_N) = f(A_{i_1}, \phi_{i_1}, A_{i_2}, \phi_{i_2}, \ldots, A_{i_N}, \phi_{i_N}) \tag{4}$$

for every permutation $(i_1, \ldots, i_N)$ of $(1, \ldots, N)$.

This assumption prevents the need for systematic transfers under the first best that we derive below and substantially simplifies the formal analysis.

3 The international planner

This section derives the first best. It will serve as a benchmark for the analysis of the market economy. The international planner has perfect freedom to allocate resources across all countries. In period 0, it chooses the national capital stocks $K_i \geq 0$, and commits to a state-contingent plan. Hence, the consumption levels in period 1 in country i are functions of the shocks: $c_i^o = c_i^o(\xi)$ and $c_i^y = c_i^y(\xi)$. The planner is bound only by the resource constraints in periods 0 and 1 which are given by, respectively,

$$N\eta_0 = \sum_{i=1}^{N} K_i, \tag{5}$$

$$\sum_{i=1}^{N} (c_i^y + c_i^o) = \sum_{i=1}^{N} \left[A_i F(K_i) + \phi_i K_i\right]. \tag{6}$$

The left-hand side of (5) denotes the world endowment in period 0, while the right-hand side is aggregate investment in physical capital. The left-hand side of (6) denotes total consumption in the second period, while the right-hand side of (6) stands for world production plus what is left over of the capital stocks after taking into account depreciation.

This international planner maximises expected weighted utility:

$$\mathcal{U} \equiv \sum_{i=1}^{N} \mathrm{E}_0 \left[\chi u_o(c_i^o) + u_y(c_i^y)\right], \tag{7}$$

where χ denotes the weight of the old generation relative to the young in the planner's welfare function. We assume that the planner treats every country the same, so they all have the same weight. Further, $\mathrm{E}_0[.]$ denotes the expectation conditional on information available in period 0.

The international planner's optimal allocation gives the following first order conditions

$$\mathrm{E}_0\left[(A_i F_K(K_i) + \phi_i)\, u_o'(c_i^o)\right] = \mathrm{E}_0\left[(A_1 F_K(K_1) + \phi_1)\, u_o'(c_i^o)\right], \quad \forall\, i \neq 1, \tag{8}$$

$$c_i^o(\xi) = c_1^o(\xi), \quad \forall\, \xi, \forall i \neq 1, \tag{9}$$

$$u_y'(c_i^y(\xi)) = \chi u_o'(c_i^o(\xi)), \quad \forall\, \xi, i, \tag{10}$$

The optimal allocation is characterised by these first order conditions and the two resource constraints (5) and (6). Equation (8) is the international arbitrage equation and determines the optimal capital allocation. Equation (9) equalises the marginal utilities of the old and — since we assume equal utility functions across countries — old-generation consumption levels for all shocks. Finally, equation (10) determines the distribution of consumption over the two generations.[3] Notice that with identical utility functions for the two generations ($u_o(\cdot) = u_y(\cdot)$) and the same treatment of the old and the young ($\chi = 1$), equation (10) reduces to

$$c_i^y(\xi) = c_i^o(\xi), \quad \forall\, \xi,\ i. \tag{11}$$

[3]Note that these equations also imply $c_i^y(\xi) = c_1^y(\xi)\ \forall\, \xi, i$, even if $\chi \neq 1$.

4 The market economy

In the previous section we studied the international planner, because it determines the allocation that yields the highest possible welfare as measured by (7) and as such it serves as a benchmark for the outcomes under a market system with pension funds. This section describes the decentralized market economy in which individuals and firms maximize their objective functions under the relevant constraints.

In a decentralized economy the old and young generations cannot directly trade risk in financial markets, because the young generations are born only after the shocks have materialized. Indeed, in the absence of pension arrangements, the old bear all the depreciation risks and cannot shift this risk toward the young. Both generations (the young through their wages and the old through their dividend incomes) are exposed to productivity risk, but it is unlikely that the allocation of this risk between the two generations is optimal. Hence, the generations would like to trade this risk but they cannot do this in financial markets. Other institutions thus have to fill the gap of the missing market for risk sharing between the old and the young generations. We shall explore to what extent the pension system can perform that role.

As we will show, the existence (or absence) of wage-indexed bonds is crucial. We abstract from "standard" real bonds (i.e., bonds that pay off a given amount of real resources when they expire) because their presence is irrelevant for the results.

4.1 Timing of events

The timing of events in the market economy is as follows:

1. Individuals from the old generations take their investment decisions.

2. The vector of shocks ξ materializes.

3. Firms take hiring and production decisions.

4.2 The pension system

The pension system in each country consists of two pillars. The first is a Pay-As-You-Go (PAYG) system composed of a lump sum part θ_i^p and a wage indexed part $\theta_i^w w_i$, where w_i is the wage rate in country i and θ_i^w is a PAYG pension contribution rate out of the wage. The second pillar of the pension system consists of a *fully-funded* pension fund that collects a mandatory contribution θ_i^f per young person in period 0, invests this contribution and pays out a benefit to this person in period 1. The (possibly negative) difference between the value of the pension fund and the benefit paid to the old generation goes to the members of the young generations. The young generations are thus the risk bearers of the defined-benefit pension system.

The pension fund of country i can invest the contributions it receives in equity and, if they exist, in wage-indexed bonds. Hence, the fund's period-0 budget constraint amounts to

$$\theta_i^f = \sum_j \left(k_{ij}^f + b_{ij}^f \right), \tag{12}$$

where, respectively, k_{ij}^f and b_{ij}^f are the fund's investments (per old generation member) in equity and wage-indexed bonds issued by country j. In the absence of wage-indexed bonds, $b_{ij}^f = 0$. The fund's value per old generation member in period 1 is

$$r_i^a \theta_i^f = \sum_j \left(r_j k_{ij}^f + r_j^w w_j b_{ij}^f \right),$$

where r_i^a denotes the *average gross* rate of return on the investments of the pension fund and r_j is the (stochastic) gross rate of return (*including* what is left over after depreciation) on the equity investment in country j. Finally, $r_j^w w_j$ is the stochastic rate of return on the investment in wage-indexed bonds issued by country j.

The pension fund is fully funded. That is, ex ante there is no redistribution. The young generations may be better off with the fund, because it potentially provides for risk sharing that they cannot achieve without the fund. Absence of ex-ante redistribution is defined in utility terms: at the margin, the old generations should in period 0 be indifferent between investing in the pension fund and in other assets. If we denote the (generally stochastic) *gross* rate of return that an old generation member receives on his pension contribution by r_i^f, the following arbitrage equation must hold for the old generations:

$$\mathrm{E}_0 \left[r_i u_o' \left(c_i^o \right) \right] = \mathrm{E}_0 \left[r_i^f u_o' \left(c_i^o \right) \right]. \tag{13}$$

This restriction states that members of the old generation are indifferent between investment in the pension fund and investment in their own country's equity.[4] This is the full funding condition for the pension fund.

Funded pension systems may differ in the way the pension benefits are defined. We focus on a defined wage-indexed (DWB) system in which the pension benefit is indexed to the wage rate in the own country. In particular, the payout to the old in period 1 is $\theta_i^{dwb} w_i$, where the scale parameter θ_i^{dwb} determines the link of the payout to the wage. Hence, the ex-post

[4] This equation is analogous to the standard arbitrage equation for investment in two different types of equities, shown below when we discuss the optimal portfolio choice of the young in period 0.

(stochastic) gross rate of return on an old generation member's pension contribution is

$$r_i^f = \frac{\theta_i^{dwb} w_i}{\theta_i^f}. \tag{14}$$

Substitution into (13) yields

$$\frac{\theta_i^{dwb}}{\theta_i^f} = \frac{\mathrm{E}_0 \left[r_i u_o' \left(c_i^o \right) \right]}{\mathrm{E}_0 \left[w_i u_o' \left(c_i^o \right) \right]} \tag{15}$$

If wage indexed bonds are present, arbitrage ensures that $r_i^w = \theta_i^{dwb} / \theta_i^f$.[5] Of course, in equilibrium, due to the symmetry of the countries, this relation will not depend on i.

As in Beetsma and Bovenberg (2008), one can also consider a defined contribution (DC) system or a defined real benefit (DRB) system. Under a DC system the old simply receive the value of the fund. Since a DC system does not provide any risk sharing, we may discard this system a priori. Under a DRB system, each old person in country i receives a pre-fixed (non-stochastic) real pension benefit θ^{drb}. The analysis with a DRB system is very similar to that with a DWB system, so we shall not consider a DRB system either.

4.3 Firms

The representative firm in country i maximizes profits $A_i F(K_i, L_i) + \phi_i K_i - w_i L_i - r_i K_i$ over L_i and K_i. The first-order conditions for L_i and K_i are:

$$A_i F_L^i = w_i \quad \text{and} \quad A_i F_K^i + \phi_i = r_i. \tag{16}$$

4.4 The old generations

In period 0, each old from country i invests a real amount k_{ij} in capital of country j, lends an amount b_{ij} in wage indexed bonds (where $b_{ij} = 0$ in the absence of such bonds), and pays a mandatory amount θ_i^f to a pension fund. The old generations take only a portfolio decision in period 0. In period 1, they simply collect the returns on their investments and consume their entire income. Hence, an old-generation individual from country i maximizes (1) over $\{k_{ij}, b_{ij}\}_{j=1\ldots N}$, subject to his budget constraints:

$$\eta_0 = \theta_i^f + \sum_{j=1}^{N} (k_{ij} + b_{ij}), \tag{17}$$

$$c_i^o = \theta_i^p + \theta_i^w w_i + r_i^f \theta_i^f + \sum_{j=1}^{N} r_j k_{ij} + \sum_{j=1}^{N} r_j^w w_j b_{ij}. \tag{18}$$

[5] De Jong (2007) analyses the valuation of non-traded wage-indexed pension liabilities.

Utility maximisation gives the first-order conditions

$$\mathrm{E}_0\left[r_j u'\left(c_i^o\right)\right] = \mathrm{E}_0\left[r_i u'\left(c_i^o\right)\right], \quad \forall\, j \neq i, \tag{19}$$

$$r_j^w = \frac{\mathrm{E}_0\left[r_i u'\left(c_i^o\right)\right]}{\mathrm{E}_0\left[w_j u'\left(c_i^o\right)\right]}, \quad \forall\, i. \tag{20}$$

Denoting $K_{ij} \equiv k_{ij} + k_{ij}^f$ and $B_{ij} \equiv b_{ij} + b_{ij}^f$ as country i's total equity, respectively wage-indexed bond, investments in j, we can rewrite the old generation's budget constraints in a more intuitive way as

$$\eta_0 = \sum_{j=1}^{N} \left(K_{ij} + B_{ij}\right), \tag{21}$$

$$c_i^o = \sum_{j=1}^{N} \left(r_j K_{ij} + r_j^w w_j B_{ij}\right) + \psi_i, \tag{22}$$

with ψ_i the generational account

$$\psi_i = \theta_i^p + \theta_i^w w_i + (r_i^f - r_i^a)\theta_i^f. \tag{23}$$

4.5 The young generations

The young in period 1 in country i receive their wage w_i. They pay the lump- sum PAYG contribution θ_i^p and the pension contribution rate θ_i^w on their wage, while they receive the residual value of the pension fund $\left(r_i^a - r_i^f\right)\theta_i^f$. Hence,

$$c_i^y = \left(1 - \theta_i^w\right) w_i - \theta_i^p + \left(r_i^a - r_i^f\right)\theta_i^f, \tag{24}$$

or using the generational account

$$c_i^y = w_i - \psi_i. \tag{25}$$

Members of the young generations face no optimisation problem, they simply consume their available resources.

4.6 Market equilibrium conditions

All markets have to clear. This is the case for the national labour markets if $L_i = 1$ for all i (a condition that we have already imposed throughout). The equilibrium conditions for physical capital and wage-indexed bonds are

$$\sum_{i=1}^{N} K_{ij} = K_j, \quad \sum_{i=1}^{N} B_{ij} = 0. \tag{26}$$

5 Markets mimicking the first best

To mimic the first best, the pension system must be such that two conditions hold:

1. Marginal utilities, weighed by the intergenerational welfare weight χ, of the young and the old generations in each country must be identical for all shocks. That is, for all shocks $u'_y(c_i^y) = \chi u'_o(c_i^o)$.
2. Consumption of the old must be equal across all countries.

Combined with equation (10), this last condition can be replaced with the condition that aggregate consumption (i.e. the sum of c_i^o and c_i^y) must be equal across countries *for all possible shock realizations*. In the special case that $\chi = 1$ and the two generations have identical utility functions, the first condition reduces to the condition that national consumption be equally divided between the two generations ($c_i^o = c_i^y$).

The individuals' budget constraints ((21), (22) and (25)) ensure that the international planner's first and second period's budget constraints always hold. Finally, the planner's international arbitrage conditions (8) hold since it is one of the first-order conditions for utility maximisation of the old generations (19).

If there exists a set of national pension arrangements for which these conditions hold, it is in principle possible to mimic the first best. However, this does not guarantee that the national authorities have an incentive to design their pension arrangements in accordance with the relevant requirements. We will focus on this potential incentive problem in Section 6.

5.1 Wage-indexed bonds exist

We now try to construct pension arrangements such that the first best is attained under the assumption that wage-indexed bonds are available. We assume that the productivity and depreciation shocks all have positive variance and (to avoid trivial outcomes) that the two shocks are not perfectly correlated. If wage-indexed bonds are available, countries may trade all risks and there is perfect sharing of risk. The following proposition formalizes this statement for a case where we can derive an explicit expression for the pension system's parameters. The appendix shows that a similar result holds for the more general case in which the utility functions of the old and the young differ or the two generations have a different weight in the welfare function.[6]

[6]If the utility functions of the old and the young differ or if the generation weight χ differs from unity, it is no longer possible with a two-pillar pension system to exactly match weighted marginal utility (i.e., to

Proposition 1. *With wage-indexed bonds, if the two generations feature the same utility functions, $\chi = 1$, and the pension systems are of the DWB type, the market economy can always mimic the first best.*

Proof. The solution

$$k_{ij} = k^f_{ij} = \frac{1}{2}\frac{\eta_0}{N}, \quad b_{ij} = b^f_{ij} = \begin{cases} \dfrac{1-N}{2r^w_j N} & i = j \\ \dfrac{1}{2r^w_j N} & i \neq j \end{cases}, \quad \theta^w_i = \frac{1}{2} - \theta^{dwb}_i, \quad \theta^f_i = \frac{1}{2}\eta_0 \text{ and } \theta^p_i = 0 \quad (27)$$

and θ^{dwb}_i determined by the fund's full-funding condition (13) equalises macroeconomic consumption and ensures that $c^y_i = c^o_i$. Hence, the above conditions 1. and 2. hold. To show that the proposed asset allocation for k_{ij} and b_{ij} is a market solution, note that the old generations' and the funds' budget restrictions hold and that due to the exchangeability of the shocks on a country-by-country basis and each country having the same capital stock ($K_i = \eta_0$) we have

$$\mathrm{E}_0\left[\left(A_j F_K(\eta_0) + \phi_j\right) u'_o\left(c^o_i\right)\right] = \mathrm{E}_0\left[\left(A_i F_K(\eta_0) + \phi_i\right) u'_o\left(c^o_i\right)\right].$$

Hence, the international arbitrage equations (19) also hold. Moreover, due to symmetry the wage-indexed bonds in all countries are equally risky and $r^w_i = r^w$. The price of these wage indexed bonds can freely adjust until the old are indifferent between holding capital and bonds. Hence the old generations have no incentive to deviate and the proposed solution is a market allocation. □

The intuition behind the proposed arrangement is that to achieve optimal risk-sharing all agents (young and old) in the world should get equal shares in human capital and physical capital in all countries. Wage shocks need to be equally distributed between the young and the old through the pension system, which is ensured by setting $\theta^w + \theta^{dwb} = \frac{1}{2}$. Further, the two generations should have equal claims on financial capital, while this financial capital must give them a claim on foreign human capital and foreign physical capital. To diversify their wage risk, they must go short in wage-indexed bonds of their own country. Finally, the lump-sum PAYG premium θ^p must be set to equalise the period-1 consumption levels of the two generations within a country. However, because the old receive half of the national wage income via the pension system, while the young receive half of the income from equity investment, also via the pension system, it is optimal to set $\theta^p = 0$.

ensure that (10) holds). The best the coordinated national pension systems can do now is to match weighted marginal utility in a most likely outcome (the median state for example) and to mimic the response of the international planner up to a first-order approximation.

5.2 Wage-indexed bonds do not exist

The above results are obtained under the assumption that the members of the old generation and pension funds can trade in wage-indexed bonds. In reality trade in these assets is virtually, if not entirely, non-existent. If there are no wage-indexed bonds, $b_{ij} = b^f_{ij} = 0$ for all i and j, and the results change dramatically. Pension arrangements can only be designed to yield the first best if either $A_i = A_j$ or $\phi_i = \phi_j$ for all j. The intuition is that ex-post international transfers are excluded, while only equity is internationally tradable. Hence, there is a shortage of instruments to diversify the effects of all shocks. In the construction of the pension portfolios a trade-off needs to be made between risk-sharing of period 1 national resources against productivity shocks or against depreciation shocks. Only if $A_i = A_j$ or $\phi_i = \phi_j$ for all j, equity positions need to spread the risks of only one type of shock so that the capital market is complete and the first best can be achieved. We formally summarize the results of this subsection as:

Proposition 2. *Without wage-indexed bonds, if the two generations feature the same utility functions, $\chi = 1$ and the pension systems are of the DWB type, the market system can always mimic the first best if and only if (a) all A-shocks are equal across the countries* or *(b) all ϕ-shocks are equal across the countries.*

Proof. Start with the 'only if' part. To mimic the first best, it is necessary that national consumption is equal across countries for all possible shock realizations. National consumption is given by

$$c^o_i + c^y_i = A_i F_L(K_i) + \sum_{j=1}^{N} \left[A_j F_K(K_j) + \phi_j\right] K_{ij}, \tag{28}$$

This must be equal to average national consumption $\frac{1}{N}\sum_j A_j F(K_j) + \phi_j K_j$. Writing $F(\cdot) = F_K K + F_L$ gives

$$A_i F_L(K_i) + \sum_j \left[A_j F_K(K_j) + \phi_j\right] K_{ij} = \frac{1}{N}\sum_j [A_j F_K(K_j) + \phi_j] K_j + \frac{1}{N}\sum_j A_j F_L(K_j) \tag{29}$$

Exchangeability of the variables on a country-by-country basis implies that country i invests the same amount in all other countries. The first period budget constraints with $B_{ij} = 0$ give $(N-1)K_{ij} + K_{ii} = \eta_0 \ \forall \ j \neq i$ which links total foreign investment to total domestic investment. Since all off-diagonal elements of the K_{ij} matrix are equal, each country invests an equal amount in domestic equity and, hence, $K_i = \eta_0$. Substitution into (29) and defining $F_L \equiv F_L(\eta_0)$, $F_K \equiv F_K(\eta_0)$ gives

$$A_i F_L + \sum_{j\neq i} (A_j F_K + \phi_j) \frac{\eta_0 - K_{ii}}{N-1} + (A_i F_K + \phi_i) K_{ii} = \frac{1}{N}\sum_j (A_j F_K + \phi_j) K_j + \frac{1}{N}\sum_j A_j F_L \tag{30}$$

Adding $(A_iF_K + \phi_i)\frac{\eta_0 - K_{ii}}{N-1}$ to the left and right hand side gives

$$A_iF_L + \sum_j (A_jF_K + \phi_j)\frac{\eta_0 - K_{ii}}{N-1} + (A_iF_K + \phi_i)K_{ii} =$$
$$\frac{1}{N}\sum_j (A_jF_K + \phi_j)K_j + \frac{1}{N}\sum_j A_jF_L + (A_iF_K + \phi_i)\frac{\eta_0 - K_{ii}}{N-1} \quad (31)$$

Collecting terms gives

$$\Big[(N-1)F_L - \eta_0 F_K + NF_KK_{ii}\Big]\sum_j (A_j - A_i) + \Big[NK_{ii} - \eta_0\Big]\sum_j (\phi_j - \phi_i) = 0 \quad (32)$$

This equation must hold for each i and K_{ii} is the same for all countries, so there is effectively just one parameter that must solve N equations. That system only has a solution if (a) $A_j = A$ for all j *or* (b) $\phi_j = \phi$ for all j.

To proof the 'if' part, note that for case (a) the solution

$$k_{ij} = k^f_{ij} = \frac{1}{2}\frac{\eta_0}{N}, \quad \theta^w_i = \frac{1}{2} - \theta^{dwb}_i, \quad \theta^f_i = \frac{1}{2}\eta_0 \quad \text{and} \quad \theta^p_i = 0 \quad (33)$$

and θ^{dwb}_i such that the pension fund's full-funding condition holds is a market allocation and that all relevant restrictions hold. A solution for case (b) is

$$k_{ij} = k^f_{ij} = \begin{cases} \dfrac{\eta_0}{2N} - \dfrac{N-1}{2N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)} & i = j \\ \dfrac{\eta_0}{2N} + \dfrac{1}{2N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)} & i \neq j \end{cases} \quad (34)$$

and θ^w_i, θ^{dwb}_i, θ^f_i and θ^p_i as in (33). □

Interestingly, for the case of equal productivity shocks and owing to identical labor shares $\omega_L \equiv \omega^i_L \equiv L_iF_L/F = F_L/F$ (recall that $L_i = 1$), each country must go short in its own capital. Linear homogeneity of the production function and $K = \eta_0$ give $\frac{F_L}{F_K} = \eta_0\frac{\omega_L}{1-\omega_L}$. Substitution into (20) results into $K_{ii} = \frac{\eta_0}{N}\frac{1-N\omega_L}{1-\omega_L}$ which is clearly negative for $N \geq 2$ and ω_L around 60-70%. The intuition is that the young's wage depends only on its own productivity shock. This creates an overexposure to this shock and, hence, the country has to go short in its own capital stock. As explained before, the domestic wage and capital shocks must be equally divided between the two generations. The capital shocks can further be diversified through equity trade with foreigners.

6 Effects of pension fund diversification

This section addresses the *incentives* of national authorities to design pension arrangements that enable the market system to replicate the international planner's optimum. Each country's authorities set up a national pension fund.[7] The previous section showed that consumption of members of the old and young generations in the market model with a DWB system can be written as

$$c_i^o = \sum_j r_j k_{ij} + \sum_j w_j \zeta_{ij}^o, \tag{35}$$

$$c_i^y = \sum_j r_j k_{ij}^f + \sum_j w_j \zeta_{ij}^y, \tag{36}$$

where

$$\zeta_{ij}^o = \begin{cases} r_i^w b_{ii} + \theta_i^w + \theta_i^{dwb} & i = j \\ r_j^w b_{ij} & i \neq j \end{cases} \qquad \zeta_{ij}^y = \begin{cases} 1 - \theta_i^w + r_i^w b_{ii}^f - \theta_i^{dwb} & i = j \\ r_j^w b_{ij}^f & i \neq j \end{cases} \tag{37}$$

The effects of pension fund portfolio diversification are determined by the sensitivity of consumption to changes in the factor prices. Specifically, we shall in this section derive the utility consequences for the young and the old if their pension fund changes the equity composition of its portfolio, while keeping the overall size of its portfolio constant. This implies that when the fund increases its equity position in one country, it must reduce its position in at least one other country.

If the pension fund increases its position in a country, then this will lower the rate of return on capital in that country. This, in turn, causes a capital outflow. However, this capital outflow will not offset the pension fund's action entirely if each country is large enough, hence, if there exist only a limited number of countries. This rules out the standard small open economy. In a small open economy, the pension fund has no market power at all and all effects of a shift in its investment allocation vanish.

More formally, we make the following assumptions about the general equilibrium effects of a change in a pension fund's portfolio composition and how the fund finances that change.

Assumption 2. *If any fund i increases its position in country j, then (1) the market does not fully offset this change, $\mathrm{d}K_j/\mathrm{d}k_{ij}^f > 0$ and (2) the fund decreases its position in all other countries, $\mathrm{d}k_{ik}^f/\mathrm{d}k_{ij}^f < 0$ for all $k \neq j$.*

6.1 Wage-indexed bonds do not exist

In the absence of wage-indexed bonds, ζ_{ij}^o and ζ_{ij}^y are 0 for $j \neq i$. For $j = i$, sensitivity is entirely determined by the pension system; $\zeta_{ii}^o = \theta_i^w + \theta_i^{dwb}$ and $\zeta_{ii}^y = 1 - \zeta_{ii}^o$. Hence,

[7] Alternatively, one could imagine the existence of a large number of relatively small (sectoral) pension funds in each country that are effectively controlled by the national authorities in their portfolio allocation. For convenience, we stick here to the assumption of a single national pension fund.

consumption of the young and the old in the market model is exposed only to domestic wage shocks, but not to foreign wage shocks.

To say something definitive about the consequences of a shift in the pension fund's equity portfolio, we employ the following assumption:

Assumption 3. *In each country i, $\frac{k_{ii}}{K_i} < \theta_i^w + \theta_i^{dwb}$ and $\frac{k_{ii}^f}{K_i} < 1 - \theta_i^w - \theta_i^{dwb}$ and $\frac{k_{ij}}{K_j}$ and $\frac{k_{ij}^f}{K_j}$ are positive for $j \neq i$.*

The intuition behind the assumptions $\frac{k_{ii}}{K_i} < \theta_i^w + \theta_i^{dwb}$ and $\frac{k_{ii}^f}{K_i} < 1 - \theta_i^w - \theta_i^{dwb}$ is that the positive consequences of an increase in domestic equity investment by the pension fund (in the form of a higher domestic wage rate) are sufficiently large relative to the negative consequences (in the form of a lower equity return). With positive foreign equity stakes there is in addition the benefit of a higher foreign equity return due to reduced equity investment abroad.

An interesting case where Assumption 3 holds is when a hypothetical national planner in each country equalises the consumption of their own old and young generations as described in Beetsma and Bovenberg (2008). Note that the international planner's optimal allocation is a special case of Beetsma and Bovenberg (2008). Equal consumption requires that the pension fund sets the parameters as

$$k_{ij}^f = k_{ij}, \quad \theta_i^p = 0, \quad \theta_i^w + \theta_i^{dwb} = \tfrac{1}{2}$$

and θ_i^{dwb} is determined by the full-funding condition. As long as the old generation and the pension fund invest a positive amount in foreign countries, but own less than 50% of their own national capital stock, Assumption 3 holds. As mentioned above, for optimal diversification the old generations and pension funds should even go short in their own capital stock to compensate for their exposure to their own productivity shock through the wage income. So we can safely assume that they own less than 50% of their own capital stock.

We can now state and discuss the propositions of this section:

Proposition 3. *Assume an initial allocation for which Assumptions 2 and 3 hold. An increase in the pension fund's domestic equity position k_{ii}^f raises the expected utility of the old generation.*

Proof. To prove the utility gain of the old, first differentiate c_i^o in (35) with respect to k_{ii}^f

$$\begin{aligned}\frac{\mathrm{d}c_i^o}{\mathrm{d}k_{ii}^f} &= \sum_j A_j \left\{ F_{KK}(K_j)k_{ij} + F_{KL}(K_j)\zeta_{ij}^o \right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} r_j \\ &= \sum_j A_j F_{KK}(K_j) K_j \left\{ \frac{k_{ij}}{K_j} - \zeta_{ij}^o \right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} r_j \qquad (38)\end{aligned}$$

where we have made use of the fact that (owing to production being constant-returns-to-scale) $KF_{KK} = -LF_{KL}$. $F_{KK}(\cdot)$ is negative by assumption. This gives for the change in expected utility

$$\frac{\mathrm{dE}_0(u_o(c_i^o))}{\mathrm{d}k_{ii}^f} = \sum_j \mathrm{E}_0\left\{u_o'(c_i^o)A_j\right\} F_{KK}(K_j)K_j \left\{\frac{k_{ij}}{K_j} - \zeta_{ij}^o\right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} \mathrm{E}_0\left\{r_j u_o'(c_i^o)\right\} \quad (39)$$

Arbitrage ensures that $\mathrm{E}_0\left\{r_j u_o'(c_i^o)\right\} = \mathrm{E}_0\left\{r_i u_o'(c_i^o)\right\}$ so we can write.

$$\frac{\mathrm{dE}_0(u_o(c_i^o))}{\mathrm{d}k_{ii}^f} = \sum_j \mathrm{E}_0\left\{u_o'(c_i^o)A_j\right\} F_{KK}(K_j)K_j \left\{\frac{k_{ij}}{K_j} - \zeta_{ij}^o\right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \mathrm{E}_0\left\{r_i u_o'(c_i^o)\right\} \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} > 0 \quad (40)$$

According to Assumption 2, the term between the curly brackets is negative if $i = j$ and positive otherwise and the derivative $\frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f}$ is positive for $i = j$ and negative otherwise, so the first summation is always positive. The second term has vanished because total private investment is constant. □

In other words, provided that the old generation has a sufficiently large share in the domestic wage risk, they want the pension fund to invest as much as possible in the domestic economy. The next proposition identifies circumstances under which this is also the case for the young generation:

Proposition 4. *Assume an initial allocation for which Assumptions 2 and 3 hold. If the pension system is such that $u_y'(c_i^y) = \chi u_o'(c_i^o)$ for all shocks, an increase in the pension fund's domestic equity position k_{ii}^f increases the expected utility of the young.*

Proof. To prove that the young profit from an equity shift to the domestic economy, differentiate (36)

$$\frac{\mathrm{d}c_i^y}{\mathrm{d}k_{ii}^f} = \sum_j A_j F_{KK}(K_j)K_j \left\{\frac{k_{ij}^f}{K_j} - \zeta_{ij}^y\right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}^f}{\mathrm{d}k_{ii}^f} r_j \quad (41)$$

Following the same steps as in the proof of Proposition 3 we find for the change in expected utility

$$\frac{\mathrm{dE}_0(u_y(c_i^y))}{\mathrm{d}k_{ii}^f} = \sum_j \mathrm{E}_0\left\{u_y'(c_i^y)A_j\right\} F_{KK}(K_j)K_j \left\{\frac{k_{ij}^f}{K_j} - \zeta_{ij}^y\right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}^f}{\mathrm{d}k_{ii}^f} \mathrm{E}_0\left\{r_j u_y'(c_i^y)\right\} \quad (42)$$

Equal marginal utility for all shocks and arbitrage ensures that $\mathrm{E}_0\left\{r_j u_y'(c_i^y)\right\} = \mathrm{E}_0\left\{r_i u_o'(c_i^o)\right\}$. Hence, we can write:

$$\frac{\mathrm{dE}_0(u_y(c_i^y))}{\mathrm{d}k_{ii}^f} = \sum_j \mathrm{E}_0\left\{u_y'(c_i^y)A_j\right\} F_{KK}(K_j)K_j \left\{\frac{k_{ij}^f}{K_j} - \zeta_{ij}^y\right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \mathrm{E}_0\left\{r_i u_o'(c_i^o)\right\} \sum_j \frac{\mathrm{d}k_{ij}^f}{\mathrm{d}k_{ii}^f} > 0 \quad (43)$$

The first sum is always positive under Assumptions 2 and 3 and the second term vanishes because total assets of the pension fund are constant. □

The intuition behind Propositions 3 and 4 is that by increasing its domestic equity investment, the pension fund raises the domestic capital stock, thereby dampening the domestic equity return and boosting the domestic wage rate. Since foreigners only share in domestic equity returns, but not in domestic wages, a larger share of the domestic production flows to the domestic population. Propositions 3 and 4 may help to rationalize why countries sometimes impose foreign investment restrictions on their pension funds (see OECD, 2008).

Proposition 4 does not require that the old and the young feature the same utility functions. It only requires that the marginal utilities of the two generations are the same for all the shocks. This equalisation of the marginal utilities is the key characteristic of the first-best solution.

Suppose that the pension funds (possibly instructed by the national authorities) aim at maximizing national welfare,

$$\mathcal{U}_i \equiv \mathrm{E}_0\left[\chi u_o\left(c_i^o\right) + u_y\left(c_i^y\right)\right], \quad (44)$$

given the restrictions imposed by their specific design (and taking into account the optimising behaviour of the private sectors). In the absence of coordination among the pension funds, each one of them selects its portfolio taking as given the portfolio choices of the other funds. With coordination, the pension funds jointly choose their portfolios to maximize $\sum_j \mathrm{E}_0\left[u_o\left(c_j^o\right) + u_y\left(c_j^y\right)\right]$.

An immediate consequence of Propositions 3 and 4 is that without international coordination markets will not mimic the first-best solution, even if this is feasible (which is the case if either all productivity shocks or all depreciation shocks are identical across countries, as shown above).

Since both generations are expected to benefit according to Propositions 3 and 4, it would be optimal for each pension fund to deviate from the first-best allocation by increasing domestic equity investment, while keeping the overall size of the portfolio unchanged. Summarising, we have:

Corollary 1. *Assume an initial allocation for which Assumptions 2 and 3 hold. Further, assume that the pension system is such that $u'_y(c_i^y) = \chi u'_o(c_i^o)$ for all shocks. Then in the absence of coordination, pension fund investment portfolios that yield the first best allocation cannot form an equilibrium and, starting from such combination of portfolios, each individual fund will increase its domestic equity position.*

This corollary does not claim that the first-best solution is achieved with coordination among the pension funds. It only asserts that under the stated conditions coordination among the pension funds is a prerequisite for reaching the first best if the countries cannot trade in their wage shocks because wage-indexed bonds are absent.

6.2 Wage-indexed bonds exist

The previous subsection showed that if wage-indexed bonds do not exist, it is optimal for individual pension funds to deviate from the first best even if it is attainable. The following proposition shows that if wage-indexed bonds do exist, the first best is stable.

Proposition 5. *Assume that wage-indexed bonds are available and that the initial allocation is one in which markets in the presence of DWB pension funds mimic the first best allocation. In this allocation, nobody benefits from a change in the pension fund's domestic equity position.*

Proof. We show that the old do not gain anything; the proof for the young's utility is analogous. Differentiate c_i^o in (35) with respect to k_{ii}^f as in (38)

$$\frac{\mathrm{d}c_i^o}{\mathrm{d}k_{ii}^f} = \sum_j A_j F_{KK}(K_j) K_j \left\{ \frac{k_{ij}}{K_j} - \zeta_{ij}^o \right\} \frac{\mathrm{d}K_j}{\mathrm{d}k_{ii}^f} + \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} r_j + \sum_j \frac{\mathrm{d}b_{ij}}{\mathrm{d}k_{ii}^f} r_j^w w_j \tag{45}$$

For the proposed solution in the proof of Proposition 1 the term between the curly brackets vanishes and we have for the change in utility

$$\frac{\mathrm{dE}(u_o(c_i^o))}{\mathrm{d}k_{ii}^f} = \sum_j \frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} \mathrm{E}\left\{ r_j u'_o(c_i^o) \right\} + \sum_j \frac{\mathrm{d}b_{ij}}{\mathrm{d}k_{ii}^f} \mathrm{E}\left\{ r_j^w w_j u'_o(c_i^o) \right\} \tag{46}$$

Arbitrage ensures that $\mathrm{E}\{r_j^w w_j u'_o(c_i^o)\} = \mathrm{E}\{r_j u'_o(c_i^o)\} = \mathrm{E}\{r_i u'_o(c_i^o)\}$ so we can write.

$$\frac{\mathrm{dE}(u_o(c_i^o))}{\mathrm{d}k_{ii}^f} = \mathrm{E}\left\{ r_i u'_o(c_i^o) \right\} \sum_j \left(\frac{\mathrm{d}k_{ij}}{\mathrm{d}k_{ii}^f} + \frac{\mathrm{d}b_{ij}}{\mathrm{d}k_{ii}^f} \right) = 0 \tag{47}$$

The last sum is zero because total assets remain constant. □

The intuition behind this proposition and its contrast with the case without wage-indexed bonds is as follows. With wage-indexed bonds, benefits or costs of factor price movements are spread equally through the world, because all countries hold investment portfolios that are sufficiently diversified in the fundamental shocks hitting the various countries. Without wage-indexed bonds such diversification is not possible and a unilateral shift in a DWB pension fund's domestic equity investment may move factor price to the benefit of the country's inhabitants. Introduction of wage-indexed bonds and the associated diversification of wage risks eliminates the benefits from exploiting market power.

7 Conclusion

This paper has explored the circumstances that lead to first-best risk sharing. First-best risk sharing requires the simultaneous optimal sharing of risks between generations and across countries. In this regard, we investigated the role of two-tier pension systems with a fully-funded second tier and of the presence of wage-indexed bonds. First, absence of wage-indexed bonds substantially limits the scope for first-best risk sharing. In particular, it reduces the scope for international risk sharing. Second, given that the young are born after the shocks have materialized, they cannot directly trade assets with the old generation and in this way share risks with this generation. Hence, the pension system potentially plays an important role facilitating intergenerational risk sharing. A fully-funded DWB second tier (in combination with a properly-designed first tier) can produce optimal intergenerational risk sharing. As a consequence, the *combination* of wage-indexed bonds and a pension system with a DWB fully-funded second tier allows for first-best risk sharing.

We also explored the incentive of national pension funds to deviate from the first-best allocation by increasing domestic equity investments. This incentive also crucially depends on the absence or presence of wage-indexed bonds. Pension funds are of non-negligible size and through their investment policies they can affect factor prices. With wage-indexed bonds, however, benefits or costs of factor price movements are spread equally through the world, because all countries hold investment portfolios that are sufficiently diversified in the fundamental shocks hitting the various countries. Without wage-indexed bonds such diversification is not possible and a unilateral shift in a DWB pension fund's domestic equity investment may move factor prices to the benefit of the country's inhabitants. In particular, starting optimal intergenerational risk sharing, both generations will benefit from an increase in domestic equity investment. While the fall in the domestic equity return affects them only to a minor extent due to the diversification of the equity positions in their investment portfolios, the induced increase in the wage rate benefits them through the stake they have in domestic wages either directly as wage earners or via the pension system. Hence, a key overall policy implication of our findings is that the introduction of wage-indexed bonds may be beneficial in a world with funded pension systems. Another general conclusion is that unlike PAYG systems, which only condition on domestic shocks, pension funds are particularly desirable when asset markets are complete, because this allows those funds to provide for international risk sharing.

The analysis can be extended into a number of directions. One extension would be to allow for small pension funds (relative to the national economy). If these funds all behave in a decentralised way, they ignore the effects of their decisions on factor prices and we may expect them to perfectly diversify their portfolios. We conjecture that in that case the first best will be achieved. Coordination of those pension funds at the national level would worsen the allocation, while full (national and international) coordination would again achieve the first best. Hence, an intermediate degree of coordination would be the least desirable situation. Another extension might to allow for endogenous labour supply. The restrictions on the pension system to reach the first best become tighter, because the wage-linked premium payment in the first pillar becomes distortionary. Another extension would allow for a richer menu of shocks. In particular, one might introduce demographic shocks (such as fertility and longevity shocks – see, for example, Auerbach and Hassett, 2002, and Andersen, 2005). With these additional shocks, the economy would ideally avail of instruments whose pay-offs are contingent on these shocks. To the best of our knowledge, such instruments do not exist, at least not on a large scale and traded on public markets. The interesting question then arises how to best design pension systems if the first best cannot be reached.

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A Appendix: Mimicking the international planner: generalisation

This appendix explores the circumstances under which a market system with pension arrangements can mimic the international planner when we allow the utility functions of the old and the young to differ or the two generations to have a different weight in the welfare function. We confine ourselves to symmetric policies, that is $k_{ii}^f = k_h^f$ (h-subscript denotes *home*), $k_{ij}^f = k_f^f$ for $i \neq j$ (f-*sub*script denotes *foreign*), $b_{ii}^f = b_h^f$, $b_{ij}^f = b_f^f$ for $i \neq j$, $\theta_i^w = \theta^w$, and $\theta_i^p = \theta^p$. Because the countries are ex-ante identical, a symmetric policy will result in a symmetric allocation $k_{ii} = k_h$, $k_{ij} = k_f$ for $i \neq j$, $b_{ii} = b_h$, $b_{ij} = b_f$ for $i \neq j$. The pension funds' full funding conditions and budget constraints determine θ_i^{dwb} and θ_i^f which are also country independent because of symmetry.

The first proposition shows that it is always (regardless of the utility functions, generational weight and the availability of wage indexed bonds) possible to approximate the social planner's intergenerational risk sharing condition.

Proposition 6. *If the pension systems are of the DWB type, then coordination of national pension systems can always equalise the marginal utilities of the old and the young within a country for a specific shock $\hat{\xi} \equiv \{\hat{A}_1, \hat{\phi}_1 \ldots \hat{A}_N, \hat{\phi}_N\}$,*

$$u_o'(c_i^o(\hat{\xi})) = \chi u_y'(c_i^y(\hat{\xi})) \tag{48}$$

and can equalise the first-order effects around this specific outcome

$$\frac{u_o'(c_i^o(\hat{\xi}))}{d\xi} = \chi \frac{u_y'(c_i^y(\hat{\xi}))}{d\xi} \tag{49}$$

regardless of the availability of wage indexed bonds.

Proof. Any symmetric policy for which the following equations hold equalises the first-order effects around $\hat{\xi}$

$$k_h^f = k_h \Omega \tag{50}$$
$$k_f^f = k_f \Omega \tag{51}$$
$$b_h^f = -b_f (N-1)(\Omega + 1) - b_h \tag{52}$$
$$b_f^f = b_f \Omega \tag{53}$$
$$\theta^w = \frac{1}{1+\Omega} - r^w[(N-1)b_f + b_h] - \theta^{dwb} \tag{54}$$

with Ω the ratio of absolute risk aversion of the old and the young evaluated for outcome $\hat{\xi}$

$$\Omega \equiv \frac{u_o''(c^o(\hat{\xi}))/u_o'(c^o(\hat{\xi}))}{u_y''(c^y(\hat{\xi}))/u_y'(c^y(\hat{\xi}))}. \tag{55}$$

Notice that for the case without wage indexed bonds $b_h = b_f = b_h^f = b_f^f = 0$. The lumpsum tax θ^p can be used to ensure that $u_o'(c_i^o(\hat{\xi}))$ equals $\chi u_y'(c_i^y(\hat{\xi}))$. □

The following propositions generalise the conditions under which perfect *international* risk sharing is possible.

Proposition 7. *If the pension systems are of the DWB type and if wage indexed bonds are available, then coordination of national pension systems can always equalise macroeconomic (i.e., national) consumption.*

Proof. Take a symmetric policy $k_h^f = k_f^f = \bar{k}^f$, $b_h^f = \bar{b}_h^f$, $b_f^f = \bar{b}_f^f$, $\theta^p = \bar{\theta}^p$, and $\theta^w = 1 + r^w \left(\bar{b}_h^f - \bar{b}_f^f\right) - \theta^{dwb}$. For this set of policy variables, private investment

$$k_{ij} = \frac{\eta_0}{N} - \bar{k}^f \tag{56}$$
$$b_{ij} = \begin{cases} -\dfrac{N-1}{r^w N} - \bar{b}_h^f & \text{for } i = j \\ \dfrac{1}{r^w N} - \bar{b}_f^f & \text{for } i \neq j \end{cases} \tag{57}$$

is a market equilibrium (the old generations' first period's budget constraint and the arbitrage equations hold, just follow the same steps as in Proposition 1). Moreover, this allocation ensures that every country has the same exposure to all shocks, hence it equalises macroeconomic consumption. □

Proposition 8. *If the pension systems are of the DWB type and if wage indexed bonds are not available, then coordination of national pension systems can equalise macroeconomic consumption if and only if (a) all A-shocks are equal across the countries* or *(b) all ϕ-shocks are equal across countries.*

Proof. The proof of the 'only if' part is equivalent to that of the 'only if' part in Proposition 2 on page 13.

To proof the 'if' part, take a symmetric policy $k_h^f = \bar{k}_h^f$, $k_f^f = \bar{k}_f^f$, $\theta^p = \bar{\theta}^p$, and $\theta^w = 1 + \left(\bar{k}_h^f - \bar{k}_f^f\right) \frac{F_K(\eta_0)}{F_L(\eta_0)} - \theta^{dwb}$. For this set of policy variables private investment

$$k_{ij} = \begin{cases} k_h \equiv \frac{\eta_0}{N} - \frac{N-1}{N} \frac{F_L(\eta_0)}{F_K(\eta_0)} - k_h^f & \text{for } i = j \\ k_f \equiv \frac{\eta_0}{N} + \frac{1}{N} \frac{F_L(\eta_0}{F_K(\eta_0)} - k_f^f & \text{for } i \neq j \end{cases} \tag{58}$$

is a market equilibrium that equalises macroeconomic consumption (just follow the same steps as in the previous proposition).

For case (b) take a symmetric policy with $k_h^f = k_f^f = \bar{k}^f$, $\theta^p = \bar{\theta}^p$, and $\theta_i^w = \bar{\theta}^w$. For this set of policy variables private investment $k_{ij} = \frac{\eta_0}{N} - k_{ij}^f$ is a market equilibrium and equalises macroeconomic consumption. □

We can combine Proposition 6 with Propostitions 7 and 8 to obtain more general versions of Propositions 1 and 2 in the main text. If utility functions of the old and the young differ or if the generational weight differs, it is no longer possible to exactly match weighted marginal utility with a two pillar pension system. The best coordination of national pension systems can do now is to match weighted marginal utility in a most likely outcome (the median state for example) and to mimic the response of the world planner up to a first-order approximation.

If markets for wage indexed bonds exist, then coordination of national pension systems can also equalise macroeconomic consumption for all shocks. Combining Propositions 6 and 7 shows that the resulting allocation is

$$k_{ij} = \frac{1}{1+\Omega} \frac{\eta_0}{N}, \qquad b_{ij} = \begin{cases} \frac{1}{1+\Omega} \frac{1}{r^w N} & \text{for } i = j \\ -\frac{1}{1+\Omega} \frac{N-1}{r^w N} & \text{for } i = j \end{cases} \qquad \theta_i^w = \frac{1}{1+\Omega} - \theta^{dwb}$$

$$k_{ij}^f = \frac{\Omega}{1+\Omega} \frac{\eta_0}{N}, \qquad b_{ij}^f = \begin{cases} \frac{\Omega}{1+\Omega} \frac{1}{r^w N} & \text{for } i = j \\ -\frac{\Omega}{1+\Omega} \frac{N-1}{r^w N} & \text{for } i = j \end{cases} \qquad \theta_i^f = \frac{\Omega}{1+\Omega} \eta_0$$

If wage indexed bonds do not exist, then coordination of national pension systems may still equalise weighted marginal utility in each country. However, in general they cannot ensure that consumption is evenly distributed across the countries, unless either TFP-shocks or financial shocks are equal in all countries. In the first case the resulting allocation is

$$k_{ij} = \begin{cases} \dfrac{1}{1+\Omega}\left[\dfrac{\eta_0}{N} - \dfrac{N-1}{N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)}\right] & \text{for } i = j \\ \dfrac{1}{1+\Omega}\left[\dfrac{\eta_0}{N} + \dfrac{1}{N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)}\right] & \text{for } i \neq j \end{cases}$$

$$k^f_{ij} = \begin{cases} \dfrac{\Omega}{1+\Omega}\left[\dfrac{\eta_0}{N} - \dfrac{N-1}{N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)}\right] & \text{for } i = j \\ \dfrac{\Omega}{1+\Omega}\left[\dfrac{\eta_0}{N} + \frac{1}{N}\dfrac{F_L(\eta_0)}{F_K(\eta_0)}\right] & \text{for } i \neq j \end{cases}$$

The latter gives

$$k_{ij} = \frac{1}{1+\Omega}\frac{\eta_0}{N}, \quad k^f_{ij} = \frac{\Omega}{1+\Omega}\frac{\eta_0}{N}$$

and θ^w_i and θ^f_i as in the complete markets case.

The ratio of absolute risk aversions of the two generations, Ω, becomes unity for the special case of identical utility functions of the two generations and an equal generational weight, and all expressions reduce to the corresponding special cases in the main text.