Statistical properties of resonances in chaotic elastic cavities: time reversal invariance and feedback
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4. STATISTICS OF RESONANCES AND TIME REVERSAL RECONSTRUCTION IN ALUMINUM ACOUSTIC CHAOTIC CAVITIES WITH FEEDBACK

4.1 Introduction

This chapter addresses the case of elastic waves in a chaotic cavity where the time reversal (TR) invariance is broken by a feedback loop. Considerable effort in acoustics has been devoted to study the influence of motion in liquids on TR invariance. For example, the rotation in a vortex motion of a liquid breaks the TR invariance for the propagation of acoustic waves [10, 11, 12, 13]. An analogous phenomenon should be present in solids and would be of practical interest in such mechanical systems as blenders, airplane engines and so on.

An alternative and appealing method we devised to study the effects of TR invariance is to detect the acoustic signal at one location on the surface of the cavity and re-inject the signal after delay and amplification into another location on the surface of the cavity. Depending on the amplification (forward and/or backward) and the signal delay this feedback loop can influence the TR invariance and reciprocity in the acoustic system. The connecting cable and the amplifier introduce a delay in the feedback. When only forward waves travelling from the pick-up source through the amplifier are re-injected, the forward and backward paths for the waves are different and the reciprocity of wave propagation and therefore TR invariance of the wave equation are broken. The wave solution with negative time is no longer a solution. However, paths not travelling through the loop still have their reciprocity intact. Similar idea of one-directional wave signal transfer was explored in work [49] by Stoffregen et al about specially designed microwave billiards.

It should be mentioned that reciprocity still can be observed in the system with TR invariance broken. Reciprocity is equality of responses obtained by sending the same signal both from point A to point B and from B to A in opposite direction. The example of preserving the reciprocity in time reversal non-invariant system is a symmetric vortex in a liquid with the source and
receiving transducers located on the opposite sides of the vortex at equal distances from the center of the vortex. The wave equation has no time reversal invariance but still the reciprocity can be observed between these two points. We consider that the above-mentioned situation is never the case in our experiment. So the feedback loop considered in this thesis breaks both TR invariance and reciprocity.

The amplification coefficient $K$ in the feedback loop strongly influences the feedback process. Self oscillation in the cavity may occur for $K$ much larger than the loss in cables and piezoelectric transducers. Recently the effects of such a strong feedback in a chaotic cavity has been studied in the context of an acoustic laser (see the work [50] by Richard Weaver, Oleg Lobkis and Alexey Yamilov).

Multi channel feedback network for radio waves has been studied in [51] using S - matrix description both in case of low amplification in the feedback loop (far from self-oscillating regime) and in self-oscillating regime.

In this chapter we will study how and to what extent the efficiency of the time reversal experiment and RMT statistics (cavity resonances) are influenced by the feedback loop with $K$ remaining below the self oscillation condition. The outcome of the TR experiments and the statistical properties of the cavity spectra obtained from cavity responses for a system with feedback will be discussed further in present chapter.

Section 4.2 gives a brief description of the experimental setup by pointing main features of the experiential setup and measurement procedures with a feedback loop. Section 4.3 tells about efficiency of excitation pulse reconstruction in TR experiments with different amplification in the feedback loop. The efficiency of the excitation pulse reconstruction in TR experiment is considered a measure of TR invariance. Section 4.4 gives statistical properties (NID, NNSD and SR) obtained from experiments with the feedback loop. Summary and conclusions are given in section 4.5.

4.2 Experimental setup for studies of the cavity with feedback loop

4.2.1 The sample and transducers

For comparison with the results described in Chapter 2 we use the same aluminum Cavity Nr. 2 as used in the experiments on TR and statistics (Fig. 4.1, left shows the shape of the cavity (aluminum block)). The time reversal invariance of elastic waves travelling inside the cavity will be broken by an additional feedback loop. The schematic of the feedback loop is
The actual shape of the sample (aluminum block) used in experiment with time reversal invariance broken by a feedback loop (left in the figure), excitation pulse (top right in the figure) played by the input transducer and schematic of the feedback loop (bottom right in the figure).

Fig. 4.1: The actual shape of the sample (aluminum block) used in experiment with time reversal invariance broken by a feedback loop (left in the figure), excitation pulse (top right in the figure) played by the input transducer and schematic of the feedback loop (bottom right in the figure).

shown in Fig. 4.1. The time reversal invariance of the wave dynamics in the cavity for waves travelling through the loop is broken because the amplifier included in the feedback connection transmits only in one direction. The frequency bandwidth of the amplifier is set constant to the frequency band of approximately 10 kHz - 400 kHz and the amplification coefficient $K$ can be changed.

To reduce losses in the system a very light low mass polystyrene support is used (Fig. 4.2). Four transducers are attached to the cavity (aluminum block). Two of them are to be used for the measurement of the cavity response in the same way as described in Chapter 2 (Experiments on the cavity without breaking of the time reversal invariance). They shall be referred to further as input and output transducers. The remaining two transducers are glued to the sample and do not change the position during the measurements. These two transducers are connected to form feedback loop between them and to break the time reversal invariance of the elastic waves in the sample.
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Fig. 4.2: Picture of the sample, transducers connected to it and transducer holders.
Transducer holders hold transducers used to measure the response of the cavity to a short excitation pulse, the same as in Chapter 2, describing the experiment without the feedback loop. An additional pair of transducers is glued to the aluminum block as shown in this figure. This pair of transducers is used to connect a feedback loop between them.

(Figures 4.1 and 4.2).

The schematics of the experimental setup mainly remains the same as in Chapter 2 (Experiments on the aluminum block without breaking the time reversal invariance). The only difference is the feedback loop connected to the sample via two additional transducers.

4.2.2 Measurements with feedback

Time reversal experiments and analysis of the statistics (NID, NNSD and SR) obtained from the cavity responses are performed for several distinct positions of input and output transducers in the same way as without a feedback loop. However a such sequence of measurements (several positions of the input and output transducers) is repeated now for the feedback loop amplifier switched off and for a few values of the feedback amplification.
coefficient $K$ (Fig. 4.1).

The range of $K$ available in experiment is limited for the following reason: for small $K$ the receiving transducer gets a decaying response composed of many modes at different resonant frequencies. When increasing the amplification $K$ further and further beyond the value of 2200 eventually the regime is reached when the receiving transducer receives an oscillation at one of the resonant frequencies with high amplitude (so that other modes contribute a negligibly small output signal). In such a case, when self-oscillation is reached, the calculation of random matrix statistics is impossible. This will be discussed again in Chapter 5, the chapter about the possible random matrix model for the cavity influenced by the feedback loop.

The case of high amplification (reaching the self-oscillation regime) has been studied in [50], although not in sense of random matrix statistics.

4.3 Efficiency of excitation pulse reconstruction in time reversal experiments with a breaking of the time reversal invariance

Fig. 4.3 shows the efficiency of the time reversal pulse reconstruction as function of the time delay between the end of the oscillation track replayed backward in time and the exciting pulse. 300 $\mu$s long oscillation tracks have been used (in the same way as explained previously in chapters 1 and 2). The circle signs show the results for the feedback loop switched off. Results are given also for the same oscillation tracks replayed backwards with active feedback loop with feedback coefficients $K$ of the voltage amplifier equal to 1000 (squares) and 2200 (crosses). Fig. 4.3 shows that the active feedback loop has a negligible effect on the TR reconstruction for small time delays (0.4-0.6 ms). But the active feedback loop noticeably suppresses the reconstruction of the excitation pulse for larger time delays (0.7-1.0 ms). Such an effect of the suppression of the TR reconstruction increases with an increase of the amplification coefficient $K$ as can be seen from Fig. 4.3. So we see explicitly from Fig. 4.3 that time reversal focusing of the part of the response back into the short excitation pulse (time reversal reversal reconstruction) becomes less and less efficient with increasing feedback: the normalized amplitude of the reconstructed pulse goes down with increasing $K$.

However, for very long time delays (about 2 ms) and a large amplification coefficient $K$ the voltage spike, representing the reconstruction of the excitation pulse in TR experiment, is obscured by the background peaks. As a result the TR reconstruction peak detection algorithm, developed to be used in this thesis and described in Chapter 1, does not work properly. Also these
background noisy peaks make the determination of the reconstructed pulse amplitude less reliable for the long time delays.

Moreover for large time delays the TR experiment becomes sensitive to many minor issues like aging and temperature dependence of transducer-oil-aluminum coupling so the daily drift is influencing the amplitude of the detected reconstruction spike (if there is not enough time to repeat the experiments at different positions of source and receiver for different amplification in the feedback loop without experiencing the effect of the daily drift). For time delays of 2 ms and larger the daily drift of the amplitude of the reconstruction spike was measured as large as 10%. For the delays shorter than 1.6 ms it never exceeded 3%.

$k \leq 1$ in Fig. 4.3 is a coefficient used to scale the input signal for lower time delays, so that the reconstructed spike will not be high enough to cause the changes in sensitivity (due to the high signal amplitude) of the receiving transducer.

4.4 Statistical properties obtained from experiments with feedback

4.4.1 Division of energy between cavity waves: distributions of intensity transmission coefficients and resonance line widths

Fig. 4.4 gives NID in 60 kHz wide frequency band. The results mainly agree with an exponential distribution corresponding to the input pulse energy being shared randomly between the cavity waves.

4.4.2 Random matrix statistics of resonance sequences obtained from experiments

The NNSD for the different amplification $K$ in the feedback loop is given in Fig. 4.5, Fig. 4.6 and Fig. 4.7. It is possible to see that the feedback loop leads to changes in the distribution more significant than the error bars shown in the figures. SR for the different amplification $K$ in the feedback loop is given in Fig. 4.8. The error bars in Fig. 4.5, Fig. 4.6, Fig. 4.7 and Fig. 4.8 are based on measurements performed for different positions of source and receiver (input and output transducers) on the surface of the aluminum block. Transducers with a feedback loop connected between them remain glued to the sample at the same locations through the full cycle of measurements.

Unfortunately the trend related to the active feedback loop is not clearly visible from NNSD given in Fig. 4.5, Fig. 4.6 and Fig. 4.7. As indicated in Chapters 1 and 2 and Appendix A, a moment analysis of the distribution may
Fig. 4.3: Normalized amplitude of the excitation pulse refocused in TR experiment depending on the time delay of the reversed oscillation track with respect to the excitation pulse. Circles correspond to a blocked feedback loop (amplification coefficient $K$ is equal to zero). Squares and crosses correspond to amplification coefficients 1000 and 2200 respectively. The area hatched with diagonal lines represents the time delays for which the reconstructed pulse is very badly detectable and the daily drift of the reconstructed pulse plays certain role.
Fig. 4.4: Normalized intensity distribution (NID) in the frequency band 210 kHz ... 270 kHz for the amplification $K=2200$ in the feedback loop. Different signs correspond to different (low/high) voltage amplitude of the exciting pulse (1.25 V and 2.5 V respectively).
Fig. 4.5: NNSD for the amplification $K = 0$ in the feedback loop (points with error bars connected with lines). Poisson case (exponential distribution) is shown by solid decaying curve. GOE model distribution is shown by solid curve with a maximum. Dashed curve shows GOE model with 25% of resonances lost.
Fig. 4.6: NNSD for the amplification $K = 1000$ in the feedback loop (points with error bars connected with lines). Poisson case (exponential distribution) is shown by solid decaying curve. GOE model distribution is shown by solid curve with a maximum. Dashed curve shows GOE model with 25% of resonances lost.
Fig. 4.7: NNSD for the amplification $K = 2200$ in the feedback loop (points with error bars connected with lines). Poisson case (exponential distribution) is shown by solid decaying curve. GOE model distribution is shown by solid curve with a maximum. Dashed curve shows GOE model with 25% of resonances lost.
Fig. 4.8: SR (delta statistics) for the amplification $K=0$ in the feedback loop (points with error bars connected by the solid line) and for amplifications $K = 1000$ and $K = 2200$ (points with error bars connected by dash and dash dotted lines respectively). Theoretical predictions for uncorrelated sequence of resonances (the same as in case of randomly chosen resonance frequencies) and GOE model are shown by dashed line and dashed curve respectively.
help to clarify the difference. In particular moments, skewness and kurtosis of the experimentally determined distributions can be calculated. Skewness is a measure of the asymmetry of the distribution around the mean and kurtosis indicates the nature of the spread around the mean (whether small or large deviations of the random variable around the mean contribute to the spread around the mean). Precise definitions of these values are given in Appendix A. The moment analysis for the main model NNSD functions (GUE, GOE and Poisson or exponential) is given in detail in the Appendix A (Chapter A).

Figures 4.9 and 4.10 show that skewness and kurtosis approach closer to the values of the Poisson model (marked as 'EXP') with increasing of the influence of the feedback loop. Hence, the NNSD is approaching the case of randomly chosen resonant frequencies (Poisson or exponential NNSD).

So it is apparent from figures 4.9 and 4.10 that although the feedback loop breaks the time reversal invariance in the system, the distribution is not approaching the GUE model. However such behavior as shown in figures 4.9 and 4.10 may be explained also by the growing amount of lost resonances with increasing influence of the feedback loop. E.g. because some of the modes are inhibited by the loop, some are on the contrary amplified and many of the modes remain uninfluenced (this will be discussed again in Chapter 5, the chapter about possible random matrix model describing the resonance statistics of the cavity with feedback). So more modes are obscured by the growth of the modes that are amplified by feedback.

### 4.5 Summary on experiments with breaking of the time reversal invariance

The TR experiments show that the feedback loop suppresses the reconstruction of the excitation pulse for different time delays of the recorded (and replayed backwards) oscillation track with respect to the original excitation pulse. The suppression of the time reversal reconstruction increases with increasing amplification coefficient $K$ of the amplifier in the feedback loop. Thus time reversal experiment becomes less efficient due to increasing influence of the feedback loop.

The NID obtained from 60 kHz wide frequency band mainly agree with
Fig. 4.9: Skewness of the NNSD obtained from experimental data for zero amplification $K$ in the feedback loop (curve 1), $K = 1000$ (curve 2) and $K = 2200$ (curve 3). Every value of "experiment #" on the x axis (from 1 to 5) corresponds to certain positions of the input and output transducers on the surface of the sample. Skewness values for NNSD due to GUE, GOE and Poisson models are shown by horizontal lines marked respectively as 'GUE', 'GOE' and 'EXP'.
Fig. 4.10: Kurtosis of the NNSD obtained from experimental data for zero amplification $K$ in the feedback loop (curve 1), $K = 1000$ (curve 2) and $K = 2200$ (curve 3). Every value of "experiment #" on the x axis (from 1 to 5) corresponds to certain positions of the input and output transducers on the surface of the sample. Kurtosis values for NNSD due to GUE, GOE and Poisson models are shown by horizontal lines marked respectively as 'GUE', 'GOE' and 'EXP'.
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the exponential distribution corresponding to the input pulse energy being shared randomly between the cavity waves.

It is found also that the active feedback loop influences the NNSD statistics. The effect of the feedback loop on SR (delta statistics) is found negligibly small compared to the error bars based on measurements at several different positions of source and receiver.

It can be seen also that skewness and kurtosis of the NNSD, determined from the data of several experiments, approach closer to the values due to the Poisson model (exponential NNSD) with an increase of the influence of the feedback loop, what implies that NNSD is approaching the case of randomly chosen resonant frequencies. This means that arrangement of the resonances becomes more random with increasing influence of the feedback loop in the experiment.

However a part of this behavior may be explained also by the growing amount of lost resonances with increasing influence of the feedback loop in case if more modes are obscured by the growth of the modes that are amplified by feedback.