Statistical properties of resonances in chaotic elastic cavities: time reversal invariance and feedback
Antoniuk, O.A.

Citation for published version (APA):
Antoniuk, O. (2011). Statistical properties of resonances in chaotic elastic cavities: time reversal invariance and feedback
6. CONCLUSIONS

Calculations based on experimental data for the case without the feedback loop (experiments on unperturbed aluminum blocks "Cavity #1" and "Cavity #2"), discussed in Chapter 2, show that the distributions of the intensity transmission coefficients studied in the narrow frequency bands confirm random division of pulse intensity between the cavity waves for both studied cavities.

Division of the frequency band into intervals by cavity resonances for the unperturbed cavity #2, characterized by the NNSD, is found in agreement with the prediction of RMT for GOE. Perfect agreement, however, is achieved when accounting for a fraction of the lost resonances (about 25%). The corresponding SR shows behavior close to the GOE model that involves logarithmic saturation of the SR. The curve that shows SR in case of GOE model with 25% of eigenvalues lost offers even better fit to the spectral rigidity determined from the experimental data of Chapter 2. So both NNSD and SR can be identified as predicted by the GOE statistics, however a relatively large fraction of the lost resonances (25%) has to be assumed to achieve such an agreement. The SR calculated from the data of the experiment on a symmetric cavity (cavity #1) is systematically larger than in the comparable case of an asymmetric cavity (cavity #2). This agrees with the concept of coexistence of odd and even independent sequences of resonances.

The normalized amplitude of the reconstructed pulse in the time reversal experiment discussed in Chapter 2 deviates from exponential dependence on the time delay if the last one is getting smaller and approaches the Heisenberg time (inverse of the average nearest neighbor resonance spacing).

We also found that moments and central moments of different order, skewness and kurtosis of the NNSD determined from experiment discussed in Chapter 2 fall close to the values corresponding to the GOE model. These values actually fall in between the GOE values and values corresponding to random arrangements of resonance frequencies, Poisson model. This may be considered as a consequence of the lost resonances.

We dealt with a considerable amount of lost resonances in our experiments discussed in Chapter 2. This amount is higher than found in earlier reports like [24] by Nogueira et al. However, these experiments [24] were performed
under idealized circumstances. Firstly, aluminum plate resonators were used instead of the volume ones in present work. Secondly, a vacuum chamber enclosing the sample was used to increase the isolation and therefore increase quality factors of the resonances. Thirdly, probably better support mechanism was used as well.

We report experiments that have been done at normal room conditions in air with support of the sample not entirely optimized. Therefore, a bit worse resonance detection conditions are indeed in place. However, our approach opens the possibility to explore experiments making use of RMT statistics on arbitrary samples under non-optimized conditions. This is important for validating the RMT statistical approach for probable future applications in mechanical engineering.

Simulation with the Wave3000 program [48] discussed in Chapter 3 allows satisfactory reproduction of the cavity responses. The spectral density of the response of the symmetric cubic aluminum resonator (without well drilled in it or corner removed) allows a correct identification of 10 consequent resonances of the cubic resonator as given by the analytical model [46].

The simulated elastic wave dynamics for cavity #2 shows noticeably larger repelling of resonances than determined from the responses in the experiment. The NNSD determined from the simulation data is better peaked around average value than predicted by the GOE distribution. SR averaged over frequency bands smaller than $10\Delta \omega_{\text{avg}}$ has a bit lower value than predicted by the GOE model. The behavior of NNSD and SR showing a fraction of lost resonances is not found in the case of the Wave3000 simulation (unlike for NNSD and SR determined from experimental data discussed in Chapter 2).

In particular, SR determined from the spectra of the Wave3000 simulated responses satisfactory follows the GOE curve for averaging over bands of size of 2 to 10 average spacings. The corresponding SR determined from experimental data (discussed in Chapter 2) has larger values than given by the GOE model and significantly deviates from the GOE curve: it falls above the GOE curve and agrees with GOE model with the lost levels. The reduced spacing value corresponding to the maximum of NNSD determined from the Wave3000 simulated responses is close to that of the pure GOE distribution (without the lost resonances). The only discrepancy between the GOE model and NNSD determined from simulated responses is the actual height of the maximum of NNSD obtained from simulated responses. This can not be explained satisfactory at the moment.

But it can be seen from the spectral density of the responses of the symmetric cube that small sharp peaks are present in addition to large (much better pronounced) ones that have been identified with 10 consequent res-
onances from work [46] for the Poisson ratio of 0.33 corresponding to aluminum. These small sharp peaks, that can be artifact of the simulation, can alter the distribution when identified as resonances. So the departure from the GOE distribution can happen if such peaks are present in the higher frequency band and are counted as resonances. But it is important to mention that NNSD determined from simulated responses in Chapter 3 does not behave as GOE distribution with randomly added resonance frequencies (considered in Chapter 1). So if the small peaks, being the artifact of the simulation, are present in higher frequency band used to study statistics then they appear around real cavity resonances in non-random fashion. This implies that they can be for example higher harmonics of the identified resonances that appear due to some kind of numerical nonlinearities.

NNSD and SR have still noticeable error bars arising from the different positions of source and receiver on the surface of the samples. The NNSD and SR as well as sequences of resonances used in calculation are different for different positions of the transducers on the surface of the sample. This was also observed in the analysis of the experimental data (Chapter 2). But in Chapter 2 different sequences of resonances for different positions of the transducers can be explained by the fraction of lost resonances. In Chapter 3 obviously the error bars of NNSD and SR appear for a different reason (e.g. small peaks in spectral density that can be an artifact of the simulation).

Indeed NNSD and SR calculated from the responses simulated with Wave3000 program do not require accounting for the fraction of lost resonances to be fitted with predictions for the GOE statistics. So the studied sample (cavity #2) fits for further studies of the resonance statistics in case of experimentally broken TR invariance, however a fraction of the lost resonances which remains an experimental issue (discussed in Chapter 2) can make the outcome of the experiment less clear.

Cavity #2 was used further to study random matrix statistical properties and time reversal experiment efficiency for the case of broken time reversal invariance (Chapter 4) which was attempted to achieve in experiment by connecting two additional transducers to the surface of the sample and connecting the feedback loop between them (so that the signal can travel only in one direction through the feedback loop).

It has been found from TR experiments discussed in Chapter 4 that the feedback loop suppresses the reconstruction of the excitation pulse for different time delays of the recorded (and replayed backwards) oscillation track with respect to the original excitation pulse. The suppression of the time reversal reconstruction increases with increasing amplification coefficient $K$ of the amplifier in the feedback loop. Thus time reversal experiment becomes less efficient due to increasing influence of the feedback loop.
The NID obtained from 60 kHz wide frequency band in case of the active feedback loop mainly agree with the exponential distribution corresponding to the input pulse energy being shared randomly between the cavity waves.

It is found also that the active feedback loop influences the NNSD statistics. The effect of the feedback loop on SR (delta statistics) is found negligibly small compared to the error bars based on measurements at several different positions of source and receiver.

It can be seen also that skewness and kurtosis of the NNSD, determined from the data of several experiments, approach closer to the values due to the Poisson model (exponential NNSD) with an increase of the influence of the feedback loop, what implies that NNSD is approaching the case of randomly chosen resonant frequencies. This means that arrangement of the resonances becomes more random with increasing influence of the feedback loop in the experiment. However a part of this behavior may be explained also by the growing amount of lost resonances with increasing influence of the feedback loop in case if more modes are obscured by the growth of the modes that are amplified by feedback.

The model matrix for the elastic cavity influenced by the feedback loop can be derived for small values of \( \tau \) (time delay of the signal travelling through the feedback loop). Imaginary parts of the eigenvalues of such a matrix derived in Chapter 5 represent eigenfrequencies of the chaotic elastic cavity influenced by the feedback loop. It can be seen from the NNSD obtained due to this model matrix for different values of simulation parameters that the number of small resonance spacings (relative to the average spacing) increases and the number of average spacings decreases due to the influence of the feedback loop. This means that the trend here is similar to the one found in Chapter 4 when distributions calculated from experimental data were discussed. Increase in small eigenvalue (resonance) spacings and decrease in average eigenvalue (resonance) spacings makes the distribution closer to exponential (earlier referred as Poisson model). However exact fitting of the distributions obtained from experimental data using the random matrix model discussed in Chapter 5 is considerably complicated due to unknown factors \( \sqrt{\langle \psi_2(D) \cdot \bar{n}_2 \rangle^2} \) and \( \sqrt{\langle \psi_1(C) \cdot \bar{n}_1 \rangle^2} \) involved in expression for the parameter B measuring the strength of the influence of the feedback loop. And unknown fraction of the lost (undetected) resonances can be also present in distributions obtained from experimental data.

The considered random matrix model also shows that the feedback loop may have influence on decay constants (linewidth, quality factor) of the individual resonances.