Three essays on banking

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Chapter 4

Capital Regulation and Tail Risks

4.1 Introduction

The policy debate on financial reform in the wake of the 2007-2008 crisis has focused on raising capital ratios of financial intermediaries. The recently adopted Basel III rules double the minimal capital ratio and, beyond that, create incentives for banks to hold excess capital in the form of conservation and countercyclical buffers (BIS, 2010). This complements moral suasion and individual targets, traditionally used by regulators to assure that banks have capital cushions above the regulatory minimum.

There are two key arguments in favor of higher capital. First is the classic notion that capital is a buffer that reduces the risk of insolvency. It also helps to reduce some systemic risk factors, such as uncertainty over counterparty risk, which had a devastating propagation effect during the recent crisis. Second is a more sophisticated argument that capital is not just a buffer, but has incentive effects. Higher capital increases shareholders' losses in bank failure, and hence reduces their incentives to take excessive risk (Jensen and Meckling, 1976, Holmstrom and Tirole, 1997).

Yet one of the lessons from the crisis is that banks are exposed to tail risks which, when realized, may trigger losses in excess of almost any plausible value of initial capital. Such risks can result from the reliance on wholesale funding (the freeze of which forced massive distress liquidation, Gorton, 2010), the underwriting of AIG-type contingent liabilities likely to be called during market panic, exposures to highly-rated senior structured debt standing to lose value in periods of extreme economic stress (Acharya and Richardson, 2009), as well as undiversified leveraged exposures to inflated housing markets (Shin, 2009).

Since tail risks can wipe out almost any initial capital, it is unclear whether traditional capital regulation is effective in addressing it. The aim of this paper is to shed light on the subject. Specifically, we focus on the relationship between bank capital and risk-taking, in the presence of tail risk projects. We show that this relationship may take unintuitive forms, with bank risk-taking being non-linear and possibly increasing in capital.

The argument is that while higher capital reduces risk-taking incentives caused by limited liability, in banks this may be dominated by an important opposite effect. Higher capital increases the distance to the minimal capital ratio, allowing the bank to take more risk without the fear of breaching regulatory requirements in case of a mildly negative project realization. If the second, excess capital-driven effect is stronger than the limited liability effect, the bank will respond to higher capital by taking more risk.

Of course, when all negative project realizations are contained by the bank's capital buffer, any additional risk-taking is fully internalized by the bank, and therefore happens only when economically efficient. However in the presence of tail risks, when high capital ratios by themselves cannot insure against all losses, a highly capitalized bank may start taking a socially excessive level of risk. Recall, excessive risk-taking will not be present in banks with lower capital. Thus, under tail risks, inefficient risk-taking may increase in bank capital. This result is consistent with the stylized fact that U.S. banks were well capitalized pre-crisis (Berger et al., 2008), yet they took significant bets on house prices and on mortgage derivatives. We show that well-capitalized banks' incentives for taking tail risks are increased in the extremeness of that tail risk – the availability of projects with heavier left tails.

The results have a number of implications for the optimal design of capital regulation. First, they caution on limits to the effectiveness of capital buffers in dealing with bank risk, particularly when capital comes in unencumbered (excess; free to put to risk) form. Second, the results suggest that the penalties for breaching conservation and countercyclical buffers of Basel III should be significant enough to avoid the highlighted 'excess capital' effects. Finally, the results shed light on the link between financial innovation and regulation. With the advent of structured debt and debt derivatives, and with the development of the wholesale business model, banks gained easier-than-before access to tail risk projects. This broke the established relationship between capital and risk-taking, and compromised the effectiveness of traditional capital regulation. The results therefore support the view that dealing with tail risks requires new regulatory tools (e.g., macro-prudential measures that address systemic risk and negative spirals, see Acharya et al., 2010, Acharya and Yorulmazer, 2007, Brunnermeier and Pedersen, 2008, and Perotti and Suarez, 2010).

The paper has two key contributions to the banking literature. First, we formalize a novel channel of bank risk-taking, based on incentives to put to risk excess capital. Second, we show that bank risk-taking may be increasing in bank capital, specifically when the bank has access to tail risk projects.

Our analysis is related to several strands of the literature on capital regulation. Earlier studies of unintented effects of bank capital regulation (Kahane 1977, Kim and Santomero 1988, Koehn and Santomero 1980) were motivated by the idea of banks as risk-averse portfolio managers, who act as price takers when choosing the composition of their assets (and liabilities) portfolio. By taking this portfolio optimization approach, these studies show how capital requirements can lead to an increase in risk of the risky part of the bank's portfolio, and hence an overall higher bank's default probability (even if the relative size of the risky part with respect to the overall portfolio decreases). The main critique of the mean-variance model employed by these earlier studies is the absence of limited liability in bank's objective function. Once this limited liability is taken into account, capital regulation in form of minimum capital requirements is shown to have a beneficial effect in preventing bank's risk-taking incentives (Keeley and Furlong,1990, Rochet 1992).

Later studies examine incentive effects. Boot and Greenbaum (1993) show that capital requirements can negatively affect assets quality due to a reduction in monitoring incentives. Flannery (1989) argues that capital adequacy requirements lead banks to prefer risky portfolio returns, with only insured banks having a rational preference for safe individual loans. Blum (1999), Caminal and Matutes (2002) and Hellman et al. (2000) suggest that higher capital can lead banks to take more risk as they attempt to compensate for its cost. The empirical evidence on whether capital requirements are efficient tools for limiting bank risk-taking is ambiguous. Basel Committee on Banking Supervision (1999) survey finds the evidence on the effects of capital requirements as being inconclusive. On the other hand, Bichsel and Blum (2004) and Angora, Distinguin and Rugemintwari (2009) provide empirical support by finding a positive correlation between levels of capital and bank risk-taking. Our paper follows this literature, with a distinct and contemporary focus on tail risks[1] and the dangers of non-binding capital.

Finally, Repullo and Suarez (2009), Peura and Keppo (2006) and Zhu (2008) explore the economic role of capital over the business cycle, highlighting that banks build up excess capital in booms and rapidly lose it in busts, possibly as a result of significant risk-taking. Ayuso, Perez and Saurina (2004) provide empirical evidence using a sample of Spanish banks on the negative correlation between business cycle and capital buffers.

The rest of the chapter is structured as follows. Section 4.2 outlines the theoretical

[1] Recent studies develop different measures for banks' tail risks. Acharya and Richardson (2009), Adrian and Brunnermeier (2009), and De Jonghe (2009) compute realized tail risks exposure over a certain period by using historical evidence of tail risks events, while Knaup and Wagner (2010) propose a forward-looking measure for bank tail risks.

model. Sections 4.3 and 4.4 solve it and offer the key result that bank risk-taking may be increasing in capital when bank has access to tail risk projects. Section 4.5 discusses the comparative statics, policy implications and alternative model's specifications. Section 4.6 concludes. The proofs are in the Appendix.

4.2 The Model

The model has three main ingredients. First, the bank is run by an owner-manager (hereafter, the banker) with limited liability, who can opportunistically engage in asset substitution. Second, the bank operates in a prudential framework based on a minimal capital ratio, where a capital adjustment cost is imposed upon the breach of the minimal ratio. Finally, the bank has access to tail risk projects. Such a setup is a stylized representation of the key relevant features of the actual banking system. There are three dates $(0, \frac{1}{2}, 1)$, no discounting, and everyone is risk-neutral.

The bank

At date 0, the bank is endowed with capital C and deposits D. For convenience, we normalize $C + D = 1$. Deposits are fully insured at no cost to the bank; they carry a 0 interest rate and need to be repaid at date 1.

The bank has access to two alternative investment projects. Both require an outlay of 1 at date 0 (all resources available to the bank), and produce return at date 1. The return of the *safe* project is certain: $R_S > 1$. The return of the *risky* project is probabilistic: high, $R_H > R_S$, with probability p; low, $0 < R_L < 1$, with probability $1 - p - \mu$; or extremely low, $R_0 = 0$, with probability μ. We consider the risky project with three outcomes in order to capture both the second (variance) and the third (skewness, or "left tail", driven by the R_0 realization) moments of the project's payoff.

In the spirit of asset substitution literature, we assume that the net present value

(NPV) of the safe project is higher than that of the risky project:

$$R_S > pR_H + (1 - p - \mu)R_L, \tag{4.1}$$

and yet the return on the safe asset, R_S, is not as high as to make the banker always prefer the safe project regardless the level of initial capital:

$$R_S < pR_H + (1 - p)\,. \tag{4.2}$$

Inequality (4.2) is equivalent to $R_S - 1 < p(R_H - 1)$, where the left-hand side is the banker's expected payoff from investing in the safe project, and the right-hand side is the expected payoff from shifting to the risky project, conditional on bank having no capital (i.e., $C = 0$). The economic intuition is as follows. When the return on the safe project is too high, bank risk-taking incentives are mitigated at any level of initial capital, since the bank is better off selecting the safe project even when it is financed only with deposits (i.e., $D = 1$). In the Appendix we show that when assumption (4.2) is relaxed, the bank selects the safe project for any level of initial capital.

We consequently study conditions under which the bank's leverage creates incentives for it to opportunistically choose the suboptimal, risky project.

The bank's project choice is unobservable and unverifiable. However, the return of the project chosen by the bank becomes observable and verifiable before final returns realize, at date $\frac{1}{2}$.[2] This enables the regulator to undertake corrective action if the bank is undercapitalized.

[2] The fact that project choice is unobservable while project returns are, is a standard approach to modelling (Hellman et al., 2000, Rochet, 2004).

Capital Regulation

Capital regulation is based on the minimal capital (leverage) ratio, a key feature of Basel regulation. We take the presence of a minimal ratio as exogenous.

At each point in time, the bank's capital ratio is defined as $(A - D)/A$, where A is the value of bank assets, D is the face value of debt, and $A - D$ is economic capital. At date 0, before the investment is undertaken, the bank's capital ratio is therefore $c = C/(C + D) = C$. At dates $\frac{1}{2}$ and 1 the bank's capital ratio is $c_i = (R_i - D)/R_i$, where $i \in \{S, H, L, 0\}$. The fact that the date $\frac{1}{2}$ capital position is defined in a forward-looking way is consistent with the practice of banks recognizing known future gains or losses.

The bank is subject to a minimal capital ratio: at any point in time, the ratio must exceed $c_{\min}$. We assume that the minimal ratio is satisfied at date 0: $c > c_{\min}$. Note that, consequently, the minimal ratio is also satisfied for realizations R_S (when the bank chooses the safe project) and R_H (when the bank chooses the risky project and is successful): $c_H > c_S > c > c_{\min}$, since $R_H > R_S > 1$. The minimal capital ratio is never satisfied for R_0 (in the extreme low outcome of the risky project), since the bank's capital is negative, $c_0 = -\infty < 0 < c$. The banks' capital sufficiency under a low realization of the risky project R_L is ambiguous. As we will observe below, depending on the bank's initial capital, it can be either positive and sufficient, $c_L > c_{\min}$, or positive but insufficient, $0 < c_L < c_{\min}$, or negative, $c_L < 0$.

The bank's capital ratio becomes known at date $\frac{1}{2}$, as it is derived from information on future returns that becomes available at that time. If the bank capital does not satisfy the minimal capital ratio — under R_0 and possibly R_L realizations of the risky project — the regulator imposes a corrective action on the bank. Specifically, the banker is given two options. One is to surrender the bank to the regulator. Then, banker's stake (equity value) is wiped out and the banker receives a zero payoff. Alternatively, the bank may attract additional capital to bring its capital ratio to the regulatory minimum, $c_{\min}$. Attracting additional capital is costly. In the main setup, we use a fixed cost of recapitalization, denoted T. In Section 4.5 we discuss a more general specification of such cost, and find that it does not affect qualitatively the results. The capital adjustment cost imposed on a bank in breach of the minimal capital ratio is the key ingredient of the model. This cost is effectively a penalty on risk-taking (as opposed to the risk-taking subsidy offered by limited liability), binding for marginally capitalized banks.

In the main setup we also abstract from the continuation value of the bank's activity (i.e., charter value) and consider that the bank is closed at date 1. As a result, the rational banker is concerned only about her short term profits since she is the only claimant of any excess returns generated by the investment projects. We address the impact of charter value on bank risk-taking in Section 4.5.

Timeline

The model outcomes and the sequence of events are depicted in Figure 1.

4.3 Bank Payoffs and Recapitalization Decision

We consider in this section the bank's payoffs, depending on its project choice, on initial capital ratio c, and, if appropriate, on recapitalization decision at date $\frac{1}{2}$.. Since we are primarily interested in the positive analysis of banking regulation, we take $c_{\min}$ as exogenous, with $c_{\min} > 0$.

With the safe project, the bank always has sufficient capital: since $R_S > 1$, $c_S > c > c_{\min}$. The banker's payoff after repaying depositors is $\Pi_S = R_S - D = R_S - (1-c)$.

The banker's payoff to the risky project is more complex. We start with two simpler outcomes. With probability p, the risky project returns a high payoff, R_H. The bank has sufficient capital: since $R_H > 1$, $c_H > c > c_{\min}$. The banker repays depositors and obtains $\Pi_H = R_H - D = R_H - (1-c)$. With probability μ, the risky project returns $R_0 = 0$. The bank is worthless, and the banker gets a 0 payoff.

With probability $1-p-\mu$ the bank obtains a low return, R_L. This case requires a more detailed analysis, because depending on the relative values of c and R_L, the bank may have three capital positions: positive and sufficient, positive but insufficient, or negative. If the bank's capital falls below the minimal value, the banker will have to either attract additional capital at a cost, or surrender the bank to the regulator. The capital falls below the regulatory minimum, $c_L < c_{\min}$, whenever project returns R_L fall below a threshold $R_{\min}$, where

$$R_{\min} = \frac{1-c}{1-c_{\min}}. \tag{4.3}$$

The above threshold is derived from the condition of a minimal capital ratio of $c_{\min}$ (i.e., $c_{\min} = (R_{\min} - D)/R_{\min}$), by solving for the value of bank's assets (i.e., $R_{\min}$), subject to balance sheet identity (i.e., $D = 1 - c$).

Therefore, the bank has positive and sufficient capital $c_L > c_{\min}$ when $R_L > R_{\min}$. In particular this happens when initial capital c is larger than a threshold $c^{Sufficient}$, where we denote

$$c^{Sufficient} = 1 - (1 - c_{\min})R_L. \tag{4.4}$$

Then, the bank continues to date 1, repays depositors, and obtains $R_L - (1 - c)$.

Below the threshold $c^{Sufficient}$, the bank can be either abandoned or recapitalized. The bank is abandoned when it has negative capital: $c_L < 0$. This happens for $R_L < D$. In this case, the bank is worthless and the banker gets a 0 payoff.

The most interesting case is the one where the bank has positive but insufficient capital, $0 < c_L < c_{\min}$. This is the case for returns R_L below the critical threshold $R_{\min}$, but above the total volume of deposits, D:

$$D < R_L < R_{\min}. \tag{4.5}$$

Since the bank is in breach of the minimal capital ratio, the banker has to either abandon the bank and get 0 payoff, or to recapitalize the bank at cost T. The banker chooses to recapitalize when her expected payoff of doing so is positive:

$$R_L - (1 - c) - T > 0. \tag{4.6}$$

Solving for the level of initial capital c satisfying both (4.5) and (4.6) we find that the banker decides to raise additional capital only when initial capital c is higher than a certain threshold we denote by $c^{\operatorname{Re} capitalize}$, where

$$c^{\operatorname{Re} capitalize} = 1 + T - R_L. \tag{4.7}$$

For values of initial equity below $c^{\operatorname{Re} capitalize}$, the banker chooses the abandonment of bank. The banker's payoff is therefore $\max\{R_L - (1 - c) - T; 0\}$. Note that both thresholds $c^{\operatorname{Re} capitalize}$ and $c^{Sufficient} \in (0, 1)$.

We focus our principal analysis on the most rich case where $c^{Sufficient} > c^{\operatorname{Re} capitalize}$, corresponding to values of T that are not too high:

$$T < c_{\min} R_L. \tag{4.8}$$

The case of T higher is discussed in the Appendix.

Overall, the banker's payoffs and recapitalization choices in the low realization of the risky project are characterized by the following two lemmas:

Lemma 8 *If initial capital c is above threshold $c^{Sufficient}$, the bank has sufficient capital when R_L is realized. Below this threshold, the banker decides between costly recapitalization and abandonment of the bank. The banker raises additional capital at cost T for $c \in (c^{\operatorname{Re} capitalize}, c^{Sufficient})$ and abandons the bank for $c < c^{\operatorname{Re} capitalize}$.*

Lemma 9 *When R_L is realized, the banker's payoff is Π_L, where*

$$\Pi_L = \begin{cases} R_L - (1-c), \textit{if } c \geq c^{Sufficient} \\ R_L - (1-c) - T, \ \textit{if } c^{\text{Re}\,capitalize} \leq c < c^{Sufficient} \\ 0, \ \textit{if } c < c^{\text{Re}\,capitalize} \end{cases}.$$

Overall, the banker's payoff to choosing the risky project is

$$\Pi_R = p[R_H - (1-c)] + (1-p-\mu) * \Pi_L. \tag{4.9}$$

The first term on the right-hand side captures the banker's expected payoff when the risky project is successful. The second term captures the banker's expected payoff when the low outcome R_L is realized, depending on bank's initial capital c, on low project realization R_L, as well as on the cost of recapitalization T.

Figure 2 illustrates our findings on the bank's recapitalization decision.

4.4 Bank Risk-Taking

In this section we solve the model and examine the relationship between bank capital and risk-taking. We first consider a setting without tail risks and show that risk-taking falls in bank capital. This classic result is the foundation of traditional bank capital regulation. We then examine bank's behavior in the presence of tail risks and show that banks with higher capital may demonstrate excess risk-taking incentives.

4.4.1 Bank Risk-Taking Without Tail Risks

We start with an illustrative benchmark of the bank's project choice without tail risks (i.e., $\mu = 0$). We consider in this section a simplified version of the risky project described above. The return on the risky project is either high, $R_H > R_S$, with probability q, or low, $0 < R_L < 1$, with probability $1 - q$. The return on the safe project is certain and identical with the one described in the previous sections, $R_S > 1$. Likewise, we make the similar two assumptions:

(1) The net present value (NPV) of the safe project is higher than that of the risky project

$$R_S > qR_H + (1-q)R_L, \tag{4.10}$$

(2) The return on the safe asset, R_S, is not as high as to make the banker always prefer the safe project regardless the level of initial capital:

$$R_S < qR_H + (1-q)\,. \tag{4.11}$$

We validate the traditional intuition that the bank's incentives to opportunistically choose the risky project monotonically decrease in its initial capital. The reason, a standard one, is that the bank with low capital does not internalize the losses in the downside R_L realization of the risky project (its limited liability is binding). In contrast, a bank with a high capital has more "skin in the game" and would therefore internalize more of the downside.

The banker's payoffs and recapitalization decision in the low realization of the risky project are characterized by Lemmas 8 and 9 from the previous section. Accordingly, the banker's payoff to choosing the risky project is

$$\Pi_R^{NTR} = q[R_H - (1-c)] + (1-q) * \Pi_L, \tag{4.12}$$

(with NTR for no tail risks).

The first term on the right-hand side captures the banker's expected payoff when the risky project is successful. The second term captures the banker's expected payoff when the low outcome R_L is realized, depending on bank's initial capital c, on low project realization R_L, as well as on the cost of recapitalization T.

Bank project choice

We now consider bank project choice at date 0, depending on its initial capital ratio, c. The bank chooses the safe project over the risky project for:

$$R_S - (1-c) \geq q[R_H - (1-c)] + (1-q) * \Pi_L. \tag{4.13}$$

The following proposition describes the bank investment decision.

Proposition 10 *The bank's project choice can be characterized by one threshold as follows: there exist $c^*_{Benchmark}$, with $0 < c^*_{Benchmark} < c^{\mathrm{Re}\,capitalization}$, and*

$$c^*_{Benchmark} = 1 - \frac{R_S - qR_H}{1-q},$$

such that the bank chooses the risky project for $0 < c < c^*_{Benchmark}$, and the safe project for $c^*_{Benchmark} < c < 1$, where R_S and R_H are the returns on the safe and on the risky project (conditional on being successful), respectively, and q is the success probability of the risky project.

Proof. *Following from Lemma 8 there are three important levels of initial capital: low (i.e., $c < c^{\mathrm{Re}\,capitalize}$), intermediate (i.e., $c^{\mathrm{Re}\,capitalize} \leq c < c^{Sufficient}$) and high (i.e., $c \geq c^{Sufficient}$). Consider first the case when $c \in (0, c^{\mathrm{Re}\,capitalize})$. The banker never finds optimal to recapitalize for low realization of the risky project. The relevant incentive compatibility condition derived from (4.13) is $R_S - (1-c) \geq q[R_H - (1-c)]$, where the left-hand side is the return on investing in the safe project and the right-hand side is the expected return on selecting the risky project. The condition can be rewritten as $c \geq 1 - \frac{R_S - qR_H}{1-q}$. We denote $c^*_{Benchmark} = 1 - \frac{R_S - qR_H}{1-q}$. The threshold $c^*_{Benchmark}$ exists if and only if the next two constraints are jointly satisfied:*

$$R_S < 1 - q + qR_H, \tag{4.14}$$

$$R_S > qR_H + (1-q)(R_L - T). \tag{4.15}$$

The former condition guarantees a positive $c^*_{Benchmark}$*, while the latter forces the threshold to be lower than* $c^{\text{Re}\,capitalize}$*, the upper limit for the interval we analyze. Assumption (4.11) implies that (4.14) is always fulfilled, hence* $c^*_{Benchmark} > 0$*. Likewise, assumption (4.10) implies that (4.15) is always fulfilled, hence* $c^*_{Benchmark} < c^{\text{Re}\,capitalize}$*. Since both constraints are simultaneously satisfied,* $\exists\; c^*_{Benchmark} \in (0, c^{\text{Re}\,capitalize})$ *such that* $\forall\; c \in (0, c^*_{Benchmark})$ *the risky project is selected and* $\forall\; c \in (c^*_{Benchmark}, c^{\text{Re}\,capitalize})$ *the safe project is chosen.*

Consider now the case when $c \in (c^{\text{Re}\,capitalize}, c^{Sufficient})$*. The banker finds optimal to recapitalize for low realization* R_L*. The relevant incentive compatibility condition is* $R_S - (1-c) \geq q[R_H - (1-c)] + (1-q)[R_L - (1-c) - T]$*, with the certain return on choosing the safe project depicted on the left-hand side, and expected return on investing in the risky project depicted on the right-hand side. Rearranging terms the condition can be rewritten as*

$$R_S > qR_H + (1-q)(R_L - T). \tag{4.16}$$

The above condition is always satisfied following from assumption (4.10). This implies that $\forall\; c \in (c^{\text{Re}\,capitalize}, c^{Sufficient})$*, the bank prefers the safe project.*

Consider now the final interval, when $c \in (c^{Sufficient}, 1)$*. For low realization* R_L *the bank always complies with the regulatory requirements. No additional capital is needed. The relevant incentive compatibility condition is* $R_S - (1-c) \geq q[R_H - (1-c)] + (1-q)[R_L - (1-c)]$*. Rearranging terms the condition becomes*

$$R_S > qR_H + (1-q)R_L. \tag{4.17}$$

Assumption (4.10) implies that the above condition is always fulfilled, thus $\forall\; c \in (c_{Sufficient}, 1)$ *the safe project is selected.* ■

The intuition is that the banker's downside is limited to the bank's initial capital, due to limited liability. A bank with smaller initial capital internalizes less of the downside of the risky project (while enjoying the full upside), and hence has incentives to take on more risk. Bank risk-taking is monotonically decreasing in initial capital.

This benchmark result validates the basic rationale for traditional capital regulation: higher capital increases banker's "skin-in-the-game" and reduces incentives for excess risk-taking. In particular, a bank with initial capital $c \geq c^*_{Benchmark}$ has no incentives to undertake the risky project. We take this intuition further in the next section, and consider the consequences of introducing tail risk projects in the traditional framework.

4.4.2 Bank Risk-Taking With Tail Risks

Consider now a bank operating under a minimal capital ratio, in an environment where tail risk projects are available for investment (i.e., $\mu > 0$). We show that, due to the existence of tail risk projects, the relationship between bank capital and risk-taking becomes more complex than under a simple limited liability-based story without tail risk projects.

Indeed, recall that under limited liability, a banker did not internalize the downside of low realizations, and that drove excess risk-taking. Higher capital made limited liability less binding, reducing bank risk-taking incentives. The minimal capital requirement has an *opposite* effect. Banks with a positive but *insufficient* capital are punished through the corrective action mechanism, because they have to either raise extra capital (at a cost) or give up the bank. Therefore, a banker may experience losses that exceed the downside of the bank's asset value. Put differently, while limited liability effectively *subsidizes* risk, the minimal capital requirement and corrective action have the capacity to *penalize* risk. Capital further above minimal capital ratio makes the corrective action less binding, potentially allowing banks to increase risk. Once the two effects of limited liability and minimal capital ratio are taken into account, the relationship between bank capital and risk-taking may become non-linear in the presence of tail risk projects.

We again solve the model backwards. We start from the banker's payoffs conditional on project realization and recapitalization decision at date $\frac{1}{2}$, which were derived in Section 4.3. We now consider bank project choice at date 0, depending on its initial capital ratio, c.

Bank project choice

The bank chooses the safe project over the risky project for:

$$R_S - (1-c) \geq p[R_H - (1-c)] + (1-p-\mu) * \Pi_L. \tag{4.18}$$

Before describing the results we introduce the following two thresholds. We denote

$$W = pR_H + (1-p)R_L - \mu c_{\min} R_L, \tag{4.19}$$

and

$$Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min} R_L), \tag{4.20}$$

A full description of the derivation of these two thresholds is given in the Appendix. The intuition is as follows. First, consider the point W. If the bank selects the risky project then, upon R_L being realized the bank has positive and sufficient capital at any levels of initial capital such that $c > c^{Sufficient}$. Conditional on bank holding this level of capital, the banker is better off selecting the safe project (i.e., condition (4.18) is fulfilled) only when the return on the safe project is high enough: $R_S \geq W$. Second, consider the point Z. Upon the realization of R_L following from an investment in the risky project, the bank ends up with positive but insufficient capital when initial capital $c \in (c^{\mathrm{Re}\, capitalize}, c^{Sufficient})$. However, for this level of initial capital the banker decides for recapitalization when subject to regulator's corrective action. The banker is better off selecting the risky project (i.e., condition (4.18) is not fulfilled) only when the return on the safe project is low enough: $R_S < Z$.

Lemma 11 *Consider* W *and* Z.

1. $Z < W$;

2. Point W *is a threshold for asset substitution, conditional on bank having a high level of initial capital (i.e.,* $c \geq c^{Sufficient}$*), that is for* $R_S < W$*, the banker chooses the risky project over the safe project in the absence of corrective action;*

3. Point Z *is a threshold for the binding impact of the corrective action, conditional on bank having an intermediate level of capital (i.e.,* $c^{\mathrm{Re}\,capitalize} \leq c < c^{Sufficient}$*), that is for* $R_S > Z$*, the bank chooses the safe project over the risky project in the presence of corrective action.*

Lemma 11 describes the main two key effects of the paper. These effects offset each other and the following proposition describes when one effect dominates the other.

Proposition 12 *The bank's project choice can be characterized by two possible thresholds as follows:*

(a) for $Z < R_S < W$*, exists* c^*, $0 < c^* < c^{Sufficient}$*, and* c^{**}, $c^{Sufficient} < c^{**} < 1$*, such that the bank's risk preference is non-monotonic in initial capital. The bank chooses the risky project for* $0 < c < c^*$*, the safe project for* $c^* < c < c^{Sufficient}$*, again the risky project for* $c^{Sufficient} < c < c^{**}$*, and the safe project for* $c > c^{**}$*;*

(b) for $R_S < Z$*, exists* c^{**}, $c^{Sufficient} < c^{**} < 1$*, such that for any levels of initial capital* $0 < c < c^{**}$*, the bank selects the risky project, while for* $c^{**} < c < 1$ *the safe project is preferred;*

(c) for $R_S > W$*, exists* c^*, $0 < c^* < c^{Sufficient}$*, such that for any levels of initial capital* $0 < c < c^*$*, the bank selects the risky project, while for* $c^* < c < 1$*, the safe project is preferred,*

where R_S *is the return on the safe project, and* W *and* Z *are defined in (4.19) and (4.20), respectively.*

Proof. *The proof is in the Appendix.* ■

Case (a) of Proposition 12 contains the key novel result of our paper. The two effects identified in Lemma11 interact as follows. When the bank has low capital (i.e., $0 < c < c^*$), it does not internalize the downside of low realizations (i.e., R_0 and R_L). Hence, limited liability subsidizes risk-taking in the presence of negative capital. As a result, the limited liability effect dominates the effect of corrective action and drives excess risk-taking. For higher levels of initial capital (i.e., $c^* < c < c^{Sufficient}$), the bank might be subject to regulator's corrective action when it has positive but insufficient capital (i.e., for realization R_L). Hence, corrective action penalizes the risk-taking and this effect dominates the limited liability effect. The banker would prefer the safe project. However, when the bank has enough initial capital such that upon realization of R_L the capital is sufficient to satisfy the minimal requirements (i.e., $c^{Sufficient} < c < c^{**}$), the bank is subject to regulator's corrective action only for extremely low realization R_0. However, in such extreme event the bank does not internalize much of the losses. Hence, limited liability effect dominates the corrective action effect, which is less binding in this case. Finally, for very high levels of capital (i.e., $c > c^{**}$), the bank absorbs more of the extremely low downside realization R_0. Hence the banker has lower incentives to take excess risk and selects the safe project.

Our findings suggest that for a certain large range of model's parameters, highly capitalized banks find optimal to take excessive risk in equilibrium. For the remainder of the paper we focus our discussion on Case (a) of Proposition 12. This case provides us with the most complex scenario which is relevant for the model's core results. A full description of thresholds c^* and c^{**} defined in Proposition 12 is given in the Appendix. For our case of interest, these thresholds are as follows:

$$c^* = 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}, \tag{4.21}$$

and

$$c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}, \tag{4.22}$$

with $c^{*} \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ and $c^{**} \in (c^{Sufficient}, 1)$, where $c^{\mathrm{Re}\,capitalize}$ and $c^{Sufficient}$ are defined in (4.7) and (4.4), respectively.

Figure 3 depicts bank's project choice in this particular case.

4.5 Discussion

In this section we first offer comparative statics and study how a change in the probability distribution of the risky project affects the model outcomes. We show that highly capitalized banks have stronger incentives to invest in tail risk projects when that risk is more extreme. Second, we offer two extensions for our model and examine the implications of charter value and of different specification for recapitalization costs. We show that our results from previous section are robust to these generalizations.

4.5.1 The Role of Left Tails

The setup of the model allows us to investigate the role played by heavier left tail, or tail risk, in the investment selection process for leveraged intermediaries. A tail risk is caused by extremely low realizations of the risky project, R_0, and is characterized by μ, its probability. Thus, higher μ reflects a heavier left tail. We investigate whether banks have greater incentives to increase the risk of their investments by shifting to the risky project when the risk profile of available investments exhibits heavy left tails. Surprisingly, we show that *better capitalized banks* are in particular the ones demonstrating risk-taking preferences when tail risk projects are available for investments.

To study this effect, we consider a change in probability distribution for the risky asset. We denote the expected value of the risky project by $E(Risky)$. We keep $E(Risky)$ constant by increasing both μ and p, other else being equal. In other words, following to an increase in μ by $\Delta\mu$, the value of p should increase by $\frac{R_L}{R_H - R_L}\Delta\mu$, in order to keep $E(Risky)$ constant. Therefore, a project displaying the same expected return as the initial risky project, but with more polarized returns (i.e., heavier left and right tails), is considered to be riskier.

This change in the return profile of the risky asset affects both thresholds c^* and c^{**}. To focus on bank's incentives to take the excessively risky strategy, we consider the area $(c^{Sufficient}, c^{**})$, corresponding to levels of initial bank capital for which the bank prefers the risky project. As $c^{Sufficient}$ is given by the exogenous regulation, the left boundary of the interval analyzed is not affected by the return profile of the risky project (see 4.4). Hence, the critical threshold for our discussion is c^{**}.

Consider the impact of the change in probability distribution of the risky project on c^{**}. Observe that the first derivative is positive:

$$\frac{\partial c^{**}}{\partial \mu}\Big|_{E(Risky)=constant} = \frac{R_S - E(Risky)}{\mu^2} > 0$$

This holds due to the fact that $R_S > E(Risky)$ by (4.1).

This means that when the left tail of the risky project becomes heavier following to an increase in μ, the interval $(c^{Sufficient}, c^{**})$ widens and the banks start choosing the risky project for a wider range of initial capital values. This result suggests that availability of investments with heavier left tails induces precisely banks with larger capital buffers to take excessive risk. Figure 4 depicts the impact of increased tail risk (i.e., heavier left tail) on bank's project choice.

The intuition is that when investment return becomes more polarized, it enables well-capitalized banks to earn higher profits in good time, while at the same time reducing the expected cost of recapitalization since the low return R_L (which triggers the recapitalization decision) is less frequent. In the extreme case when $\mu + p = 1$, the risky project never returns R_L, so it either produces a high return R_H, or a large loss R_0, when the bank is worthless and recapitalization is useless. A less capitalized bank will earn a return too infrequently in this case, and will avoid the risky project, unlike a bank with larger buffers.

4.5.2 Alternative Capital Regimes

In Section 4.2 we considered a simple fixed cost of recapitalization. We now show that results are robust to a more general specification of this cost function.

In this section we discuss a variation of the model in which the cost of recapitalization has a fixed component and a variable component as well. The variable component is proportional to the amount of new capital that the bank has to raise in order to comply with regulatory requirements. If the bank does not satisfy minimal capital ratio (under R_L realization of the risky project), the bank has to attract additional capital to bring its capital ratio to the regulatory minimum, $c_{\min}$. Specifically, the bank has to raise a capital level $R_{\min} - R_L$, where $R_{\min}$ is given in (4.3).[3] In this new setting, the recapitalization cost is concave in capital level, and has the following specification:

$$Cost(c, R_L) = T + \beta\left(\frac{1-c}{1-c_{\min}} - R_L\right). \tag{4.23}$$

We assume that the marginal cost of recapitalization (i.e., β) is not as low as to make the banker to abandon the bank regardless the level of initial capital:

$$T < R_L(1+\beta) \tag{4.24}$$

The banker's payoff from the safe project, as well as the realizations of the risky project are the same as in the basic model. However, when the low realization R_L is

[3] Consider the following example. Assume that the bank has to raise δ units of capital to satisfy the regulatory minimum when R_L is realized. Hence, $c_{\min} = \frac{R_L-(1-c)+\delta}{R_L+\delta}$.This implies that $R_L + \delta = \frac{1-c}{1-c_{\min}}$, which equals $R_{\min}$ according to (4.3). We can conclude that $\delta = R_{\min} - R_L$.

obtained, the bank is abandoned for higher levels of initial capital than in the basic model. The bank is closed when $c < c_{CC}^{\operatorname{Re}capitalize}$, where

$$c_{CC}^{\operatorname{Re}capitalize} = 1 + \frac{T - R_L(1+\beta)}{1 + \frac{\beta}{1-c_{\min}}}, \tag{4.25}$$

(with CC for concave cost).

Under assumption (4.8), $c_{CC}^{\operatorname{Re}capitalize} > c^{\operatorname{Re}capitalize}$. On the other hand, the level of capital which guarantees that the bank satisfies ex-post the regulatory minimal upon realization of R_L (i.e., $c^{Sufficient}$) remains unchanged. Hence, the interval $(c_{CC}^{\operatorname{Re}capitalize}, c^{Sufficient})$ shrinks, suggesting that the bank is less likely to raise additional capital under corrective action of the regulator for a concave cost of recapitalization.

We denote

$$B_{CC} = pR_H + (1-p)\frac{R_L(1+\beta) - T}{1 + \frac{\beta}{1-c_{\min}}}. \tag{4.26}$$

The next proposition characterizes the project's choice in the presence of concave recapitalization cost.

Proposition 13 *There exist two thresholds c_{CC}^{*} and c_{CC}^{**} for the level of initial bank capital such that under assumption (4.8) and for level of return on the safe project satisfying $Z < R_S < B_{CC}$, with Z and B_{CC} defined in (4.20) and (4.26), respectively, the bank's investment preference is as follows:*

(a) the bank prefers the risky project for $0 < c < c_{CC}^{*}$, while for $c_{CC}^{*} < c < c^{Sufficient}$ the safe project is preferred, with $c_{CC}^{*} \in (c_{CC}^{\operatorname{Re}capitalize}, c^{Sufficient})$, where $c_{CC}^{\operatorname{Re}capitalize}$ and $c^{Sufficient}$ are defined in (4.25) and (4.4), respectively, and

$$c_{CC}^{*} = 1 - \frac{R_S - pR_H - (1-p-\mu)[R_L(1+\beta) - T]}{\mu - \frac{\beta}{1-c_{\min}}(1-p-\mu)} \tag{4.27}$$

(b) the bank prefers the risky project for $c^{Sufficient} < c < c_{CC}^{**}$, and the safe project for $c > c_{CC}^{**}$, where $c_{CC}^{**} \in (c^{Sufficient}, 1)$ and $c_{CC}^{**} = c^{**}$, with c^{**} defined in (4.22).

Proof. *The proof is similar with the proof for Proposition 12.* ■

Observe that the introduction of a variable component for recapitalization cost leaves both boundaries of the interval $(c^{Sufficient}, c^{**}_{CC})$ unchanged. Thus, our model is robust and a concave cost of recapitalization does not affect the risk-taking preferences of well-capitalized banks, when they are allowed to invest in projects exhibiting heavier left tails.

4.5.3 Charter Value

In Section 4.2 we have assumed that there is no charter value for the continuation of bank's activity. In this section we introduce a positive charter value $V > 0$ and show that our results are robust to this extension. Our model suggests that low competition in banking, which provides a high charter value, leads to investment in the efficient safe project even by well-capitalized banks.

The role of banks' franchise values have been shown relevant in other studies. Hellmann et al. (2000) and Repullo (2004) argue that prudent behavior can be facilitated by increasing banks' charter value. They study the links between capital requirements, competition for deposits, charter value and risk-taking incentives, and point out that banks are more likely to gamble and to take more risk in a competitive banking system, since competition erodes profits and implicitly the franchise value. A similar idea is put forward by Matutes and Vives (2000). They argue that capital regulation should be complemented by deposit rate regulation and direct asset restrictions in order to efficiently keep risk-taking under control. Acharya (2002) explores how continuation value affects risk preferences in the context of optimal regulation, and demonstrates the disciplinating effect of charter value on bank risk-taking. Finally, Keeley (1990) and Furlong and Kwan (2005) explore empirically the relation between charter value and different measures of bank risk, and find strong evidence that bank charter value disciplined bank risk-taking.

In the new setting, the banker's payoff to the safe project after repaying depositors becomes $\Pi_S^V = R_S - (1-c) + V$. The banker's payoff to the risky project is as follows: when R_H is realized, the banker gets $\Pi_H^V = R_H - (1-c) + V$, while the payoff is 0 for extremely low realization R_0. When the low return R_L is realized and capital is positive but insufficient ex-post, the banker prefers to recapitalize at a cost T for lower levels of initial capital c. The reason for this is that banker's expected payoff depicted in equation (4.6) increases by V if the bank is not closed by the regulator. Hence, the bank raises additional capital when initial capital c is higher than $c_V^{\mathrm{Re}\,capitalize}$, where

$$c_V^{\mathrm{Re}\,capitalize} = 1 + T - R_L - V, \tag{4.28}$$

and $c_V^{\mathrm{Re}\,capitalize} < c^{\mathrm{Re}\,capitalize}$. On the other hand, the threshold point $c^{Sufficient}$ does not change since it is given by the exogenous regulation.

We make the simplifying assumption that the charter value is not larger than a certain threshold:

$$V < 1 + T - R_L. \tag{4.29}$$

This makes threshold $c_V^{\mathrm{Re}\,capitalize}$ positive and assures the existence for the area $(0,\, c_V^{\mathrm{Re}\,capitalize})$ where the bank is abandoned for low realization of the risky asset. Consider the area $(c_V^{\mathrm{Re}\,capitalize}, c^{Sufficient})$. When initial capital c is in this range, a bank which is subject to regulator's corrective action prefers to raise additional capital. Since $c_V^{\mathrm{Re}\,capitalize} < c^{\mathrm{Re}\,capitalize}$, while right boundary of the interval is left unchanged by any increase in V, we can argue that any reduction in banking competition, which increases bank charter value, makes the decision to raise fresh capital more likely.

We introduce the following two thresholds:

$$Z_V = pR_H + (1-p)(R_L - T) + \mu(T - V - c_{\min} R_L), \tag{4.30}$$

as the new threshold for the binding impact of the prompt corrective action (with $Z_V < Z$), and

$$B = pR_H + (1-p)(R_L - T). \tag{4.31}$$

The next proposition characterizes the project's choice in the presence of charter value.

Proposition 14 *There exist two thresholds c_V^* and c_V^{**} for the level of initial bank capital such that under assumption (4.8) and for levels of return on the safe project satisfying $Z_V < R_S < B$, with Z_v and B defined in (4.30) and (4.31), respectively, the bank's investment preference is as follows:*

(a) the bank prefers the risky project for $0 < c < c_V^*$, while for $c_V^* < c < c^{Sufficient}$ the safe project is preferred, with $c_V^* \in (c_V^{\mathrm{Re}\, capitalize}, c^{Sufficient})$, where $c_V^{\mathrm{Re}\, capitalize}$ and $c^{Sufficient}$ are defined in (4.28) and (4.4), respectively, and

$$c_V^* = 1 - V - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}, \tag{4.32}$$

(b) the bank prefers the risky project for $c^{Sufficient} < c < c_V^{**}$, and the safe project for $c > c_V^{**}$, where $c_V^{**} \in (c^{Sufficient}, 1)$ and

$$c_V^{**} = 1 - V - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}, \tag{4.33}$$

Proof. *The proof is similar with the proof for Proposition 12.* ■

Observe that a positive charter value has a negative impact on all relevant thresholds which drive bank's preferences (i.e., $c_V^{\mathrm{Re}\, capitalize}$, c_V^*, and c_V^{**}), except for $c^{Sufficient}$. Hence, we can argue that higher charter value plays the role of a counterbalancing force to the risk-taking incentives generated by the presence of risky projects with heavy left tails. This means that when the continuation value of bank's activity is high enough, both intervals $(0, c_V^*)$ and $(c^{Sufficient}, c_V^{**})$ shrink. This suggests that low competition in the banking industry induces banks with larger capital buffers to take less risk. Figure 5 illustrates the impact of charter value on bank's project choice.

In summary, the results of our basic model are therefore robust to the introduction of charter value, conditional on the fact that this value is not too large. For large values of franchise value V, there are no risk-taking problems in banks, regardless the level of initial capital.

4.6 Concluding Remarks

This paper examined the relationship between bank capital and risk-taking, in the presence of tail risk projects. We have identified a novel channel of bank risk-taking, based on incentives to put to risk excess capital. While a poorly capitalized bank may act risk-averse to avoid breaching the minimal capital ratio (which would force a costly recapitalization), a bank with higher capital may take more risk as it has a lower probability of breaching the ratio. Still, in the presence of tail risks, the highly capitalized banks do not internalize all the consequences of its risk-taking. The key result is that when banks have access to high tail risk projects, this can lead to excess risk-taking by highly capitalized banks in equilibrium. This demonstrates the limits of traditional capital regulation in mitigating banks' incentives to take tail risks. The capital requirements alone may be insufficient to control banks' preferences when tail risk projects are available to them. In fact, in the presence of both limited liability effect and corrective action effect (triggered by minimal capital requirement), the relationship between bank capital and risk-taking may become non-linear. We related our results with stylized facts about pre-crisis bank behavior, and discussed implications for capital regulation.

4.A Proofs

Proof of Proposition 12 under the assumption of low cost of recapitalization (i.e., $T < c_{\min} R_L$)

We consider two scenarios in turn. We start by analyzing a scenario in which the cost of recapitalization is such that $\frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$. Subsequently we show that our results are similar for $T < \frac{\mu}{1-p} c_{\min} R_L$.

$\frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$

We study bank's behavior for three levels of initial capital: low (i.e., $c < c^{\operatorname{Re} capitalize}$), intermediate (i.e., $c^{\operatorname{Re} capitalize} \leq c < c^{Sufficient}$) and high (i.e., $c \geq c^{Sufficient}$).

Consider first the case when $c \in (0, c^{\operatorname{Re} capitalize})$.The banker never finds optimal to recapitalize for low realization of the risky project. The relevant incentive compatibility condition derived from (4.18) is $R_S - (1-c) \geq p[R_H - (1-c)]$, where the left-hand side is the return on investing in the safe project and the right-hand side is the expected return on selecting the risky project. The condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H}{1-p}$. We denote $c_1^* = 1 - \frac{R_S - pR_H}{1-p}$.The threshold c_1^* exists if and only if the next two constraints are jointly satisfied:

$$R_S < 1 - p + pR_H, \tag{T1a'}$$

$$R_S > pR_H + (1-p)(R_L - T). \tag{T1a}$$

The former condition guarantees a positive c_1^*, while the latter forces the threshold to be lower than $c^{\operatorname{Re} capitalize}$, the upper limit for the interval we analyze. If (T1a') is not fulfilled, then $c_1^* < 0$ and $\forall\ c \in (0, c^{\operatorname{Re} capitalize})$, the bank prefers the safe project. If (T1a) is not fulfilled, then $c_1^* > c^{\operatorname{Re} capitalize}$ and $\forall\ c \in (0, c^{\operatorname{Re} capitalize})$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\ c_1^* \in (0, c^{\operatorname{Re} capitalize})$ such that $\forall\ c \in (0, c_1^*)$ the risky project is selected and $\forall\ c \in (c_1^*, c^{\operatorname{Re} capitalize})$ the safe project is chosen. Assumption (4.2) implies that (T1a') is always fulfilled.

Consider now the case when $c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$. The banker finds optimal to recapitalize for low realization R_L. The relevant incentive compatibility condition is $R_S - (1-c) \geq p[R_H - (1-c)] + (1-p-\mu)[R_L - (1-c) - T]$, with the certain return on choosing the safe project depicted on the left-hand side, and expected return on investing in the risky project depicted on the right-hand side. Rearranging terms the condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. We denote $c_2^* = 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. Similarly with the previous case, the threshold c_2^* exists if and only if it is simultaneously higher and lower than the lower and the higher boundary of the analyzed interval, respectively. The conditions are as follows:

$$R_S < pR_H + (1-p)(R_L - T), \tag{T2a}$$

$$R_S > pR_H + (1-p)(R_L - T) + \mu(T - c_{\min} R_L). \tag{T2b}$$

Condition (T2a) is the opposite of (T1a). Thus, a satisfied condition (T1a) implies that (T2a) is not fulfilled. Condition (T2a) not satisfied implies that $c_2^* < c^{\mathrm{Re}\,capitalize}$ and $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$, the bank prefers the safe project. If the second condition is not fulfilled, then $c_2^* > c^{Sufficient}$ and $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\ c_2^* \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ such that $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c_2^*)$ the risky project is selected. The safe project is preferred $\forall\ c \in (c_2^*, c^{Sufficient})$.

Consider now the final interval, when $c \in (c^{Sufficient}, 1)$. For low realization R_L the bank always complies with the regulatory requirements. No additional capital is needed. The relevant incentive compatibility condition is $R_S - (1-c) \geq p[R_H - (1-c)] + (1-p-\mu)[R_L - (1-c)]$. Rearranging terms the condition becomes $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We denote $c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold c^{**} exists if and only if $c^{**} > c^{Sufficient}$ and $c^{**} < 1$. The later is always fulfilled following from the assumption (4.1) of higher NPV for the safe project. The former condition is depicted in (T3a) below. When (T3a) is not satisfied, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. Otherwise, $\forall$ $c \in (c^{Sufficient}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall$ $c \in (c^{**}, 1)$.

$$R_S < pR_H + (1-p)R_L - \mu c_{\min} R_L. \tag{T3a}$$

Next we discuss the process of project selection. Recall that $Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min}R_L)$ and $W = pR_H + (1-p)R_L - \mu c_{\min}R_L$, from (4.20) and (4.19), respectively. We also denote $B = pR_H + (1-p)(R_L - T)$. Under assumption (4.8), $Z < B < W$. We distinguish among four possible scenarios.

Scenario S1: $R_S < Z$. As a consequence, condition (T2b) is not satisfied and $\forall$ $c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the bank selects the risky project. $R_S < Z$ also implies that $R_S < B$ and $R_S < W$.Condition (T1a) is not satisfied but (T3a) is. As a result, the bank invests risky $\forall\ c \in (0, c^{\mathrm{Re}\,capitalize}) \cup (c^{Sufficient}, c^{**})$, and the bank invests safe $\forall\ c \in (c^{**}, 1)$.

Scenario S2: $Z \leq R_S \leq B$. The right-hand side implies that condition (T1a) is not satisfied. For initial capital c lower than $c^{\mathrm{Re}\,capitalize}$ the bank prefers the risky project. The left hand side implies that condition (T2b) is fulfilled. Also condition (T2a) is satisfied being the opposite of (T1a). Hence, we can conclude that $\exists\ c^* \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^*, c^{Sufficient})$. Condition (T3a) is also satisfied. Similarly with the previous scenario, the bank invests risky $\forall\ c \in (c^{Sufficient}, c^{**})$, and safe $\forall\ c \in (c^{**}, 1)$.

Scenario S3: $B < R_S < W$. The left hand-side implies that condition (T1a) is satisfied. We can argue that $\exists\ c^* \in (0, c^{\mathrm{Re}\,capitalize})$ with $c^* = c_1^*$, such that $\forall\ c \in (0, c^*)$ the risky project is selected, while the safe project is preferred $\forall$ $c \in (c^*, c^{\mathrm{Re}\,capitalize})$. Condition (T1a) implies that (T2a) is not satisfied. Thus, $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the safe project will be selected. The bank investment decision is identical with the one from previous scenarios when the level of capital is high enough (i.e., c larger than $c^{Sufficient}$).

Scenario S4: $R_S \geq W$. Neither condition (T3a) nor condition (T2a) are satisfied anymore. The bank selects the safe project $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, 1)$. However, condition (T1a) is fulfilled. Hence, $\exists\ c^* \in (0, c^{\mathrm{Re}\,capitalize})$ with $c^* = c_1^*$, such that $\forall\ c \in (0, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^*, c^{\mathrm{Re}\,capitalize})$.

The values for thresholds c^* and c^{**} for Case (a) of Proposition 12, are derived under Scenario S2 above, for $Z \leq R_S \leq B$, with Z and B defined in (4.20) and (4.31), respectively.

$T < \frac{\mu}{1-p} c_{\min} R_L$

We consider now the scenario under which the cost of recapitalization is very low. Lowering T has no quantitative impact on $c^{Sufficient}$ and $c^{\mathrm{Re}\,capitalize}$, the thresholds in initial capital which trigger bank's decision between raising additional capital or letting the regulator to overtake the bank. Their relative position is unchanged: $c^{Sufficient}$ is larger than $c^{\mathrm{Re}\,capitalize}$ following from easily verifiable identity $\frac{\mu}{1-p} c_{\min} R_L < c_{\min} R_L$ combined with our restriction on T. However, the process of project selection under assumption (4.8) is marginally affected. In this scenario $Z < W < B$, as a consequence of lower T. As discussed before, we distinguish among four possible scenarios (S1') $R_S < Z$, (S2') $Z \leq R_S < W$, (S3') $W \leq R_S < B$ and (S4') $R_S > B$. Discussions for scenarios S1', S2' and S4' are identical with our previous discussion for scenarios S1, S2, and S4. We discuss scenario S3' next. $W \leq R_S$ implies that condition (T3a) is not satisfied. Hence, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. $R_S < B$ implies that condition (T1a) is not satisfied. For initial capital c lower than $c^{\mathrm{Re}\,capitalize}$ the bank prefers the risky project. However, condition (T2a) is satisfied being the opposite of (T1a), and also condition (T2b) is implied by the fact that $W > Z$. Hence, we can conclude that $\exists$ $c^* \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^*, c^{Sufficient})$.

Proof of Proposition 12 under the assumption of high cost of recapitalization (i.e., $T > c_{\min} R_L$)

We relax assumption (4.8) and explore the case of high cost of recapitalization: $T > c_{\min} R_L$. Although the main results are not qualitatively affected, the modified assumption has a quantitative impact on our results. Therefore, we start by deriving the new conditions which drive the results. As was previously explained, the bank is subject to corrective action and receives a request to raise additional capital after investing in the risky project whenever bank capital is below minimal ratio as a result of a low return R_L. The bank has incentives to attract new costly equity only if the payoff of doing so is positive. It is optimal for the bank to raise additional capital (if this was demanded by the regulator) when conditions (4.5) and (4.6) are simultaneously satisfied. The former condition implies that $c < 1 - R_L(1 - c_{\min})$, while from the latter $c > 1 + T - R_L$. Under our modified assumption of high cost of recapitalization T, these conditions can not be satisfied simultaneously. For any levels of initial capital c below $1 - R_L(1 - c_{\min})$, the bank receives a request for adding extra capital but she never finds optimal to do so because such an action will not generate positive payoffs. As a result, the bank is closed and the shareholder expropriated. Conversely, when the level of initial capital is above $1-R_L(1-c_{\min})$ the banking authority doesn't take any corrective action against the bank since returns R_L are above the critical level $R_{\min}$. We denote

$$c_{NEW}^{\mathrm{Re}\,capitalize} = 1 - R_L(1 - c_{\min}), \tag{4.34}$$

where $c_{NEW}^{\mathrm{Re}\,capitalize} \in (0, 1)$. Next, we explore the bank's project choice for levels of initial capital below and above this critical threshold.

Consider first the case when $c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$.The bank never recapitalizes for the low realization of the risky project. The bank would have incentive to select the safe project when $R_S - (1 - c) \geq p[R_H - (1 - c)]$, which implies that initial capital c

to be larger than $1-\frac{R_S-pR_H}{1-p}$. We previously denoted $c_1^*=1-\frac{R_S-pR_H}{1-p}$. This threshold exists if and only if (T1a') and the following condition are jointly satisfied:

$$R_S > pR_H + (1-p)R_L(1-c_{\min}). \tag{T1a NEW}$$

The second condition guarantees that c_1^* is lower than $c_{NEW}^{\mathrm{Re}\,capitalize}$, the upper boundary for the interval we analyze. For large returns on the safe project (i.e., condition (T1a') is not fulfilled), $\forall\ c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$, the bank prefers the safe project. If (T1a NEW) is not fulfilled, then $\forall\ c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ the bank invests risky. Otherwise, when both constraints are simultaneously satisfied, $\forall\ c \in (0, c_1^*)$ the risky project is selected and $\forall\ c \in (c_1^*, c_{NEW}^{\mathrm{Re}\,capitalize})$ the safe project is chosen. Our assumption (4.2) implies that (T1a') is always fulfilled.

Consider now the second case when $c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, 1)$. The bank always complies with the regulatory requirements when R_L is obtained due to high initial capital. No additional capital is needed. The bank would have incentive to select the safe project when $R_S-(1-c) \geq p[R_H-(1-c)]+(1-p-\mu)[R_L-(1-c)]$, which implies $c \geq 1-\frac{R_S-pR_H-(1-p-\mu)R_L}{\mu}$. We previously denoted $c^{**}=1-\frac{R_S-pR_H-(1-p-\mu)R_L}{\mu}$. The threshold c^{**} exists if and only if condition (T3a) is satisfied. The safe project is preferred for any level of initial capital larger than $c_{NEW}^{\mathrm{Re}\,capitalize}$ whenever (T3a) is not satisfied. Otherwise, $\forall\ c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^{**}, 1)$.

Recall that $W = pR_H + (1-p)R_L - \mu c_{\min} R_L$. We also denote $Q = pR_H + (1-p)R_L(1-c_{\min})$. It is easy to show that $Q < W$ due to the identity $1-p-\mu > 0$. We distinguish among only three possible scenarios.

Scenario S1": $R_S \leq Q$. As a consequence, condition (T1a NEW) is not satisfied and $\forall\ c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ the bank selects the risky project. $R_S < Q$ implies that $R_S < W$.Condition (T3a) is satisfied. As a result, the bank invests risky $\forall\ c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$, while she prefers the safe project $\forall\ c \in (c^{**}, 1)$.

Scenario S2": $Q < R_S < W$. The left hand-side implies that condition (T1a

NEW) is satisfied. This implies that $\exists\ c^* \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ with $c^* = c_1^*$, such that $\forall\ c \in (0, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^*, c_{NEW}^{\mathrm{Re}\,capitalize})$. Similarly with the previous scenario, the bank invests risky $\forall\ c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$, and safe $\forall\ c \in (c^{**}, 1)$. This result is implied by R_S being lower than W.

Scenario S3": $R_S \geq W$. Condition (T1a NEW) is satisfied while condition (T3a) is not. Hence, the bank selects the risky projects $\forall\ c \in (0, c^*)$, with $c^* = c_1^*$, and she selects the safe project $\forall\ c \in (c^*, 1)$.

To conclude, we can argue that the qualitative results of Proposition 2 are valid under the relaxed assumption. Nevertheless, condition (T2b) has to be replaced by the relevant condition (T1a NEW).

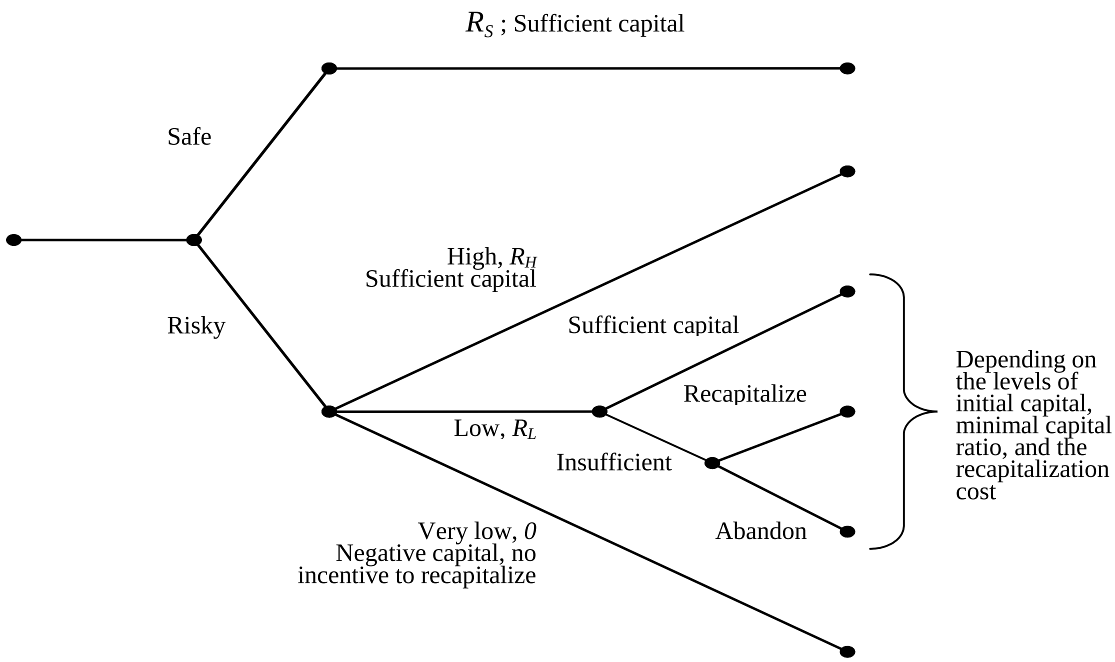
Bank's choice when the return on safe project is large

Let us consider here that the return on the safe asset is large, that is $R_S > 1 - p + pR_H$. This drives the following results under assumption (4.8): (1) condition (T1a') is not satisfied, implying that $\forall\ c \in (0, c^{\mathrm{Re}\,capitalize})$, the bank prefers the safe project; (2) condition (T1a) is satisfied, which implies that condition (T2a) is not and as a result $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$, the bank prefers the safe project; (3) condition (T3a) is not satisfied and as a consequence $\forall\ c \in (c^{Sufficient}, 1)$, the bank invests in the safe project. Summing up, for any levels of initial capital c, the bank prefers the safe project when the certain return R_S is high enough.

4.B Figures

Figure 1.

The timeline



Date 0

- Bank is endowed with capital C and deposits D
- Bank selects one project

Date 1/2

- Information about future returns becomes available
- Regulator takes corrective action (if necessary)
- Bank decides for recapitalization if subject to regulatory penalty

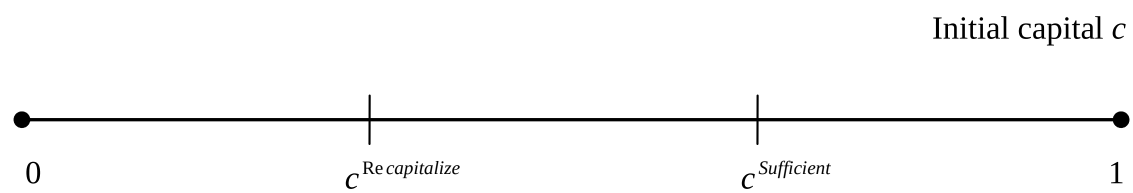
Date 1

- Returns are realized and distributed

Figure 2.

Bank’s recapitalization decision and payoffs

The figure illustrates the bank's recapitalization decision and banker’s payoffs as a function of initial capital c, upon the realization of low return R_L. For $c > c^{Sufficient}$, the bank has positive and sufficient capital at date ½. The bank continues to date 1, repays depositors and obtains a positive payoff. For $c < c^{Sufficient}$, the bank has positive and insufficient or negative capital. The bank can be either abandoned or recapitalized. The bank is abandoned for $c < c^{recapitalize}$. As a result the bank is closed and the banker gets a zero payoff. The bank is recapitalized at a cost for $c^{\mathrm{Re}\,capitalize} < c < c^{Sufficient}$. The bank continues to date 1, repays depositors, pays the recapitalization cost, and obtains a positive payoff.



- No recapitalization;
- Bank is abandoned;
- Banker gets zero payoff.

- The bank is recapitalized at cost T;
- Banker gets a positive payoff $R_L - (1-c) - T$

- Capital is sufficient;
- Banker gets positive payoff

Figure 3.

Bank's project choice

The figure depicts the bank's project choice depending on the level of initial capital, in Case (a) of Proposition 12. The relationship between bank capital and risk-taking is non-linear and is characterized by two thresholds as follows. When the level of capital is low ($c < c^{*}$), the bank prefers the risky asset, while for high level of capital ($c > c^{**}$) the safe asset is chosen. For intermediate level of capital ($c^{*} < c < c^{**}$), the bank prefers either the safe asset (for c^{*} <c< $c^{Sufficient}$) or the risky one (for $c^{Sufficient} < c < c^{**}$).

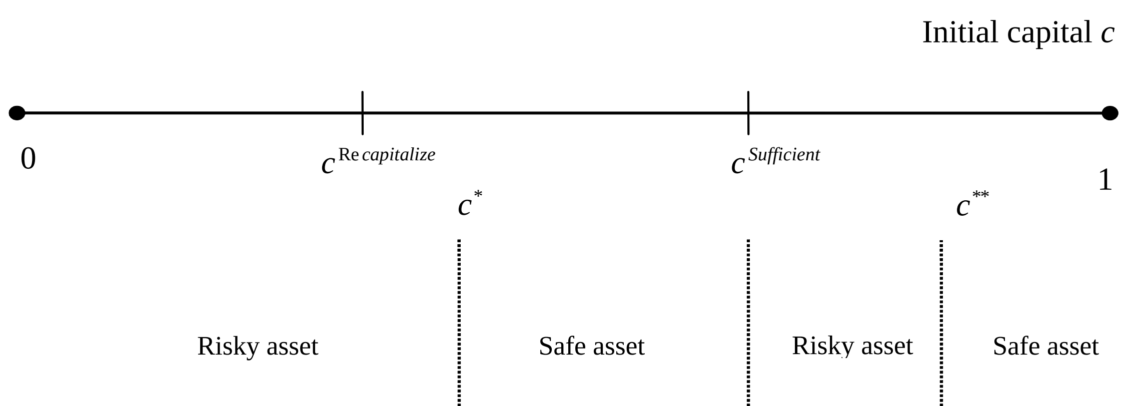


Figure 4.

Bank's project choice when the risky project has a heavier left tail

A heavier left tail is characterized by a higher probability for the extremely low outcome (i.e., a higher μ). A change in the return profile of the risky asset following a change in probability distribution (i.e., both p and μ are increased, other else equal, such that the expected value of the risky project remains the same), affects both thresholds c^{*} and c^{**}. The interval ($c^{Sufficient}$, c^{**}) widens, suggesting that well-capitalized banks which behave prudently in absence of tail risk projects, have a strong incentive to undertake more risk, if projects with heavier left tail are available in economy.

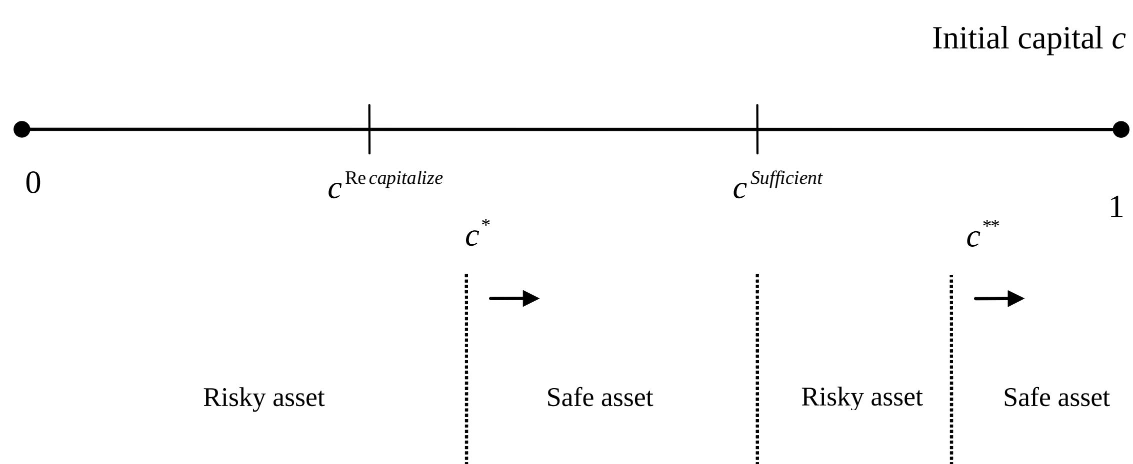


Figure 5.

Bank’s project choice when the charter value is positive

The figure shows that charter value has a disciplinating effect on bank risk-taking. It has a negative impact on the thresholds driving risk-shifting, and as a result, both intervals $(0, c_V^*)$ and $(c^{Sufficient}, c_V^{**})$ shrink.