

## Overview

The contents of this document contain the Online Supplementary materials to "Interrater reliability for Multilevel Data: A Generalizability Theory Approach". Additional materials, such as the Stan models, the R syntax from the simulations and the applied example, and the data used for the applied example are available through the Open Science Framework <URL masked for review>.

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**S1. Population Values ICCs Simulation 1**

Table 1

*Population Values of Subject-Level ICCs across Conditions*

$k$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
	ICC <sub>s</sub> (A, 1)				ICC <sub>s</sub> (A, k)			
2	0.60	0.50	0.60	0.50	0.75	0.67	0.75	0.67
5	0.60	0.50	0.60	0.50	0.88	0.83	0.88	0.83
10	0.60	0.50	0.60	0.50	0.94	0.91	0.94	0.91
	ICC <sub>s</sub> (C, 1)				ICC <sub>s</sub> (C, k)			
2	0.67	0.67	0.67	0.67	0.80	0.80	0.80	0.80
5	0.67	0.67	0.67	0.67	0.91	0.91	0.91	0.91
10	0.67	0.67	0.67	0.67	0.95	0.95	0.95	0.95

ICC = Intraclass correlation;  $k$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects.

Table 2  
*Population Values of Cluster-Level ICCs across Conditions*

$k$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
	ICC <sub>c</sub> (A, 1)				ICC <sub>c</sub> (A, $k$ )			
2	0.33	0.20	0.61	0.43	0.50	0.33	0.76	0.60
5	0.33	0.20	0.61	0.43	0.71	0.55	0.89	0.79
10	0.33	0.20	0.61	0.43	0.83	0.71	0.94	0.88
	ICC <sub>c</sub> (C, 1)				ICC <sub>c</sub> (C, $k$ )			
2	0.50	0.50	0.76	0.76	0.67	0.67	0.86	0.86
5	0.50	0.50	0.76	0.76	0.83	0.83	0.94	0.94
10	0.50	0.50	0.76	0.76	0.91	0.91	0.97	0.97

ICC = Intraclass correlation;  $k$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects.

Table 3

*Relative Difference in Population Values of Conflated ICCs compared to Multilevel ICCs across Conditions*

$k$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
Subject-Level								
	ICC <sub>s</sub> (A, 1)				ICC <sub>s</sub> (A, k)			
2	2.51	4.88	9.27	15.35	1.00	1.00	5.60	5.60
5	2.51	4.88	9.27	15.35	1.00	1.00	5.60	5.60
10	2.51	4.88	9.27	15.35	1.00	1.00	5.60	5.60
	ICC <sub>s</sub> (C, 1)				ICC <sub>s</sub> (C, k)			
2	1.45	3.05	5.22	9.25	0.55	0.55	3.03	3.03
5	0.64	1.44	2.26	4.22	0.24	0.24	1.28	1.28
10	0.33	0.76	1.16	2.21	0.12	0.12	0.65	0.65
Cluster-Level								
	ICC <sub>c</sub> (A, 1)				ICC <sub>c</sub> (A, k)			
2	57.03	156.80	6.26	35.21	-6.27	-6.27	-11.20	-11.20
5	57.03	156.80	6.26	35.21	-6.27	-6.27	-11.20	-11.20
10	57.03	156.80	6.26	35.21	-6.27	-6.27	-11.20	-11.20
	ICC <sub>c</sub> (C, 1)				ICC <sub>c</sub> (C, k)			
2	33.02	98.04	3.53	21.22	-3.47	-3.47	-6.07	-6.07
5	14.59	46.16	1.53	9.68	-1.48	-1.48	-2.56	-2.56
10	7.56	24.52	0.79	5.08	-0.76	-0.76	-1.30	-1.30

ICC = Intraclass correlation;  $k$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects. Let  $ICC_{conf}$  denote the conflated ICC, and let  $ICC_{ML}$  the multilevel ICC. Relative difference is computed as  $\frac{ICC_{conf} - ICC_{ML}}{ICC_{ML}} * 100$ .

**S2. Results Simulation 1**

Table 4  
*Relative MAP Bias of Subject-Level ICCs across Conditions*

$N_c$	$N_s$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>s</sub> (A, 1)				ICC <sub>s</sub> (A, k)			
20	10	2	-0.03	0.06	-0.02	0.06	-0.01	0.04	-0.01	0.04
		5	-0.01	0.03	0.00	0.02	0.00	0.01	0.00	0.01
		10	-0.01	0.01	-0.01	0.01	0.00	0.00	0.00	0.00
	30	2	0.00	0.10	0.02	0.10	0.00	0.07	0.01	0.07
		5	0.00	0.03	0.01	0.03	0.00	0.01	0.00	0.01
		10	0.00	0.02	0.00	0.02	0.00	0.00	0.00	0.00
40	10	2	-0.02	0.06	-0.02	0.06	-0.01	0.05	-0.01	0.05
		5	0.00	0.03	0.00	0.02	0.00	0.01	0.00	0.01
		10	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00
	30	2	0.00	0.09	0.02	0.09	0.00	0.06	0.01	0.06
		5	0.00	0.03	0.01	0.03	0.00	0.01	0.00	0.01
		10	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00
			ICC <sub>s</sub> (C, 1)				ICC <sub>s</sub> (C, k)			
20	10	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	30	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
40	10	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	30	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects. Relative bias was computed as  $\frac{\bar{\theta} - \theta}{\theta}$ , where  $\bar{\theta}$  denotes the average MAP estimate of a parameter (or derived ICC) across replications in a condition, and  $\theta$  denotes the population parameter in that condition.

Table 5  
*Relative MAP Bias of Cluster-Level ICCs across Conditions*

$N_c$	$N_s$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>c</sub> (A, 1)				ICC <sub>c</sub> (A, k)			
20	10	2	-0.54	-0.41	-0.22	-0.11	-0.42	-0.31	-0.13	-0.05
		5	-0.25	-0.23	-0.06	-0.05	-0.11	-0.13	-0.01	0.00
		10	-0.14	-0.15	-0.04	-0.04	-0.02	-0.04	0.00	0.00
	30	2	-0.53	-0.38	-0.14	-0.04	-0.4	-0.29	-0.06	0.00
		5	-0.15	-0.12	-0.04	-0.02	-0.05	-0.04	-0.01	0.00
		10	-0.08	-0.07	-0.03	-0.02	-0.01	-0.01	0.00	0.00
40	10	2	-0.48	-0.33	-0.13	-0.02	-0.36	-0.25	-0.06	0.01
		5	-0.18	-0.16	-0.04	-0.01	-0.06	-0.06	-0.01	0.01
		10	-0.09	-0.10	-0.02	-0.02	-0.01	-0.02	0.00	0.00
	30	2	-0.41	-0.26	-0.05	0.04	-0.27	-0.15	-0.01	0.06
		5	-0.10	-0.05	-0.02	0.01	-0.03	0.00	0.00	0.02
		10	-0.06	-0.04	-0.01	-0.01	-0.01	0.00	0.00	0.00
			ICC <sub>c</sub> (C, 1)				ICC <sub>c</sub> (C, k)			
20	10	2	-0.13	-0.13	-0.02	-0.02	-0.08	-0.08	-0.01	-0.01
		5	-0.09	-0.09	0.00	-0.01	-0.02	-0.02	0.00	0.00
		10	-0.05	-0.05	0.00	0.00	0.00	0.00	0.00	0.00
	30	2	-0.16	-0.16	-0.01	-0.02	-0.10	-0.11	0.00	-0.01
		5	-0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00
		10	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
40	10	2	-0.09	-0.08	0.00	0.00	-0.05	-0.05	0.00	0.00
		5	-0.04	-0.04	0.00	0.00	-0.01	-0.01	0.00	0.00
		10	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00
	30	2	-0.03	-0.03	0.01	0.00	-0.01	-0.02	0.01	0.00
		5	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
		10	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects. Relative bias was computed as  $\frac{\hat{\theta} - \theta}{\theta}$ , where  $\hat{\theta}$  denotes the average MAP estimate of a parameter (or derived ICC) across replications in a condition, and  $\theta$  denotes the population parameter in that condition.

Table 6  
*Percentage of 95% BCI Coverage of Subject-Level ICCs across Conditions*

$N_c$	$N_{s:c}$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>s</sub> (A, 1)				ICC <sub>s</sub> (A, k)			
20	10	2	95	98*	95	98*	95	98*	95	98*
		5	95	96	95	96	95	96	95	96
		10	95	95	95	95	95	95	95	95
	30	2	97*	98*	97*	98*	97*	98*	97*	98*
		5	94*	95	94	95	94*	95	94	95
		10	94	96	95	95	94	96	95	95
40	10	2	96*	98*	96	98*	96*	98*	96	98*
		5	95	97*	95	97*	95	97*	95	97*
		10	95	95	95	95	95	95	95	95
	30	2	97*	99*	98*	99*	97*	99*	98*	99*
		5	95	97*	95	97*	95	97*	95	97*
		10	95	96	96	96	95	96	96	96
			ICC <sub>s</sub> (C, 1)				ICC <sub>s</sub> (C, k)			
20	10	2	94	95	94	94	94	95	94	94
		5	95	95	95	95	95	95	95	95
		10	95	95	95	95	95	95	95	95
	30	2	97*	96	97*	96*	97*	96	97*	96*
		5	96	95	96	95	96	95	96	95
		10	95	94	94	94	95	94	94	94
40	10	2	96	96	96	96	96	96	96	96
		5	94	94	95	94	94	94	95	94
		10	94	94	94	94	94	94	94	94
	30	2	96	96	96	96	96	96	96	96
		5	94	95	95	95	94	95	95	95
		10	95	95	95	95	95	95	95	95

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_s^2$  = Variance of subject effects; \* Outside 95% Agresti-Coull confidence interval.



Table 7  
*Percentage of 95% BCI Coverage of Cluster-Level ICCs across Conditions*

$N_c$	$N_{s:c}$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>c</sub> (A, 1)				ICC <sub>c</sub> (A, k)			
20	10	2	96	98*	94	97*	96	98*	94	97*
		5	95	96	96	96	95	96	96	96
		10	96	96	96*	96	96	96	96*	96
	30	2	95	98*	94	97*	95	98*	94	97*
		5	95	96	95	96	95	96	95	96
		10	95	95	95	95	95	95	95	95
40	10	2	94	96	95	97*	94	96	95	97*
		5	94	95	95	96*	94	95	95	96*
		10	94	94	95	96	94	94	95	96
	30	2	96	97*	96	98*	96	97*	96	98*
		5	96	97*	96*	97*	96	97*	96*	97*
		10	95	95	95	95	95	95	95	95
			ICC <sub>c</sub> (C, 1)				ICC <sub>c</sub> (C, k)			
20	10	2	97*	96*	94*	94*	97*	96*	94*	94*
		5	95	95	95	95	95	95	95	95
		10	96	96	96	96*	96	96	96	96*
	30	2	96	96	94	95	96	96	94	95
		5	95	95	96	95	95	95	96	95
		10	95	95	95	96	95	95	95	96
40	10	2	95	95	94	94	95	95	94	94
		5	95	94	96	95	95	94	96	95
		10	95	94	95	95	95	94	95	95
	30	2	94	95	94	95	94	95	94	95
		5	95	94	96	96	95	94	96	96
		10	95	95	95	95	95	95	95	95

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_s^2$  = Variance of subject effects; \* Outside 95% Agresti-Coull confidence interval.

Table 8  
*Relative Efficiency of Subject-Level ICCs across Conditions*

$N_c$	$N_s$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>s</sub> (A, 1)				ICC <sub>s</sub> (A, k)			
20	10	2	1.98	1.51	1.96	1.51	2.15	1.62	2.13	1.62
		5	1.37	1.22	1.37	1.22	1.58	1.40	1.58	1.39
		10	1.13	1.08	1.12	1.08	1.21	1.20	1.20	1.19
	30	2	2.22	1.61	2.22	1.62	2.39	1.72	2.39	1.73
		5	1.37	1.18	1.38	1.18	1.59	1.37	1.59	1.37
		10	1.15	1.07	1.14	1.07	1.24	1.18	1.24	1.18
40	10	2	2.15	1.59	2.13	1.59	2.33	1.71	2.31	1.71
		5	1.44	1.25	1.44	1.25	1.66	1.43	1.66	1.44
		10	1.21	1.14	1.20	1.14	1.30	1.26	1.29	1.26
	30	2	2.24	1.66	2.25	1.66	2.44	1.78	2.45	1.79
		5	1.46	1.24	1.46	1.24	1.69	1.44	1.70	1.43
		10	1.24	1.15	1.24	1.15	1.35	1.28	1.35	1.28
			ICC <sub>s</sub> (C, 1)				ICC <sub>s</sub> (C, k)			
20	10	2	0.98	0.99	0.98	0.98	0.98	0.99	0.98	0.98
		5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		10	1.01	1.02	1.02	1.02	1.01	1.01	1.01	1.01
	30	2	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
		5	1.02	1.02	1.02	1.02	1.01	1.01	1.02	1.02
		10	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
40	10	2	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03
		5	0.99	0.99	0.98	0.99	0.99	0.99	0.98	0.99
		10	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97
	30	2	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		5	1.03	1.03	1.03	1.02	1.03	1.03	1.03	1.02
		10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects. Relative efficiency was computed as the ratio of the average posterior  $SD$  relative to the  $SD$  of the posterior means. Preferably, this ratio equals 1.

Table 9  
*Relative Efficiency of Cluster-Level ICCs across Conditions*

$N_c$	$N_s$	$K$	$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$		$\sigma_c^2 = 0.16$		$\sigma_c^2 = 0.50$	
			$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$	$\sigma_r^2 = 0.16$	$\sigma_r^2 = 0.50$
			ICC <sub>c</sub> (A, 1)				ICC <sub>c</sub> (A, k)			
20	10	2	1.38	1.26	1.40	1.24	1.41	1.29	1.42	1.26
		5	1.06	1.02	1.11	1.06	1.09	1.06	1.16	1.12
		10	1.04	1.04	1.04	1.04	1.00	1.01	0.97	1.00
	30	2	1.40	1.26	1.53	1.32	1.43	1.29	1.58	1.37
		5	1.10	1.04	1.15	1.07	1.09	1.06	1.22	1.15
		10	1.03	1.01	1.03	1.02	0.96	0.99	1.01	1.03
40	10	2	1.33	1.18	1.57	1.31	1.37	1.22	1.68	1.39
		5	1.09	1.03	1.20	1.10	1.11	1.07	1.34	1.22
		10	1.01	0.99	1.05	1.04	0.95	0.97	1.06	1.07
	30	2	1.47	1.27	1.78	1.43	1.52	1.32	1.91	1.53
		5	1.13	1.05	1.24	1.11	1.21	1.12	1.42	1.26
		10	1.05	1.01	1.10	1.06	1.06	1.06	1.15	1.13
			ICC <sub>c</sub> (C, 1)				ICC <sub>c</sub> (C, k)			
20	10	2	1.14	1.14	0.96	0.97	1.21	1.21	0.97	0.97
		5	1.00	0.99	0.92	0.93	1.05	1.04	0.88	0.88
		10	0.99	0.99	0.94	0.93	1.00	0.99	0.81	0.77
	30	2	1.10	1.08	1.02	1.02	1.14	1.13	1.03	1.03
		5	0.97	0.97	0.97	0.96	0.90	0.90	0.92	0.92
		10	0.99	0.99	0.96	0.97	0.85	0.84	0.90	0.91
40	10	2	1.03	1.03	0.97	0.98	1.06	1.06	0.96	0.97
		5	0.97	0.96	0.96	0.96	0.93	0.92	0.92	0.91
		10	0.95	0.94	0.97	0.98	0.87	0.84	0.91	0.92
	30	2	1.01	1.00	1.00	1.00	1.02	1.01	1.00	1.00
		5	0.99	0.99	0.99	1.00	0.94	0.94	0.97	0.98
		10	1.01	1.00	1.00	1.00	0.93	0.93	0.97	0.97

ICC = Intraclass correlation;  $N_c$  = Number of clusters;  $N_s$  = Number of subjects per cluster;  $K$  = Number of raters;  $\sigma_c^2$  = Variance of cluster effects;  $\sigma_r^2$  = Variance of rater effects. Relative efficiency was computed as the ratio of the average posterior  $SD$  relative to the  $SD$  of the posterior means. Preferably, this ratio equals 1.

**S3. Results Simulation 2**

Table 10  
*Relative MAP Bias of ICCs Across Conditions*

$k_s$	$K$	$ICC_s(A, 1)$	$ICC_s(A, k)$	$ICC_s(C, 1)$	$ICC_s(C, k)$
2	5	-0.02	-0.01	0.00	0.00
	10	-0.02	-0.01	0.00	0.00
3	5	-0.02	-0.01	0.00	0.00
	10	-0.01	-0.01	0.00	0.00

  

$k_s$	$K$	$ICC_c(A, 1)$	$ICC_c(A, k)$	$ICC_c(C, 1)$	$ICC_c(C, k)$
2	5	-0.29	-0.22	-0.08	-0.05
	10	-0.26	-0.20	-0.06	-0.03
3	5	-0.21	-0.12	-0.04	-0.01
	10	-0.21	-0.12	-0.04	-0.01

ICC = Intraclass correlation;  $k_s$  = Number of raters per subject;  $K$  = Total number of raters (size of the rater pool).  
 Relative bias was computed as  $\frac{\bar{\theta} - \theta}{\theta}$ , where  $\bar{\theta}$  denotes the average MAP estimate of a parameter (or derived ICC) across replications in a condition, and  $\theta$  denotes the population parameter in that condition.

Table 11  
*Percentage of 95% BCI Coverage of ICCs Across Conditions*

$k_s$	$K$	$ICC_s(A, 1)$	$ICC_s(A, k)$	$ICC_s(C, 1)$	$ICC_s(C, k)$
2	5	95	95	96	96
	10	95	95	96	96
3	5	95	95	96	96
	10	95	95	96	96

  

$k_s$	$K$	$ICC_c(A, 1)$	$ICC_c(A, k)$	$ICC_c(C, 1)$	$ICC_c(C, k)$
2	5	96	96	95	95
	10	95	95	95	95
3	5	95	95	94	94
	10	95	95	96	96

ICC = Intraclass correlation;  $k_s$  = Number of raters per subject;  $K$  = Total number of raters (size of the rater pool).

Table 12  
*Relative Efficiency of ICCs Across Conditions*

$k_s$	$K$	$ICC_s(A, 1)$	$ICC_s(A, k)$	$ICC_s(C, 1)$	$ICC_s(C, k)$
2	5	1.29	1.36	1.03	1.04
	10	1.29	1.36	1.03	1.03
3	5	1.37	1.50	1.04	1.03
	10	1.32	1.44	1.00	1.00

  

$k_s$	$K$	$ICC_c(A, 1)$	$ICC_c(A, k)$	$ICC_c(C, 1)$	$ICC_c(C, k)$
2	5	1.09	1.11	1.04	1.07
	10	1.10	1.11	1.04	1.06
3	5	1.06	1.05	0.97	0.96
	10	1.07	1.08	1.02	1.02

ICC = Intraclass correlation;  $k_s$  = Number of raters per subject;  $K$  = Total number of raters (size of the rater pool). Relative efficiency was computed as the ratio of the average posterior  $SD$  relative to the  $SD$  of the posterior means. Preferably, this ratio equals 1.

#### S4. Maximum Likelihood Estimates and additional MCMC Estimates

##### Application

Table 13  
*Variance Decomposition as Estimated with MLE*

	Conflated	MLM
Student	1.63	1.40
Teacher	–	0.23
Rater	0.26	0.26
Student $\times$ Rater	0.48	0.44
Teacher $\times$ Rater	–	0.03

MLM = Multilevel Model.

Table 14  
*IRR coefficients as Estimated with MCMC*

	ICC(A,1)		ICC(A, $k$ )		ICC(C,1)		ICC(C, $k$ )	
	MAP	BCI	MAP	BCI	MAP	BCI	MAP	BCI
Conflated ( $k = 3$ )	.68	[.49, .75]	.87	[.74, .90]	.77	[.73, .81]	.91	[.89, .93]
Student Level ( $k = 3$ )	.66	[.46, .74]	.85	[.72, .89]	.76	[.71, .81]	.91	[.88, .93]
Teacher Level ( $k = 3$ )	.39	[.10, .68]	.71	[.24, .86]	.90	[.62, .96]	.96	[.83, .98]
Teacher Level ( $k = 5$ )	–	–	.80	[.35, .91]	–	–	.98	[.89, .99]

ICC = Intraclass correlation; MAP = maximum a posteriori; BCI = Bayesian credible interval;  $k$  = number of raters.



Table 15  
*IRR coefficients as Estimated with MLE*

	ICC(A,1)	ICC(A, $k$ )	ICC(C,1)	ICC(C, $k$ )
Conflated ( $k = 3$ )	.69	.87	.77	.91
Student Level ( $k = 3$ )	.67	.86	.76	.90
Teacher Level ( $k = 3$ )	.45	.71	.88	.96
Teacher Level ( $k = 5$ )	–	.80	–	.97

ICC = Intraclass correlation;  $k$  = number of raters.