Puzzles in quantum gravity: what can black hole microstates teach us about quantum gravity?

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CHAPTER 1

INTRODUCTION

In this thesis we will consider a diverse range of physical phenomena and theories explored in the works \[1\] \[2\] \[3\] \[4\] \[5\]. Our ultimate goal will be to enhance our understanding of the structure of quantum gravity by trying to learn general lessons from specific but exotic phenomena.

1.1 MOTIVATION

In the accumulated lore of string or M-theory we now have the makings of a theory of quantum gravity. Unfortunately, lacking either experimental underpinnings or even a full, non-perturbative formulation these theories may seem, at first instance, to be on rather unsound footing.

Despite these flaws string/M-theory has had profound successes on several fronts. Within this framework it has been possible to address several outstanding issues in black hole physics such as the black hole entropy and the essentially thermodynamic nature of black holes (and perhaps even gravity itself). In giving birth to and being an integral part of AdS/CFT these theories have also provided the first instance of a fully non-perturbative theory of quantized gravity that is manifestly holographic and that demonstrates that gravity emerges from very different underlying degrees of freedom. This correspondence has also been turned on its heels, allowing us to use string theory to study theories closely related to (and perhaps, in some sense, within the same universality class as) well-tested theories like QCD.

In and of themselves string and M-theory also exhibit a rich and beautiful structure, deeply interwoven with supersymmetry, which has been very fruitful for mathematics and also
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suggests, contingent upon the existence of this symmetry, that the theories are very natural.

Ideally we would like to find a first principles formulation of M-theory (with string theory as a derivative) that unifies both our perturbative and non-perturbative understandings of the theories. Doing so would also require elucidating various foundational questions that emerge in any theory of quantum gravity such as the role of boundary conditions, so-called “background independence”, and the structure of the Hilbert space. Of course Matrix theory and AdS/CFT provide us with some insights or even examples of what such a theory would look like but these formulations are somewhat removed from our original notions of strings and spacetime and do not have the range of applicability we would like.

Given this state of affairs it is important to improve our understanding of these theories, in particular the ways they differ from non-gravitational quantum theories. In this thesis we hope to build upon the accumulated wisdom in the literature by contributing some new examples of somewhat exotic quantum gravitational phenomena that, moreover, yield potential resolutions to issues in black hole physics.

While our understanding of string theory still lacks the depth and completeness that might provide for a satisfactory and convincing resolution of all the puzzles associated with quantum gravity there is a growing sense that the general conceptual outline of this theory is beginning to reveal itself. Through a combination of diverse insights a general picture has emerged of gravity as an emergent or effective theory that has, as one manifestation of itself, a non-Abelian field theory in one dimension lower. Moreover string theory challenges our intuition, built on the foundation of effective field theory, as gravity seems essentially holographic and non-local and a new, still quite coarse, intuition is slowly emerging by incorporating these lessons.

It is our goal here to build on this emerging picture. We would like to understand, for instance, the structure of the gravitational spectrum, what super-selection sectors exist, how perturbative and non-perturbative states combine to generate e.g. a black hole ensemble, and what these states look like in spacetime. We will be able to address these questions to some degree in very specific contexts but, as our findings will be quite surprising from the point of view of standard field theory, we hope to tease from them some general features of the theory that will one day be string theory.

Part I

String theory is a quantum theory of gravity. Whether it is the theory of quantum gravity is not yet clear. As suggested above, here we take the attitude that any consistent theory of quantum gravity provides a framework to extract qualitative features of gravity associated with black hole paradoxes. Thus we would like to claim that the results in the first part of
this thesis are of considerable interest to black hole physics somewhat independently of whether or not string theory proves to be the final theory of gravity.

The motivation for this perhaps surprising claim is the following. Black holes exhibit mysterious behaviour in a wide range of theories including the standard, well-tested, theory of four dimensional general relativity (Einstein gravity). The essential point is that black holes exhibit (almost) exactly the same paradoxes in more exotic theories and even higher dimensional theories like string theory and supergravity. Fortunately the latter are more symmetric and hence, to some degree, under better control than standard Einstein gravity. By studying black holes in such theories we can hope to use our enhanced technical control to resolve some of these issues, at least at a conceptual level. While it is possible, it seems highly implausible that the resolution to ubiquitous black hole quantum gravity puzzles, whose existence hinges on only the most basic features common to any gravitational theory, would somehow depend very sensitively on the details of the theory considered. In some sense then, the robustness of the information loss paradox is exactly what allows us to probe it, with some degree of confidence, using theories seemingly very different than the one where it was first discovered. That is to say, since the puzzle seems to emerge in any theory which has, as a consistent limit, low-energy Einstein gravity one would imagine that if this theory is a consistent theory of quantum gravity it has to capture, at least in broad strokes, the conceptual outlines of the “universal” resolution to this paradox. It is to the task of finding these universal principles that we turn in Part I of this thesis, working in the specific context of four and five dimensional black holes in supersymmetric string theory.

Another essential theme or lesson gleaned from string theory is that open degrees of freedom provide the non-perturbative completion of the theory. From black hole considerations it seems likely that gravity, in any UV completion, is an effective description of unknown (perhaps holographic) microscopic degrees of freedom and the lesson in string theory is that these degrees of freedom are non-perturbative in nature and enjoy some representation as non-Abelian degrees of freedom in a non-gravitational theory. It is of great interest to understand if and how these degrees of freedom emerge in a description where spacetime is manifest. In a weak-coupling limit we know they can be represented by branes but the interplay between non-Abelian degrees of freedom and spacetime is still quite obscure. We will not touch on this very directly in this thesis but it provides a backdrop for much of the work here and is somewhat evident in our considerations of the supergravity spectrum and its relation to black holes.

**PART II**

In a similar vein there we would like to use a vastly simplified version of string theory to extract lessons about the fundamental structure of the theory itself. Unlike Part I the
lessons learnt here are unlikely to apply to general theories of quantum gravity as they are fundamentally intertwined with supersymmetry (so only apply if the latter is an essential part of quantum gravity).

Simplified, topological versions of string theory seem to exhibit many of the same structures as their full-blown counterparts with the former being far more amenable to detailed study and treatment. For instance, closed-open duality seems to be a feature of both physical and topological string theory but this duality is far easier to demonstrate in the topological version of the theory [7]. The relationship between the closed and open version of string theory seems to provide an underpinning for addressing non-perturbative aspects of the theory simply because open strings encode non-perturbative degrees of freedom from the closed perspective. Thus by better understanding this relationship, even in a simplified version of the theory (that is at least a derivative of the full physical theory) we may learn something about the way the full, non-perturbative version of string theory should be formulated.

Another non-perturbative duality is the relationship between strings and membranes in string/M-theory. A potential role for this duality has also recently emerged in topological string theory [8] as there are hints that topological A/B string theory may actually be united under the auspices of “topological M-theory”. It is with the latter that we will concern ourselves in the second part of this thesis.

Although it is not clear what “topological M-theory” should be an immediate guess would be a topological membrane sigma model on $G_2$ manifolds that reduces, for manifolds of the form $\text{CY}_3 \times S^1$, to some combination of the A and B model. Such a direct analogy with the physical theory has proven somewhat difficult from a computational point of view as membrane theories seem hard to define and work with so we focus instead on a topological string theory on $G_2$ manifolds and hope to learn how such a theory may relate to the well studied topological A and B models and what lessons such a theory may yield for physical string theory. One particular hope, unfortunately unmet, is a clear understanding of how “topological M-theory” unifies, non-perturbatively, the A and B model and how the partition functions of the latter display a wavefunction like behaviour as a consequence of this.

### 1.2 Results

Here we would like to high-light, for the readers convenience, the major results of this thesis.
ADs/CFT for Multicentered Configurations

In Chapter 3, we find a decoupling limit for a large class of solutions to solutions of $\mathcal{N} = 1$ supergravity in five dimensions (which descend to all solutions to $\mathcal{N} = 2$ supergravity in four dimensions). This limit places the centers in asymptotically $\text{AdS}_3 \times S^2 \times \text{CY}_3$ space making them amenable to study via AdS/CFT, a task which we also embark upon.

Quantization of Multicentered Configurations

In Chapters 4 and 5, we determine a procedure to quantize large families of the aforementioned solutions, correctly reproducing the degeneracy found in [9] using split attractor arguments. Our quantization is made possible by restricting to the BPS locus of the phase space (and hence also the solution space) thereby avoiding the problems generally associated with quantizing gravity. Moreover, we exploit a non-renormalization theorem relating some gravitational (closed string) quantities to open string quantities in order to simplify our computation. This quantization also provides the groundwork for several of our other results.

Macroscopic Quantum Fluctuations

In Chapter 5, we exploit the quantum structure of the solution space described above to find the following fascinating result. Certain classical solutions, corresponding to points in the phase space of the theory (see Section 2.3), do not seem to support semi-classical quantum states localized on them. This is because the symplectic form is very sparse in a region of solution space so a large family of solutions that look macroscopically quite distinct nevertheless all sit in one “unit” of phase space once the latter is quantized. Because quantum states can be localized, at most, on and around a cell in phase space (but never within one) there are no semi-classical quantum states (with low variance) describing the very different solutions within the cell.

As this is a very important discovery let us go into somewhat more detail. The solutions described above have, as a defining characteristic, arbitrary long, deep throats that are nonetheless entirely smooth without any regions of high curvature. According to standard effective field theory intuition applied to general relativity these are good saddle-points to the path integral and can be trusted as classical solutions, accessible to macroscopic observers and having vanishingly small variance in the large charge limit. In our quantization a large family of such solutions sit in a region of solution space, which can be mapped to the phase space according to the arguments of Section 2.3, and are parameterized by a small number associated with the depth of the throat. The novel physics that
emerges is that the symplectic form is somewhat insensitive to these many macroscopically different solutions; the symplectic volume associated with this region of the phase space is vanishingly small. Thus when quantum effects are included these geometries will fluctuate wildly into each other defying our intuition that classical solutions do not suffer from (significant) quantum fluctuations. Most of the differences between these geometries lie in the throat region so a general phase space density (the analog of a wavefunction) localized in this region of the phase space will have vanishingly small variance away from the throat but, at some depth down the throat, will start exhibiting wild quantum fluctuations over the macroscopic distances associated with the throat. This is exactly the kind of novel physical phenomena one might hope helps resolve puzzles such as information loss so we take this as an important qualitative lesson from string theory.

(Partial) Resolution of the Entropy Enigma

A somewhat new “enigma” that emerged after the discovery of the above mentioned multicentered black hole solutions was the realization that some two-centered black holes seem to have, in a particular region of charge space, parametrically more entropy than a single centered black hole with the same total charge. This is enigmatic from several perspectives. As the entropy of a black hole usually depends quadratically (or with some power greater than one) on its charges there is a general expectation that entropy can be maximized by having a single black hole. This is also consistent with the idea that a single thermodynamic system has more entropy than its two separated components. Finally, this seems to directly contradict the entropy scaling of the $\mathcal{N} = 2$ partition function conjectured by OSV [10].

In this thesis we find a partial resolution that essentially addresses all the points above except the last one. While a two centered configuration can have a parametrically larger entropy than a single centered one there is actually a large family of such two centered solutions and the dominant configuration in this family has all the entropy localized at just one of the two centers. In this configuration one center is entirely smooth and horizon free carrying no entropy while the other center is a black hole carrying (almost) all the entropy of the system. Thus in all cases it is a single black hole configuration that is most entropic but the kind of black hole that is dominant depends on the value of the charges. This dependence still seems to be in contradiction with OSV but various loopholes have been proposed for this in the literature [9][11].

Insufficient Entropy in Supergravity

A further application of our quantization procedure is the determination of the number of black hole states accessible via direct quantization of only supergravity fields. This is an
important question as the latter are well understood and it turns out, perhaps surprisingly, sufficient in the case of so-called “small” $\mathcal{N} = 4$ black holes (preserving 8 supercharges) to account for all the entropy of the black hole. As a result there was some hope that something similar might occur for more realistic $\mathcal{N} = 2$ black holes, thereby providing access to the spacetime structure of a generic black hole microstate. The latter is of course interesting as it promises to shed light on the information loss paradox and indeed any quantum process associated with a black hole.

Our results, however, suggest that we are not so fortunate and that supergravity can only account for a parametrically small fraction of the black hole entropy. In particular, while the entropy of a large black hole grows as the square-root of the quartic invariant of the charges, the entropy of supergravity states grows only as a cube-root. That supergravity is capable of providing an exponential number of states justifies the initial hope at least but the difference in powers makes the latter very subleading at large charge.

**DEFINITION AND SPECTRUM OF OPEN $G_2$ STRINGS**

In Part II we extend the results of [12] by defining the open version of topological string theory on $G_2$ manifolds. The branes in this theory turn out to be the same as those in a physical theory on $G_2$ manifolds, namely associative three cycles and co-associative four cycles as well as zero branes and branes wrapping the whole manifold. We determine the spectrum of open string excitations for strings with any of these boundary conditions and find that they correspond to gauge fields on the branes and scalars encoding calibration-preserving fluctuations in the transverse space.

**WORLDVOLUME THEORIES ON TOPOLOGICAL $G_2$ BRANES**

Extending the above results we study the open string field theory for these topological theories and see that they reduce to Chern-Simons-like theories on associative and co-associative membranes in $G_2$ manifolds. In fact both these theories descend from a seven-dimensional theory, defined on the entire manifold, which is essentially the $G_2$ analog of holomorphic Chern-Simons theory.

For associative cycles this theory is nothing more than ordinary Chern-Simons theory coupled to essentially non-interacting matter (scalar degrees of freedom from the normal modes which couple minimally to the Chern-Simons gauge field). This is a powerful result as it provides a non-perturbative definition of the open theory which may, as in [7], be related to the closed version of the theory via a geometric transition. One could then hope to put the closed theory, presently defined only at genus zero, on more firm footing.
One-loop Comparison of Hitchin Functional and $G_2$ String

Although we do not include detailed results from [2] in this thesis, we do review it briefly so we include the results in this list. The primary result is that, unlike the case of the B-model [13], the one-loop partition functions of the (closed) topological $G_2$ string and the generalized Hitchin functional for $G_2$ manifolds do not match but, in fact, differ by power of the Ray-Singer torsion of the manifold.

In a somewhat different vein we do find agreement between the degrees of freedom of the dimensional reduction of the (original and the generalized) Hitchin functional on $G_2$ manifolds of the form $CY_3 \times S^1$ and the six-dimensional Hitchin functionals of [13]. This may seem to contradict the idea that topological M-theory, and indeed the $G_2$ Hitchin functional itself, encode both Kähler and complex structure deformations of the CY since the Hitchin functionals considered in [13] only incorporate complex structure deformations. A possible explanation for this discrepancy is the fact that the Kähler deformations are generally encoded in a purely topological gauge theory (such as that of [14]) and we neglect the topologically non-trivial sector of the $G_2$ theory in our analysis. A more careful analysis of the $G_2$ functional may therefore be needed to find a unification of the A and B models.