Statistical properties of resonances in chaotic elastic cavities: time reversal invariance and feedback
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SUMMARY

The statistical properties of wave propagation in classical chaotic systems are of fundamental interest in physics. They can be used as the basis for diagnostic tools in materials science [1, 2]. One successful statistical approach originating from quantum mechanics [3, 4] is to describe properties of eigenvalues (i.e. resonant frequencies or energy levels) and eigenfunctions of a complex chaotic system by modelling the Hamiltonian of the system with an ensemble of random matrices with certain properties. The statistical properties of the eigenvalues of this ensemble of random matrices give the statistical properties of the considered resonant frequencies or energy levels. The method can be applied to interpret the properties of acoustic waves in complex mechanical systems.

The statistical properties of the resonance frequencies depend also on the presence of time reversal invariance in the system. The role of time reversal invariance can be verified independently by time reversal experiments. In time reversal experiments part of the recorded (elastic) response is played backwards in time in order to refocus the strongly scattered signal back into a short pulse by back-propagation of the waves in the system.

As a model system to test the statistical properties of resonances with the ability to perform time reversal reconstruction, we investigated chaotic systems with time reversal invariance using ultrasonic waves in aluminum blocks (cavities). After excitation of the samples with a short acoustic pulse the reverberation responses were recorded and analyzed.

The statistical properties of resonance frequencies of the cavities were obtained from the spectral density of the reverberant responses. The distribution of the transmitted intensities displays a random division of intensity between cavity waves in narrow frequency bands. The distribution of frequency spacing between neighboring cavity resonances and the Spectral Rigidity agree with the predictions for a Gaussian Orthogonal Ensemble of random matrices. In the analysis of the spectral density of the recorded responses we explicitly included the fact that not all resonances are detected. The agreement with predictions for the Gaussian Orthogonal Ensemble is achieved if we a assume that a fraction of typically 25 percent of the resonances is not detected in the experiment. We also found that moments and
central moments of different order, skewness and kurtosis for nearest neighbor resonance spacing distributions determined from experimental data fall close to the values corresponding to the Gaussian Orthogonal Ensemble. These values actually fall in between the values for the Gaussian Orthogonal Ensemble and the values corresponding to a random arrangement of resonance frequencies, the Poisson model. This may be considered a consequence of the lost resonances.

Reversibility of the excited wave dynamics in the cavity after a given time delay was studied by reconstruction of the excitation pulse in time reversal experiments. The normalized amplitude of the reconstructed pulse decays exponentially with the time delay between the original excitation pulse and the end of the reversed oscillation track. The exponential behavior exists for time delays longer than the inverse of the nearest neighbor resonance spacing.

The statistical properties of the chaotic cavity were determined both from the experimental cavity responses as well as from simulations of the elastic cavity responses (using Wave3000 simulation program [48]). In this thesis it was confirmed that the simulation correctly predicts the spectral density of the elastic responses of an aluminum cube in the low frequency limit [46].

The distribution of frequency spacing between neighboring cavity resonances and the Spectral Rigidity calculated from the responses simulated with Wave3000 program do not show behavior predicted for the fraction of the lost resonances in the Gaussian Orthogonal Ensemble. However, small peaks, being the artifact of the simulation, may be present in the higher frequency band used to study statistics. Such spurious peaks that appear around real cavity resonances may influence the statistics and cause the deviation from the prediction for Gaussian Orthogonal Ensemble.

The time reversal invariance of waves in the chaotic cavity was broken experimentally by connecting an amplified feedback loop between the two additional transducers on the surface of the aluminum block (cavity). We repeated the time reversal experiments and the statistical analysis of the spectral density of the cavity responses. Thus we did prove that the feedback loop inhibits time reversal reconstruction of the excitation pulse in time reversal experiment. The effect of the feedback loop on the nearest neighbor resonance spacing statistics has been observed. The experimental results show that the skewness and kurtosis of the distribution of the spacing between the neighboring resonances approach the values due to the Poisson model (exponential distribution) when the influence of the feedback loop is increased. This implies that the distribution approaches the case of randomly chosen resonant frequencies with increasing influence of the feedback loop in the experiment.

A random matrix model was constructed within this thesis to describe
the statistical properties of the resonance frequencies of the aluminum block (chaotic cavity) influenced by the feedback loop. Predictions for the nearest neighbor resonance spacing distribution due to this random matrix model have been made. It can be seen from these predictions that the number of small eigenvalue (resonance) spacings increases and the number of average resonance spacings decreases with increasing influence of the feedback loop. This makes the distribution closer to exponential (earlier referred as Poisson model). The calculations confirm the trend of the experimental data.

It makes sense to further investigate similar systems with (and without) the feedback loop using chaotic and regular resonators of different shapes made out of different materials. Such results would help to assess how random matrix statistics can be used in nondestructive testing techniques in material science. A good improvement would also be to improve resonance detection in case of spurious peaks in the spectral density. An important improvement to the random matrix model for the elastic chaotic cavity influenced by feedback can be made by including the linewidth (quality factors) of the individual resonances and distributions of these values.

A short overview of the content of chapters of the present thesis is given below.

Chapter 1 gives an overview of the statistics of Random Matrix Theory and Time Reversal Experiments. The Gaussian Orthogonal Ensemble, Nearest Neighbor eigenvalue Spacing Distribution and Spectral Rigidity are introduced. Distributions resulting from the coexistence of independent resonance sequences and from incomplete sequences of resonances are treated as well. The general scheme of the time-reversal experiments is also described in Chapter 1.

Experiments on and statistical properties of the elastic chaotic cavity without breaking the time reversal invariance are discussed in detail in Chapter 2. The efficiency of the time reversal experiments using this cavity is discussed as well.

In chapter 3 the statistical properties of the chaotic cavity are calculated from simulations using Wav3000 software [48].

Experimental investigation of the statistical properties and efficiency of the time reversal experiment when the time reversal invariance in a chaotic cavity is broken by a feedback loop is discussed in Chapter 4.

Chapter 5 describes a novel random matrix model for the description of a chaotic cavity influenced by a feedback loop. It discusses the corresponding Nearest Neighbor eigenvalue Spacing Distribution obtained with this model.

The following new results are claimed to be obtained within the scope of the present thesis:

- Calculation of the Nearest Neighbor eigenvalue Spacing Distributions,
its moments and spectral rigidity from experiments on a chaotic elastic volume resonator (Chapter 2). Correspondence of the found Nearest Neighbor eigenvalue Spacing Distribution and Spectral Rigidity to those of a Gaussian Orthogonal Ensemble with a fraction of lost resonances.

- Determination of the Nearest Neighbor eigenvalue Spacing Distribution and Spectral Rigidity from simulations of the acoustic wave propagation in an elastic chaotic resonator (Chapter 3).

- Measurements of the acoustic responses of the cavity influenced by the feedback loop. Calculation of the Nearest Neighbor eigenvalue Spacing Distribution, its moments and the Spectral Rigidity from these measured responses. Time reversal experiments performed on such a system (Chapter 4).

- Construction of the random matrix model of an elastic chaotic cavity influenced by a feedback loop and prediction of the behavior of the Nearest Neighbor eigenvalue Spacing Distribution behavior within this model (Chapter 5).