Three essays on banking

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Chapter 2

Collective Strategic Defaults: Bailouts and Repayment Incentives

2.1 Introduction

This paper looks at the possibility of collective strategic default, leading to bank collapse. There is much anecdotal evidence of coordinated non repayment in emerging markets, such as Eastern European countries during transition, and banking crises in Mexico and East Asia. In many cases penalties for delaying repayments were usually lower than the cost of borrowing. This occurred in particular when bailouts were funded by monetary creation, leading to a massive inflation and thus devaluation of loan repayments. Moreover, the delays in the legal procedure to recover the loans were huge. Very often the governments in these countries decided to clear debtor firms obligations in order to avoid tough and unpopular social measures as in Eastern Europe and Russia (1992). While analyzing the strategies and policies implemented by Mexico to resolve their 1994 banking crisis, De Luna-Martinez (2000) identifies the reluctance of borrowers to repay their loans as one of the causes that exacerbated the crisis. A large part of borrowers faced not only the lack of capability, but also the lack of incentives to repay their loans. Referring to the credit crunch Mexico experienced in mid 1990s, Krueger and Tornell (1999) also find that the lack of transparent and effective bankruptcy procedures combined with the fact that the crisis increased the number of insolvent borrowers created the incentives for some debtors with the capacity to service their debts not to do so, since non payment would be hardly punished. Hence, as evidence suggests, in circumstances when financial environment is characterized by inadequate bankruptcy laws, inefficient judiciary system and poor disclosure and accounting rules, where bankruptcy and restructuring frameworks are deficient and creditor rights are poorly defined or weakly enforced, potentially solvent firms would have a strong incentive to mimic the behavior of a distressed firm. They understand that the lending bank will be able to fully pursue non paying solvent firms only if it survives. Therefore, solvent firms action may depend on their beliefs about other firms actions and not only on the information regarding how strong the bank fundamentals are. This behavior may be a different determinant for low level of credit and high interest rate differential between deposit and loan rates in emerging economies beside the known factors that are studied in banking literature such as poor quality of corporate sector, low competition in banking, poor law enforcement or political favors in lending.[1]

Although the claim in strict form predicts that many severe banking crises episodes have been exacerbated by the strategic default of solvent firms, particularly in those emerging economies where poor supervision and regulation of banking system were in place, I would still expect that this non repayment tendency to be met in developed western economies too. In the Financial Times' article (May 2008) discussing the subprime meltdown, Martin Feldstein was suggesting that structured

[1] Haber (2005) and Haber and Maurer (2006) show that in Mexico bankers face large difficulty in enforcing loan contracts and therefore tend not to make many loans. Related lending practicies are described by La Porta, Lopez-de-Silanes and Zamarripa (2003), Laeven (2001) and Claessens, Feijn and Laeven (2006).

finance and securitization has created a coordination failure among borrowers which might cause the deepest and longest recession in the US in the last decades. Due to the fact that commercial banks, acting like lenders, have no recourse to the house's owner beyond the value of the house, and given that more and more people see the value of their mortgages exceeding the value of their homes, individuals with negative equity, most of them real estate speculators, have a strong incentive to default. This temptation to turn in the keys and walk away is aggravated by the difficulty in voluntary negotiations between creditors and borrowers, because most of these mortgages have been securitized and a renegotiation with the mortgage originator proves impossible. According to Hull (2008), the downward trend in house prices during the credit crisis of 2007 was reinforced by the action of many borrowers who exercised their "*implicit put options and walked away from their houses and their mortgage obligations*". Guiso et al. (2009) find that 26% of mortgage defaults in US are strategic. They use a survey to study American households likelihood to default when they have negative equity. They argue that even if those households can afford to pay their mortgages, they prefer not to do so if the equity shortfall is high enough. They also show that the most important factors affecting the incentive to default are moral and social considerations (i.e., social stigma).

Anecdotal evidence is also provided by the interbank payment system. Kahn and Roberds (1998) examine the effects of settlement rules on banks' tendencies to honor interbank commitments rather than default. They found that default probability and the costs associated with potential defaults is higher when net settlement system is in place, while gross settlement increases the costs associated with holding reserves. Banks with large net debt position relative to their capital find tempting to default or to delay the sending of payment messages to other banks, these decisions being exacerbated by the imperfect monitoring by the managers of the payment network or governmental regulators. The recent credit market events demonstrated that the failure of one bank to meet payment obligations can have a negative impact on the ability of other banks to meet their own payment obligations, particularly when interbank exposures are very large. The collapse of Lehman Brothers and the legal claims pursued by its creditors against JPMorgan Chase support to some extent the Kahn and Roberds (1998) argument that a bank might face a liquidity crisis when one of its main counterparties in the interbank market delays the sending of the payment message. JPMorgan had more than $17 billion of Lehman's cash and securities three days before the investment bank filed for bankruptcy on Sept. 15, 2008. Lehman's creditors accused JPMorgan Chase to create an immediate liquidity crisis that could have been avoided if Lehman's access to these assets wouldn't have been denied on Sept. 12, 2008. As we have also seen in March 2008, when the US Fed decided to bail out Bear Stearns, regulators have a crucial role in solving the conflict between the interests of an individual bank and the social interest of the payment network.

The main goal of this paper is to evaluate the effect of Central Bank intervention policy as a Lender of Last Resort under opportunistic behavior from borrowers. I study a model in which a monopolistic commercial bank receives funds from depositors and invests them in a continuum of identical risky loans granted to risk-neutral firms. The bank faces a liquidity shortage which is aggravated by the strategic default of some solvent firms. Within this framework, the Central Bank acts as the only regulator. The Central Bank intervention policy should, on one hand, to minimize the ex-ante moral hazard problem for all parties involved and, on the other hand, to minimize the cost of intervention when it acts as a Lender of Last Resort (or LOLR, as I will refer to it from here). The commercial bank understands that its current assets choice will affect the Central Bank intervention policy, while the Central Bank recognizes the opportunity cost of forgone intermediation if the commercial bank is closed. I examine the borrowers' and commercial bank's behavior under two scenarios. In the first one, the Central Bank is *inactive*. I consider a Central Bank as being inactive if its ex-ante stated decision of non intervention is consistent with its ex-post adopted policy. Under this scenario, the closure of the illiquid bank is the only alternative. In the second scenario, the Central Bank is *active*. I consider a Central Bank as being active when it might step in once the commercial bank is in trouble and provide help under some specific market conditions. In both scenarios the paper requires that any closure threats by the Central Bank be credible. Behind the main findings there are three crucial ingredients. First, the bank fundamentals are not common knowledge. The borrowers hold common prior beliefs about the state of fundamentals and receive private signals about its realization. The bank's fundamentals are the measure of insolvent debtor firms. Second, the lending bank will be able to fully pursue non paying solvent firms only if it survives. Third, the commercial bank and the borrowers know ex-ante the Central Bank's cost of intervention (it is common knowledge for all the players), while the Central Bank decision to bail out or not is not known ex-ante.

This paper derives the probability of a run by bank borrowers, while depositors are passive players.[2] Since banking sector problems and particularly bank runs represent a threat in both emerging and developed economies, a voluminous literature is dedicated to this topic.[3] Nevertheless, the existing literature is centered around two traditional approaches. In one approach, bank runs are based on panics (i.e., coordination problems among depositors). Bryant (1980), Diamond and Dybvig (1983), Goldstein and Pauzner (2005) and Rochet and Vives (2005) are models which share

[2]The conventional explanation for a bank run is given by analyzing the liabilities side of bank balance sheet. Standard bank deposit contracts allow depositors to withdraw a nominal amount on demand. When depositors observe large withdrawals from their bank, they fear bankruptcy and respond by withdrawing their own deposits. Bank's probability of failure will increase due to the negative externality induced by withdrawals in excess of the current expected demand for liquidity.

[3]Lindgren, Garcia and Saal (1996) identify 133 countries facing banking problems between 1980 and 1996. Gorton (1988) analyzes panics during US National Banking Era from 1865 to 1914. Honohan and Laeven (2005) document banking crises throughout the world since 1970. Caprio and Klingebiel (1996), Lindgren et. al.(1999), Hoggarth, Reis and Saporta (2001), Honohan and Klingebiel (2003), Ongena et al. (2003), Calomiris and Mason (2003), Dell'Ariccia et al. (2008) document the costs of these banking problems. Comprehensive surveys on banking crises are Bhattacharya and Thakor (1993), Freixas and Rochet (1997), Gorton and Winton (2003), and Allen and Gale (2007).

this approach. In the other approach, bank runs are based on poor fundamentals and a result of asymmetric information among depositors regarding these fundamentals. Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988) and Calomiris and Kahn (1991) are models which share this second approach. In all these models, the focus is on the liabilities side of bank balance sheet.[4] I complement this literature by showing that collective strategic default of borrowers induces financial fragility when the bank is weak and financial environment is characterized by a poor quality of corporate sector.

The problem I study calls for a specific form of global games. In standard global games the number of agents who might coordinate is independent of fundamentals. In my model, the realization of fundamentals translates directly in the number of active agents who can play effectively the game. The value of the bank's fundamentals depends on the measure of firms that are in genuine financial distress and can not repay their loans. Hence, the number of solvent firms able to coordinate their actions is also a random variable.

The main findings are the following. Firstly, I derive an ex-post optimal solution for Central Bank intervention as a LOLR which has an influence ex-ante on commercial bank and firms behavior. This intervention policy is not known ex-ante and depends on the coordination of solvent firms. Secondly, I show that an active Central Bank can mitigate the strategic behavior of debtor firms. It allows commercial banks to survive more often when they face opportunistic behavior from borrowers. Debtor firms will behave strategically only when bank fundamentals are very poor, because in that case Central Bank intervention is very costly and thus improbable while bankrupt banks do not pursue failed debtors. Thirdly, the cost of intervention faced by the Central Bank has a double-edge effect. On one hand

[4] Most of the models on bank runs are concerned with an equilibrium selection problem. The depositors play a static simultaneous game, in which a coordination failure denies the players to participate in a higher equilibrium payoff due to the fact that they decide to withdraw their money early.

it reduces the moral hazard problem at the commercial bank level. A higher cost of intervention incurred by an active Central Bank reduces the commercial bank's risk-taking incentives. This translates ex-ante into a higher screening effort and, as a result, a better assets quality. On the other hand it can precipitate bank failure by lowering the fundamentals threshold that triggers collective strategic default. Anticipating that the active Central Bank will be reluctant to intervene when the cost of intervention is high, the solvent borrowers behave aggressively. Nevertheless, the threshold in bank fundamentals which triggers collective strategic default for the case of an active Central Bank is always higher than the threshold characterizing the case of an inactive Central Bank. Finally, I provide a different interpretation for the high interest rate differential between deposit and loan rates in emerging economies. I show that high expected profitability reduces the likelihood of collective strategic defaults.

Related literature

The modelling approach in this paper is related to various strands of literature. One strand studies strategic default as an individual borrower strategy. Townsend (1979) and Gale and Hellwig (1985) show that firms behave strategically under asymmetric information on firm profits.[5] The cash diversion problem may be severe when contracts are incomplete in the sense that cash flows are not verifiable (Hart and Moore 1988, 1994, and Bolton and Scharfstein, 1990).[6] A documented path for cash diversion is the tunneling transfer which is described by Akerlof and Romer (1993) and by Johnson, La Porta, Lopez-de-Silanes and Shleifer (2000).

[5] They study a costly state verification model in which the lender cannot observe the cashflow obtained by the borrower, unless a costly audit is performed. They show that the efficient incentive compatible contracts ensuring the truthful reporting by borrowers are standard debt contracts.

[6] Bolton and Scharfstein (1990) show that when the returns of the borrower's investment are not verifiable by a third party (and thus are noncontractible) the threat of termination (not to lend in the future) provides the incentive to repay. They argue that borrowing from multiple lenders decreases the incentive to strategically default since the firm manager must coordinate a restructuring plan with multiple claimants.

The second strand studies different aspects of regulators' intervention policies during banking crises. There is a growing literature on the regulators' choices between rescuing and closing troubled banks. The classical argument by Bagehot (1873) regarding the idea of the Central Bank as a LOLR is that the Central Bank should lend at a penalty rate to illiquid but solvent banks, against good collateral. Goodfriend and King (1988) criticize this view by arguing that a solvent bank will be able to find liquidity in an efficient interbank and money market. By using a 'too big to fail' approach Freixas (1999) argues that the LOLR should bail out an insolvent bank, while solvent banks are assumed to be bailed out by the interbank market. Rochet and Vives (1994) support Bagehot's doctrine by showing that even sophisticated interbank markets will not provide liquidity due to a potential coordination failure between investors which might have different opinions about bank solvency. Goodhart and Huang (2003) show that the Central Bank should act as a LOLR to avoid contagion during a banking crisis. Acharya and Yorulmazer (2006) argue that when the number of bank failures is low, the optimal ex-post policy is not to intervene, but when this number is sufficiently large, the regulator should choose randomly which banks to assist. The rationale behind this intervention mechanism is that the regulator sets a liquidity target which limits banks' assets sales and prevents the decrease in assets prices which might induce more bank failures. In this paper we abstract from contagion issue and we focus on an intervention mechanism based on the reporting and disclosure of non-performing assets. Commercial banks have to inform the Central Bank about their non-performing loans, and, by using this information, the Central Bank decides if its role as LOLR is requested. A similar approach is used by Mitchell (2001). She founds that bank managers have incentives to underestimate the size of non-performing loans under a tough intervention policy and show that this leads to inefficient liquidation of bad loans. Aghion, Bolton and Fries (1999) analyze both tough and soft recapitalization policies, arguing that soft intervention mechanism induces bank managers to exaggerate the recapitalization needs. They also suggest that bank's incentives to misreport can be mitigated by an efficient bailout scheme which is conditional on the liquidation of firms in default.

Third, this paper complements the theoretical literature on bank runs. The analysis differs from ex-ante literature by examining a potential coordination problem between borrowers, in particular examining the possibility of a bank failure as result of asymmetric information among borrowers regarding bank fundamentals. To the best of my knowledge there are few formal models that are trying to determine the probability of a bank failure due to strategic coordination of its borrowers. Bond and Rai (2008) investigate the effect of borrower run in microfinance, for an environment where the threat of credit denial is an important source of repayment incentives. Unlike my paper they study only the relation between lenders and borrowers and try to identify the best lending policies which allow lenders to survive borrower run. My purpose is to understand the role of the Central Bank as a LOLR under opportunistic behavior from borrowers. Both Rochet and Vives (2004) and Naqvi (2006) endogenize the asset side of the bank balance sheet. Rochet and Vives (2004) model coordination failure on the interbank market. They find that there is a critical value of banks' assets above solvency threshold such that, whenever the value of the banks' assets falls below this threshold, the banks will not have access to liquidity. In a bank run model Naqvi (2006) shows that the presence of a perfectly informed LOLR can avoid costly liquidations and thus it is Pareto improvement. However, these papers abstract from the moral hazard problem between the borrowers and the bank, an issue which I take explicitly into account. The main novelty in my paper relative to these papers is that it focuses on the borrowers collective strategic default, while depositors are passive players.

Finally, from the methodological point of view, this paper is related with global games literature. My work complements this literature by looking to a particular coordination game, used to explain collective strategic default on the asset side of a commercial bank balance sheet. I form the model in the context of the global games methodology first introduced by Carlsson and van Damme (1993) and later refined by Morris and Shin (1998). This realistic approach does not depend on common knowledge and helps to resolve the issue of multiple equilibria. Common knowledge, introduced in theoretical models through a perfect public information, can create self-fulfilling beliefs equilibria which might destabilize an economy. Sudden crises without any fundamental reason can arise in such unstable economy due to changes in beliefs of market participants. The presence of multiple equilibria in many macroeconomic models makes any policy analysis very difficult because is problematic to attach probabilities to different outcomes.[7] The central assumption of the global games methodology is that individual actions are strategic complements: an agent's incentive to take a particular action increases as more and more agents take the same action.[8] In this approach, a small amount of noise in fundamentals can be stabilizing and can pin down a unique equilibrium with agents playing threshold strategies. By using an iterated deletion of strictly dominated strategies, the unique Nash equilibrium can be derived for games with incomplete information. The theory of global games has been useful in modeling various economic applications.[9] Fukao (1994) and Morris and Shin (1998) use this approach in modeling speculative currency attacks in the presence of identical speculators.[10] Morris and Shin (2004) examine pricing of debt. Corsetti et al.(2004) and Peydro-Alcalde (2005) show how the presence of a large player affects the coordination problem in forex market and in a creditor's decision to renew its credit, respectively. Shin (1996) studies asset trading. Postlewaite and Vives (1987), Goldstein (1999), Morris and Shin (2000), Rochet and

[7] Diamond and Dybvig (1983) model bank runs; Diamond (1982), Hart (1982), Bryant (1983) examine unemployment or underemployment; currency crises are analyzed by Obstfeld (1996) in a model of a balance-of-payments crisis.

[8] See Morris, Rob and Shin (1995) and Kaji and Morris (1997) for generalizations of the logic behind the result of Carlsson and van Damme (1993).

[9] Morris and Shin (2003b) is a comprehensive review of the literature on global games. See also Vives (2005) for a review of recent applications to finance, macroeconomics and industrial organization.

[10] See also Heinemann (2000) comment on Morris and Shin (1998).

Vives (2004), Dasgupta (2004), Iyer and Peydro-Alcalde (2004), and Goldstein and Pauzner (2005) applied the theory of global games to model bank runs and to investigate contagion in the interbank market. Morris and Shin (2003a) and Corsetti, Guimaraes and Roubini (2004) use global games to study the impact of an international LOLR on adjustment policies of borrower countries. Atkeson (2000) and Edmond (2004) employ this method for explaining riots and political change. More advanced models allow not only for noisy private signals about fundamentals, but also for public signals and discuss their impact on the unique equilibrium.[11] Recent studies find empirical evidence for the financial fragility generated by strategic complementarities. Chen et al. (2008) use data on mutual fund outflows and find that when complementarities are stronger (i.e., funds with illiquid assets), the response of investors are more sensitive to fundamentals than in funds with liquid assets. Heinemann et al. (2004) use experimental methods to show that the predictions of global games theory are accurate.

The remainder of the chapter is organized as follows. Section 2.2 describes the basic model, the agents and their payoffs. Section 2.3 discusses the equilibrium and the thresholds derivation under imperfect information. Section 2.4 shows the comparative statics and the predictions of the model. Finally, Section 2.5 concludes and provides some directions for further research. The Appendix contains the mathematical detailed solutions for the main results.

[11] Morris and Shin (1999) and Hellwig (2001) show that uniqueness of equilibria is preserved only if the private information is precise enough when compared with public information. Angeletos and Werning (2004) endogenize public information by allowing individuals to observe financial prices or other noisy indicators of aggregate activity.

2.2 The Model

I consider a static economy over three periods: $0, 1, 2$. The economy is populated by a single bank which has no capital of its own, a continuum of identical risk-neutral firms of measure one, uniformly distributed over $[0, 1]$ and indexed by i, and a Central Bank.

2.2.1 Agents

The bank

The bank accumulates at date 0 uninsured deposits for a total amount q, which mature at date 1. Focusing on uninsured deposits avoids the moral hazard problem from any deposit insurance scheme with respect to bank incentives.[12] The bank has no cash or other reserves.[13] The nominal return on deposits at date 1 is $Q > q$. Bank sets Q before choosing the investment strategy. The choice of investment is unobservable to depositors. In this case, the depositors will require a nominal return that provides them with (at least) zero expected return. As I focus on the assets side of the bank's balance sheet, I model depositors as passive players without alternative investment opportunities besides costless storage. Since depositors are passive players we abstract from the monitoring role of depositors.

Let the riskless interest rate be 0. The bank invests at date 0 the total amount of its funds q in a continuum of identical risky loans of size 1 each, granted to risk-neutral firms. Each loan matures in the next period and the nominal returns on

[12]The banking literature suggests that when depositors are uninsured, a deterioration in the quality of a bank's asset portfolio may trigger a run (Diamond and Dybvig 1993, Demirguc-Kunt and Detragiache 1998, 2000). In our model, as all deposits mature next period there is no intermediate period when a run might occur. Besides its positive attribute (elimination of self-fulfilling panics), both an implicit and an explicit deposit insurance creates incentives for excessive risk-taking by banks. The distortions and bank failures are more likely in the presence of full insurance, because depositors have no incentive in this case to monitor their banks. A comprehensive survey on deposit insurance schemes is Bhattacharya, Boot and Thakor (1998).

[13]The qualitative nature of our results is unchanged if the bank has initial capital, and it holds a certain amount of cash.

these loans at date 1 is $D > 1$. If all loans are repaid, the bank is able to repay depositors:

$$Q < qD \tag{2.1}$$

The bank's balance sheet at date 0 is the following:

TABLE 1. BANKS'S BALANCE SHEET AT DATE 0.

No cash	No Equity
$\sum$Loans $= q$	$\sum$Deposits $= q$

Before granting risky loans at date 0 the bank has to set up its loan screening strategy. Specifically, the bank has to implement a costly effort e to assure repayment by its borrowers. The screening effort is costly because is difficult to identify good firms to lend to. The effort lies in the interval $[0, 1]$ and it is exercised through activities such as extensively screening loans applications, hiring better loan agents or writing better contracts that can be easily enforced in a court. I assume the cost function is quadratic in effort and proportional to the returns on the loans, $c(e) = \frac{e^2}{2}qD$. Put differently, the screening cost is increasing with volume of loans q, and also it increases in the loan rate D since more effort is required in identifying the appropriate rate for risky loans. Once e has been chosen, it is common knowledge among all agents.

I assume that upon insolvency, the bank can no longer enforce contracts. As a result, once bank assets are below liabilities, the enforcement technology is lost if liquidity support is not provided to bank. Empirical evidence that supports this assumption comes from Perotti (1998). He shows that during the transition period in Eastern European countries and in Russia in early '90s, the bank credit was made more scarce because the banks couldn't pursue non-paying firms, leading to an increase in trade credit. As the accumulation of trade arrears also increased, the defaulted firms benefited mostly due to higher expectations for a collective bailout.

He concludes that in a weak institutional context characterized by inadequate bankruptcy laws and unreliable enforcement of contractual obligations, firms' perverse incentives and collusive behavior are generated by the impossibility to discriminate across viable and distressed firms.

The value of the bank's fundamentals in this model depends on d, the measure of firms that are in genuine financial distress and can not repay their loans. This random variable is normally distributed with mean $(\mu - e)$ and variance $1/\alpha$ (precision α). Here μ stands for *unconditional bank fundamentals* and it is interpreted as a measure for the health of economic environment. A healthy economy, in which the prospects for corporate sector performance are high, it is characterized by a lower μ. In such an environment most of the debtor firms are expected to generate positive cash flows. I assume that $0 \leq \mu \leq 1$. The level of effort directly affects the quality of assets the bank holds. Thus higher the effort e exerted by the commercial bank, better are the lending policies and lower the probability of insolvency. The bank fundamentals d are not common knowledge among market participants. From these insolvent firms the bank can not extract any liquidation value, whatever its effort.

Debtor firms have no information about bank value. Alternatively, the bank may not be listed on a stock exchange.[14] This is not a very restrictive assumption since financial environment of most emerging economies is characterized by poor capital markets and even in well developed economies small banks might not be listed. There is no other source of financing for the bank in the short run (i.e., the bank has no access to interbank loans). A motive could be that fear of contagion might lead to low level of liquidity in the interbank system (Dasgupta 2004, Iyer and Peydro-Alcalde 2004, Allen and Gale 2000, Calomiris and Mason 2003).[15]

[14] Atkenson (2000) questioned the decision of agents to take different actions on the basis of their private signals when a publicly observed asset price accurately reflect which outcome will occur. This issue was examined by Morris and Shin (1999), Hellwig (2001) and Angeletos and Werning (2004).

[15] As the subprime loans crisis of 2007 showed, liquidity will dry up if mutual confidence falls in the interbank market.

I assume that the bank is the most efficient agent at extracting cash from the debtor firms due to its collection skills (Diamond and Rajan 2000, 2001). Its loans are not tradeable and selling the entire loans portfolio to other financial agents is not optimal while the bank can not insure itself against the costs of forcing a defaulting firm to repay. As evidence suggests, securitization and complex tools such as CDO or CDS reduce the quality of credit originated. Allowing for these tools will make the main results of this paper stronger by creating incentives for excessive risk-taking by commercial bank. I also implicitly assume that equity can not be raised overnight if the bank faces liquidity shortage.

I abstract from issues of renegotiation and lending against a collateral. Including both aspects together in the model is simply redundant. Although the loan agreement may stipulate that the collateral can be seized should the borrower not repay, it is expected if borrowers goes insolvent, the loan to be renegotiate, as banks are typically less efficient managers. While important, explicitly allowing only for the existence of collateral would complicate the model without significantly changing the main conclusions. With respect to renegotiation, although it is common for commercial banks to renegotiate non-performing loans the reduction is not explicitly stated. The new agreement relaxes the payment process through reduction of interest rates and extension of payment periods and more important, takes time to be implemented. A bank facing liquidity shortage and deposits withdrawals has no time to renegotiate its non-performing loans. It needs funds to pay its depositors, and needs those funds quickly. On the other hand, this paper does not study the perverse incentives borrowers might have under the possibility of contracts renegotiation. Hence, there is no credible avenue for renegotiation within this paper framework.

Firms

There are two types of firms in this economy. One one hand, there are firms which expect a positive cash flow in period 1 (henceforth, *good firms*). For simplicity I assume that the cash flow is equal to D, which is firm's obligation to the bank. A good firm may take one of two actions. It may decide *not to repay* its loan, thus mimicking the situation of a firm in real financial distress. Alternatively it may decide *to repay* it in full. The second type are distressed firms. I assume that an insolvent firm (henceforth, a firm in genuine financial distress, or a *bad firm*) has zero cash, thus it has no option but to default. The critical question is whether the potentially solvent firms will choose to repay. They understand that the lending bank will be able to fully pursue non paying solvent firms only if it survives, otherwise the recovery process takes time due to the fact that a regulator (the Central Bank in this model) will be in charge with this process. Hence, it is possible that borrowers are less willing to honour their obligations in due time. However, if the bank survives it can enforce all contracts it has with solvent borrowers, and also it will break off the relation with strategic defaulters. In this model, the act of payment represents a decision. Kahn and Roberds (2009) argue that the choice of whether, when and how to pay depends on a variety of characteristics of the agents involved in the trade and on the environment, such as differential information, legal structure enforcing the contracts, importance of reputation and ease of damaging it among many others. In this model, solvent firms decisions to repay or not may depend on their beliefs about other firms' actions and not only on the information regarding how strong the bank fundamentals are. I assume that a good firm which is indifferent between repaying and not will choose not to repay.

The Central Bank

The Central Bank decides on its intervention policy based on the total number of non-performing loans reported by the commercial bank. Henceforth I make the assumption that the commercial bank can not misreport the non-performing loans volume (the Central Bank can verify at zero cost this reported number).[16] Note that the real number of non paying firms include all distressed firms plus those good firms mimicking the behavior of a distressed one. Within this framework I examine two polar cases. In the first one I assume no Central Bank intervention. This case corresponds to an inactive Central Bank. The Central Bank declares ex-ante that it will not bail out commercial bank if it faces trouble and it is consistent ex-post with this decision. Then I consider the case of an active Central Bank. An active Central Bank might step in and provide additional funds if necessary, by lending at a zero interest rate, in order to avoid the bank default. If the bank is rescued, its enforcement technology remains intact and the bank can identify the strategic defaulters and extract payments from them. The Central Bank decides on the *optimal BailOut Amount* (BOA) by balancing the cost of a successful intervention against the social cost of doing nothing. This social cost is the opportunity cost of forgone intermediation if the commercial bank is closed, while the administrative costs of closure are ignored. The bailout amount is unknown ex-ante to firms and commercial banker and since the Central Bank itself can not differentiate between bad firms and good firms that are not repaying, it will choose between two possible actions: full bail out or no bail out. This uncertainty may induce the bank to take costly effort to reduce the incidence of strategic default by debtor firms. I assume that if the active Central Bank is indifferent between helping the decapitalized bank and not, it will choose to bail out the bank. In both scenarios any closure threats by the Central Bank are credible.

Next I describe the payoff functions for market participants and the structure of the game they play.

[16] Mitchell (1997) looks at a specific too-many-to fail problem and shows that, if distressed but solvent banks expect the regulator to apply a policy of closure and if the probability of detection of loan rollovers is high enough, then banks will reveal and deal with their bad loans. See also Rajan (1994) and Aghion, Bolton and Fries (1999).

2.2.2 Payoff Functions

Firms' payoffs

The payoff structure for a debtor firm is as follows. If a healthy financial firm doesn't repay its loan and the bank fails, then it saves the repayment, producing a positive payoff of D, minus an amount X representing the positive value that the Central Bank can extract ex-post from non repaying good firm. This difference $D - X$ is positive and indicates that the original lender is the most efficient at extracting any hidden cash from borrower firms. Alternatively, this positive value suggests that the commercial bank is a more efficient user of its assets than outsiders (James, 1991). Without loss of generality I assume for the reminder of the paper that X is zero (i.e., the Central Bank recovers nothing from strategic good firms). If a good firm does not repay and the bank survives, the bank can force at no cost repayment D and also can force a contractual fine $F > 0$. This contractual fine represents penalties for non repaying or delaying the repayment. If a good firm decides to repay its loan, it will get either 0 if the bank fails or $V > D$ otherwise, where V represents the present value of future long term relation with the bank.[17] The banking literature argues that long term interactions between a bank and its borrowers lower the cost of asymmetric information for the lender and improve the credit terms for the debtors. The coordination failure among good firms might deny those firms to participate in the higher payoff V due to the fact that they decide to default on their loans and to keep the entire amount D. I also assume that the value of relation with the bank is not as low as to make debtor firms to prefer to default on their obligation regardless

[17] Diamond (1991) shows that firms that have been successful in the past are able to obtain better credit terms, since they are more likely to be successful in the future. Fama (1985) points out that the value of intermediation and the firm-bank relationship is central to what makes a bank 'special'. According to Mayer (1988) firms and financial intermediaries develop long-term relationships. Boot (2000) and Ongena and Smith (2000) provide detailed surveys of the relationship banking literature.

the level of bank fundamentals:

$$V > F + \frac{QD}{qD - Q} \tag{2.2}$$

The payoff structure for a good firm is illustrated in the next table.

TABLE 2. GOOD FIRM'S PAYOFF.

	Firm repays loan	Firm does not repay
Bank survives	V	-F
Bank defaults	0	D

I abstract from issues such as good firms coordinating and bailing out either the commercial bank or the bad firms, or Central Bank monitoring the debtor firms. These issues will give a different twist to this paper and will also challenge the role played by commercial banks as financial intermediaries and delegated monitors.

Default condition

For both financially distressed and healthy firms, a default is triggered by non repayment at date 1. I introduce here the following function of d, representing a measure of solvent firms which should strategically default on their loans such that the bank becomes insolvent:

$$z(d) = 1 - d - \frac{Q}{qD}. \tag{2.3}$$

This function is strictly decreasing in d. The bank becomes insolvent if the value of its liabilities Q is higher than the value of its assets:

$$[1 - d - \underbrace{(1 - d)n(d)}_{z(d)}] * qD < Q \tag{2.4}$$

At date 1 the bank's assets depends on the measure of firms in genuine financial distress, d, and also on the measure of strategic firms, $z(d)$. The function $z(d)$ can be written as $(1-d)n(d)$, where the function $n(d)$ indexes the fraction of good firms which chooses the action *not repay*. I denote by $z'_d(d)$ the first derivative of $z(d)$ with respect to bank fundamentals d. Thus $z'_d(d) < 0$, meaning that higher the number of firms in real financial distress, the lower is the threshold number of mimicking good firms which should not repay such that the bank fails. I denote by $\overline{d}$ the point at which $z(d) = 0$ and by $\underline{d}$ the point at which $z(d) = 1$. The intuition for the solution of the model *if we assume* common knowledge about bank fundamentals is as follows.

The state of bank fundamentals affects the degree of coordination among good firms and thus the payoff from successful collective default. If the bank fundamentals are strong, only a high degree of coordination can undermine the bank capacity to collect unpaid loans. Therefore all good firms will repay their loans on time. On the other hand, when bank fundamentals are very poor, few firms which do not repay their loans can trigger the bank failure. In this situation the dominant strategy for all firms will be non repaying because this strategy will generate a positive payoff. While in these two regions there is one pure Nash equilibrium, in the intermediate region the model has multiple equilibria under a common knowledge assumption. With self-fulfilling beliefs two pure strategy equilibria will coexist: a 'bad' equilibria in which all firms decide not to repay because each one believes in bank's failure, and the 'good' equilibria in which all firms decide to repay because each one believes in bank's strength.[18] We assume that there exists a level of extremely good fundamentals $\underline{d} > 0$, such that good firms always repay when they know that bank fundamentals are such that $d \leq \underline{d}$, independently of what they think other good firms would do. The idea behind this assumption is similar to Goldstein and Pauzner (2004), namely that for extreme situations when fundamentals are very good, an external

[18]This classification of fundamentals under common knowledge assumption was emphasized by Morris and Shin (1998) in their paper about currency attacks, by Obstfeld (1996) in his paper about balance-of-payments crises, and Rochet and Vives (2004) in their work on the coordination failure on the interbank market.

large enough investor exists and she is willing to buy the bank given the high returns which she can be sure of making if bank's enforcement ability is preserved. This assumption generates a supersolvency or upper dominance region which is crucial for selecting a unique equilibrium. To summarize, under common knowledge the classification of fundamentals will be:

- for $d \leq \underline{d}$ we have $z(d) > z(\underline{d}) = 1$. In this case all good firms repay because the bank's enforcement ability is preserved even if all borrowers default. Thus a good firm would obtain a higher payoff repaying the loan than strategically defaulting.

- for $\overline{d} \leq d$ we have $z(d) < z(\overline{d}) = 0$. In this case all good firms not repay because the bank goes bankrupt even if no firm defaults. This range of fundamentals is called insolvency or lower dominance region.

- for $\underline{d} < d < \overline{d}$ we have multiple equilibria. The outcome depends on firms' expectations of what other firms will do, and not on the underlying bank's fundamentals.

The rest of the paper assumes no common knowledge about fundamentals. The debtor firms hold common prior beliefs about the state of fundamentals d and receive private signals about its realization $x_i = d + \epsilon_i$, where ϵ_i is normally distributed with mean 0 and variance $1/\beta$ (precision β). Moreover, ϵ_i is independent of d and identically and independently distributed across firms. A firm's signal can be thought of as its private opinion regarding the prospects of the corporate sector and the impact that these may have on bank's assets. Based on their private signals, borrowers can infer under these circumstances a conditional distribution for bank fundamentals.

Bank's payoff

The model assumes that upon insolvency, the bank can no longer enforce contracts and thus it can not extract the available cash from good firms, conditional on liquidity not being provided by the Central Bank. This gives to the Central Bank the incentive to keep the commercial bank alive, since the bank failure is socially costly. Since the bank has no other sources for financing at date 1, only the Central Bank's intervention can allow it to preserve enforcement technology and to collect payments from strategic defaulters in order to its obligations. Insolvency condition is captured by (2.4).

The bank's expected payoff if it survives is given by:

$$qD * \underbrace{E_d[(1-d)*(1-n(d))]}_{\text{expected measure of repaying firms}} + q(F+D) * \underbrace{E_d[(1-d)*n(d)]}_{\text{expected measure of mimicking firms}} - Q - c(e), \tag{2.5}$$

while the expected payoff if the bank goes bankrupt is: $-c(e)$.

If the bank survives, it can differentiate ex-post bad firms from good firms, and thus can extract a repayment from good firms (from bad firms, as I assumed earlier, it can extract nothing). Ex-ante, the bank tries to infer the measure of good firms repaying, conditional on the prior distribution of d. The payoff depends on the expected measure of good firms repaying their loans and on the expected measure of good firms that will be fined, adjusted for the cost of effort and depositors claim. When the bank fails, the payoff is negative and equals the cost of incurred effort (if any).

Central Bank's payoff

The Central Bank values the long term relation between good firms and their bank. The Central Bank recognizes the opportunity cost of forgone intermediation if the commercial bank is closed as all good firms which repay their loans will lose a positive value V if the bank fails. The Central Bank tries to minimize the social cost induced by the failure of the bank, trading it off against the cost of full intervention. Let γ be the Central Bank's marginal cost of intervention. We can interpret this cost from two different perspectives. The most straightforward interpretation is to consider the intervention cost as being the fiscal cost of providing funds to the falling bank. Using this approach, this cost can be linked either with the negative effects of tax increases required to fund the bailout, or with the similar effect of government deficits on other macro variables such as exchange rate. An alternative interpretation is to consider the intervention cost as a proxy for Central Bank's independence and its commitment to maintain price stability.

The concept of independence means that, once appointed, the central banker is able to set policy without interference or restriction of the political authorities (Rogoff, 1985). The central banks' credibility depends on being prepared to do two unpopular things: raising interest rates despite the economic pain, and letting financial institutions to fail. For industrialized economies it has been shown that different measures of Central Bank independence are negatively correlated with average inflation, suggesting that, highly independent central banks are presumed to place more weight on achieving low inflation.[19] Unlike in industrial countries, where nowadays domestic central banks are considered to be more independent and where they can limit the degree of instability in the banking system through LOLR operations, in emerging markets the credibility of central banks as an inflation-fighter is still in doubt. Mishkin (2007) argues that central banks intervention as LOLR in these economies, in which debt contracts are typically short term and denominated in foreign currencies, more likely exacerbates financial crises by rising inflation fears and by causing currency depreciation. Following this approach and considering the intervention cost γ as a proxy for Central Bank's independence and its commitment to maintain price stability, we can assume that an independent Central Bank, which is committed to deal with inflationary pressures, will have a higher γ.

Moving one step further from the independence issue and allowing the focus to be only on central banks' commitment to maintain price stability, an interesting ques-

[19] Comprehensive surveys on Central Bank independence are Cukierman (1992) and Eijffinger and de Haan (1996).

tion might be explored, namely what is the impact of a central bank's main mandate on the preference it has toward bailing out falling commercial banks. As we know, central banks are of two types: those concerned with both inflation and economic growth, and those targeting only inflation rate. Two of the most important Central Banks in the world, the US Federal Reserve (FED) and the European Central Bank (ECB), belong to different types. While the ECB has a single mandate - maintaining price stability by controlling inflation - and is not concerned with maximizing economic growth, the FED has two mandates - to preserve the value of the US dollar and to maintain full employment - the two being incompatible. To exemplify the distinction between these two central banks we can look to their policies adopted during 2007/2008 credit crisis. On one hand, the ECB was reluctant to cut interest rates because inflation was consistently above its target. On the other hand, FED governors chose to try to boost up the economy and let the inflation surge by reducing interest rates and they also flooded the market with liquidity (e.g. bailing out Bear Stearns, a primary broker which was not under direct supervision of the FED, setting up Term Lending Facilities and lengthen their maturity profiles periodically). According to the previous example, the ECB, which places a higher weight on inflation objectives due to its mandate, has a higher cost of intervention γ than the FED, which was concerned more about economic contraction during the credit crisis.

Further, I assume common knowledge about γ. It is a positive constant and a higher γ means higher cost of intervention. If the Central Bank decides to intervene and to bail out the commercial bank, the cost of intervention is proportional with lender's liquidity needs:

$$-\gamma[Q - \underbrace{qD * (1-d) * (1-n(d))}_{\text{repaid loans}}], \tag{2.6}$$

while the social cost incurred if the Central Bank allows the bank to go bankrupt is proportional with the destroyed relational value for those firms who choose to repay:

$$-qV * \underbrace{(1-d) * (1-n(d))}_{\text{measure of repaying firms}}. \tag{2.7}$$

Thus, the Central Bank's cost function is given by:

$$min\left\{qV * (1-d) * (1-n(d)),\ \gamma[Q - qD * (1-d)(1-n(d))]\right\}$$

If the Central Bank decides to step in, it provides all the necessary funds to recapitalize the bank (i.e., an amount equal to $Q - qD(1-d)(1-n(d))$. Otherwise, it provides no money, allowing the bank to fail.

The set of available information the Central Bank has when it takes the intervention decision is different than the one possessed by the commercial bank when it sets its loan screening strategy. While the commercial bank has information only about the prior distribution of its fundamentals d, the Central Bank is informed about the measure of reported non-performing loans, $d + (1-d) * n(d)$, as well.

The extensive form of the game. We can now summarize the sequence of events:

$t = 0$

- The bank collects uninsured deposits;
- The bank exerts the screening effort which becomes common knowledge, and invests.

$t = 1$

- Nature draws the bank fundamentals d according to the prior normal distribution; d is not common knowledge;
- Each debtor firm receives a private noisy signal x_i;
- Based on their signals, the debtor firms update their beliefs about the bank fundamentals and simultaneously decide to repay their loans or not;

- The bank reports the volume of non-performing loans to the Central Bank.

$t = 2$

- Central Bank decides on the optimal bailout amount;
- The payoffs of the game are revealed and distributed.

2.3 Equilibrium and Thresholds Derivation

Next I derive the probability of a run by bank borrowers, measuring a collective strategic default. Since a firm's signal provides information not only about bank fundamentals, but also about other firms' signals, it allows debtor firms to infer the beliefs of the others. Thus, observing a high signal induces a debtor firm to believe that other firms also received a high signal. Hence it assigns a high probability to an attack on the bank, while a low signal suggests exactly the opposite.

We solve the model backwards. First, we derive the optimal Central Bank's intervention policy at date 2. Then, we analyze the behavior of debtor firms at date 1, and finally we derive the optimal bank's effort at date 0.

In solving the game and deriving the unique equilibrium, I use the global games methodology first introduced by Carlsson and van Damme (1993) and later refined by Morris and Shin (1998). Two scenarios are considered in turn. First scenario assumes that the Central Bank is inactive and never intervenes, while in the second one I explore the borrowers' and commercial bank's behavior when the Central Bank is active.

2.3.1 Inactive Central Bank Case

Firms repayment incentives

A good firm has a dominant strategy to not repay its loan, when the expected payoff of doing so, conditional on the available information, is higher than the expected payoff of repaying the loan:

$$P(BankSurvives \mid x_i) * (-F) + P(BankFails \mid x_i) * D > \qquad (2.8)$$
$$P(BankSurvives \mid x_i) * V + P(BankFails \mid x_i) * 0$$

On the left side in (2.8), the expected payoff for a firm deciding not to repay is described. The first term denotes the firm's return when the bank survives, times the probability that the bank does not go bankrupt, while the second term denotes the return in the case of bank collapse, times the probability of bankruptcy. On the right side, the expected payoff for a firm deciding to repay is computed. The first term denotes the firm's expected return when the bank survives, while the second term denotes the expected return in the case of bankruptcy.

For each realization of the private signal x_i, a good firm i should decide for a specific action (either repay, or not). This decision rule which connects signals with actions represents the strategy of each good firm. Beside his own decision rule, the outcome for each good firm depends on all the other good firms' strategies. An equilibrium is characterized by a set of strategies maximizing the expected payoff for each good firm, conditional on the available information, given that each good firm is adopting a strategy from this set.

I focus on threshold strategies in which each debtor firm decides not to repay if and only if its signal is above some threshold level. Nevertheless, by focusing our attention to this type of strategies is without loss of generality. Morris and Shin (2000, 2003b) show that, when there is a unique symmetric equilibrium in thresholds strategies, there can be no other equilibrium. This switching equilibrium is the only set of strategies that survives iterated elimination of strictly dominated strategies. Since my model satisfies the symmetry condition due to the fact that all debtor firms are identical and also there are strategic complementarities between firms' actions, I can employ global games methodology for deriving the unique equilibrium.

Let us suppose that there are two thresholds x^* and d^*, such that all the firms which see a signal $x > x^*$ will not repay their loans, while d^* represents the threshold in bank fundamentals at which the bank fails for values of $d > d^*$. I prove the existence and uniqueness of these two thresholds by using an algebraic solution similar with Morris and Shin (1998). The detailed mathematical derivations are in the Appendix.

The equilibrium thresholds x^* and d^* are:

$$x^* = \frac{\alpha + \beta}{\beta} d^* - \frac{\alpha}{\beta}(\mu - e) - \frac{\sqrt{\alpha + \beta}}{\beta} \Phi^{-1}(\frac{D}{V + F + D}), \tag{2.9}$$

where x^* is the threshold signal at which a good firm is indifferent between repaying the loan or not, and

$$d^* + \frac{Q}{qD} = \Phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu - e) - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1}(\frac{D}{V + F + D}))), \tag{2.10}$$

where d^* is the threshold value for commercial bank fundamentals above which the bank fails when the Central Bank is not active (*see the Appendix for detailed derivations.*). Φ represents the cumulative distribution function of the standard normal distribution. The solution to equation (2.10) should belong to the region of fundamentals which is characterized by multiple equilibria under common knowledge assumption. Thus, d^* should lie in $[\underline{d}, \overline{d})$.

The right side of equation (2.10) is a cumulative normal distribution

$$N((\mu - e) + \tfrac{\sqrt{\alpha+\beta}}{\alpha} \Phi^{-1}(\tfrac{D}{V+F+D}),\ \tfrac{1}{\frac{\beta}{\alpha^2}}).$$

Thus, we may conclude that d^* is the intersection point between the cumulative normal distribution just described and a straight line (with a slope of 1) and positive intercept $\frac{Q}{qD}$. This intersection point exists and it is unique if the slope of cumulative normal distribution is less than one everywhere. This slope equals

$$\tfrac{\alpha}{\sqrt{\beta}} \phi(\tfrac{\alpha}{\sqrt{\beta}}(d^* - (\mu - e) - \tfrac{\sqrt{\alpha+\beta}}{\alpha} \Phi^{-1}(\tfrac{D}{V+F+D}))),$$

where ϕ is the density function of the standard normal distribution. From statistical properties of standard normal density function $\phi \leq \frac{1}{\sqrt{2\pi}}$, thus a sufficient condition for a unique solution for d^*, under assumption (2.1), is given by:

$$\frac{\alpha}{\sqrt{\beta}} \leq \sqrt{2\pi} \tag{2.11}$$

Proposition 1 *When the precision of the private signal of debtor firms (β) is large enough relative to prior precision (α) so as to satisfy (2.11), there is a unique d^* defined in (2.10) such that, in any equilibrium of the game with imperfect information, the bank fails if and only if $d > d^*$.*

Proof. *See the Appendix. The proof is along the lines of Morris and Shin (2000).*
■

This proposition implies that by relaxing the assumption of common knowledge, one can eliminates the multiple equilibria only if the precision of private signals is large relative to the precision of the prior (i.e., β large when compared to α). This condition is sufficient for uniqueness of equilibrium because only this equilibrium survives iterated deletion of strictly dominated strategies (see Morris and Shin 2000, 2003b). The intuition behind this result is as follows. In the highly stylized banking model described, there is a unique value for the bank fundamentals, denoted d^*, which generates a distribution of private signals in such a way that there is only one signal, denoted x^*, which makes a good firm receiving this signal indifferent between repaying the loan or not. If all good firms with signals higher than x^* decide not to repay their loans, then the threshold d^* generates a proportion of mimicking firms that is sufficient to make the commercial bank insolvent.

In order to keep the model tractable and to derive closed form solutions I analyze the equilibrium values under the assumption that the private signal's precision is very high ($\beta \to \infty$). This approach is standard in the literature of symmetric binary global games. Importantly, Morris and Shin (2003b) show that this limiting assumption will not restore common knowledge. The intuition for this result is as follows. As information concerning fundamentals become more precise and the noise smaller, the actions in equilibrium resemble the behavior when the uncertainty regarding the actions of other agents become more diffuse. Hence, strategic uncertainty regarding the actions of other agents is higher for $\beta \to \infty$ and limiting behavior can be identified independently of the prior beliefs and the shape of noise. This result holds in the framework of this paper. Under very high precision of the private signals, the marginal solvent debtor firm believes that the measure of good debtor firms choosing the action not repay is a uniform distributed random variable over $[0,1]$. Under this result equation (2.10) becomes $d^* + \frac{Q}{qD} = \Phi(-\Phi^{-1}(\frac{D}{V+F+D}))$, which implies that:

$$d^* + \frac{Q}{qD} = \frac{V+F}{V+F+D} \tag{2.12}$$

This result shows that the decision to default or not strategically depends on the bank's expected profitability ($\frac{Q}{qD}$), the benefits of the on-going relation with the bank (V), and the penalty for shirking (F). According to (2.12), expected profitability plays an important role in preventing a strategic behavior of debtor firms, even when the Central Bank is inactive. A lower deposit-to-assets ratio for the bank ($\frac{Q}{qD}$) implies a higher threshold above which the bank fails when facing collective strategic default. Nevertheless, this result holds always when the decrease in this ratio is due to a decrease in Q, the nominal value of deposits at date 1, or an increase in q, the volume of loans. The impact of D, the returns on loans at date 1, on equilibrium threshold is ambiguous. Differentiating (2.12) with respect to D yields the following result:

$$\frac{\partial d^*}{\partial D} = \frac{Q}{qD^2} - \frac{V+F}{(V+F+D)^2}$$

If the difference on the right side of the above equation is positive, then keeping all other factors constant an increase in returns on loans implies a stronger position for the commercial bank facing strategic behavior from debtor firms, which is characterized by a higher value for the equilibrium threshold d^* above which the bank fails. Otherwise, if the difference is negative, the increase in nominal returns on loans has the opposite effect, weakening bank position in front of a strategic attack of debtor firms.

The effects of changes in the contractual fine (F) that can be enforced by the surviving commercial bank, or in the present value of future long term relation between a good firm and the bank (V), are straightforward and intuitive. An increase in these parameters has a positive impact on the equilibrium threshold d^*, reducing the likelihood of successful strategic default. I interpret higher values of F as better loan contracts written by the bank, while higher values for V might be interpreted as an increased importance of the banking sector in the overall economy. Following to an increase in any of these two parameters the good firms will behave less aggressively when deciding to repay their loans or not because any potential gain driven by non repayment is reduced.

Bank optimal effort

Taking as given the optimal strategy for debtor firms, the bank chooses its optimal effort by maximizing its expected payoff, conditional on the available information. First, I present the expected payoff of the bank in the general case and then I derive the closed form solution for bank optimal effort in the limiting case. An increase in exerted effort e implies a lower value for average weakness of bank fundamentals ($\mu - e$). Thus, by reducing its portfolio's risk through costly activities (e.g. extensively screening loans applications, hiring better loan agents, or writing better contracts that can be easily enforced in a court), the bank might make a strategic attack from debtor firms less likely. This happens because by lowering the average weakness of its fundamentals the bank actually increases the probability of lower signals received by debtor firms while decreasing the measure of genuine non-performing loans. The expected payoff for the bank is given by:

$$
\begin{aligned}
&P(BankSurvives) * qD * E_d[(1-d)*(1-n(d))]+ \\
&+P(BankSurvives) * q(F+D) * E_d[(1-d)*n(d)]+ \\
&+P(BankSurvives) * (-Q-c(e))+ \\
&+P(BankFails) * (-c(e))
\end{aligned} \tag{2.13}
$$

The first term represents the bank's return from the good firms which repay their loans, times the probability that the bank does not go bankrupt, times the expected number of repaying good firms; the second term denotes the bank's return from the good firms which do not repay their loans, times the probability that bank survives, times the expected number of non repaying good firms; the third term represents bank's outflow to depositors and for funding the exerted effort, times the probability that the bank does not go bankrupt; finally, the last term denotes the return in the case of bankruptcy, times the probability of bankruptcy.

If bank survives it will be able to collect D and also to fine with F all those good firms which choose not to repay their loans because its enforcement technology is preserved. This amount adds to all the repaid loans by the good firms which decided not to attack the bank. Out of this cash available at date 1 the commercial bank has to repay its depositors with notional amount Q and it also has to fund its cost of effort $c(e)$. If bank fails, the loss is given by the cost of incurred effort (if any). Ex-ante, the bank will try to infer the measure of good firms which will be repaying, conditional on the prior distribution of d.

The commercial bank has information only about the prior distribution of fundamentals. Thus, the probability of bank survival is given by:

$$P(BankSurvives) = P(d \leq d^*) = \Phi(\sqrt{\alpha}(d^* - (\mu - e))).$$

The bank anticipates the behavior of debtor firms and thus expects a measure of non repaying good firms equal to:

$$E_d[(1-d)*n(d)] = E_d[P(x > x^*] = 1 - H(x^*),$$

where $H(x^*)$ is the cumulative normal distribution function for signal x. Now we can determine the expected measure of repaying firms as:

$$E_d[(1-d)*(1-n(d))] = H(x^*) - (\mu - e).$$

See the Appendix for detailed derivations.

Finally, the expected payoff for commercial bank is given by:

$$\begin{gathered}\Phi(\sqrt{\alpha}(d^* - (\mu - e))) * qD * [H(x^*) - (\mu - e)] + \\ +\Phi(\sqrt{\alpha}(d^* - (\mu - e))) * q(F + D) * [1 - H(x^*)] \\ +\Phi(\sqrt{\alpha}(d^* - (\mu - e))) * (-Q - c(e)) + \\ +(1 - \Phi(\sqrt{\alpha}(d^* - (\mu - e)))) * (-c(e))\end{gathered} \tag{2.14}$$

In order to have an explicit solution I take first limit with respect to β, and afterwards with respect to α. When $\alpha \to \infty$, the information about prior distribution of fundamentals is very precise.A very precise prior translates in less noise regarding the bank's fundamentals. The solution for optimal choice of effort in the limiting case is:

$$e^* = \begin{cases} 1, & \frac{V+F}{V+F+D} - \frac{Q}{qD} \leq \mu \leq 1 + \frac{V+F}{V+F+D} - \frac{Q}{qD} \\ 0, & otherwise \end{cases} \tag{2.15}$$

See the Appendix for detailed derivations.

According to (2.15), the commercial bank exerts *no effort* when its unconditional fundamentals (μ) are below $\frac{V+F}{V+F+D} - \frac{Q}{qD}$, in which case a strategic attack from debtor firms is not likely, or when the unconditional fundamentals are very high (above $1 + \frac{V+F}{V+F+D} - \frac{Q}{qD}$). Both thresholds are positive given (2.2). A poor quality of loans portfolio characterized by a high average weakness of bank fundamentals $\mu - e$ increases the probability for a successful strategic attack from debtor firms. I analyze this in depth in Section 2.4.

The choice of effort is unobservable to depositors. As a result, depositors require at date 0 a nominal return Q that provides them with (at least) zero expected return in the worst case (i.e., bank chooses a risky portfolio and $e = 0$). Therefore, depositors participate only if

$$Q * P(BankSurvives \mid d, e = 0) \geq q, \tag{2.16}$$

where $P(BankSurvives \mid d, e = 0) = \Phi(\sqrt{\alpha}(d^* - \mu))$, and d^* is given by (2.10).

2.3.2 Active Central Bank Case

Central Bank intervention decision

I consider now the case when the Central Bank is active. The Central Bank intervention policy in the case of an illiquid bank should be designed to meet two challenges. First, it has to minimize the ex-ante moral hazard problem for all parties involved. This translates in a reduction of borrowers' incentive to default strategically and in higher incentives for banks to mitigate strategic behavior of debtor firms by exerting costly effort. Second, it has to minimize the cost of intervention.

The Central Bank compares the cost implied by a full intervention described in (2.6) with the social cost expected if the bank goes bankrupt, which is captured by (2.7)

Goodhart and Schoenmaker (2009) find that the recapitalization of a failing bank is efficient if the social benefits exceed the cost of recapitalization. Following a similar approach, the Central Bank in this model is more likely to intervene under high opportunity cost of forgone intermediation caused by the bank closure, and is more likely to allow bank failure in the case of high intervention costs. I denote the measure of non-performing loans by $NPL(d) = d + (1 - d)n(d)$. Following from the bank's insolvency condition (2.4) the Central Bank's cost function is given by:

$$C(NPL(d)) = min\left\{qV * (1 - d) * (1 - n(d)),\ \gamma[Q - qD * (1 - d)(1 - n(d))]\right\} \tag{2.17}$$

The firms and the commercial bank know ex-ante the Central Bank's preference (i.e., γ is common knowledge) between helping the bank and letting it go. Although the cost of intervention γ is common knowledge for all the participants, ex-post intervention policy is affected by the degree of coordination between debtor firms. A high degree of coordination increases the cost of intervention while decreasing the social cost caused by the bank closure. Hence, the Central Bank decision to bail out or not is not known ex-ante. By implementing such a bailout policy which is focused on the above objective function, Central Bank introduces a lot of ambiguity regarding the bailout amount. Hence, its final decision induces the commercial bank to exert maximum of effort ex-ante even when fundamentals are strong and also helps to mitigate the strategic behavior of debtor firms. With respect to its cost function, the Central Bank decides for a full bailout when the social cost is higher than intervention cost. Alternatively, it chooses no intervention:

$$BOA \in \{0, Q - qD * [1 - NPL(d))]\} \tag{2.18}$$

Thus, given the above cost function, the Central Bank intervenes and saves the bank if $C(NPL(d)) = \gamma \{Q - qD * (1 - d)(1 - n(d))]\}$. This translates in the following necessary condition for a full bailout:

$$NPL^* \leq \frac{qV + \gamma qD - \gamma Q}{qV + \gamma qD} \tag{2.19}$$

This expression is decreasing in γ. A higher cost of intervention implies a lower probability for a full bailout. The Central Bank steps in and provides the necessary amount only for a reduced measure of non-performing loans reported, given the higher cost of intervention γ. According to this inverse relation between the cost of intervention and the probability for a Central Bank bailout, we can argue that in countries where the central bankers are able to set policy without interference or restriction of the political authorities (i.e., independent central bankers) and where their commitment to deal with inflationary pressures is credible (e.g. the main mandate is to maintain price stability by controlling inflation), the Central Bank's incentive to close the illiquid banks are higher. This result shed some light on the distinction between FED and ECB and between their expected policies towards banks closure. There are voices who claim that due to the fact that the Federal Reserve Act is easier to be changed than the Maastricht Treaty, the FED's independence is tenuous than the ECB's, particularly because in US political pressure is easy to exert. On the other hand, the FED has to preserve both full employment and price stability, whereas stable prices are the ECB's sole mandate. The policies adopted by these two central banks during 2007/2008 credit crisis support both the previous claims and the insights provided by this model with respect to the cost of intervention incurred by a regulator. While the ECB was reluctant to cut interest rates for a long period after the crisis' onset, the FED lowered the interest rate aggressively and lengthen the term on direct loans to banks from the FED's discount window and, when banks were slow to respond, the FED introduced its term auction facilities to make loans at the discount window cheaper.

Next I consider the impact that deposit-to-assets ratio has on the Central Bank's intervention policy. A high expected profitability increases the threshold NPL^* below which the Central Bank intervention allow commercial bank to avoid failure. This result holds for both a decrease in the nominal value of deposits at date 1, Q, or an increase in the volume of loans, q, and for an increase in the returns on loans at date $1, D$. The positive relation between the expected profitability of the commercial bank and the threshold NPL^* suggests that, in the case of an active Central Bank, deposit-to-assets ratio plays an important role in preventing a strategic behavior of debtor firms because a lower ratio makes the bailout decision of Central Bank more likely. A similar result is generated by an increase in V, interpreted as the present value of future long term relation between a good firm and the bank. An increase in the opportunity cost of forgone intermediation if the commercial bank is closed implies a higher probability for a bailout, thus a more relaxed policy towards closure.

Taking as given this optimal strategy for the Central Bank we can now explain the equilibrium strategies for both the debtor firms and the commercial bank.

Firms repayment incentives

When deciding its action, each good firm should try to infer when the Central Bank decides to step in and bail out the bank. Thus, the probability of bank survival in this case is given by:

$$P(NPL(d) \leq NPL^* \mid x)$$

If the measure of non-performing loans is below the threshold accepted by the Central Bank, then the commercial bank will be bailed out (if necessary) and a strategic attack from debtor firms will be contained. I start by deriving the equilibrium in threshold strategies. Let suppose as before that there is a threshold x^{**} such that all the firms which see a signal $x > x^{**}$ will not repay their loans to bank. Given NPL^* derived in (2.19) and following the same reasoning as when the Central Bank was inactive, we may derive the new thresholds x^{**}, the threshold signal at which a good firm is indifferent between repaying his loan or not, and d^{**}, the threshold value for commercial bank fundamentals above which the bank fails when Central Bank is active. Note however that the default condition in the presence of an active Central Bank is different than (2.4). Insolvency triggers are those values of fundamentals satisfying (2.4), but due to potential intervention of the Central Bank the bank might survive for some values of d satisfying the insolvency condition. Nevertheless, for values of d satisfying

$$d + (1 - d)n(d) \geq NPL^* \tag{2.20}$$

the bank is insolvent (since NPL^* derived in (2.19) satisfies insolvency condition (2.4)) and is not subject of liquidity injection. Hence, under default condition (2.20), our thresholds are as follows:

$$x^{**} = NPL^* \frac{\alpha + C}{C} - \frac{\alpha(1 - d^{**} + \mu - e)}{C} - \sqrt{\frac{\alpha + C}{C\beta}} \Phi^{-1}(\frac{D}{V + F + D}), \tag{2.21}$$

$$d^{**} + 1 - NPL^* = \Phi(\sqrt{\beta} \frac{\alpha - C}{C}(d^{**} + NPL^* \frac{\alpha + C}{\alpha - C} - \frac{\alpha(1 + \mu - e)}{\alpha - C} - \frac{C}{\alpha - C} \sqrt{\frac{\alpha + C}{C\beta}} \Phi^{-1}(\frac{D}{V + F + D}))), \tag{2.22}$$

where $C = \alpha + \beta + 2\rho\sqrt{\alpha\beta}$. ρ is the correlation coefficient between random variables d and $(1-d)n(d)$. The detailed mathematical derivations are in the Appendix.

The right side of equation (2.22) is a cumulative normal distribution:

$$N(-NPL^* \tfrac{\alpha+C}{\alpha-C} + \tfrac{\alpha(1+\mu-e)}{\alpha-C} + \tfrac{C}{\alpha-C}\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D}), \tfrac{C^2}{\beta(\alpha-C)^2}).$$

Thus, we may conclude that d^{**} is the intersection point between the cumulative normal distribution just described and a straight line (with a slope of 1) and positive intercept $1 - NPL^*$. This intersection point exists and it is unique if the slope of cumulative normal distribution is less than one everywhere. The sufficient conditions for a unique solution for d^{**} are given by:

$$\begin{aligned} &\frac{\sqrt{\beta}}{\alpha + \beta + 2\rho\sqrt{\alpha\beta}}(-\beta - 2\rho\sqrt{\alpha\beta}) \leq \sqrt{2\pi} \\ &\rho < 0 \\ &0 < \beta \leq -2\rho\sqrt{\alpha\beta} \\ &\alpha > -\beta - 2\rho\sqrt{\alpha\beta} \end{aligned} \tag{2.23}$$

Proposition 2 *When the precision of the private signal of debtor firms (β) and the prior precision (α) satisfy the conditions described by (2.23), there is a unique d^{**} defined in (2.22) such that, in any equilibrium of the game with imperfect information, the bank fails if and only if $d > d^{**}$.*

Proof. *See the Appendix.* ∎

Put differently, Proposition 2 says that when the Central Bank is active, we can still have an unique equilibrium which survives iterated deletion of strictly dominated strategies. Nevertheless, from the individual borrower point of view, the inference regarding the probability of bank failure is much more complex compared with the case of an inactive Central Bank. A debtor firm cares not only about the true realization of fundamentals d, but she also cares about the measure of good firms deciding not to repay, $(1-d)n(d)$. Hence, not only the precision of private signals relative to the precision of the prior (β relative to α) plays a determinant role in eliminating multiple equilibria, but also the negative correlation between random variables d and $(1-d)n(d)$.

The solution in the limiting case when private signal's precision is very high ($\beta \rightarrow \infty$) is :

$$d^{**} = NPL^{*}. \tag{2.24}$$

Proposition 3 *The threshold d^{**} above which the bank fails even if the Central Bank is active is always larger than d^{*}, the threshold above which the bank fails if Central Bank is inactive.*

Proof. *See the Appendix.* ■

The intuition is straightforward. An active Central Bank can mitigate strategic default behavior of debtor firms, as it allows commercial banks to survive more often, preserving its loan enforcement capacity so that firms choose to repay more often. Actually it can provide an extra liquidity buffer under the condition that providing this is not too costly. Debtor firms will behave strategically only when commercial bank fundamentals are very poor, in which case they assign a low probability for an intervention by the Central Bank as it would be very expensive.

Deposit-to-assets ratio plays an important role in preventing a strategic behavior of debtor firms in this case too. A lower deposit-to-assets ratio for the bank $(\frac{Q}{qD})$ implies a higher threshold above which the bank fails when facing collective strategic default. This result holds always both for a decrease in Q, the nominal value of deposits at date 1, and for an increase in D, returns on loans at date 1. This result is intuitive. By increasing expected profitability the commercial bank improves the threshold of non-performing loans (NPL^*) below which the Central Bank might intervene, hence making the bailout decision of Central Bank more likely. Anticipating this, good firms will behave less aggressively when deciding to repay their loans or not.

Bank optimal effort

Taking as given the optimal strategies for the Central Bank and for debtor firms, the bank chooses its optimal effort by maximizing its expected payoff, conditional on the available information. The equilibrium decision in this case can be derived using the same methodology as in the previous case. The bank knows that by exerting effort ex-ante it may reduce the probability for a strategic behavior on the debtor firms side, and also might reduce the measure of non-performing loans, making in this way the Central Bank intervention more likely (if it is necessary). The expected payoff for bank is given by:

$$\begin{gathered} P(BankSurvives) * qD * E_d[(1-d) * (1-n(d))] + \\ +P(BankSurvives) * q(F+D) * E_d[(1-d) * n(d)] + \\ +P(BankSurvives) * (-Q - c(e)) + \\ +P(BankFails) * (-c(e)) \end{gathered}$$

Next I study how an active Central Bank influences the behavior of commercial bank.

The only difference from the situation when Central Bank is inactive, comes from the building mechanism of probability of bank survival. Namely, when deciding its action, the bank should try to infer when the Central Bank decides to step in. Thus, the probability of bank survival in this case is given by:

$$P(BankSurvives) = P(NPL \leq NPL^*) = \Phi(\sqrt{\tfrac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d))).$$

The choice of effort is unobservable to depositors. Therefore, depositors participate only if the nominal return Q provides them with (at least) zero expected return in the worst case (i.e., bank chooses a risky portfolio and $e = 0$). The participation constraint for depositors is as follows

$$Q * P(BankSurvives \mid d, e = 0) \geq q,$$

where $P(BankSurvives \mid d, e = 0) = \Phi(\sqrt{\tfrac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu + 1 - d)))$, and NPL^* is given by (2.19).

The optimal choice of effort for the limiting case (i.e., $\beta \to \infty$, and $\alpha \to \infty$) is:

$$e^{**} = \begin{cases} 1, & 1 - \frac{2\gamma Q}{qV+\gamma qD} \leq \mu \leq 2 - \frac{2\gamma Q}{qV+\gamma qD} \\ 0, & otherwise \end{cases} \tag{2.25}$$

See the Appendix for detailed derivations.

The optimal results for bank's effort suggest that bank's behavior is influenced in this case by its unconditional fundamentals (μ) and by the Central Bank cost of intervention (γ). From (2.25), the commercial bank exerts *no effort* when its unconditional fundamentals (μ) are below $1 - \frac{2\gamma Q}{qV+\gamma qD}$, in which case a strategic attack from debtor firm is less likely, or when the unconditional fundamentals are very high (above $2 - \frac{2\gamma Q}{qV+\gamma qD}$), which makes any effort useless in avoiding closure. Both thresholds should be positive in order to have economic significance due to the fact that fundamentals are defined in this model as a positive measure. It is straightforward to see that the lower threshold $(1 - \frac{2\gamma Q}{qV+\gamma qD})$ is positive only for values of γ below the threshold

$$\gamma_M = \frac{qV}{2Q - qD}. \tag{2.26}$$

Since the cost of intervention is always positive, I restrict the analysis according to the following condition:

$$Q > \frac{qD}{2} \tag{2.27}$$

This restriction allow us to have a positive and economically significant cost γ_M.

Proposition 4 *The optimal effort e^{**} has the same binary values as in the case with an inactive Central Bank, namely is either* 1 *or* 0*.*

Proof. *Following from (2.15) and (2.25), this is a direct result of my limiting assumptions, driven by the functional form of the screening cost, $c(e)$.* ■

Next I investigate how the Central Bank intervention mechanism influences the commercial bank behavior with respect to its fundamentals. As we will see, from the point of view of moral hazard induced at the commercial bank level, for some values of γ the presence of an active Central Bank is beneficial, while for others it is not.

In order to identify those γ values for which the presence of an active Central Bank mitigates the moral hazard not only for borrowers, but also for the commercial bank, I compare the two polar cases studied so far (active Central Bank and inactive Central Bank) focusing the attention on the impact that marginal cost of intervention has on the behavior of commercial bank.

I denote the thresholds in unconditional fundamentals identified in (2.15) and (2.25) such that $\mu_1 = \frac{V+F}{V+F+D} - \frac{Q}{qD}$, $\mu_2 = 1 - \frac{2\gamma Q}{qV+\gamma qD}$, $\mu_3 = 1 + \frac{V+F}{V+F+D} - \frac{Q}{qD}$ and $\mu_4 = 2 - \frac{2\gamma Q}{qV+\gamma qD}$. It is straightforward to prove that higher γ has a negative impact for μ_2 and μ_4. When marginal cost of intervention γ is strictly lower than a certain threshold γ^*, with

$$\gamma^* = \frac{qVD^2 + VQ(V+F+D)}{D[Q(V+F+D) - qD^2]}, \tag{2.28}$$

the following classification for the unconditional bank fundamentals is met:

$$\mu_1 < \mu_2 < \mu_3 < \mu_4$$

The threshold γ^* is positive and $\gamma^* < \gamma_M$ from (2.2) and (2.27). Following this classification, the optimal effort exerted by the commercial bank is:

$$e = \begin{cases} 0, & \mu < \mu_1, & \text{with or without an active CB} \\ 1 & \mu_1 \leq \mu < \mu_2, & \text{with an inactive CB} \\ 1 & \mu_2 \leq \mu \leq \mu_3, & \text{with or without an active CB} \\ 1 & \mu_3 < \mu \leq \mu_4, & \text{with an active CB} \\ 0 & \mu > \mu_4, & \text{with or without an active CB} \end{cases}$$

When the Central Bank is inactive, the possibility of a collective strategic default induces the commercial bank to exert maximum optimal effort $e = 1$ even when the economic environment is healthy and the corporate sector prospects for good performance are high (i.e., unconditional fundamentals μ are low, $\mu_1 \leq \mu < \mu_2$). This happens for values of μ lower than μ_2, the level which triggers a change in the behavior of commercial bank when Central Bank is active.

Figure 1 shows the level of effort exerted by the commercial bank for different values of its unconditional fundamentals when the cost of intervention is low ($\gamma < \gamma^*$). In this case, an inactive Central Bank induces commercial bank to exert maximum of effort sooner (with respect to the values of unconditional fundamentals) than when it is active.

<< FIGURE 1 HERE >>

On the other hand, a higher cost of intervention ($\gamma > \gamma^*$) justifies an active Central Bank presence and mitigates the moral hazard problem. When the cost of intervention is high enough, the following classification for the unconditional bank fundamentals will be met:

$$\mu_2 < \mu_1 < \mu_4 < \mu_3$$

Following this classification, the optimal effort exerted by the commercial bank is:

$$e = \begin{cases} 0, & \mu < \mu_2, & \text{with or without an active CB} \\ 1 & \mu_2 \le \mu < \mu_1, & \text{with an active CB} \\ 1 & \mu_1 \le \mu \le \mu_4, & \text{with or without an active CB} \\ 1 & \mu_4 < \mu \le \mu_3, & \text{with an inactive CB} \\ 0 & \mu > \mu_3, & \text{with or without an active CB} \end{cases}$$

Figure 2a shows the level of effort exerted by commercial bank for different values of its unconditional fundamentals when cost of intervention is high ($\gamma > \gamma^*$), but not extremely high ($\gamma < \gamma_M$). In this case, an active Central Bank induces commercial bank to exert maximum of effort sooner (with respect to the values of unconditional fundamentals) than when it is inactive. An extreme case for this scenario is depicted in Figure 2b which shows the level of effort exerted by commercial bank for very high cost of intervention ($\gamma > \gamma_M > \gamma^*$). In this particular case, an active Central Bank induces commercial bank to exert maximum of effort even when μ goes to zero.

<< FIGURE 2a HERE >>

<< FIGURE 2b HERE >>

I summarize these findings in the following three propositions.

Proposition 5 *For low cost of intervention ($\gamma < \gamma^*$), an active Central Bank induces moral hazard in commercial bank behavior. When Central Bank is inactive, the commercial bank finds optimal to exert maximum of effort ($e = 1$) when its unconditional fundamentals are stronger ($\mu_1 \le \mu \le \mu_2$) than when Central Bank is active. When the Central Bank is active the commercial bank exerts maximum of effort only when the unconditional bank fundamentals are above μ_2. This result holds given that the exogenous parameters satisfy (2.2) and (2.27).*

Proof. *See the Appendix.* ■

The intuition behind this result is as follows. When the Central Bank is active in the economy and the cost of intervention is very low, either because the lack of credibility in Central Bank's commitment to maintain price stability, or due the political inference in setting the intervention policy, the commercial bank's expectation for a bailout is very high. Under these circumstances it prefers to exert no effort and to bet on the preference that the Central Bank might have in avoiding tough and unpopular social measures such as closing the bank and denying access to credit to good firms which have been repaying their loans. Nevertheless, as bank unconditional fundamentals characterizing the health of economic environment worsen ($\mu \geq \mu_2$), the commercial bank starts to exert higher costly effort in screening loan applications, in order to improve the quality of its assets, knowing that, even for a very low cost of intervention, the Central Bank will not always decide for a bailout. A different behavior the commercial bank has when the Central Bank is inactive. In this situation, because the Central Bank never intervenes, the commercial bank has only one weapon against the possible collective strategic behavior of its borrowers, namely better screening. Hence, it prefers to exert costly effort even when its unconditional fundamentals are very strong ($\mu_1 \leq \mu \leq \mu_2$) in order to make the coordination between firms more difficult.

Proposition 6 *A high enough cost of intervention ($\gamma > \gamma^*$) mitigates the moral hazard generated by an active Central Bank when commercial bank unconditional fundamentals are strong enough ($\mu_2 \leq \mu \leq \mu_1$). This result holds given that the exogenous parameters satisfy (2.2) and (2.27).*

Proposition 7 *When cost of intervention is very high ($\gamma > \gamma_M > \gamma^*$), the presence of an active Central Bank has a double-edge effect. On one hand determines the commercial bank to exert maximum of effort for very strong unconditional fundamentals ($0 \leq \mu \leq \mu_1$), while on the other hand it induces commercial bank to exert no effort for poor unconditional fundamentals ($\mu > \mu_4$).This result holds given that the exogenous parameters satisfy (2.2) and (2.27).*

Proof. *See the Appendix.* ■

The first proposition asserts that a higher cost of intervention supports the presence of an active Central Bank as long as moral hazard mitigation is its main concern. The higher cost of intervention implies that strategic debtor firms behave more aggressively. They assign a lower probability for a bailout under this condition. The commercial bank understands this and decides to exert maximum of effort even under strong unconditional fundamentals. This chain effect translates in our model in the negative impact of intervention cost γ on the value of unconditional fundamentals which characterize the change in behavior for commercial bank (μ_2 and μ_4). To conclude, a higher cost of intervention, together with the ambiguity introduced by the Central Bank intervention mechanism allow commercial bank to survive more often when it faces opportunistic behavior from borrowers and in the same time induce the commercial bank to improve loans quality by exerting costly effort in good states of the economy.

The key implication of the second proposition is that under a very high cost of intervention, the ambiguity of Central Bank's intervention decision has a very strong positive impact on commercial bank incentives under strong ($0 \leq \mu \leq \mu_1$) unconditional fundamentals, while having a negative impact for very poor ($\mu > \mu_4$) unconditional fundamentals. The first part of this result implies that, under the prospect of collective strategic default, and knowing that Central Bank has a strong preference not to provide liquidity, commercial bank finds optimal to reduce its risk taking (e.g. by exerting maximum of effort) even when genuine distress is absent (μ is close to 0). The second part suggests that commercial bank assigns a low probability of intervention under poor unconditional fundamentals, because in these circumstances the intervention is very expensive for the Central Bank. Since it expects a high non repayment, it finds optimal to exert no effort.

2.4 Comparative Statics

2.4.1 Changes in Unconditional Bank Fundamentals

The effort exerted by the bank in period 0 has a direct impact on the loan quality. An increase in effort e implies a lower value for average weakness of bank fundamentals $(\mu - e)$. A higher average weakness of bank fundamentals caused either by a lower effort e exerted by the bank, or by a higher unconditional bank fundamentals μ, has a double impact: it lowers the thresholds which trigger strategic default, and it increases the probability of bank failure. I start by examining the changes in the exogenous variable μ, when effort e is given. I continue by investigating the changes in variable μ, when effort e is the optimal effort.

Firstly, when effort e is given, an increase in unconditional fundamentals μ lowers the equilibrium threshold in bank fundamentals (d^* or d^{**}) below which the bank survives when facing a strategic attack of debtor firms. This result comes from differentiating (2.10) and (2.22), respectively. Hence, $\frac{\partial d^*}{\partial \mu} < 0$ and $\frac{\partial d^{**}}{\partial \mu} < 0$. These results suggest that when the unconditional fundamentals become worse, commercial banks may be subject to risk of failure more often due to a coordination problem among debtors.

On the other hand, an increase in the unconditional bank fundamentals increases the probability of a bank failure. For the case when Central Bank is inactive, given the prior distribution of d, the probability of bank failure for a given value d^* is:

$$P(BankFails) = P(d > d^*) = 1 - \Phi(\sqrt{\alpha}(d^* - (\mu - e))).$$

Differentiating the above probability with respect to μ yields

$$\tfrac{\partial P(d > d^*)}{\partial \mu} = -\phi(\sqrt{\alpha}(d^* - (\mu - e))) * \sqrt{\alpha} * (\tfrac{\partial d^*}{\partial \mu} - 1),$$

which is positive given the negative impact of unconditional bank fundamentals on the equilibrium threshold d^*.

The result holds for the alternative case when Central Bank is active in the economy. In this case the probability of bank failure for a given value d^{**} is:

$$P(BankFails) = P(NPL > NPL^{*}) =$$
$$= 1 - \Phi\left(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^{*} - (\mu - e + 1 - d))\right).$$

Differentiating the above probability with respect to μ yields

$$\frac{\partial P(NPL>NPL^{*})}{\partial \mu} = -\phi\left(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^{*} - (\mu - e + 1 - d))\right) * \sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}} * \left(-1 + \frac{\partial d^{**}}{\partial \mu}\right),$$

which is positive given the negative impact of unconditional bank fundamentals on the equilibrium threshold d^{**}.

To sum up these findings I can argue that the thresholds for successful collective strategic default and the probability of bank failure are directly related. The lower the threshold below which an attack is contained, the higher the prior probability of a bank collapse.

Secondly I examine the changes in the exogenous variable μ, when effort e is the optimal effort. The effects described above hold when all agents play their optimal strategies. Nevertheless, as the next three figures illustrate, due to the fact that the optimal effort e is not a continuous variable, we meet jumps for both the probability of bank failure and equilibrium thresholds in bank fundamentals. These jumps are generated by a change in strategy followed by the commercial bank.

Figures 3 to 5 illustrates what happens for a specific set of parameters by plotting the equilibrium thresholds in bank fundamentals and the probability of default as a function of unconditional fundamentals. The case of an inactive Central Bank is captured in Figure 3 for $\frac{Q}{qD} = 0.5$, $\Phi^{-1}(\frac{D}{V+F+D}) = 1.293$, $\alpha = 9$,and $\beta = 160$, while for the case of an active Central Bank parameters are $\frac{Q}{qD} = 0.5$, $\Phi^{-1}(\frac{D}{V+F+D}) = 1.293$, $\alpha = 10$, $\beta = 0.3$ and $\rho = -0.1$. Figure 4 illustrates a scenario with a low cost of intervention γ which translates in a higher threshold in non-performing loans below which the Central Bank intervenes ($NPL^{*} = 0.8$). Figure 5 illustrates a scenario with a high cost of intervention γ which translates in a lower threshold in non-performing loans below which the Central Bank intervenes ($NPL^* = 0.2$).

<< FIGURE 3 HERE >>

<< FIGURE 4 HERE >>

<< FIGURE 5 HERE >>

The above results suggest that the economic environment has a very serious impact on the ability that commercial banks have to survive when facing strategic default. I interpret the unconditional bank fundamentals μ as a measure for development of corporate sector or health of economic environment. A high μ can be interpreted as weak corporate sector with poor performance (poor asset quality). A healthy economy, in which the prospects for corporate sector performance are high, it is characterized by a lower μ. The commercial banks can not influence directly this macro variable. Nevertheless, the level of effort chosen when screening loan applications affects the quality of assets the bank ends up holding. Thus, in countries where financial environment is characterized by poor quality of corporate sector, commercial banks are very exposed to the risk of collective strategic default.

Next I examine the impact that an increase in unconditional fundamentals μ has on the incidence of collective strategic default, measured here by the variable $(1-d)n(d)$. Firstly I examine the changes in the exogenous variable μ, when effort e is given and afterwards I examine the case when effort e is the optimal effort. When the unconditional bank fundamentals increase, keeping the effort exerted by the commercial bank constant, the probability that good firms receive a higher signal increases. This implies that more solvent firms choose the action not repay. As we have seen previously, the unconditional bank fundamentals have a negative impact on the equilibrium thresholds. Intuition suggests that, due to the fact that bank fails for lower levels of fundamentals, the necessary number of good firms which should choose the action non repay such that the bank fails should increase. The result holds true in both cases we have analyzed. The derivative $\frac{\partial(1-d)n(d)}{\partial\mu}$ is positive for the case of an inactive/active Central Bank. Complete derivations can be found in the Appendix.

When effort e is the optimal effort, and thus a change in unconditional bank fundamentals triggers a change of strategy from commercial bank, the effects described above hold when all agents play their optimal strategies. As we have seen before, due to the fact that the optimal effort e is not a continuous variable, we meet jumps in the incidence of collective strategic default. For the case of an inactive Central Bank I illustrate this in figure 6 via simulation by setting $\frac{Q}{qD} = 0.5$, $\Phi^{-1}(\frac{D}{V+F+D}) = 1.293$, $\alpha = 9$,and $\beta = 160$, and varying μ from 0 to 1. The case of an active Central Bank is captured in figure 7 for $\frac{Q}{qD} = 0.5$, $\Phi^{-1}(\frac{D}{V+F+D}) = 1.293$, $\alpha = 10$, $\beta = 0.3$, $\rho = -0.1$ and $NPL^{*} = 0.8$.

<< FIGURE 6 HERE >>

<< FIGURE 7 HERE >>

The simulation suggests that in both cases, when we compute the necessary measure of good firms which should behave strategically in order to trigger bank's default with respect to total number of debtor firms and also with respect to total numbers of good firms, these ratios are increasing with respect to bank unconditional fundamentals. I conclude that higher values for unconditional bank fundamentals increase the measure of good firms behaving strategically in equilibrium due to the negative impact on equilibrium thresholds d^{*} and d^{**}. Put differently, a higher μ implies an increase in the required degree of coordination among these good firms that can lead to bank failure.

2.4.2 Effect Changes in the Intervention Cost

Three directions are considered in turn: impact that changes in intervention cost have on commercial bank behavior; on the behavior of good debtor firms; and on the degree of coordination between borrowers which triggers bank's failure. When the cost of intervention γ increases, the threshold values for unconditional bank fundamentals μ at which the commercial bank chooses to exert maximum of effort are reduced. This result comes directly from Proposition 6 which captures the switch between thresholds μ_1 and μ_2. Higher cost of intervention has a negative impact for the value of μ_2. This result suggests that moral hazard introduced by the Central Bank as a LOLR is mitigated by a higher cost of intervention. When the Central Bank is active in the economy and the cost of intervention is high, either because of the credible Central Bank's commitment to maintain price stability, or due the lack of political inference in setting the intervention policy, the commercial bank's expectation for a bailout is low. Under these circumstances it prefers to exert costly effort even when its unconditional fundamentals are very strong ($\mu \leq \mu_1$) in order to make the coordination between firms more difficult. Thus countries in which monetary authorities are concerned with the high inflationary costs induced by printing money, or countries in which central bankers are independent, create good incentives for banks to be pro-active and to reduce the risk of their portfolios by exerting maximum of effort in order to avoid collective strategic default of borrowers.

Another effect of intervention cost is on the thresholds in fundamentals above which the bank fails. By differentiating (2.22), we obtain a negative value for $\frac{\partial d^{**}}{\partial \gamma}$. This implies a lower threshold below which the bank survives when Central Bank is active. Thus, an increase in marginal cost of intervention translates into a lower equilibrium value for commercial bank fundamentals above which the bank fails. Good debtor firms behave more aggressive when the cost of intervention is high, in which case they assign a low probability for an intervention by the Central Bank as it would be very expensive. Hence, the commercial bank might fail even for strong fundamentals. Nevertheless, as Proposition 3 shows, the threshold in bank fundamentals which triggers collective strategic default for the case of an active Central Bank is always higher than the threshold characterizing the case of an inactive Central Bank.

The third effect of intervention cost I examine is on the degree of coordination between borrowers required to make a collective strategic default successful. By differentiating the necessary measure of firms which should decide not to repay in order to trigger the bank's default, we find a positive value for $\frac{\partial(1-d)n(d)}{\partial\gamma}$. Hence, the impact is identical with the effect that an increase in unconditional bank fundamentals has on the incidence of a strategic attack. A higher cost of intervention implies a higher degree of coordination between borrowers.

Figure 8 illustrates what happens for a specific set of parameters by plotting the equilibrium thresholds in bank fundamentals and the probability of default as a function of unconditional fundamentals for different values in cost of intervention. The cost of intervention γ and the measure of non-performing loans NPL^* changes from 0.1 to 0.2 and from 0.4 to 0.2, respectively, while other parameters are kept constant: $\frac{Q}{qD} = 0.05$, $\Phi^{-1}(\frac{D}{V+F+D}) = 1.67$, $\alpha = 10$, $\beta = 0.3$, and $\rho = -0.1$.

<< FIGURE 8 HERE >>

Thus, we may conclude that higher cost of intervention (γ) has a double-edge effect: on one hand it helps in reducing bank's moral hazard due to the fact that commercial bank has strong incentive to exert maximum of effort when its unconditional fundamentals are strong (μ is low); on the other hand it lowers the threshold in fundamentals that triggers collective strategic default (d^{**}) and increases the degree of coordination between good firms, thus precipitating bank failure.

2.5 Concluding Remarks and Further Research

In this paper I examine the impact of Central Bank intervention policy as a LOLR on borrowers' and commercial bank's incentives and I derive the probability of a run by bank borrowers. I study a model in which a monopolistic commercial bank faces a liquidity shortage which is aggravated by the strategic default of solvent firms. The main assumption behind the model is that the bank fundamentals are not common knowledge. The borrowers hold common prior beliefs about the state of fundamentals and receive private signals about its realization. Within this framework, the Central Bank acts as the only regulator. The commercial bank understands that its current asset choice will affect the Central Bank intervention policy, while the Central Bank recognizes the opportunity cost of forgone intermediation if the commercial bank is closed. The Central Bank tries to minimize the social cost induced by the failure of the bank, trading it off against the cost of full intervention. The paper's main findings are the following.

First, I show that banks may be subject to risk of failure even when fundamentals are strong due to a coordination problem among debtors. As a result of collective strategic default a financially sound firm may claim inability to repay if it expects a sufficient number of other firms to do so as well, thus reducing bank's enforcement ability. This occurs in particular when financial environment is characterized by poor quality of corporate sector.

Second, I find that an active Central Bank can mitigate the strategic behavior of debtor firms. I distinguished between two types of intervention policies: a tough policy, when the Central Bank is inactive, and a semi-tough policy, when the Central Bank is active. I show that an active Central Bank allows commercial banks to survive more often when they face opportunistic behavior from borrowers. Debtor firms behave strategically only when bank fundamentals are very poor, because in that case Central Bank intervention is very costly and thus improbable.

Third, I find that under specific market conditions an active Central Bank induces commercial banks to affect loan quality ex-ante, which indirectly reduces debtors' incentives for strategic default. This result contradicts the idea that an ex-post bail-out policy often reduces bank incentives to exert effort and to improve the quality of its assets portfolio. The market conditions I refer to are represented by the marginal cost of intervention. I interpret this intervention cost as a proxy for Central Bank's independence and its commitment to maintain price stability. I argue that the cost of intervention faced by the Central Bank has a double-edge effect. On one hand a higher cost of intervention reduces the moral hazard problem at the commercial bank level. On the other hand, it can precipitate bank failure by lowering the threshold in fundamentals that triggers collective strategic default and by increasing the degree of coordination between good firms. Put differently, a higher cost of intervention makes the debtors to behave more aggressively and also it makes the commercial bank to behave more prudently. Nevertheless, the threshold in bank fundamentals which triggers collective strategic default for the case of an active Central Bank is always higher than the threshold characterizing the case of an inactive Central Bank.

Fourth, I show that high bank expected profitability reduces the likelihood of collective strategic default. As a measure of expected profitability I use the deposit-to-asset ratio. I show that this ratio plays an important role in preventing a strategic behavior of debtor firms, even if the Central Bank never intervenes to bail out a defaulting bank. A lower deposit-to-assets ratio implies a higher threshold above which the bank fails when facing collective strategic default. This result supports the existing empirical research which has shown that in developing economies high interest rate differential between deposit and loan rates is the main result of a poor development of corporate sector and lack of competition in banking industry. This paper adds a new interpretation, namely that in developing economies high interest rate differential between deposit and loan rates can be seen as a risk management mechanism which helps commercial banks to protect themselves against a collective strategic default.

The model provides some testable implications. Three areas are considered in turn: related lending, portfolio diversification and business cycle. The backbone of related lending literature is that a large proportion of bank lending is granted to related parties. Under this condition, if borrowers are minority shareholders of bank, they may have an incentive to default as loan default hurts them as shareholders less than the gain from not repaying loans. One implication of this paper is that banks are fragile because related parties default strategically precisely when outside borrowers are in financial distress. Second, if one believes in the predictions of this analysis, particularly in the result that bank failure might be induced by strategic coordination of borrowers, the immediate implication is that in order to increase financial stability, regulators should force banks to make small loans in order to make the coordination between debtors more difficult. Thus, loan portfolio diversification is good because reduces the chances for a successful collective attack against the bank. Finally, a natural interpretation of the result that the higher measure of genuinely distressed firms implies a higher possibility for successful strategic defaults, is in terms of business cycle. In recessions one would expect more strategic defaults, particularly due to the poor quality of economic environment.

The model in this paper is highly stylized. Behind the main findings there are three crucial ingredients. First, the bank fundamentals are not common knowledge. Second, the lending bank will be able to fully pursue non paying solvent firms only if it survives. Third, the commercial bank and the borrowers know ex-ante the Central Bank's cost of intervention, while the Central Bank decision to bail out or not is not known ex-ante. Hence, I have studied the role of the Central Bank as LOLR under opportunistic behavior from borrowers in a highly simplified way in order to highlight the main points. The model is open to several interesting extensions. The natural directions for future research are the following. First, the assumption of common knowledge about Central Bank's cost of intervention can be relaxed. The intuition suggests that in this case, the coordination between debtor firms will be much more difficult to attain. Second, the assumption that the Central Bank is the only regulator might be relaxed. It would be interesting to examine how a policy maker will allocate the LOLR responsibilities to that regulator (e.g. either a Central Bank, or a deposit insurer, or any other financial authority) whose lending decision minimizes the intervention cost.

This model ignores many complexities of real world banking system that affect both borrower and lender, such as the existence of the interbank market, or the presence of complex balance sheets, or the renegotiation between lender and borrowers issue. Relaxing the assumption of a monopolistic bank would allow us to study the impact of strategic default on systemic banking crises. It is possible to extract more policy implications if we allow for a complex balance sheet. The existing banking literature argues that liquidity requirements reduce the impact of strategic uncertainty on the bank conditions since they allow the bank to withstand larger runs from depositors. The same logic applies here: more liquidity (i.e., cash reserves) or more capital increase the threshold at which collective strategic default happens, thus reducing the probability of bank failure. Some particular forms of renegotiation are interesting to be explored too. A first form of renegotiation assumes that the Central Bank bails out some bad borrowers, leaving the commercial bank with a healthy balance sheet. This should be stabilizing because it reduces the measure of firms in genuine distress and increases the threshold at which collective strategic default happens. The critical question here will be to compare the costs of bailing out banks with the costs of identifying and bailing out firms in genuine financial distress. A second form of renegotiation assumes that the good firms which choose to repay will decide also to bail out their bank if necessary. The immediate result of such an action is that bank's enforcement ability is preserved. Both forms of renegotiation might be preferable to closing the bank.

Further research should also focus on empirical tests of the implications of this analysis. To implement the tests, however, more detailed data are required than are available from sources such as anecdotal evidence.

2.A Derivations and Proofs

Signal and fundamentals thresholds derivation. The case of inactive Central Bank.

Let us suppose that there are two thresholds x^* and d^*, such that all the firms which see a signal $x > x^*$ will not repay their loans, while d^* represents the threshold in bank fundamentals at which the bank will fail for values of $d > d^*$.

The distribution of signals x_i across firms conditional on the realization of fundamentals d is given by cumulative distribution function (cdf) $P(x \leq x^* \mid d)$. This cumulative normal distribution function is decreasing in d, positive and continuous for any value of x^*. The higher d, the lower the probability that signal x lies below any threshold x^*. Given the normality assumption, we may derive this cdf:

$$P(x \leq x^* \mid d) = \Phi(\sqrt{\beta}(x^* - d)), \tag{2.29}$$

where Φ is the cumulative distribution function of the standard normal distribution.

This is a straightforward result of the following variable transformation:

$$P(x \leq x^* \mid d) = P(d + \epsilon \leq x^* \mid d) = P(\epsilon \leq x^* - d) = P(\tfrac{\epsilon - 0}{\sqrt{1/\beta}} \leq \tfrac{x^* - d}{\sqrt{1/\beta}}) = \Phi(\sqrt{\beta}(x^* - d)).$$

This means that a critical number of good firms should attack the bank in order to cause the bank's failure. This value is given by the cumulative mass of good firms who have seen a signal above the threshold signal x^*. Since we have assumed that a good firm which is indifferent between attacking and not will choose not to attack,

$$P(x > x^* \mid d) = z(d) = (1 - d)n(d) \tag{2.30}$$

By plugging the distribution of signals x_i across firms conditional on the realization of fundamentals d^* and the measure of good firms which choose not to repay (in 2.29 and 2.4, respectively), one obtains the main equilibrium condition:

$$\Phi(\sqrt{\beta}(x^* - d^*)) = \frac{Q}{qD} + d^* \tag{2.31}$$

Following from (2.4), the bank insolvency (and subsequent default) is triggered by

$$[1 - d^* - (1 - d^*)n(d^*)] * qD = Q,$$

where d^* represents the threshold in bank fundamentals at which the bank fails for values of $d > d^*$.

Applying Bayesian inference under a normal distribution conditional on another normal distribution, we may derive the posterior distribution over bank's fundamentals d for a firm who has seen a signal x, as the following cdf: $P(d \leq d^* \mid x)$.[20] This function is decreasing in x, positive and continuous for any values of d^*. The higher x, the lower the probability that fundamentals d lies below any threshold d^*:

$$P(d \leq d^* \mid x) = \Phi(\sqrt{\alpha + \beta}(d^* - \frac{\alpha(\mu - e) + \beta x}{\alpha + \beta})) \tag{2.32}$$

This result is derived using the same method of variable transformation. As a result of Bayesian inference, each borrower firm who sees signal x has a posterior distribution over d that is normal with mean $\frac{\alpha(\mu - e) + \beta x}{\alpha + \beta}$ and variance $\frac{1}{\alpha + \beta}$.

By replacing in (2.8) the probability of bank survival with the value we have found in (2.32), we may built the second main equilibrium condition:

$$d^* - \frac{\alpha(\mu - e) + \beta x^*}{\alpha + \beta} = \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(\frac{D}{V + F + D}) \tag{2.33}$$

[20] To keep model tractable we start only with this level of inference. Borrower firms can calculate under these circumstances a conditional distribution based on their private signals only. We would obtain similar results for this model if we examined a more complicated structure for this game. In such a structure the first inference made by a good firm will be to update its beliefs about true value of fundamentals given the fact that it knows it belongs to the group of good firms (firms with cash that can repay their loans). The first Bayesian inference would be then $P(d \,|\, \text{Cash} > 0)$. The fact that firm has cash can be interpreted as a private signal.

By solving the system of equations formed by (2.31) and (2.33), equilibrium thresholds d^* and x^*can be found. Solving (2.33) for x^*, we obtain the threshold signal at which a good firm is indifferent between repaying the loan or not:

$$x^* = \frac{\alpha+\beta}{\beta}d^* - \frac{\alpha}{\beta}(\mu - e) - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\frac{D}{V+F+D})$$

Solving now (2.31) for d^* and using the value we have just derived above for equilibrium signal x^*, gives the threshold value for bank's fundamentals above which the bank fails:

$$d^* + \frac{Q}{qD} = \Phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu - e) - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{D}{V+F+D})))$$

The right side of above equation is a cumulative normal distribution

$$N((\mu - e) + \tfrac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\tfrac{D}{V+F+D})),\ \tfrac{1}{\frac{\beta}{\alpha^2}}).$$

Thus, we may conclude that d^* is the intersection point between the cumulative normal distribution just described and a straight line (with a slope of 1) and positive intercept $\frac{Q}{qD}$. This intersection point exists and it is unique if the slope of cumulative normal distribution is less than one everywhere. This slope equals $\frac{\alpha}{\sqrt{\beta}}\phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu - e) - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{D}{V+F+D})))$, where ϕ is the density function of the standard normal distribution. From statistical properties of standard normal density function $\phi \leq \frac{1}{\sqrt{2\pi}}$, thus a sufficient condition for a unique solution for d^* is given by:

$$\frac{\alpha}{\sqrt{\beta}} \leq \sqrt{2\pi}$$

Proof of Proposition 1

In order to prove this proposition I use iterated deletion of dominated strategies. Let's denote by $\underline{x_1}$ the signal for which a good firm is confident in her posterior beliefs that $d \leq \underline{d}$ and find it a dominant strategy to repay regardless of what other good firms do, and denote by $\overline{x_1}$ the signal for which a good firm is confident in her posterior beliefs that $d > \overline{d}$ and find it a dominant strategy to default regardless of what other firms do. The signal $\underline{x_1}$ is the highest value of x such that

$$P(d \leq \underline{d} \mid x) * (-F) + P(d > \underline{d} \mid x) * D \leq P(d \leq \underline{d} \mid x) * V.$$

On the left side the expected payoff when not repaying is described, and on the right side the expected payoff for repaying is depicted. The interpretation is as follows. Even if one good firm believes that all other good firms will stop repaying, and thus any individual firm which repays would get a payoff 0 if $d > \underline{d}$, the posterior probability that $d \leq \underline{d}$, for the firm who saw $x \leq \underline{x_1}$, makes repaying a dominant action. Analogously, the signal $\overline{x_1}$ is the smallest value of x such that

$$P(d \leq \overline{d} \mid x) * (-F) + P(d > \overline{d} \mid x) * D \geq P(d \leq \overline{d} \mid x) * V.$$

For a firm who saw $x > \overline{x_1}$, the posterior probability that $d \leq \overline{d}$ makes not repaying a dominant action even though one good firm believes that all other good firms will repay, and thus that they would get a high payoff V if $d \leq \overline{d}$. Hence, the first round of deletion of dominated strategies implies the following: all good firms receiving signals $x \leq \underline{x_1}$will repay their loans, while good firms receiving signals $x > \overline{x_1}$ will choose to default.

Taken as given this restriction on dominated strategies, next step is to determine the thresholds in fundamentals above (below) which the bank fails (survives). The measure of good firms which choose to default when fundamentals are d, is given by the cumulative mass of good firms who have seen a signal above $\overline{x_1}$. Given the assumption on signal's normal distribution, this fraction $P(x > \overline{x_1} \mid d)$ equals $1 - \Phi(\sqrt{\beta}(\overline{x_1} - d))$, and is always positive, continuous and increasing in d. Since $\Phi(\sqrt{\beta}(\overline{x_1} - d))$ is decreasing in d, there is a maximum value $\overline{d_1} < \overline{d}$ such that the bank fails when good firms receiving signals $x > \overline{x_1}$ default $(\Phi(\sqrt{\beta}(\overline{x_1} - \overline{d_1})) < \frac{Q}{qD} + \overline{d_1})$.

Similarly, the measure of good firms which choose to repay when fundamentals are d is given by the cumulative mass of good firms who have seen a signal below

$\underline{x_1}$. Hence, at least a fraction $P(x \leq \underline{x_1} \mid d)$ will repay, and the bank will fail when fundamentals are higher than a threshold $\underline{d_1} > \underline{d}$, where $\underline{d_1}$ is the maximum value of d such that $\Phi(\sqrt{\beta}(\underline{x_1} - \underline{d_1})) < \frac{Q}{qD} + \underline{d_1}$.

The equilibrium strategy described so far can be summarized as follows: rational good firms receiving signals $x \leq \underline{x_1}$will repay their loans, while good firms receiving signals $x > \overline{x_1}$ will default, and as a result the bank will fail when fundamentals $d > \overline{d_1}$, and it will survive for fundamentals $d < \underline{d_1}$.

This gives us the second round of deletion of dominated strategies. Good firms understand that for large value of fundamentals $(d > \overline{d_1})$ enough firms will behave strategically and as a result the bank enforcement ability will be lost. Hence, any good firm which receives a signal $x > \overline{x_2}$ finds as dominant strategy not to repay its loan. The signal $\overline{x_2}$ is the smallest value of x such that

$$P(d \leq \overline{d_1} \mid x) * (-F) + P(d > \overline{d_1} \mid x) * D \geq P(d \leq \overline{d_1} \mid x) * V.$$

Analogously, repay is a dominant strategy for a good firm when her signal x is lower than $\underline{x_2}$, where the signal $\underline{x_2}$ is the highest value of x such that

$$P(d \leq \underline{d_1} \mid x) * (-F) + P(d > \underline{d_1} \mid x) * D \leq P(d \leq \underline{d_1} \mid x) * V.$$

By doing the same calculations as for the first round of deletion of dominated strategies, we can generate two sequences of restrictions on the equilibrium strategies. The "smallest" equilibrium strategy is defined by an increasing sequence $\underline{x_1}$, $\underline{d_1} \leq \underline{x_2}$, $\underline{d_2} \leq ... \leq \underline{x_n}$, $\underline{d_n} \leq ... \leq \underline{x_\infty}$, $\underline{d_\infty}$. The "largest" equilibrium strategy is defined by a decreasing sequence $\overline{x_1}, \overline{d_1} > \overline{x_2}, \overline{d_2} > ...\overline{x_\infty}, \overline{d_\infty}$.

Any equilibrium strategy must be a solution to a couple of equations (2.31, and 2.33) in two unknowns (a signal threshold x^* and a failure threshold d^*). Since any limit points of the two sequences must be also a solution to these equations, if we show that these equations have a unique solution then we can conclude that there is a unique Bayesian equilibrium. For complete set of derivations please refer to the **Signal and fundamentals thresholds derivation. The case of inactive Central Bank** section from Appendix. There is only one strategy remaining after eliminating all iteratively dominated strategies. In equilibrium, good firms use a threshold strategy: good firms with signals $x \leq x^*$ repay, while good firms with signals $x > x^*$ default, and the bank fails for fundamentals $d > d^*$, and survives otherwise.

Expected measure of non repaying good firms. Commercial bank inference. The case of inactive Central Bank

Taking as given the optimal strategy for debtor firms and knowing the prior distribution of d, the bank will infer $E_d[(1-d) * n(d)]$ as

$$E_d[P(x > x^*] = E_d[1 - \Phi(\sqrt{\beta}(x^* - d)] =$$
$$= 1 - E_d[\Phi(\sqrt{\beta}(x^* - d)]$$

Recall that d is normally distributed and that Φ is the cumulative density function of the standard normal distribution. I denote by h the normal density function and by H the cumulative normal function for signal x, with mean $\mu - e$ and variance $\frac{1}{\alpha} + \frac{1}{\beta}$. Hence:

$$E_d[\Phi(\sqrt{\beta}(x^* - d)] = \int H(x^* \mid d) * h\ (d)\ dd =$$
$$= \int (\int_{-\infty}^{x^*} h(y \mid d) dy) * h\ (d)\ dd = \int \int_{-\infty}^{x^*} \underbrace{h(y \mid d) * h(d)}_{h(y,d)}\ dy\ dd =$$
$$\underset{by\ changing\ limits}{=} \int_{-\infty}^{x^*} \int h(y, d)\ dd\ dy = \int_{-\infty}^{x^*} h(y)\ dy = H(x^*),$$

Thus, we can conclude that: $E_d[(1-d) * n(d)] = 1 - H(x^*)$.

Expected measure of good firms repaying. Commercial bank inference. The case of inactive Central Bank

Further to our intermediate result when we derived the expected measure of non repaying good firms under commercial bank inference we can write the expected measure of repaying good firms as being

$$E_d[(1-d)*(1-n(d))] = E_d[1-d-(1-d)n(d)] =$$
$$= 1 - E_d[d] - E_d[(1-d)n(d)] =$$
$$= 1-(\mu-e)-(1-H(x^*)) = H(x^*)-(\mu-e).$$

Bank optimal effort derivation. The case of inactive Central Bank

Taking as given the optimal strategy for debtor firms, the bank will choose its optimal effort by maximizing its expected payoff, conditional on the available information. The maximization problem with respect to e is:

$$\Phi(\sqrt{\alpha}(d^*-(\mu-e)))*qD*[H(x^*)-(\mu-e)]+$$
$$+\Phi(\sqrt{\alpha}(d^*-(\mu-e)))*q(F+D)*[1-H(x^*)]$$
$$+\Phi(\sqrt{\alpha}(d^*-(\mu-e)))*(-Q-c(e))+$$
$$+(1-\Phi(\sqrt{\alpha}(d^*-(\mu-e))))*(-c(e))$$

The explicit result is derived by assuming that both prior and private signal are very precise. This limiting assumption translates in allowing $\beta \to \infty$ and then $\alpha \to \infty$. When solving the maximization problem we have to take into account the relation between the equilibrium threshold d^* and the average weakness of bank fundamentals $(\mu - e)$. We have to distinguish between two cases:

1. $d^* > \mu - e$, which implies that $\Phi(\sqrt{\alpha}(d^*-(\mu-e))) \underset{\alpha,\beta\to\infty}{\to} 1$. The simplified maximization problem is:

$$\underset{e}{max}\ \{-qD*(\mu-e)+qD*H(x^*)+q(F+D)-q(F+D)*H(x^*)-Q-c(e)\}$$

with solution $e = 1$. Recall that $c(e) = \frac{e^2}{2}qD$. Also in the limiting case when the precision of private signals is high the signal threshold is very precise and it does not depend on e. In this case $x^* = d^* = \frac{V+F}{V+F+D} - \frac{Q}{qD}$.

2. $d^* < \mu - e$, which implies that $\Phi(\sqrt{\alpha}(d^* - (\mu - e))) \underset{\alpha,\beta\to\infty}{\to} 0$. The simplified maximization problem is:

$$\underset{e}{max}\ \{-c(e)\}$$

with solution $e = 0$.

Combining the last two results,

$$e^* = \begin{cases} 1, & \mu - e < d^* \\ 0, & otherwise \end{cases} \Leftrightarrow e^* = \begin{cases} 1, & -1 \leq d^* - \mu \leq 0 \\ 0, & otherwise \end{cases} \Leftrightarrow$$

$$\Leftrightarrow e^* = \begin{cases} 1, & -1 \leq \frac{V+F}{V+F+D} - \frac{Q}{qD} - \mu \leq 0 \\ 0, & otherwise \end{cases}$$

Optimal strategy for an active Central Bank

The Central Bank is more likely to intervene under high social cost caused by the bank closure, and is more likely to allow bank failure in the case of high inflationary costs. Recall that the measure of non-performing loans was denoted by $NPL(d) = d + (1-d)n(d)$. Following from bank's default condition (equation (2.4)) the Central Bank's expected cost function is given by:

$$C(NPL(d)) = min\,\{qV * (1-d) * (1-n(d)),\ \gamma[Q - qD * (1-d)(1-n(d))]\}$$

An active Central Bank indifferent between helping the decapitalized bank and not, is assumed to prefer bailing out the bank. Given the above cost function, the Central Bank will intervene and save the bank if $C(NPL(d)) = \gamma[Q - qD * (1-d)(1-n(d))]$. This implies following necessary condition for a full bailout:

$$qV * (1-d) * (1-n(d)) \geq \gamma[Q - qD * (1-d)(1-n(d))] \Leftrightarrow$$
$$\Leftrightarrow qV * (1 - NPL(d)) \geq \gamma[Q - qD * (1 - NPL(d))] \Leftrightarrow$$
$$\Leftrightarrow qV + \gamma qD - \gamma Q \geq qV * NPL(d) + \gamma qD * NPL(d) \Leftrightarrow$$
$$\Leftrightarrow NPL(d) \leq \frac{qV + \gamma qD - \gamma Q}{qV + \gamma qD}$$

Signal and fundamentals thresholds derivation. The case of active Central Bank

When deciding its action, each good firm should try to infer when the Central Bank decides to step in and bail out the bank. Thus, the probability of bank survival in this case will be given by:

$$P(NPL(d) \leq NPL^* \mid x)$$

Let suppose as before that there is a threshold x^{**} such that all the firms which see a signal $x > x^{**}$ will not repay their loans to bank . The distributions of signals x_i across firms conditional on the realization of non-performing loans $NPL(d)$ is given by cumulative distribution function (cdf) $P(x \leq x^{**} \mid NPL(d))$. Since this function depends on d, and from the distribution of d we can easily infer the distribution of $(1-d)n(d)$ and hence the distribution of $NPL(d)$, we may conclude that:

$$P(x \leq x^{**} \mid NPL(d)) = P(x \leq x^{**} \mid d)$$

This cumulative normal distribution function is decreasing in d, positive and continuous for any value of x^{**}. The higher d, the lower the probability that signal x lies below any threshold x^{**}. Given the normality assumption, we may derive this cdf:

$$P(x \leq x^{**} \mid d) = \Phi(\sqrt{\beta}(x^{**} - d)), \tag{2.34}$$

where Φ is the cumulative distribution function of the standard normal distribution.

We can derive now the cumulative mass of good firms who have seen a signal above the threshold signal x^{**}. Since we have assumed that a good firm which is indifferent between attacking and not will choose not to attack,

$$P(x > x^{**} \mid d) = z(d) = (1-d)n(d) \tag{2.35}$$

Following from (2.20), the bank default will be triggered for

$$d^{**} + (1 - d^{**})n(d^{**}) = NPL^{*},$$

By plugging the distribution of signals x_i across firms conditional on the realization of fundamentals d^{**} and the measure of good firms which choose not to repay (in (2.34) and (2.20), respectively), one obtains the main equilibrium condition:

$$d^{**} + 1 - NPL^{*} = \Phi(\sqrt{\beta}(x^{**} - d^{**})) \tag{2.36}$$

where d^{**} represents the threshold in bank fundamentals at which the bank will fail for values of $d > d^{**}$.

Given the fundamentals d's distribution is $N(\mu - e, \frac{1}{\alpha}))$ and the distribution for the measure of good firms who have seen a signal higher than x^{**} (from (2.34) we can infer that $(1-d)n(d)$ is $N(1-d, \frac{1}{\beta}))$, the distribution of $NPL(d)$ can be computed as a normal one with mean $(\mu - e + 1 - d)$ and variance $(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{2\rho}{\alpha\beta})$, where ρ is the correlation coefficient between random variables d and $(1-d)n(d)$.

Applying again Bayesian inference under a normal distribution conditional on another normal distribution, we derive the posterior distribution over the measure of non-performing loans $NPL(d)$ for a firm who has seen a signal x, as the following cdf:

$$P(NPL(d) \leq NPL^{*} \mid x)$$

If for the case of an inactive Central Bank we have used as random variables $x \mid d$ (which was $N(d, \frac{1}{\beta}))$ and d (which was $N(\mu - e, \frac{1}{\alpha}))$, for the case of an active Central Bank we have to use as random variables $x \mid NPL$ (which is $N(d, \frac{1}{\beta}))$ and NPL (which is $N(\mu - e + 1 - d, \frac{1}{\alpha} + \frac{1}{\beta} + \frac{2\rho}{\alpha\beta}))$.

The probability of bank survival in this case will be given by:

$$P(NPL(d) \leq NPL^{*} \mid x) = \Phi(\sqrt{\frac{\beta(\alpha + C)}{C}}(NPL^{*} - \frac{\alpha(1 - d + \mu - e) + xC}{\alpha + C})), \tag{2.37}$$

where $C = \alpha + \beta + 2\rho\sqrt{\alpha\beta}$. This function is decreasing in x, positive and continuous for any values of NPL^*. The higher x, the lower the probability that the measure of non-performing loans NPL lies below any threshold NPL^*.As a result of Bayesian inference, each borrower firm who sees signal x has a posterior distribution over NPL that is normal with mean $\frac{\alpha(1-d+\mu-e)+xC}{\alpha+C}$ and variance $\frac{C}{\beta(\alpha+C)}$.

Given NPL^* derived in (2.19) and following the same rationament we have used in the previous case when we have studied an economy in which Central Bank was inactive, we may derive the new thresholds x^{**} and d^{**} following the next steps:

By replacing in (2.8) the probability of bank survival with the value we have found in (2.37), we may built the second main equilibrium condition:

$$\frac{C}{C+\alpha}x = NPL^* - \frac{\alpha(\mu - e + 1 - d)}{C+\alpha} - \sqrt{\frac{C}{\beta(C+\alpha)}}\Phi^{-1}(\frac{D}{V+F+D}) \qquad (2.38)$$

By solving the system of equations formed by (2.36) and (2.38), equilibrium thresholds d^{**} and x^{**}can be found. Solving (2.38) for x^{**}, we obtain the threshold signal at which a good firm is indifferent between repaying his loan or not:

$$x^{**} = NPL^*\tfrac{\alpha+C}{C} - \tfrac{\alpha(1-d^{**}+\mu-e)}{C} - \sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D}),$$

Solving now (2.36) for d^{**} and using the value we have just derived above for equilibrium signal x^{**}, gives the threshold value for bank's fundamentals above which the bank fails:

$$d^{**}+1-NPL^* = \Phi(\sqrt{\beta}\frac{\alpha - C}{C}(d^{**}+NPL^*\frac{\alpha+C}{\alpha-C} - \frac{\alpha(1+\mu-e)}{\alpha-C} - \frac{C}{\alpha-C}\sqrt{\frac{\alpha+C}{C\beta}}\Phi^{-1}(\frac{D}{V+F+D}))).$$

The right side of threshold equation is a cumulative normal distribution:

$$N(-NPL^*\tfrac{\alpha+C}{\alpha-C} + \tfrac{\alpha(1+\mu-e)}{\alpha-C} + \tfrac{C}{\alpha-C}\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D}), \tfrac{C^2}{\beta(\alpha-C)^2}).$$

Thus, we may conclude that d^{**} is the intersection point between the cumulative normal distribution just described and a straight line (with a slope of 1) and positive intercept $1 - NPL^*$. This intersection point exists and it is unique if the slope of cumulative normal distribution is less than one everywhere. This slope equals

$$\sqrt{\beta}\frac{\alpha-C}{C}\phi(\sqrt{\beta}\frac{\alpha-C}{C}(d^{**} + NPL^*\frac{\alpha+C}{\alpha-C} - \frac{\alpha(1+\mu-e)}{\alpha-C} - \frac{C}{\alpha-C}\sqrt{\frac{\alpha+C}{C\beta}}\Phi^{-1}(\frac{D}{V+F+D}))),$$

where ϕ is the density function of the standard normal distribution. Recall that from statistical properties of standard normal density function $\phi \leq \frac{1}{\sqrt{2\pi}}$.Thus, a sufficient condition for a unique solution is :

$$\sqrt{\beta}\frac{\alpha-C}{C} \leq \sqrt{2\pi} \Leftrightarrow \sqrt{\beta}\frac{-\beta-2\rho\sqrt{\alpha\beta}}{\alpha+\beta+2\rho\sqrt{\alpha\beta}} \leq \sqrt{2\pi}$$

Also $\sqrt{\beta}\frac{-\beta-2\rho\sqrt{\alpha\beta}}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}$ should be positive. We have to distinguish between two cases:

1. $-\beta - 2\rho\sqrt{\alpha\beta} \geq 0$ AND $\alpha + \beta + 2\rho\sqrt{\alpha\beta} > 0$, which implies $\rho < 0$ and $0 < \beta \leq -2\rho\sqrt{\alpha\beta}$ and $\alpha > -\beta - 2\rho\sqrt{\alpha\beta} > 0$.

2. $-\beta - 2\rho\sqrt{\alpha\beta} \leq 0$ AND $\alpha + \beta + 2\rho\sqrt{\alpha\beta} < 0$, which implies EITHER $\rho > 0$ and $\beta \geq -2\rho\sqrt{\alpha\beta}$ and $\alpha + \beta < -2\rho\sqrt{\alpha\beta} < 0$ $(FALSE)$, OR $\rho < 0$ and $\beta \geq -2\rho\sqrt{\alpha\beta}$ and $\alpha + \beta + 2\rho\sqrt{\alpha\beta} < 0$ $(FALSE)$.

Thus the sufficient conditions for a unique solution for d^{**} is given by:

$$\frac{\sqrt{\beta}}{\alpha + \beta + 2\rho\sqrt{\alpha\beta}}(-\beta - 2\rho\sqrt{\alpha\beta}) \leq \sqrt{2\pi}$$
$$\rho < 0$$
$$0 < \beta \leq -2\rho\sqrt{\alpha\beta}$$
$$\alpha > -\beta - 2\rho\sqrt{\alpha\beta} \geq 0$$

Proof of Proposition 2

In order to not overload the paper, I do not provide this proof. It follows the lines of Proof for Proposition 1.

Proof of Proposition 3

This proposition states that the threshold d^{**} above which the bank fails even if the Central Bank is active is always larger than d^*, the threshold above which the bank fails if Central Bank is inactive. Recall that when $\beta \to \infty$, $d^* = \frac{V+F}{V+F+D} - \frac{Q}{qD}$ and $d^{**} = NPL^*$, with both $d^*$and d^{**} in $[0,1]$. We have to show that $d^* < d^{**}$, which is $\frac{V+F}{V+F+D} - \frac{Q}{qD} < \frac{qV+\gamma qD-\gamma Q}{qV+\gamma qD}$.We can rewrite it as $\frac{\gamma Q}{qV+\gamma qD} - \frac{Q}{qD} < 1 - \frac{V+F}{V+F+D} \Leftrightarrow \frac{-qVQ}{(qV+\gamma qD)qD} < \frac{D}{V+F+D}(TRUE)$.

Bank optimal effort derivation. The case of active Central Bank

Expected measure of non repaying good firms and expected measure of good firms repaying were derived using the same inference mechanism as in the case of an inactive Central Bank. These measures are $1 - H(x^{**})$ and $H(x^{**}) - (\mu - e)$, respectively.

Taking as given the optimal strategy for Central Bank and debtor firms, the bank will choose its optimal effort by maximizing its expected payoff, conditional on the available information. The maximization problem with respect to e is:

$$\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d^{**}))) * qD * [H(x^{**}) - (\mu - e)] +$$
$$+\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d^{**}))) * q(F+D) * [1 - H(x^{**})]$$
$$+\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d^{**}))) * (-Q - c(e)) +$$
$$+(1 - \Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d^{**}))) * (-c(e))$$

The explicit result is derived by assuming that both prior and private signal are very precise. This limiting assumption translates in allowing $\beta \to \infty$ and then $\alpha \to \infty$. When solving the maximization problem we have to take into account the relation between the equilibrium threshold d^{**}, the non-performing loans threshold NPL^* and the average weakness of bank fundamentals $(\mu - e)$. We have to distinguish again between two cases:

1. $NPL^* > \mu - e + 1 - d^{**}$, which implies that $\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu -$

$e + 1 - d^{**}))) \underset{\alpha,\beta\to\infty}{\to} 1$. The simplified maximization problem is:

$$\underset{e}{max}\ \{-qD*(\mu - e) + qD*H(x^{**}) + q(F+D) - q(F+D)*H(x^{**}) - Q - c(e)\}$$

with solution $e = 1$. Recall that $c(e) = \frac{e^2}{2}qD$.

2. $NPL^* < \mu - e + 1 - d^{**}$, which implies that $\Phi(\sqrt{\frac{\alpha\beta}{\alpha+\beta+2\rho\sqrt{\alpha\beta}}}(NPL^* - (\mu - e + 1 - d^{**}))) \underset{\alpha,\beta\to\infty}{\to} 0$.The simplified maximization problem is:

$$\underset{e}{max}\ \{-c(e)\}$$

with solution $e = 0$.

Combining the last two results,

$$e^{**} = \begin{cases} 1, & \mu - e + 1 - d^{**} < NPL^* \\ 0, & otherwise \end{cases} \Leftrightarrow$$

$$\Leftrightarrow e^{**} = \begin{cases} 1, & -1 \leq NPL^* + d^{**} - \mu - 1 \leq 0 \\ 0, & otherwise \end{cases} \Leftrightarrow$$

$$\Leftrightarrow e^{**} = \begin{cases} 1, & -1 \leq \frac{2(qV+\gamma qD-\gamma Q)}{qV+\gamma qD} - \mu \leq 0 \\ 0, & otherwise \end{cases} \Leftrightarrow$$

$$\Leftrightarrow e^{**} = \begin{cases} 1, & -1 \leq 1 - \mu - \frac{2\gamma Q}{qV+\gamma qD} \leq 0 \\ 0, & otherwise \end{cases}$$

Proof of Proposition 5

This proposition states that when cost of intervention is low ($\gamma < \gamma^* = \frac{qVD^2+VQ(V+F+D)}{D[Q(V+F+D)-qD^2]}$), an active Central Bank will induce moral hazard in commercial bank behavior. When Central Bank is not active, the commercial bank will decide to exert optimal effort $e = 1$ when its unconditional fundamentals are stronger ($\mu_1 \leq \mu \leq \mu_2$) than when Central Bank is active. When the Central Bank is active the commercial bank exerts effort only when the unconditional bank fundamentals are above μ_2. The thresholds in unconditional bank fundamentals μ_1 and μ_2 are given by $\frac{V+F}{V+F+D} - \frac{Q}{qD}$ and $1 - \frac{2\gamma Q}{qV+\gamma qD}$, respectively. Given the fact that commercial bank will exert effort when the Central Bank is inactive only for values of unconditional fundamentals higher than μ_1, while it will exerts effort when the Central Bank is active for values of unconditional fundamentals higher than μ_2, the proof reduces to show that $\frac{V+F}{V+F+D} - \frac{Q}{qD} < 1 - \frac{2\gamma Q}{qV+\gamma qD}$ holds true for $\gamma < \gamma^*$.

$$\frac{V+F}{V+F+D} - \frac{Q}{qD} < 1 - \frac{2\gamma Q}{qV+\gamma qD} \Leftrightarrow \frac{2\gamma Q}{qV+\gamma qD} - \frac{Q}{qD} < 1 - \frac{V+F}{V+F+D} \Leftrightarrow$$
$$\Leftrightarrow \gamma < \frac{qVD^2+VQ(V+F+D)}{D[Q(V+F+D)-qD^2]}.$$

Assumptions the restrictions (2.2) and (2.27) imply that the denominator $D[Q(V+F+D)-qD^2]$ is positive.

Proof of Proposition 6

This proposition states that a high enough cost of intervention ($\gamma > \gamma^*$) mitigates the moral hazard problem introduced by an active Central Bank when commercial bank unconditional fundamentals are strong enough ($\mu_2 \leq \mu \leq \mu_1$). The proof follows the lines of the proof given for Proposition 5. The only difference consists in the fact that we have to show that $\frac{V+F}{V+F+D} - \frac{Q}{qD} > 1 - \frac{2\gamma Q}{qV+\gamma qD}$ holds true for $\gamma > \gamma^*$.

Proof of Proposition 7

This proposition states that when cost of intervention is very high ($\gamma > \gamma_M > \gamma^*$), an active Central Bank will not convince the commercial bank to exert maximum of effort when its unconditional fundamentals are very poor ($\mu > \mu_4$). The thresholds in unconditional bank fundamentals μ_2 and μ_4 are given by $1 - \frac{2\gamma Q}{qV+\gamma qD}$ and $2 - \frac{2\gamma Q}{qV+\gamma qD}$, respectively. These values are decreasing in γ. For $\gamma > \gamma_M$ it is straightforward to prove that μ_2 is negative and μ_4 is less than 1. Since our attention is focused only on unconditional fundamentals μ in the range $[0, 1]$, and due to the fact that μ_1 and μ_3 are not influenced by the value of γ, where μ_1 is less than 1 and μ_3 is higher than 1 always, we have to analyze the commercial bank choice of effort when the following classification for fundamentals is met: $0 < \mu_1 < \mu_4 < 1$. The proof reduces to show that $2 - \frac{2\gamma Q}{qV+\gamma qD} < 1$ holds true for $\gamma > \gamma_M$.

$$2 - \frac{2\gamma Q}{qV+\gamma qD} < 1 \Leftrightarrow 1 - \frac{2\gamma Q}{qV+\gamma qD} < 0 \Leftrightarrow$$
$$\Leftrightarrow \gamma > \frac{qV}{2Q-qD} = \gamma_M.$$

Derivation for changes in unconditional bank fundamentals

Differentiating (2.10) and (2.22) with respect to μ, when effort e is given, yields:

$$\frac{\partial d^*}{\partial \mu} = \phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu - e) - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{D}{V+F+D}))) * \frac{\alpha}{\sqrt{\beta}} * (\frac{\partial d^*}{\partial \mu} - 1) \Rightarrow$$
$$\frac{\partial d^*}{\partial \mu} = \frac{\frac{\alpha}{\sqrt{\beta}}\phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu-e) - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{D}{V+F+D})))}{\frac{\alpha}{\sqrt{\beta}}\phi(\frac{\alpha}{\sqrt{\beta}}(d^* - (\mu-e) - \frac{\sqrt{\alpha+\beta}}{\alpha}\Phi^{-1}(\frac{D}{V+F+D}))) - 1},$$

which is less than 0 given (2.11), and

$$\frac{\partial d^{**}}{\partial \mu} = \phi(\sqrt{\beta}\frac{\alpha-C}{C}(d^{**} + NPL^*\frac{\alpha+C}{\alpha-C} - \frac{\alpha(1+\mu-e)}{\alpha-C} - \frac{C}{\alpha-C}\sqrt{\frac{\alpha+C}{C\beta}}\Phi^{-1}(\frac{D}{V+F+D}))) *$$
$$\sqrt{\beta}\frac{\alpha-C}{C}(\frac{\partial d^{**}}{\partial \mu} - \frac{\alpha}{\alpha-C}) \Rightarrow$$
$$\frac{\partial d^{**}}{\partial \mu} = \frac{\sqrt{\beta}\frac{\alpha}{C}\phi(\sqrt{\beta}\frac{\alpha-C}{C}(d^{**} + NPL^*\frac{\alpha+C}{\alpha-C} - \frac{\alpha(1+\mu-e)}{\alpha-C} - \frac{C}{\alpha-C}\sqrt{\frac{\alpha+C}{C\beta}}\Phi^{-1}(\frac{D}{V+F+D})))}{\sqrt{\beta}\frac{\alpha-C}{C}\phi(\sqrt{\beta}\frac{\alpha-C}{C}(d^{**} + NPL^*\frac{\alpha+C}{\alpha-C} - \frac{\alpha(1+\mu-e)}{\alpha-C} - \frac{C}{\alpha-C}\sqrt{\frac{\alpha+C}{C\beta}}\Phi^{-1}(\frac{D}{V+F+D}))) - 1},$$

which is less than 0 given (2.23).

When Central Bank is not active $(1-d^*)n(d) = P(x > x^* \mid d^*) = 1 - \Phi(\sqrt{\beta}(x^* - d^*))$, where x* and d* are equilibrium values described in (2.9) and (2.10). Thus, we can imply that, when effort e is constant,

$$(1-d^*)n(d) = 1 - \Phi(\sqrt{\beta}(\frac{\alpha}{\beta}d^* - \frac{\alpha}{\beta}(\mu - e) - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\frac{D}{V+F+D}))) \underset{\text{by differentiating wrt to } \mu}{\Rightarrow}$$
$$\frac{\partial(1-d^*)n(d)}{\partial \mu} = -\phi(\sqrt{\beta}(\frac{\alpha}{\beta}d^* - \frac{\alpha}{\beta}(\mu - e) - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\frac{D}{V+F+D}))) * \frac{\alpha}{\sqrt{\beta}} * (\frac{\partial d^*}{\partial \mu} - 1)$$

which is positive for the case of an inactive Central Bank because $\frac{\partial d^*}{\partial \mu} - 1 < 0$.

The case when the Central Bank is active is analogous:

$$(1-d^{**})n(d) = P(x > x^{**} \mid NPL^*) = P(x > x^{**} \mid d^{**}) = 1 - \Phi(\sqrt{\beta}(x^{**} - d^{**})),$$

where x** and d** are equilibrium values described in (2.21) and (2.22). Thus, we can imply that

$$(1-d^{**})n(d) =$$
$$1-\Phi(\sqrt{\beta}(NPL^*\tfrac{\alpha+C}{C}-\tfrac{\alpha(1-d^{**}+\mu-e)}{C}-\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D})-d^{**})) \underset{\text{by differentiating wrt to } \mu}{\Rightarrow}$$
$$\tfrac{\partial(1-d^{**})n(d)}{\partial\mu} = -\phi(\sqrt{\beta}(NPL^*\tfrac{\alpha+C}{C}-\tfrac{\alpha(1-d^{**}+\mu-e)}{C}-\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D})-d^{**}))*$$
$$(-\tfrac{\alpha\sqrt{\beta}}{C}-\tfrac{\partial d^{**}}{\partial\mu}*\sqrt{\beta}*\tfrac{C-\alpha}{C}) =$$
$$=\phi(.)*\tfrac{\sqrt{\beta}}{C}*(\alpha+\tfrac{\partial d^{**}}{\partial\mu}*(\beta+2\rho\sqrt{\alpha\beta}))$$

which is positive for the case of an active Central Bank because $\frac{\partial d^{**}}{\partial\mu}-1<0$ and $\alpha>-\beta-2\rho\sqrt{\alpha\beta}\geq 0$.

Derivation for changes in intervention cost

Differentiating (2.22) with respect to γ, when effort e is given, yields:

$$\tfrac{\partial d^{**}}{\partial\gamma}-\tfrac{\partial NPL^*}{\partial\gamma} = \phi(\sqrt{\beta}\tfrac{\alpha-C}{C}(d^{**}+NPL^*\tfrac{\alpha+C}{\alpha-C}-\tfrac{\alpha(1+\mu-e)}{\alpha-C}-\tfrac{C}{\alpha-C}\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D})))*$$
$$\sqrt{\beta}\tfrac{\alpha-C}{C}(\tfrac{\partial d^{**}}{\partial\gamma}+\tfrac{\alpha+C}{\alpha-C}\tfrac{\partial NPL^*}{\partial\gamma})\Rightarrow$$
$$\tfrac{\partial d^{**}}{\partial\gamma} = \phi(.)*\sqrt{\beta}\tfrac{\alpha-C}{C}(\tfrac{\partial d^{**}}{\partial\gamma}-\tfrac{\alpha+C}{\alpha-C}\tfrac{qQV}{(qV+\gamma qD)^2})+\tfrac{\partial NPL^*}{\partial\gamma}\Rightarrow$$
$$\tfrac{\partial d^{**}}{\partial\gamma} = \frac{\sqrt{\beta}\frac{C-\alpha}{C}\phi(.)\frac{\alpha+C}{\alpha-C}\frac{qQV}{(qV+\gamma qD)^2}}{1-\sqrt{\beta}\frac{\alpha-C}{C}\phi(.)}-\frac{\frac{qQV}{(qV+\gamma qD)^2}}{1-\sqrt{\beta}\frac{\alpha-C}{C}\phi(.)},$$

which is less than 0 given (2.23). Then, we can imply that:

$$(1-d^{**})n(d) =$$
$$1-\Phi(\sqrt{\beta}(NPL^*\tfrac{\alpha+C}{C}-\tfrac{\alpha(1-d^{**}+\mu-e)}{C}-\sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D})-d^{**})) \underset{\text{by differentiating wrt to } \gamma}{\Rightarrow}$$
$$\tfrac{\partial(1-d^{**})n(d)}{\partial\gamma} = -\phi(.)\tfrac{\sqrt{\beta}}{C}*[\underbrace{(\alpha+C)\frac{-qQV}{(qV+\gamma qD)^2}}_{\text{negative}}+\underbrace{\frac{\partial d^{**}}{\partial\gamma}}_{\text{negative}}*\underbrace{(-\beta-2\rho\sqrt{\alpha\beta})}_{\text{positive}}]>0$$

Derivation for changes in fundamentals

When Central Bank is not active $(1-d)n(d)=P(x>x^*\mid d)=1-\Phi(\sqrt{\beta}(x^*-d))$, where x* is equilibrium value described in (2.9). Thus, we can imply that

$$(1-d)n(d)=1-\Phi(\sqrt{\beta}(\tfrac{\alpha}{\beta}d-\tfrac{\alpha}{\beta}(\mu-e)-\tfrac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\tfrac{D}{V+F+D}))) \underset{\text{by differentiating wrt to } d}{\Rightarrow}$$
$$\tfrac{\partial(1-d)n(d)}{\partial d} = -\phi(\sqrt{\beta}(\tfrac{\alpha}{\beta}d-\tfrac{\alpha}{\beta}(\mu-e)-\tfrac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\tfrac{D}{V+F+D})))*\tfrac{\alpha}{\sqrt{\beta}}$$

which is negative.

The case when the Central Bank is active is analogous:

$$(1-d)n(d) = P(x > x^{**} \mid NPL) = P(x > x^{**} \mid d) = 1 - \Phi(\sqrt{\beta}(x^{**} - d)),$$

where x** is equilibrium value described in (2.21). Thus, we can imply that

$$(1-d)n(d) =$$
$$1 - \Phi(\sqrt{\beta}(NPL^* \tfrac{\alpha+C}{C} - \tfrac{\alpha(1-d+\mu-e)}{C} - \sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D}) - d)) \underset{\text{by differentiating wrt to } d}{\Rightarrow}$$
$$\tfrac{\partial(1-d)n(d)}{\partial d} = -\phi(\sqrt{\beta}(NPL^* \tfrac{\alpha+C}{C} - \tfrac{\alpha(1-d+\mu-e)}{C} - \sqrt{\tfrac{\alpha+C}{C\beta}}\Phi^{-1}(\tfrac{D}{V+F+D}) - d)) * \tfrac{-\beta-2\rho\sqrt{\alpha\beta}}{C}$$

which is negative for the case of an active Central Bank because $-\beta-2\rho\sqrt{\alpha\beta} \geq 0$.

2.B Figures

Figure 1. Optimal effort for low cost of intervention ($\gamma<\gamma^*$).

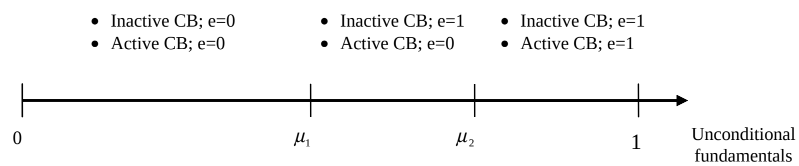


Figure 2a. Optimal effort for high cost of intervention ($\gamma_M > \gamma > \gamma^*$).

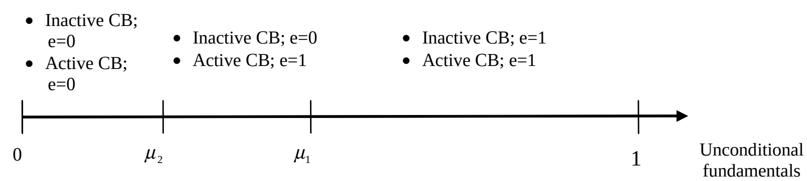


Figure 2b. Optimal effort for the highest cost of intervention ($\gamma>\gamma_M > \gamma^*$).

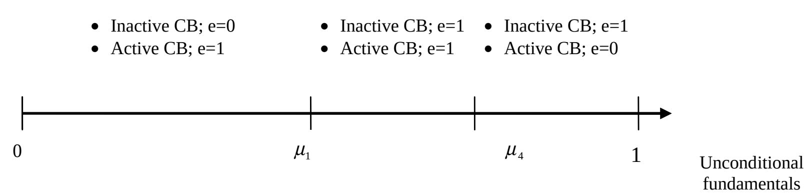


Figure 3a. Chance of collective strategic default when CB is inactive

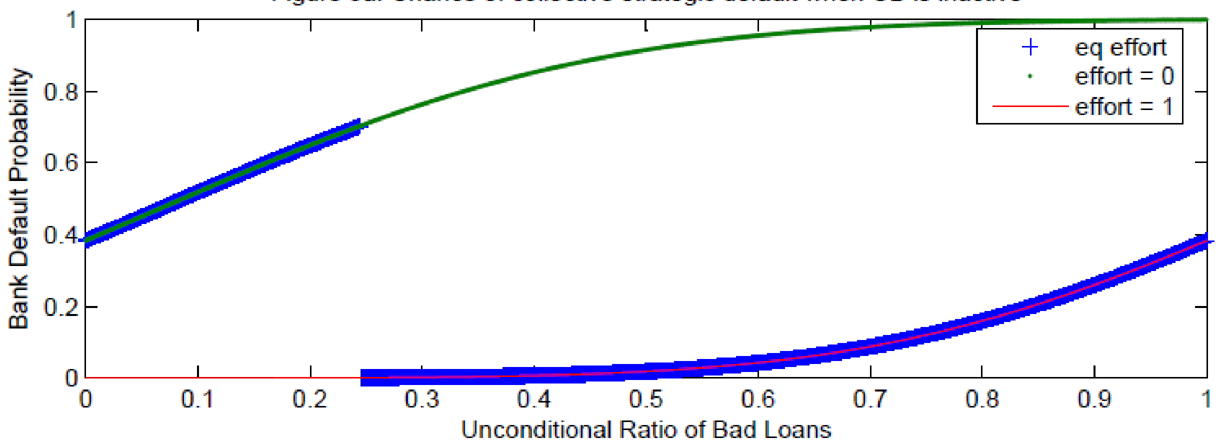


Figure 3b. Threshold for Collective Strategic Default when CB is Inactive

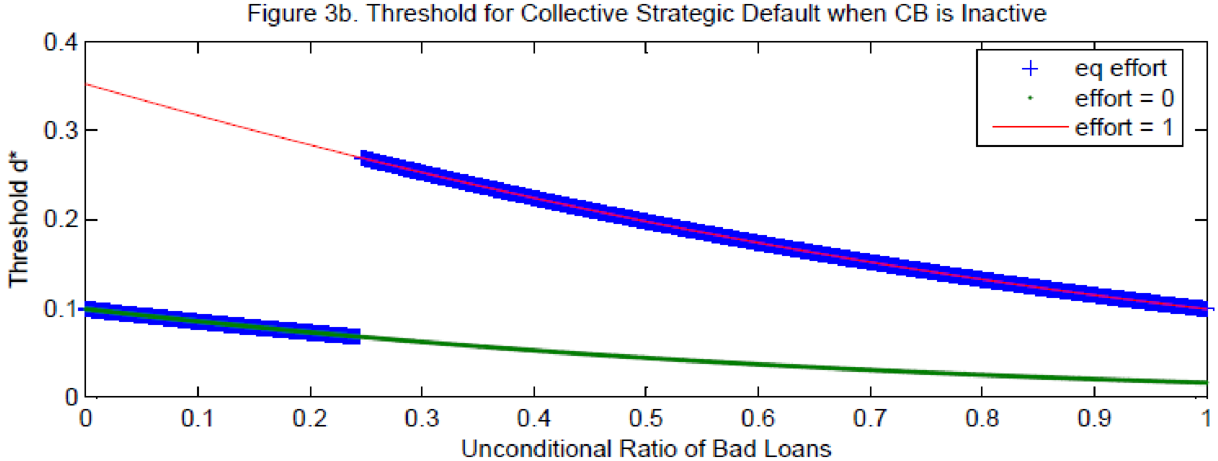


Figure 4a. Chance of collective strategic default when CB is Active.
Cost of intervention is low. NPL* = 0.8

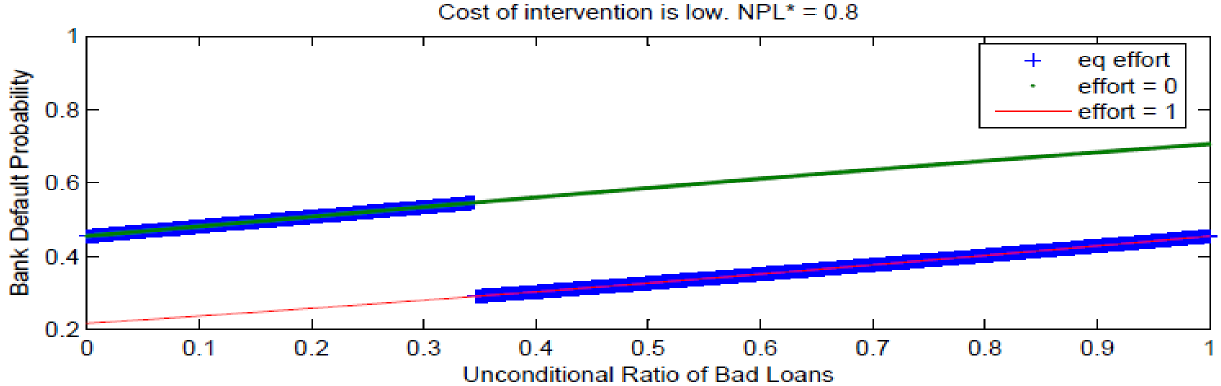


Figure 4b. Threshold for Collective Strategic Default when CB is Active.
Cost of intervention is low. NPL* = 0.8

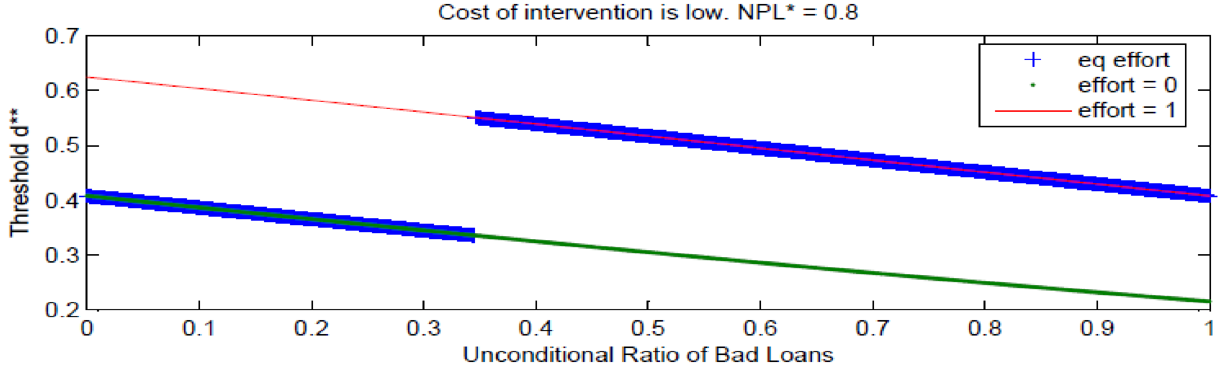


Figure 5a. Chance of collective strategic default when CB is Active.
Cost of intervention is high. NPL* = 0.2

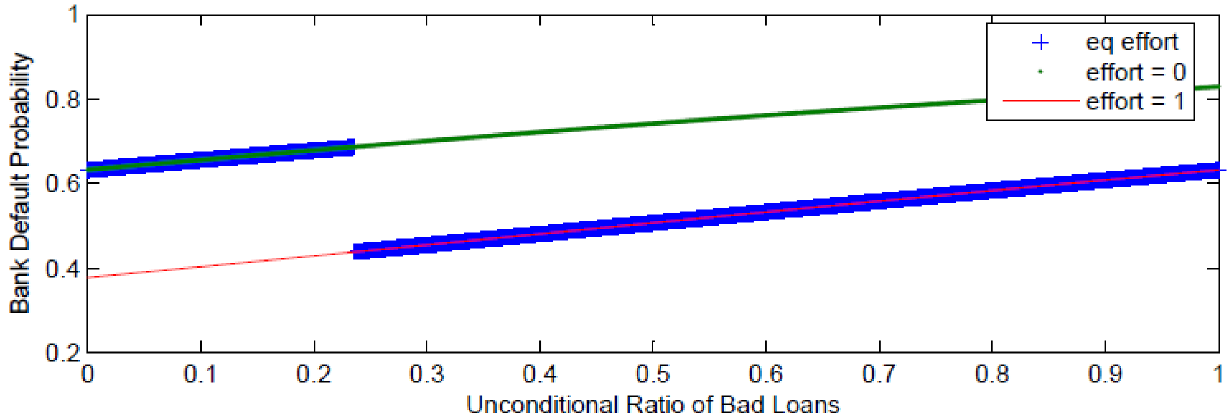


Figure 5b. Threshold for Collective Strategic Default when CB is Active.
Cost of intervention is high. NPL* = 0.2

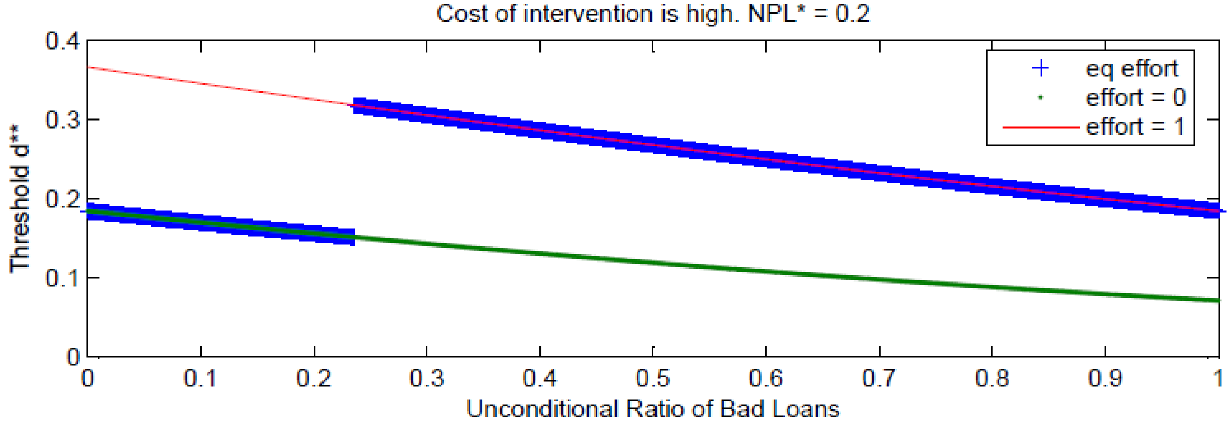


Figure 6a. Required Ratio for Succesfull Collective Strategic Default when CB is Inactive
The Ratio is out of Total No of Firms

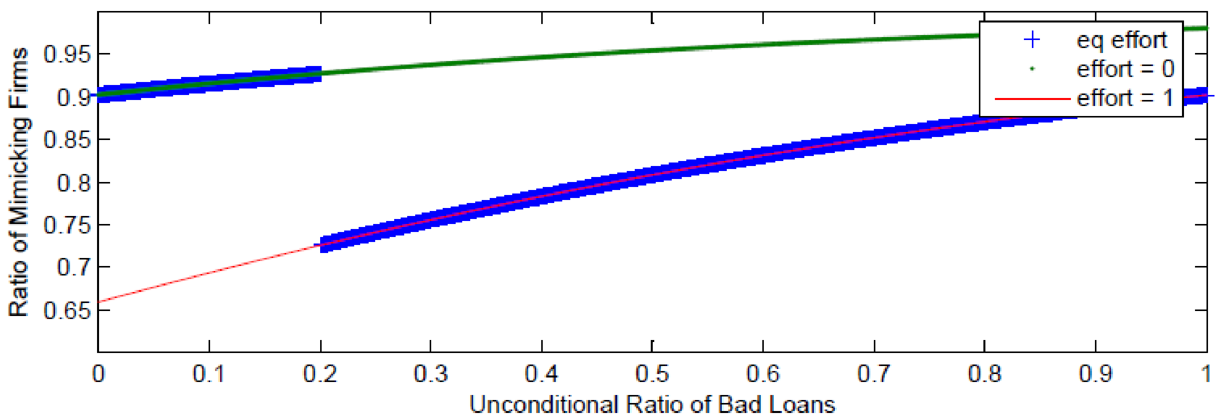


Figure 6b. Required Ratio for Succesfull Collective Strategic Default when CB is Inactive
The Ratio is out of Total No of Good Firms

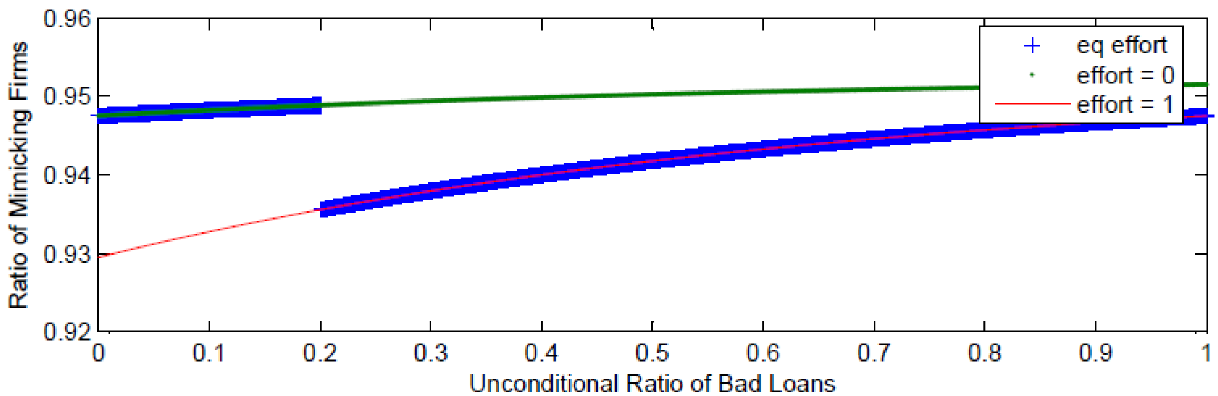


Figure 7a. Required Ratio for Succesfull Collective Strategic Default when CB is Active
Cost of intervention is low. NPL* = 0.8
The Ratio is out of Total No of Firms

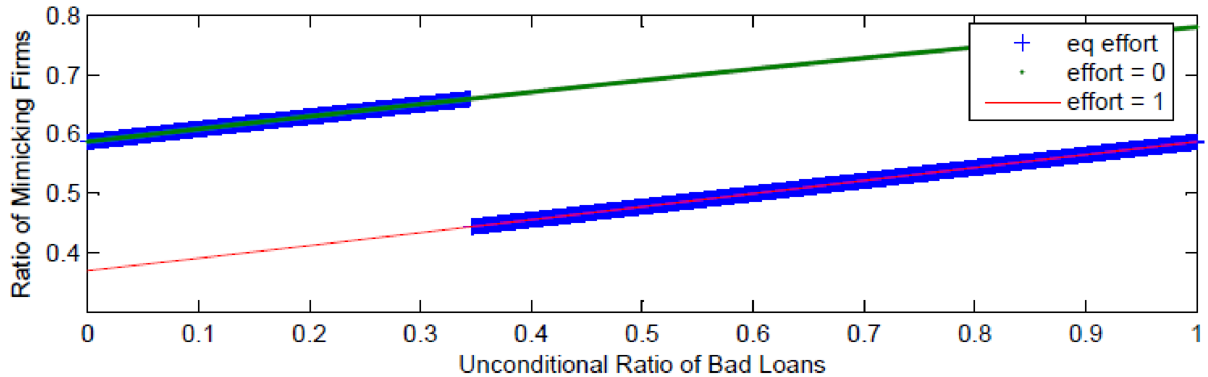
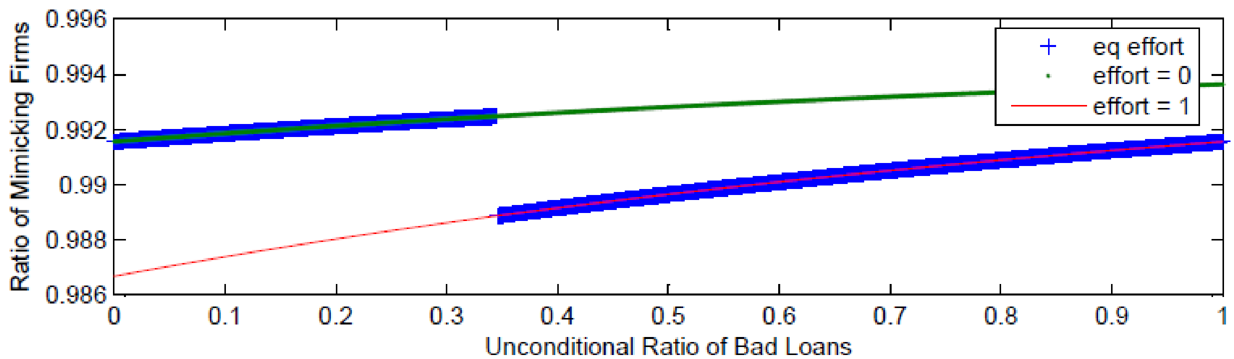


Figure 7b. Required Ratio for Succesfull Collective Strategic Default when CB is Active
Cost of intervention is low. NPL* = 0.8
The Ratio is out of Total No of Good Firms



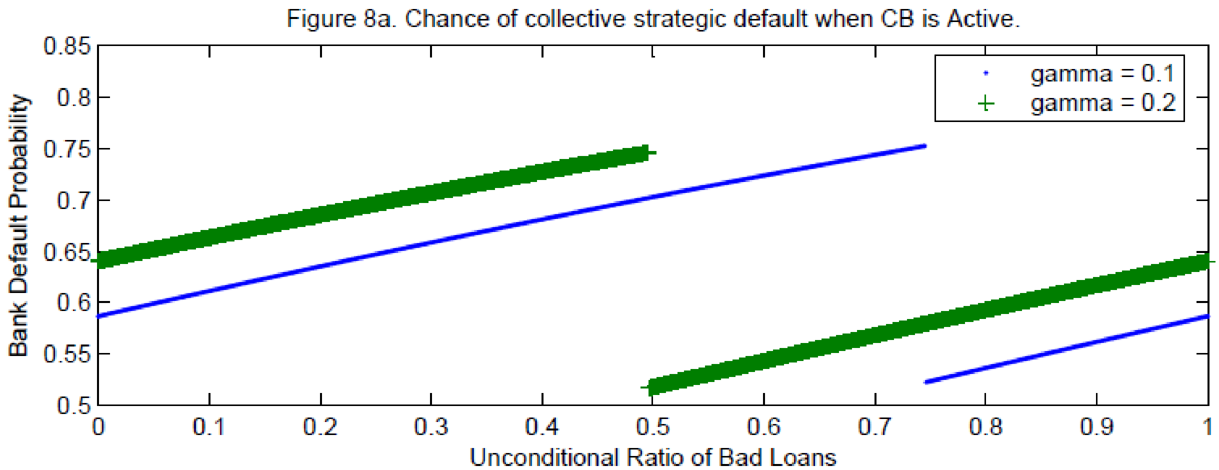


Figure 8a. Chance of collective strategic default when CB is Active.
Bank Default Probability
Unconditional Ratio of Bad Loans
gamma = 0.1
gamma = 0.2

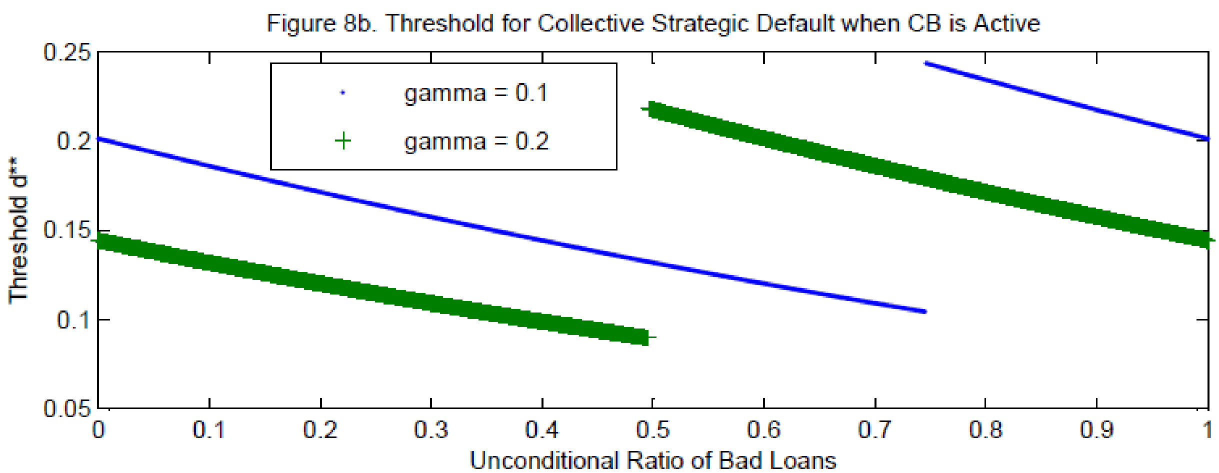


Figure 8b. Threshold for Collective Strategic Default when CB is Active
Threshold d**
Unconditional Ratio of Bad Loans
gamma = 0.1
gamma = 0.2