You must keep money moving

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You Must Keep Money Moving

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Abstract
Physics teachers often like to discuss physics concepts with their students on the basis of simple demonstration experiments. In this article we discuss an experiment for which one only needs a few coins of the same denomination and a wooden stick: the coins are laid alongside the stick on a horizontal surface, the stick is rotated about one of its endpoints, and when the stick stops, the coins move away and, once they have stopped moving, the coins seem to lie on a parabola. The students' task is to validate and understand this. They will discover by video analysis that the experimental results may not be as obvious as initially thought and that the most plausible explanation of the real phenomenon must be slightly modified to get measurements and theory in agreement. This does not just lead to a considerable amount of effort either correcting or verifying experimental flaws, but it actually gives ample opportunity to experience how physics, mathematics, and technology use can jointly help to achieve a good understanding of the real phenomenon.

(*a*) (*b*) (*c*)

Fig. 1. Frames from a video-recorded experiment with moving money.
(*a*) Coins leaving the wooden stick. (*b*) Moving coins. (*c*) End position of coins.

The experiment

The experiment discussed in this article is simple and takes little time (the second author has often used it in a 50-minutes physics lesson at the secondary school where he works as physics teacher and he always had ample time to do the experiment and to have a good and deep discussion on the physics with his students). Six coins of the same denomination are lined up, at equal distances, against a wooden stick. This stick, making a circular movement, pushes the coins over a flat piece of paper on a horizontal table. The origin of this circle is at one side of the row of coins (in this case, near the upper left corner of paper in the pictures in Figure 1 taken from a video recording of the experiment). The stick stops and the coins leave the stick (Figure 1a), having a velocity perpendicular to the stick. Once the coins have stopped moving they seem to lie on a parabola; at least Figure 1c suggests this conclusion. The students' task is to validate and explain this.

The explanation

The mathematics and physics needed for the explanation of the phenomenon is not beyond high school level. It might go in the following way and lay the foundation of experimental verification. At the point when the coins leave the stick, friction comes into play. We will assume that the force of friction does not depend on the velocity of a coin and is the same for all coins because they are similar, but this could be a further point of discussion. The work done by the frictional force F on a coin that moves over a distance s after leaving the stick is given by the expression $W = F \cdot s$. This means that the distance that a coin travels after leaving the stick is proportional to the kinetic energy at the moment that the coin loses contact with the stick. In symbolic form: let v denote the velocity of the coin at the moment that it leaves the stick, then

$$F \cdot s = \frac{1}{2} m v^2 \qquad (1)$$

Since F and m are constants in Equation (1), s is proportional to v^2. Because the stick makes a circular movement, the velocity v of each coin is proportional to the radial distance r of the coin to the centre of rotation, say $v = \omega r$. This means that the distance travelled by a coin is proportional to the square of its radial distance to the centre of rotation. In other words, we expect that the coins, once they have stopped moving, lie on a parabola.

Are we sure and how can we find out?

One way to monitor the motion of the coins and to check whether the coins, once they have stopped moving, really lie on a parabola is to carry out a video analysis. We have used the video tool of the Coach 6 computer learning and authoring environment [1,2]. This tool choice has to do with the following three practical problems that come quickly into play:

1. Depending on the depth of the analysis that one has in mind, the motion is too quick for recording with a normal camera or web cam;
2. In practice, it may not be possible to make a movie from the top view and we obtain a video in perspective distortion.
3. Manual collection of position data of six coins during their motion (approximately for 1 second) may be a bit time-consuming, boring, and error-prone.

Although we have successfully analysed the experiment on the basis of a video clip made with a webcam at 30 frames per second, our experience was that for the determination of the velocity curves it was better to have more data points available. We solved this problem by using a high-speed camera: we used one in the low budget range to record at a speed of 100 frames per second. In 2008, with the advent of the CASIO Exilim high-speed cameras, the costs of a decent camera that can record at a speed of 1000 fps has dropped to about 600 US$. So, a high-speed camera is not unreachable anymore at consumer level and at high school level.

The second problem can be solved by correction of the perspective distortion of a plane of motion (see Figure 2). Our plane of interest is the rectangular paper on the horizontal table on which the coins move. In reality the edges of the piece of paper are two pairs of parallel lines. However, this is not the case in the video clip shown in Figure 2a. The image transformation that restores this property is determined by mapping the four corners of the rectangular paper with a projective transformation to the corners of a rectangle in a new image. The frames in the video clip are all transformed such that the overlay becomes a rectangle. Actually, the orthogonality of two pairs of parallel lines determines the image rectification up to an unknown aspect ratio. This scaling can be specified in Coach 6, if necessary. We have rescaled the rectified video clip shown in Figure 2b to the one shown in Figure 1 in order to make dimensions better match reality, i.e., to obtain a video clip in which the paper has been rescaled to real proportionality.

This rectification process is within the capabilities of high school students: we have seen fourteen-years-old pupils in prevocational secondary school doing this in the classroom without any difficulties [3].

(*a*) (*b*)

Fig. 2. Video clip before (*a*) and after (*b*) correction of perspective distortion.

Collection of data about the motion of six coins in a recorded video clip that consists of 90 video frames is more easily done by tracking the motion of the coins and automatically recording the coordinates of the objects in subsequent frames of the video clip. The algorithm used in the tracking filter in Coach 6 is composed of two parts: (1) finding the best match of a given model template in a subsequent frame, i.e., locating the area that resembles most a specified area, and (2) limiting the search area in order to reduce computing time or to avoid ambiguity. The template that will be tracked and is selected by the user at the start of the tracking process has been chosen to be an area bounded by a circle with a user-specified radius. The search area for the subsequent frame has been chosen to be a rectangle, of user specified dimensions, centred on the position that is found for the current frame. This explains the circular areas and rectangular boxes close to the coins in the video window in Figure 3.

Correction of perspective distortion and point-tracking of objects in video measurement are powerful facilities of the video tool of Coach 6 and many applications have been reported [3,4] that currently surpass in strength other popular video analysis tools for education like VideoPoint [5] or Vernier's Logger Pro 3 [6].

In what follows, we will check two statements made in the previously described model for the experiment.

Statement 1. *The stick stops, and the coins leave the stick, having a velocity perpendicular to the stick.*

Because there are no other forces besides frictional force, the track of each coin will be a straight line once it has left the stick. Under these circumstances it seems natural to take the centre of rotation as the origin of the coordinate system and to align the *x*-axis with the stick when it stops moving and the coins are leaving the stick. This would mean that we align the coordinate axis with the front side of the wooden stick as shown in Figure 1a. But is it then not strange that the coins seem to move parallel to the edge of the paper? This would mean that the above statement is incorrect or that the experiment was not carried out carefully enough. Using the selected coordinate settings and the feature of point tracking, we get the next screen shot of part of a Coach 6 activity (Figure 3). Please note that the *x*-coordinate axis shown in Figure 3 is not aligned anymore with the wooden stick because the frame shows the coins in the final position and the wooden stick has been moved a little bit further after the coins had left it (you cannot manually stop the stick at once).

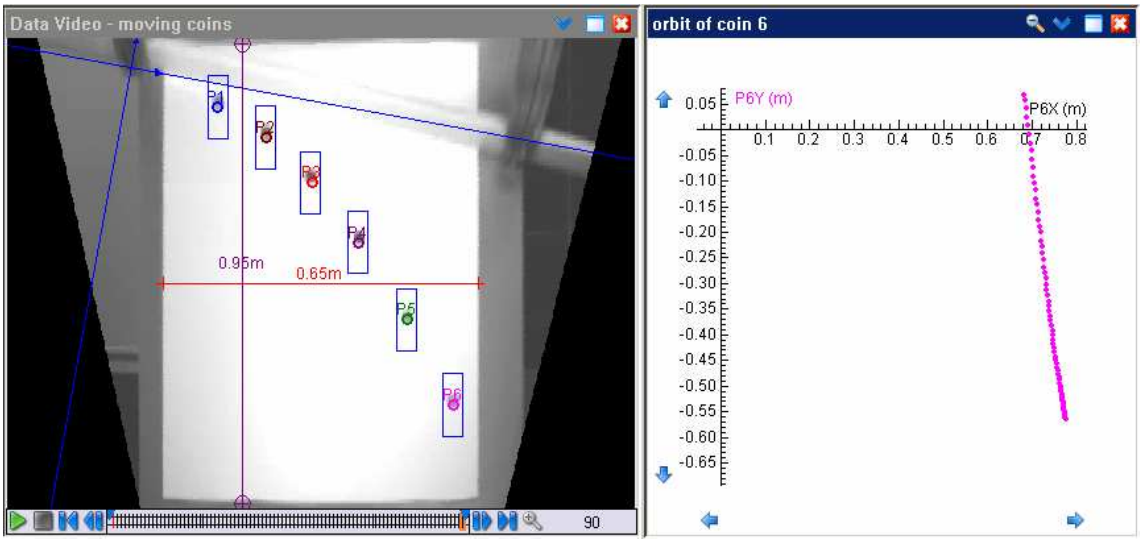
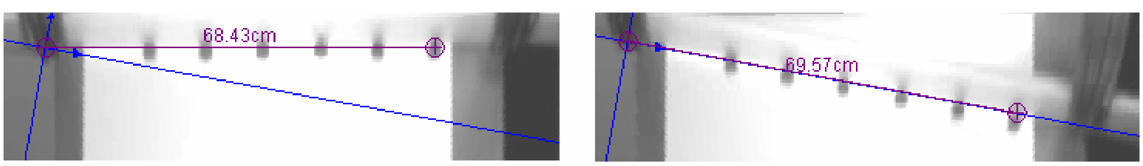


Fig. 3. Tracking of coins and the orbit of the sixth coin.

To the left, you see the axis system pointed out just like the stick was at the moment that the coins lost contact. To the right, you see the track of the sixth coin. Clearly, with these coordinate settings the *x*-component of the velocity is not equal to zero. This means that the velocity at the moment when the coin lost contact was not perpendicular to the stick.

We can take a closer look on this velocity component parallel to the stick. We rewind the movie and play it frame by frame. While the stick is rotating and is in contact with the coins, the coins turn out to move along the stick. For example, one can measure the distance between the sixth coin and the origin in various frames. Analyzing the frame at the start of the experiment (Figure 4a) and the frame when the coins leave the stick (Figure 4b), one finds a slightly different value for the distance from the coin to the origin: values are 68.4 cm and 69.6 cm, respectively. It is a small difference, but not an experimental flaw.



(*a*) (*b*)

Fig. 4. Measurement of the distance of the 6th coin in different frames of the video clip. (*a*) coins at the start of the experiment. (*b*) coins leaving the wooden stick.

This means that the coins have indeed a velocity parallel to the stick. In other words, the velocity when a coin loses contact with the stick is in reality not perpendicular to the stick. An open question remains why and how this motion of the coins along the rotating stick takes place. What forces are involved? Do coins roll or glide outwards? Has the rim of a coin everywhere the same roughness? And so on. This may elicit a nice classroom discussion on experimental setups and physics concepts involved in the motion study.

Statement 2. … *we expect that the coins, once they have stopped moving, lie on a parabola.*

The first thing we do is to orientate the coordinate system in the video window such that the 6th coin moves in a straight vertical line. Figure 5 shows clearly that this horizontal axis is not aligned to the orientation of the stick at the moment that the coins loose contact. Our new choice of coordinate system is driven by mathematics, and less by physics or the experiment. We only want to achieve that the coins move in a vertical line. Because this coordinate system in Figure 5 is only rotated about 2.5 degrees with respect to the paper sheet we can understand that the human eye cannot distinguish between the motion with respect to this coordinate system and the motion with respect to the paper sheet. The coins only appear to be moving parallel to the edge of the paper, but in reality they move perpendicular to a slightly rotated coordinate system.

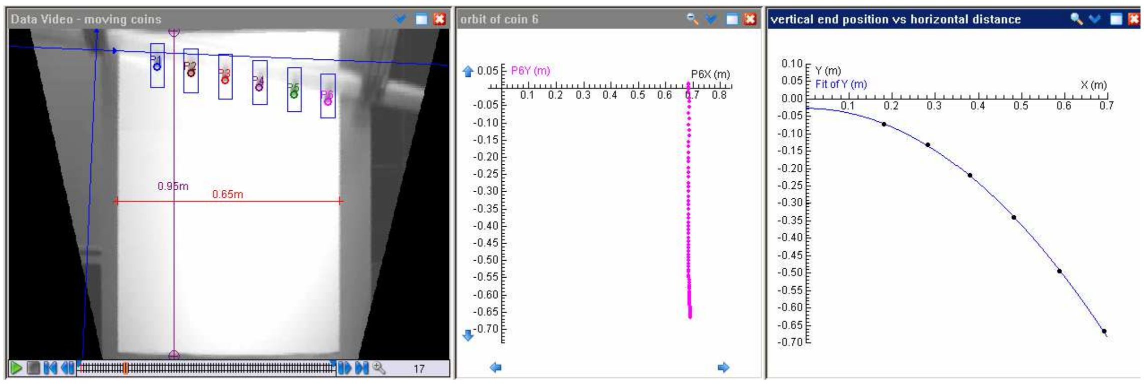


Fig. 5. A more convenient choice of the coordinate system.

The diagram to the right in Figure 5 shows the final positions of the six coins in the new coordinate system. The coins seem to lie on a parabola and regression analysis supports this view. The curve in the diagram shows the best quadratic fit calculated by the computer program (unit of length = meter):

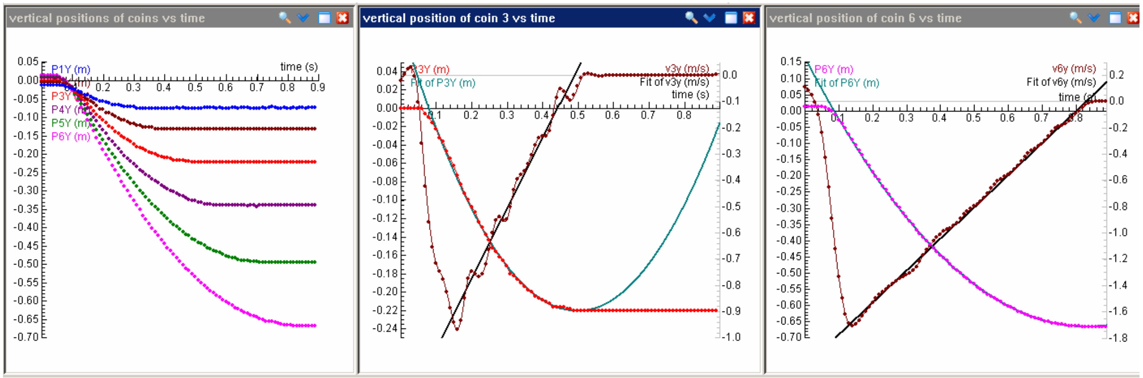
$$y = \text{-}1.3414x^2 - 0.0009x - 0.0271,$$

or in factorized form

$$y = -1.3414(x + 0.0003)^2 - 0.0271.$$

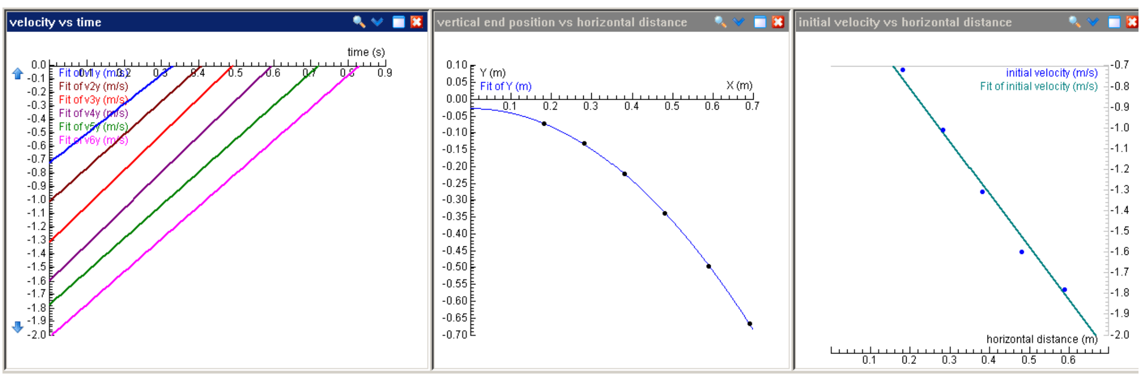
Translation of the origin of the current coordinate system over the vector (-0.0003, -0.0271) leads to a simple relationship of the form $y = -c \cdot x^2$. But it would be mere luck if we had immediately chosen the coordinate system as such. Actually we may expect some vertical shift because we measured the position of the front side of the moving coins. In our case, a 1-euro coin, which is the currency in many countries of Europe, has a diameter of 0.023 m.

The position-time (*y*-*t*) graphs of the coins, as long as they move on the table away from the stick, also look like parabolas. See the diagrams in Figure 6b and 6c for the quadratic regression curves of the 3rd and the 6th coin, respectively. In each diagram is also displayed the numerical derivative of the vertical position of the coin together with the linear regression curve of the velocity as long as the coin moves on the table away from the stick. This numerical derivative is computed via automatic spline smoothing to get the best results. From the position-time graph as well as from the velocity-time graph we can determine at what time the particular coin stopped moving. A good discussion point for students is to find out which method works best and for what reason. An explanation of the parabolic shape of the *y*-*t* graph describing the motion of a coin once it has left the stick may in our opinion be included in classroom discussions too.



(*a*) (*b*) (*c*)

Fig. 6. (*a*) *y-t* graphs of all moving coins.
(*b*) *y-t* and *v-t* graphs of the 3rd coin.
(*c*) *y-t* and *v-t* graphs of the 6th coin.



(*a*) (*b*) (*c*)

Fig. 7. (*a*) *v-t* graphs of all moving coins.
(*b*) a quadratic fit of the final positions of the coins.
(*c*) a linear fit of the initial velocity plotted against the horizontal distance.

The diagram in Figure 7c is the scatter plot of the initial velocities, i.e., the velocities when the coins leave the stick, against the radial distance to the centre of rotation. Indeed a straight line fits these data well and it confirms that the velocity of a coin at the time that it leaves the stick is proportional to the radial distance. in Figure 7a we show the linear approximations of the velocity-time curves of the coins in one diagram: with a few exceptions it seems that the coins move such that their velocity curves during motion away from the stick are parallel equidistant lines, as theory could explain. This is also reflected in the diagram in Figure 7b, which shows a quadratic fit of the final positions of the coins.

Various reasons could be given for the small differences in parallelism of the line fits of the velocity-time curves and for the deviations in the parabolic arrangement of the coins at the end of their motion. The circumference edge of a 1-euro coin has six alternating segments, viz., three smooth and three finely ribbed segments, but did we place each coin with a ribbed segment against the stick? Maybe frictional forces were not equally large for all coins because there might have been a difference between head or tail down, some coins may have been greasier than other, and the paper underground was perhaps not a surface with uniform roughness. With hindsight, did we pay attention to this in the experiment and would it have made a difference?

Reality may turn out to be more complex than can be foreseen and theory predicts. We think it is a good experience for students to deal with such subtle issues in linking theory and practice, and that it is wise to discuss at least these issues on the basis of their knowledge and experience. It may also elicit designs of other experimental setups, which one expects to allow more simple explanations that are less obscured by technical details. For example, one could think of and carry out an experiment with a wooden stick whose point of rotation is fixed by a perpendicular axis and that is stopped by a rigid barrier at a specific location. Alternatively, one can experiment with a wooden stick in which semicircular grooves prevent the coins from gaining a radial velocity. Anyway, it turned out that our data were still in good agreement with theory after we had a close look, with the help of technology, on what really occurred in the experiment. This was not mere luck because the experiment can be repeated over and over again and always the same kinds of results are obtained.

Conclusion

The experiment was simple and it confirmed at first sight the predictions made on the basis of mathematics and physics taught at high school level. Taking a closer look with the help of appropriate ICT tools, brought up some peculiarities initiating a search for a more satisfactory explanation. A more thorough investigation of the phenomenon by video analysis and a sound reasoning based on physics and mathematics that was still a high school level brought measurements and theory again in agreement.

As in every good physics experiment at high school, the phenomenon under investigation and the modelling of it offered great opportunities to students and teachers to discuss physics concepts, scientific methods of study, and the importance of experimental validation in science. Both the physics of uniform motion and of rotational motion were present in the study of moving coins. Interpreting motion graphs, numerical differentiation, and regression techniques were some of the mathematical aspects that came into play in the discussion of the problem situation. In conclusion, physics and mathematics were natural ingredients in the students' process of coming to grips with the posed problem.

References

[1] *Coach 6* is a versatile computer learning and authoring environment that provides integrated tools for MBL-based measurement, control activities, digital image and video analysis, and computer modeling. It has been translated into many languages, it is used in many countries, and it is distributed by the CMA Foundation. For more information: www.cma.science.uva.nl

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