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Abstract

Should the central bank prevent "excessive" asset price dynamics or should it wait until the boom spontaneously turns into a crash and intervene only afterwards? The debate over this issue goes back at least to the exchange between Bernanke-Gertler (BG) and Cecchetti but has not settled yet. In their 1999 paper BG claimed that price stability and financial stability are 'highly complementary and mutually consistent objectives' in a flexible inflation targeting regime which 'dictates that central banks ... should not respond to changes in asset prices, except insofar as they signal changes in expected inflation.' (BG, 1999, p.18). This conclusion is straightforward within the variant of the NK-DSGE framework used by BG in which asset inflation shows up as a factor 'augmenting' the IS curve. In the present paper, we pursue a different modelling strategy so that, in the end, asset price dynamics will be incorporated into the NK Phillips curve. In our context it is not true anymore that by focusing on inflation the central bank is also checking an asset price boom. We put ourselves, therefore, in the best position to obtain a significant stabilizing role for asset price targeting. It turns out, however, that inflation volatility is higher in the asset price targeting case. After all, therefore, targeting asset prices may not be a good idea.

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1 Introduction

Should the central bank prevent “excessive” asset price dynamics, raising interest rates to halt an asset price boom or should it wait until the boom spontaneously turns into a crash and inject liquidity afterwards to attenuate the fallout on the real economy?

The debate over this crucial issue is at least a decade old – if we date it from the Bernanke-Gertler (1999,2001) (BG hereafter) vs Cecchetti et al. (2000) exchange – but it has not been settled yet. BG got a point at the time, with the authoritative (at the time) endorsement of Alan Greenspan. Their conclusion according to which central banks should not attempt to stabilize asset prices has been the consensus view in the first half of the decade. Following the 2007-08 financial crisis, however, the conventional wisdom has changed dramatically. Nowadays – expecially on the media and in policy circle – it seems to be that asset prices should indeed be stabilized by the central bank to avoid vicious booms and busts with remarkably negative real effects on the macroeconomy.

In this paper we face the same issue with a different theoretical framework. In a sense we put ourselves in the best (theoretical) position to answer a resounding "yes" to the research question posed above. It turns out, however – somewhat to our surprise – that also in this new framework targeting asset prices may be destabilizing. Asset prices booms and busts should be mitigated but a modified Taylor rule, augmented by asset price dynamics, may not be the best policy response.

In their 1999 paper BG made essentially the following point: "Central banks should view price stability and financial stability as highly complementary and mutually consistent objectives...the best policy framework for attaining both objectives is a regime of flexible inflation targeting...The inflation-targeting approach dictates that central banks should adjust monetary policy actively and pre-emptively to offset incipient inflationary or deflationary pressures." (BG, 1999, p.18).

The main rationale for this claim is that "by focusing on the inflationary or deflationary pressures generated by asset price movements, a central bank effectively responds to the toxic side effects of asset booms and busts...Inflation targeting ...implies that interest rates will tend to rise during (inflationary) asset price booms and fall during (deflationary) asset price busts." (ibidem).

This conclusion is straightforward within the BG variant of the New Keynesian (NK) DSGE framework. In their model, asset price inflation shows up as a factor "augmenting" the IS curve: An asset price shock, in fact, yields a *net worth or balance sheet effect* on investment (the reference model is Bernanke, Gertler and Gilchrist, 1999). Essentially the same approach has been adopted by Carlstrom and Fuerst (2007), Iacoviello (2005) and Monacelli (2008).[1] A different approach is followed by Airaudo et al. (2007) who stress the role of the *wealth effect* on consumption. Also in this case, however, asset prices affect aggregate demand and lead to an *"Augmented" (optimizing) IS curve.*

In the BG framework, therefore, a Stock market boom shows up as a demand shock so that asset prices and inflation move in the same direction. As a consequence, following the BG modelling strategy, one is led naturally to conclude that if the central bank follows an inflation targeting approach there is no need to specifically target asset prices above and beyond inflation. By stabilizing the latter, it will stabilize also the former.

In this paper we follow a different route. In our model in fact, asset price dynamics will be eventually incorporated into the NK Phillips curve. In the simplified economy we consider, in fact, firms have to anticipate wages to workers before they can cash in sales proceeds. Assuming that firms do not accumulate internal finance, they need funds at the moment wages have to be paid. In other words, we explore the same environment as in Ravenna and Walsh (2006) model of the *cost channel.* For simplicity, however, we assume that, in order to raise external finance, firms issue new equities ("equity only" financing) instead of asking for bank loans.

In our model the return on shares, which is determined by asset price dynamics, is in turn a determinant of the firms' marginal cost so that in the end we obtain an *"Augmented" NK Phillips curve.*

While in Ravenna-Walsh monetary policy impacts on inflation *directly* because the interest rate (on loans) is a determinant of the firm's cost, in our setting the cost channel is activated *indirectly* whenever monetary policy affects – through changes in the interest rate – asset price inflation.

As a consequence of this modelling strategy, in our framework a Stock market boom shows up as a positive supply shock – in fact, in a rational expectations equilibrium it yields a reduction of the return on shares – leading

[1] Iacoviello and Monacelli build rich models in which also the price of real assets (on the housing market) plays a role. In a sense they blend the BG approach to the Kiyotaki and Moore (1997) emphasis on endogenous borrowing constraints.

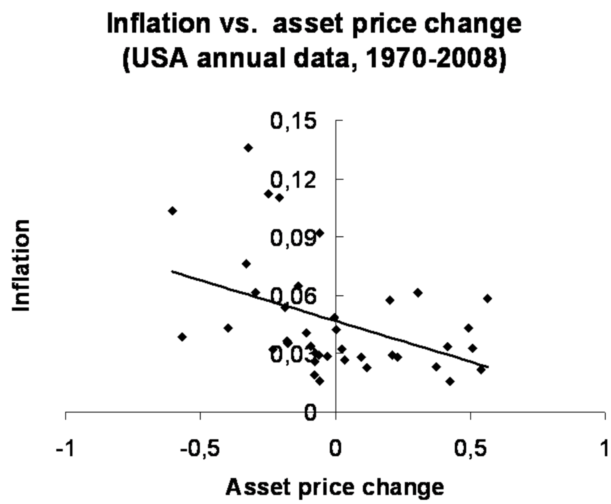

Figure 1: Inflation and asset price change (USA)

to lower inflation: asset prices and inflation move in opposite directions. [2] It is not true any more, in this context, that by focusing on inflation the central bank is also checking an asset price boom. On the contrary, if the central bank adopts an inflation targeting approach, in the attempt to stabilize inflation it will boost Stock prices even further.

The empirical evidence on the correlation between inflation and asset price changes is mixed and certainly not overwhelmingly in favour of the BG approach. In figure 1 we report the scatter diagram of inflation[3] and the change in real asset prices[4] in the USA from 1970 to 2008.

Linear interpolation returns a negatively sloped regression line. The correlation is -0.4, in line with the assumption proposed in the present paper.

[2] Also De Grauwe (2008) implicitly assumes a negative correlation between asset price dynamics and inflation (along the NK Phillips curve). In his framework, the marginal cost is decreasing with the firms' net worth (because of the external finance premium). An increase of the asset price pushes up net worth and brings down the external finance premium, marginal cost and inflation.

[3] Inflation is here defined as the rate of change of the Consumer Price Index.

[4] Asset price change is defined as the percent deviation of the real asset price (nominal asset price deflated by the Consumer Price Index) from a linear trend.

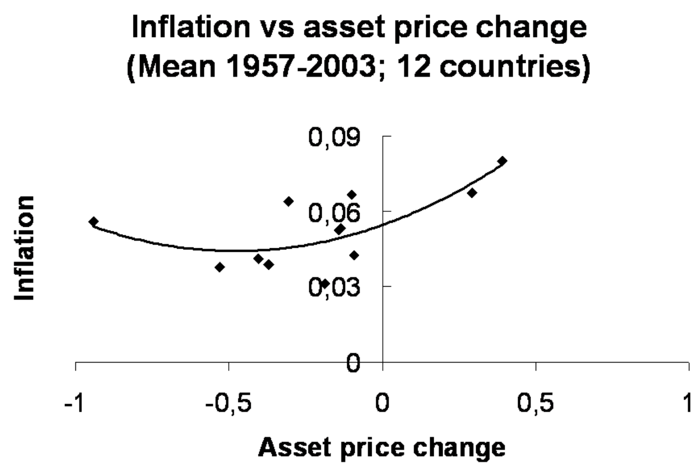

Figure 2: Inflation and asset price change (12 countries)

In figure 2 we focus on cross sectional evidence summarized by the scatter diagram of the mean inflation rate and the mean asset price change[5] over the period 1957Q1-2003Q1 for 12 countries [6] as reported in Chih-Chuan Yeh and Ching-Fang Chi (2009).

Linear interpolation returns a positively sloped regression line (not shown in the figure). The linear correlation index is greater than 0,5 in line with BG. Notice, however, that a quadratic interpolation fits the data much better [7]. The relationship between inflation and asset price change over the long run, therefore, seems to be non-monotonic.

The toy economy we consider is of course a far cry from reality. For reasons of tractability and as a very preliminary step towards a more satisfactory – and necessarily more complicated – setting, in fact, we abstract from a wide range of crucial imperfections of financial markets. The implications of the

[5] The asset price change is defined as the natural logarithm of the nominal stock index divided by the consumer price index.

[6] Australia, Canada, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Spain, United States.

[7] R^2 is 0,26 in case of a linear interpolation while it is 0,55 in case of a quadratic interpolation.

model, however, are surprisingly far reaching.

We analyse the design and the transmission mechanism of monetary policy in two regimes:(a) an instrument rule with no-reaction to asset prices (Strict Inflation Targeting, SIT),(b) an instrument rule with reaction to asset prices (Asset augmented Inflation Targeting, AIT).

In the case of a supply shock, the central bank reacts to inflation by raising the interest rate, asset prices fall, the output gap turns negative and dividends fall, the return on shares increases (even if dividends fall) to match the increase in the real interest rate. The central bank therefore faces a trade off: An aggressive policy stance aiming at stabilizing inflation would make the asset price bust even worse. If it takes into account asset price changes – i.e. in the AIT case – the central bank will ease a bit and therefore in the end it will mitigate (with respect to the SIT case) the impact on output of its contractionary policy. On the other hand, however, it will exacerbate the impact of the shock on inflation. The AIT regime, therefore, is characterized by milder variations in output but larger changes in inflation.

Consider now a demand shock, which has a positive effect on inflation and the output gap (and dividends). As in the previous case, the central bank reacts to inflation by raising the interest rate. The asset price tends to increase because of the increase in dividends but the increase of the real interest rate prompts a flight from equities which depresses asset prices. In our model the second effect prevails over the first one so that in the end asset prices fall. If the central bank takes into account asset price changes – i.e. in the AIT case – the fall in asset prices will induce a monetary easing so that the central bank will amplify the impact on output of the demand shock. On the other hand, it will exacerbate the impact of the shock on inflation. In the AIT regime, therefore, output grows more than in the SIT case but inflation will be higher.

In the AIT case, therefore, inflation volatility is always higher while output volatility is higher (lower) in case of a demand (supply) shock. After all, therefore, targeting asset prices may not be a good idea. At first sight, this is surprising because we put ourselves in the best position to obtain a significant stabilizing role for asset price targeting. Why is it so? In the end, as we will show in section 4, in the AIT case the central bank is "too accommodating". Targeting asset prices makes the central bank particularly "wet". This may be welfare-reducing.

The paper is organized as follows. Sections 2 and 3 describe households' and firms' decision rules. In section 3.1 we derive the Augmented NK Phillips curve. In section 4 we evaluate the impact of a Taylor type instrument rule for monetary policy, with and without asset prices (i.e. in the AIT and SIT cases). In this section the comparison between the two regimes in case a demand or a supply shock occur is spelled out in detail. Section 5 is devoted to some welfare considerations. Finally, section 6 concludes.

2 Households

The economy is populated by households and firms. The former decide on consumption, asset holdings (money, bonds, shares) and labour supply.

There is a continuum of unit mass of infinitely lived identical households which discount the future at the factor β. Period utility is represented by a standard CRRA function:

$$U(C_t, m_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} m_t^{1-\zeta} - \chi \frac{N_t^{1+\eta}}{1+\eta}$$

where σ, γ, ζ, χ, η are positive parameters with the usual interpretation, C_t is a CES aggregator of consumption goods,[8] $m_t := M_t/P_t$ are real money balances [9] and N_t represents hours worked. Real money balances show up in the utility function because they provide liquidity services.

The households' portfolio consists of money, bonds and shares. The nominal value in t of money balances (resp. Government bonds) carried over from the past is denoted by M_{t-1} (B_{t-1}). Moreover the household owns A_{t-1} shares, whose price is Q_t. In period t the household receives a flow of interest payments on Government bonds $i_{t-1}B_{t-1}$ where i_{t-1} is the nominal interest rate in t-1. Moreover we assume that firms pay in t (nominal) dividends equal to D_t per share held in t-1.

The household employs "resources" consisting of wage income, interest payments, and dividends to consume and increase money, bond and share-

[8] C_t consists of differentiated consumption goods produced by monopolistically competitive firms and is defined as follows:

$$C_t = \left[\int_0^1 c_{jt}^{\frac{e-1}{e}} dj\right]^{\frac{e}{e-1}}$$

where $e > 1$ turns out to be the price elasticity of demand of each good.

[9] The price level is a CES aggregator of the individual prices: $P_t = \left[\int_0^1 p_{jt}^{1-e} dj\right]^{\frac{1}{e-1}}$

holdings according to the following budget constraint in real terms:

$$C_t+m_t+b_t+q_tA_t = w_tN_t+\frac{1}{1+\pi_t}\left[m_{t-1}+(1+i_{t-1})\,b_{t-1}\right]+(q_t+d_t)\,A_{t-1} \quad (1)$$

where $b_t := B_t/P_t$ are real bond holdings; $q_t := Q_t/P_t$ is the real price of each share (asset price or Stock price for short in the following); $w_t := W_t/P_t$ is the real wage; $\pi_t := \frac{P_t}{P_{t-1}} - 1$ is the inflation rate and d_t are dividends per share.

Liquidity injections (withdrawals) are implemented (by the central bank) by means of open market purchases (sales) of bonds: $M_t - M_{t-1} = -\left[B_t - (1+i_{t-1})\,B_{t-1}\right]$.

Taking into account this procedure, the budget constraint of the representative household boils down to: $P_tC_t + Q_t\,(A_t - A_{t-1}) = W_tN_t + D_tA_{t-1}$. In the present context, the wage bill W_tN_t is financed by means of equity issues Q_tA_t (see next section). Hence $Q_tA_t = W_tN_t$. Using this equality, it turns out that

$$P_tC_t = (Q_t + D_t)\,A_{t-1} \quad (2)$$

In period t, the representative household maximizes:

$$E_t\sum_{s=0}^{\infty}\beta^s\left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma}+\frac{\gamma}{1-\zeta}\,(m_{t+s})^{1-\zeta}-\chi\frac{N_{t+s}^{1+\eta}}{1+\eta}\right] \quad (3)$$

subject to a sequence of budget constraints of the form (1). From the first order conditions (see appendix A for details) one can derive the usual optimal relations, i.e. the Euler equations for consumption, money and labour supply:

$$C_t^{-\sigma} = \beta\,(1+i_t)\,E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma} \quad (4)$$

$$\frac{i_t}{1+i_t} = \gamma\frac{C_t^{\sigma}}{m_t^{\zeta}} \quad (5)$$

$$\chi C_t^{\sigma} N_t^{\eta} = w_t \quad (6)$$

Moreover we get one additional optimal relation that we interpret as a

No-Arbitrage Condition

$$\frac{1+i_t}{1+E_t\pi_{t+1}} = \frac{E_t\left(q_{t+1}+d_{t+1}\right)}{q_t} \tag{7}$$

Equation (7) establishes the equality between the return on bonds, i.e. the real interest rate, and the return on equities, i.e. the sum of the dividend yield and the capital gain (in real terms). By simple algebra, this condition can be turned into an *asset price equation*:[10]

$$q_t = \frac{E_t\left(q_{t+1}+d_{t+1}\right)}{1+i_t}\left(1+E_t\pi_{t+1}\right) \tag{8}$$

From the consumption Euler equation (4) through linearization around the steady state and taking into account the equilibrium condition $C_t = Y_t$ we get

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1}\right) \tag{9}$$

where x_t denotes the output gap, i.e. the difference between current output and flexible price equilibrium output (derived in appendix B), while i_t denotes the deviation of the nominal interest rate from the steady state.[11]

From the asset price equation (8) through linearization we get the Asset Price (AP) schedule:

$$\hat{q}_t = -(i_t - E_t\pi_{t+1}) + \left[\beta E_t\hat{q}_{t+1} + (1-\beta)\,E_t\hat{d}_{t+1}\right] \tag{10}$$

where hatted variables represent percent deviations from the steady state. In our framework, technology is linear (see next section): $Y_t = N_t$. Moreover, in equilibrium $C_t = Y_t$. Using these equalities to rewrite the optimality condi-

[10] Consolidating the No-arbitrage condition and the Consumption Euler equation we get:

$$C_t^{-\sigma} q_t = \beta E_t C_{t+1}^{-\sigma}\left(q_{t+1}+d_{t+1}\right)$$

This optimality condition states the equality between the marginal utility the agent gives up by saving in order to purchase one share and the present value of the marginal utility the agent will gain one period ahead by transforming the dividend and the capital gain into consumption.

[11] In a zero-inflation steady state, the steady state nominal interest rate is equal to the real interest rate, which in turn is anchored to the rate of time preference (see again appendix B).

tion (6) and rearranging we get $w_t = \chi Y_t^{\eta+\sigma}$. Log-linearizing this expression around the steady state we get:

$$\hat{w}_t = (\eta + \sigma)\, x_t \tag{11}$$

We assume that firms' real profits are paid out to households in the form of dividends: $d_t = Y_t - w_t Y_t$. Substituting the real wage w_t as defined above into this expression and log-linearizing around the s.s. we get:

$$\hat{d}_t = (1 + \delta)\, x_t \tag{12}$$

where $\delta := \dfrac{\beta}{\mu - \beta}(\eta + \sigma)$ where μ is the mark up. [12] Hence dividends are an increasing linear function of the output gap.

3 Firms

As in the standard New Keynesian model the corporate sector consists of J firms, indexed by j, which produce differentiated goods in a monopolistically competitive setting à la Dixit and Stiglitz using only labour. Therefore firms incur only the production cost represented by the wage bill.

We depart from the standard setting in assuming the following

a) *Financing gap:* Technology is represented by a one-to-one production function $Y_{jt} = N_{jt}$. Since firms hire workers at the beginning of period t and sell output at the end of the period, they cannot pay wages out of sales proceeds: at the beginning of each period they have to anticipate the wage bill to employees.This is the financing gap.

b) *No internal funds*: firms do not accumulate internal finance so that the financing gap coincides with the wage bill. They have to raise external finance to fill the financing gap.

In order to concentrate on the role of asset prices in macroeconomic performance, we adopt the following simplifying shortcut:

[12] $\mu = \frac{e}{e-1}$; $e > 1$.Of course $\mu > \beta$. The ratio $\dfrac{\beta}{\mu - \beta}$ is the steady state ratio of wages to dividends.

c) *"Equity only" financing:* there is only one source of external funds, the Stock market.

Assumptions b) and c) allow us to get rid, in the following, of the complications due to the accumulation of net worth and to ignore the credit market. This is patently unrealistic. We consider the present framework as only a first step towards a more satisfactory and realistic model.

From the *"equity only" financing* assumption follows that the j-th firm raises funds issuing new shares and the amount of shares sold is equal to the wage bill: [13]

$$w_t N_{jt} = q_t A_{jt} \tag{13}$$

d) *Dividend and buy-back policy:* Shareholders are remunerated by means of dividends (distributed in $t+1$ on shares held in t), which represent the cost of external funds for the firms. Furthermore firms buy back all the shares outstanding in $t+1$.

The time schedule can be summarized as follows. At the beginning of period t, the firm issues equities and uses the proceeds to hire workers and start production. Since production takes an entire period, output will be available for sale in $t+1$. Sale proceeds are used in $t+1$ to pay dividends and buy back shares issued in t. In fact, as shown above – see (2) – $P_{t+1}C_{t+1} = (Q_{t+1} + D_{t+1}) A_t$. At the beginning of period $t+1$, the cycle starts again.

In the end, therefore, we are assuming that in the same period $(t+1)$ the firm is (i) paying dividends and reimbursing shareholders for the shares they bought in t and (ii) it is issuing new equities to finance production in $t+1$. This is clearly unrealistic but simplifies the analysis to a great extent.

From the standard microfoundations of the NK Phillips curve (see subsection 3.1, below for technical details), after linearization we get $\pi_t = k\hat{\phi}_t + \beta E_t \pi_{t+1}$ where $k = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.Substituting (17) and rearranging we

[13] In principle, each firm issues its own shares so that there should be an entire range of heterogeneous asset prices, one for each firm. In order to simplify the argument, we will impose symmetry among firms so that *the asset price is uniform across equity-issuing firms.* Alternatively, one can think of q as the average Stock market index and assume that each individual share prices q_j is not too far from the average. In the end, however, firms will behave uniformly – they are essentially identical – so that the individual share price will coincide with the average.

get

$$\pi_t = \lambda x_t + k\left[\beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1} - \hat{q}_t\right] + \beta E_t \pi_{t+1} \tag{14}$$

with $\lambda := k(\eta + \sigma)$. Equation (14) is the NK Phillips curve in the new setting.

The difference with respect to the canonical NK-PC is the term in brackets, i.e. $\widehat{ROS}$ (see equation (16)). In fact, the cost channel and the equity-only financing assumptions imply that the cost of external finance, which coincides with the ROS, is affecting the firms' pricing decisions and therefore inflation. This is the reason why we will define the equation above the *Augmented* New Keynesian-Phillips Curve (A-NKPC).

3.1 The "augmented" NK Phillips curve

The firm's total disbursement occur in $t+1$ but are related to operating costs incurred in t. The firm's total cost in real terms, therefore, is $TC_{jt} = E_t(q_{t+1} + d_{t+1}) A_{jt}$. [14]Substituting (13) into this expression we obtain: $TC_j = \dfrac{E_t(q_{t+1} + d_{t+1})}{q_t} w_t N_{jt}$. Hence the real marginal cost is:

$$\phi_t = \frac{E_t(q_{t+1} + d_{t+1})}{q_t} w_t \tag{15}$$

The expression

$$\frac{E_t(q_{t+1} + d_{t+1})}{q_t} = ROS$$

is the novelty of this approach. With respect to the standard setting, whereby $\phi_t = w_t$, the marginal cost must be augmented by a term which represents the *cost of external finance* for the firm. This, in turn, coincides with the *Return On Shares (ROS)*, i.e. the sum of the dividend yield $\dfrac{E_t d_{t+1}}{q_t}$ and the capital gain $\dfrac{E_t q_{t+1}}{q_t}$.

[14]Since disbursement will occur one period ahead, in t the firm has to form expectations on the total gross return in t+1 of each share issued in t. This gross return in real terms is the sum of the asset price and dividends in t+1.

From the linearization of (15) around the s.s. we get

$$\hat{\phi}_t = \hat{w}_t + \left[\beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1} - \hat{q}_t\right]$$

where the expression

$$\beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1} - \hat{q}_t = \widehat{ROS} \tag{16}$$

is the deviation of the ROS from the steady state.[15] In a *symmetric equilibrium with flexible prices* all the firms charge the same price P_t equal to a markup μ over nominal marginal cost $P_t\phi_t$. Therefore $\phi_t = \frac{1}{\mu}$. Recalling (15) we get $w_t = \frac{q_t}{\mu E_t (q_{t+1} + d_{t+1})}$. Plugging (11) into the expression for $\hat{\phi}_t$ above and rearranging we get:

$$\hat{\phi}_t = (\eta + \sigma) \left\{ x_t + \frac{1}{\eta + \sigma} \left[\beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1} - \hat{q}_t\right] \right\} \tag{17}$$

In each period a fraction ω of firms is unable to adjust its price. As usual in a Calvo pricing context ω is a measure of the degree of *nominal rigidity.* The j-th firm's pricing decision problem therefore is

$$\max_{p_{jt}} E_t \sum_{s=0}^{\infty} \omega^s \Delta_{s,t+s} \left[\left(\frac{p_{jt}}{P_{t+s}}\right)^{1-e} - \phi_{t+s} \left(\frac{p_{jt}}{P_{t+s}}\right)^{-e} \right] C_{t+s}$$

where $\Delta_{s,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\sigma}$ is the consumption based discount factor, $\left(\frac{p_{jt}}{P_{t+s}}\right)^{-e} C_{t+s} = Y_{jt}$ is demand for the j-th firm's product and ϕ_t is the marginal (and average) cost.

The optimal relative price of the good produced by the adjusting firm in period t, therefore, takes into account the stream of future marginal costs, which, in our framework, depends on *current and future asset prices and*

[15] From the No-arbitrage condition it is immediate to infer that the steady state ROS $ROS_s = \frac{q_s + d_s}{q_s}$ must be equal to the steady state real return on bonds β^{-1}. Hence $\beta = \frac{q_s}{q_s + d_s}$.

dividends (see (15)).

4 Monetary policy rules

We will explore the transmission mechanism of monetary policy in the case in which the central bank adopts a simple Taylor-type *instrument rule*. In sub-section 4.1 we will assume that the central bank responds only to inflation (Strict Inflation Targeting, SIT). In section 4.2 we will augment the instrument rule taking into account asset price dynamics (Asset augmented Inflation targeting, AIT).

4.1 Strict inflation targeting (model I-1)

For the sake of simplicity and without loss of generality, in this sub-section we assume that the instrument rule is activated exclusively by the feedback from inflation (in other words, the central bank does not take into account the output gap in devising its policy). Hence, the rule specifies to:

$$i_t = \gamma_\pi \pi_t \tag{18}$$

is of the *strict inflation targeting* (SIT) type.

The structural form of the macroeconomic model consists of the IS curve (9), No-Arbitrage Condition (10), dividend policy (12), Augmented NK Phillips curve (14) and Taylor rule (18) which we reproduce here for the reader's convenience.

$$\begin{gathered}
x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + g_t \\
\hat{q}_t = -\left(i_t - E_t \pi_{t+1}\right) + \beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1} \\
\hat{d}_t = (1+\delta) x_t \\
\pi_t = \lambda x_t + k\left[\beta E_t \hat{q}_{t+1} + (1-\beta) E_t \hat{d}_{t+1}\right] + \beta E_t \pi_{t+1} + u_t \\
i_t = \gamma_\pi \pi_t
\end{gathered} \tag{M I-1}$$

Notice that we have appended a demand shock g_t to the IS curve and a supply shock u_t to the Phillips curve to avoid the "divine coincidence". As usual g_t and u_t follow an AR(1) process:$g_t = \psi g_{t-1} + \widetilde{g}_t$ with $\widetilde{g}_t$ ~iid$(0, \sigma_g^2)$; $u_t = \rho u_{t-1} + \widetilde{u}_t$ with $\widetilde{u}_t$ ~iid$(0, \sigma_u^2)$.

(M I-1) is a system of five linear difference equations in five state variables, $x_t, \hat{q}_t, \pi_t, i_t, \hat{d}_t$.

The model is *recursive*. Using the no-arbitrage condition, in fact, we obtain:

$$\begin{aligned} \pi_t &= \lambda x_t + k\left(i_t - E_t\pi_{t+1}\right) + \beta E_t\pi_{t+1} + u_t \\ x_t &= E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1}\right) + g_t \\ i_t &= \gamma_\pi \pi_t \end{aligned} \tag{M I-0}$$

These equations form "the core model" with SIT or model I-0, a system of three equations in x_t, π_t, i_t.

We can solve for these variables without any reference to $\widehat{ROS}$ and therefore to asset prices and dividends. In fact, we have replaced $\widehat{ROS}$ with the real interest rate $i_t - E_t\pi_{t+1}$, exploiting the no-arbitrage condition. In other words

Remark 1 *If the economy is described by model I-1 the determination of the asset price and dividends can be separated from the determination of all the other state variables. The equilibrium values of x_t, π_t, i_t can be logically determined by solving model I-0 before determining asset prices and dividends.*[16]

The Rational Expectations (RE) equilibrium of model I-0 and the conditions for determinacy are computed in appendix C.

In order to solve the system by the method of undetermined coefficient, we guess $s_1 = s_1 u_t + s_2 g_t$ for each variable $s = \pi, x, i$. Therefore $E_t s_{t+1} = s_1 \rho u_t + s_2 \psi g_t$. For the sake of simplicity, we will adopt the following

Assumption 1: $\rho = \psi$.

This assumption is of course restrictive and may entail a modest loss of generality. It greatly simplifies the calculations, however, and yields very neat results since $E_t s_{t+1} = \rho\left(s_0 u_t + s_1 g_t\right) = \rho s_t$ for each and every variable.

Because of assumption 1, the model-consistent (i.e. rational) expectation of a variable taken in t for $t+1$ is a fraction of the current value of the variable. [17]

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[16] A similar dichotomy occurs also in Carlstrom and Fuerst (2007) albeit in a different context.

[17] The expected rate of change therefore is decreasing with the current value: $E_t s_{t+1} -$

The RE solutions of the system can be represented as follows:

$$\begin{aligned} x_t &= a_1 u_t + a_2 g_t \\ \pi_t &= b_1 u_t + b_2 g_t \\ i_t &= \gamma_\pi b_1 u_t + \gamma_\pi b_1 g_t \end{aligned}$$

where a_i and $b_i; i = 1, 2$ are polynomials of the "deep parameters" $\beta, \eta, \gamma_\pi, k, \rho, \sigma$. It turns out that $a_1 < 0, a_2 > 0, b_1 > 0, b_2 > 0$ (see appendix M I-1).

In the following we illustrate the transmission of shocks within model I-0 by means of simple diagrams.

From assumption 1 follows that $E_t x_{t+1} = \rho x_t, E_t \pi_{t+1} = \rho \pi_t$. Hence we can define the real interest rate as

$$i_t - E_t \pi_{t+1} = (\gamma_\pi - \rho)\, \pi_t \tag{19}$$

Using (19), M I-0 boils down to:

$$x_t = -\frac{\gamma_\pi - \rho}{\sigma (1 - \rho)} \pi_t + \frac{1}{1 - \rho} g_t \tag{20}$$

$$\pi_t = \frac{\lambda}{1 - \beta\rho - k(\gamma_\pi - \rho)} x_t + \frac{1}{1 - \beta\rho - k(\gamma_\pi - \rho)} u_t \tag{21}$$

Equation (20) can be conceived of as a policy induced AD schedule in the present setting. Equation (21) represents therefore the AS schedule. Monetary policy affects the AS schedule through the cost channel.

Assumption 2. We assume $\sigma < \eta$ and

$$\begin{aligned} 1 &< \gamma_\pi < \hat{\gamma}_\pi \\ \hat{\gamma}_\pi &: = \rho + \frac{1 - \beta\rho}{k} \end{aligned} \tag{22}$$

The inequality on the LHS of (22) – i.e. $\gamma_\pi > 1$ – is the *Taylor principle.* Thanks to this inequality the RE solution of model I-0 is determinate if $\sigma < \eta$ (see appendix C for a discussion of determinacy) and the AD schedule is "well

$s_t = -(1 - \rho)\, s_t$. This implicitly determines a *mean reverting* behaviour of that variable. If a shock hits a variable, causing a departure from the steady state, a negative (stabilizing) feedback is activated.

behaved", i.e. downward sloping on the (x_t, π_t) plane. [18]

The inequality on the RHS of (22) – i.e. $\gamma_\pi < \hat{\gamma}_\pi$ – assures, on the other hand, that the AS schedule is "well behaved", i.e. upward sloping. When the AD and the AS curves are both well behaved (i.e. they have the "appropriate slopes"), the solutions of (M I-0) are "realistic" in the precise sense that equilibrium inflation and the output gap respond to shocks in the usual way. In other words, the reaction of the central bank to current inflation must be neither too weak ($1 < \gamma_\pi$) **nor too strong** ($\gamma_\pi < \hat{\gamma}_\pi$) to assure well behaved (i.e. realistic) model solutions.

The RHS of (22) is the truly novel feature of this setting. In the absence of the cost channel, in fact, model M I-0 would boil down to the following "canonical" model which we will label M I-0(c):

$$\begin{aligned} \pi_t &= \lambda x_t + \beta E_t \pi_{t+1} + u_t \\ x_t &= E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + g_t \\ i_t &= \gamma_\pi \pi_t \end{aligned} \tag{M I-0(c)}$$

which, after plugging the monetary policy rule into the IS curve and incorporating model-consistent expectations, becomes:

$$x_t = -\frac{\gamma_\pi - \rho}{\sigma(1-\rho)}\pi_t + \frac{1}{1-\rho} g_t \tag{23}$$

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t \tag{24}$$

Hence only $1 < \gamma_\pi$ must be assumed to assure both determinacy and a downward sloping AD schedule. The AS curve in model M I-0(c), in fact, is upward sloping **for any** value of γ_π. Notice, moreover, that the AS schedule in the canonical case is flatter than in the presence of the cost channel.

4.1.1 The effect of a supply shock

We are now ready to examine the transmission of shocks.

Suppose initially there are no shocks: $g_t = u_t = 0$. In figure 3 we represent the AD and the AS schedules in the present setting (in bold) and in the

[18] Notice that $1 < \gamma_\pi$ is a *necessary* condition for determinacy (if $\sigma < \eta$) and a *sufficient* condition for a well behaved AD schedule. In fact, the slope of the AD curve is negative for $\rho < \gamma_\pi$.

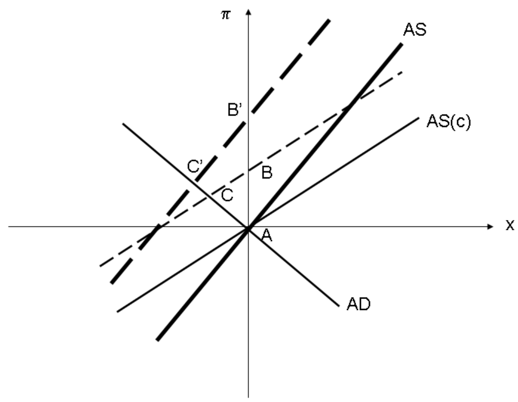

Figure 3: Effects of a supply shock in model I-1

canonical one.

Suppose a (temporary) *supply shock* hits the economy. In a canonical setting, inflation goes up by $\frac{1}{1-\beta\rho}u_t$ on impact (see point B).

In the presence of the cost channel, the reaction of the central bank to the increase in inflation – i.e. the increase of the interest rate – adds to inflation on impact. This is the reason why inflation goes up by $\frac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}u_t$on impact (see point B'). In other words, the AS curve augmented with the cost channel shifts up more than in the canonical case.[19]

In equilibrium the economy will converge to C'. In the end, therefore, there will be more inflation and a more acute recession than in the canonical case (compare with C). In the case of a supply shock, the cost channel works therefore as an *amplification mechanism* of the shock.[20] Of course, since the

[19] It is easy to see, however, that the intercepts on the x-axis of the AS and AS(c) schedules after the shock coincide.

[20] In fact, in the RE solution the coefficients of inflation and the output gap with respect to the supply shock are greater in absolute value in the presence of the cost channel. In symbols: $b_1 > b_1^c$, $|a_1| > |a_1^c|$ as shown in appendix C where the superscript c refers to the canonical NK-DSGE model.

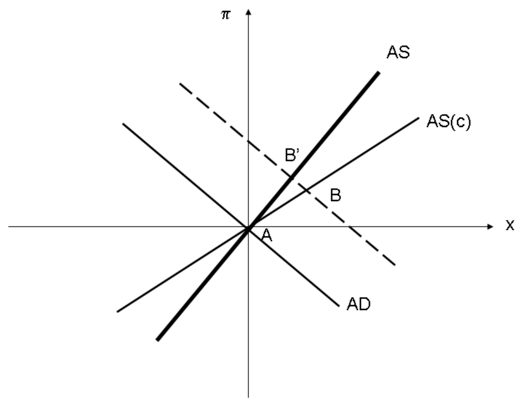

Figure 4: Effects of a demand shock in model I-1.

shock is temporary, with the passing of time the economy will move back to point A.

4.1.2 The effect of a demand shock

In the case of a *demand shock*, the new (short run) equilibrium will be B' as shown in figure 4. The output gap turns positive but, in the presence of the cost channel, the expansion is weaker and inflation is higher than in the canonical case (compare with B).

What happens to the stock price? Since the system is recursive we can solve for the asset price after having solved for the output gap, inflation and the interest rate. In order to do so, we have to start from dividends. Iterating (12) one period ahead and taking the expected value we get

$$E_t\hat{d}_{t+1} = (1+\delta)\,E_t x_{t+1} \tag{25}$$

When expected dividends are defined as in (25), the no-arbitrage condition (10) becomes:[21]

$$\hat{q}_t = -(i_t - E_t\pi_{t+1}) + \beta E_t\hat{q}_{t+1} + (1-\beta)(1+\delta) E_t x_{t+1} \tag{26}$$

Due to assumption 1, $E_t\hat{q}_{t+1} = \rho\hat{q}_t$ and $E_t x_{t+1} = \rho x_t$. Hence using (19) the expression above boils down to:

$$\hat{q}_t = -\frac{\gamma_\pi - \rho}{1-\beta\rho}\pi_t + \frac{1-\beta}{1-\beta\rho}(1+\delta)\rho x_t \tag{27}$$

Hence the asset price (i) falls when there is a burst of inflation and (ii) goes up in a boom, i.e. when the output gap goes up. The reason for (i) is simple: When the economy is hit by an inflationary shock, the central bank raises the interest rate prompting a flight from equities; asset prices fall bringing about an increase of the return on shares such as to match the increase of the interest rate. This process re-establishes the no-arbitrage condition. The reason for (ii) is even more straightforward: An increase of the output gap yields an increase in profits and dividends, which translates into a higher asset price.

As a consequence,the RE solution for $\hat{q}$ is a linear function of the shocks: $\hat{q}_t = c_1 u_t + c_2 g_t$ (see appendix C) where $c_1 < 0$ while c_2 has uncertain sign. The reason why $\hat{q}_t$ is a decreasing with u is obvious: A supply shock, in fact, yields an increase of inflation and a decrease of the output gap. As to g, things are more complicated. A demand shock brings about an increase of inflation – which is detrimental for the Stock market – but also an increase of the output gap, which makes dividends (and asset prices) go up.

The net effect of these two contrasting tendencies will depend on the strength of the response of the central bank to inflation. It turns out (see appendix C for details) that the net effect is negative – i.e. asset prices fall

[21] Taking (25) into account, the $\widehat{ROS}$ becomes

$$\widehat{ROS} = \beta E_t\hat{q}_{t+1} + (1-\beta)(1+\delta) E_t x_{t+1} - \hat{q}_t$$

Taking model consistent expectations into account we get:

$$\widehat{ROS} = (1-\beta)(1+\delta)\rho x_t - (1-\beta\rho)\hat{q}_t$$

Therefore $\widehat{ROS}$ is not only decreasing with $\hat{q}_t$ (because of the capital gain) but also increasing with x_t (because of the distribution of dividends).

in the presence of a demand shock – if

$$\begin{aligned} \gamma_\pi &> \check{\gamma}_\pi \\ \check{\gamma}_\pi &:= \rho + \frac{1-\beta\rho}{k + \dfrac{\lambda}{(1-\beta)(1+\delta)\rho}} \end{aligned} \tag{28}$$

i.e. if the response of the central bank is relatively "strong", greater than a threshold $\check{\gamma}_\pi$. Notice that this threshold is smaller than the upper limit $\hat{\gamma}_\pi$ of condition (22).

4.2 Asset augmented inflation targeting (model I-2)

Let's consider now an *augmented* interest rate rule for monetary policy which takes into account not only inflation but also the asset price deviation from the steady state (asset inflation for short):

$$i_t = \gamma_\pi \pi_t + \gamma_q \hat{q}_t \tag{29}$$

with $\gamma_q > 0$. We will characterize this rule as Asset augmented Inflation Targeting (AIT). In this case, the macroeconomic model in structural form consists of the IS curve (9), No-Arbitrage Condition (10), dividend policy (12), Augmented NK Phillips curve (14) and Taylor rule (29). In order to solve the model it is useful to incorporate dividend policy into the asset price equation, replacing (10) with (26). In the end we get:

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + g_t \\ \hat{q}_t &= -(i_t - E_t\pi_{t+1}) + \beta E_t \hat{q}_{t+1} + (1-\beta)(1+\delta) E_t x_{t+1} \\ \pi_t &= \lambda x_t + k(i_t - E_t\pi_{t+1}) + \beta E_t \pi_{t+1} + u_t \\ i_t &= \gamma_\pi \pi_t + \gamma_q \hat{q}_t \end{aligned} \tag{M I-2}$$

This is "model I-2", a system of four linear difference equations in four variables, $x_t, \hat{q}_t, \pi_t, i_t$.

This system is **not recursive.** In other words, when the central bank reacts to the asset price, the system does not dichotomize into 2 independent subsystems (one for x_t, π_t, i_t and the other for $\hat{q}_t$) as in model I-1.

The RE solutions of model I-2 can be represented as follows:

$$\begin{aligned} x_t &= a_1^q u_t + a_2^q g_t \\ \pi_t &= b_1^q u_t + b_2^q g_t \\ \hat{q}_t &= c_1^q u_t + c_2^q g_t \\ i_t &= \left(\gamma_\pi b_1^q + \gamma_q c_1^q\right) u_t + \left(\gamma_\pi b_2^q + \gamma_q c_2^q\right) g_t \end{aligned}$$

where a_i^q, $b_i^q, c_i^q; i = 1, 2$ are polynomials of the "deep parameters" (see appendix D). It turns out that $a_1^q < 0, a_2^q > 0, b_1^q > 0, b_2^q > 0, c_1^q < 0$. The sign of c_2^q is uncertain. [22]

In the following we will illustrate and discuss these results in a simple modified AD-AS framework in order to compare the transmission mechanism of the shocks and contrast it with the previous SIT case.

In the AIT setting, using assumption 1 (so that $Es_{t+1} = \rho s_t$, $s_t = x_t, \pi_t, \hat{q}_t, i_t$) we can write the ex ante real interest rate as follows:

$$i_t - E_t \pi_{t+1} = (\gamma_\pi - \rho)\,\pi_t + \gamma_q \hat{q}_t \tag{30}$$

In order to solve this model, it is convenient to plug (30) into (26). Using model consistent expectations one gets

$$\hat{q}_t = -\frac{\gamma_\pi - \rho}{1 + \gamma_q - \beta\rho}\pi_t + \frac{1-\beta}{1 + \gamma_q - \beta\rho}(1+\delta)\,\rho x_t \tag{31}$$

It is worth noting that plugging (31) into (29) one gets the following rule for monetary policy, which we label indirect instrument rule:

$$\begin{aligned} i_t &= \gamma_\pi' \pi_t + \gamma_x' x_t \\ \gamma_\pi' &= \gamma_\pi - \frac{\gamma_q (\gamma_\pi - \rho)}{1 + \gamma_q - \beta\rho} \\ \gamma_x' &= \frac{\gamma_q (1-\beta)}{1 + \gamma_q - \beta\rho}(1+\delta)\,\rho \end{aligned} \tag{32}$$

The indirect rule (32) shows that adding asset inflation to a strict inflation targeting rule, in the end, is equivalent to targeting both inflation and the

[22] For the configuration of numerical values of the parameters specified below (see section 4.3), the Taylor principle $\gamma_\pi > 1$ is a sufficient condition for determinacy (assuming, of course, that $\gamma_q > 0$).

output gap, i.e. to an instrument rule of the *flexible inflation targeting* type. It is worth noting that $\gamma'_\pi < \gamma_\pi$. Moreover $\gamma'_x > 0$.

Remark 2 *In the AIT case, the response of the central bank to inflation is weaker than in the SIT case. Moreover, the central bank is indirectly targeting the output gap.*

When the prices of goods and services go up, in fact, the price of assets goes down as shown by (31). In the AIT case, the contraction of the asset price will induce a monetary easing, i.e. a reduction of the interest rate with respect to the case in which the central bank is not concerned with asset inflation.

Using assumption 1 and substituting (29), model M I-2 boils down to:

$$\begin{aligned}
\pi_t &= \frac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}x_t+\frac{k\gamma_q}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}\hat{q}_t+\frac{1}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}u_t \\
x_t &= -\frac{\gamma_\pi-\rho}{\sigma\left(1-\rho\right)}\pi_t-\frac{\gamma_q}{\sigma\left(1-\rho\right)}\hat{q}_t+\frac{g_t}{1-\rho} \\
\hat{q}_t &= -\frac{\gamma_\pi-\rho}{1+\gamma_q-\beta\rho}\pi_t+\frac{1-\beta}{1+\gamma_q-\beta\rho}\left(1+\delta\right)\rho x_t
\end{aligned} \tag{M I-2bis}$$

Substituting the third equation, i.e. the asset price equation (31), into the other equations we get:

$$x_t = -a\pi_t + cg_t \tag{33}$$

$$\pi_t = dx_t + fu_t \tag{34}$$

$$\begin{aligned}
a &= \frac{\left(1-\beta\rho\right)\left(\gamma_\pi-\rho\right)}{\sigma\left(1-\rho\right)\left(1+\gamma_q-\beta\rho\right)+\gamma_q\left(1-\beta\right)\left(1+\delta\right)\rho} \\
c &= \frac{\sigma\left(1+\gamma_q-\beta\rho\right)}{\sigma\left(1-\rho\right)\left(1+\gamma_q-\beta\rho\right)+\gamma_q\left(1-\beta\right)\left(1+\delta\right)\rho} \\
d &= \frac{\lambda\left(1+\gamma_q-\beta\rho\right)+k\gamma_q\left(1-\beta\right)\left(1+\delta\right)\rho}{\left(1-\beta\rho\right)\left[\left(1+\gamma_q-\beta\rho\right)-k\left(\gamma_\pi-\rho\right)\right]} \\
f &= \frac{1+\gamma_q-\beta\rho}{\left(1-\beta\rho\right)\left[\left(1+\gamma_q-\beta\rho\right)-k\left(\gamma_\pi-\rho\right)\right]}
\end{aligned}$$

Equation (33) represents the (policy induced) AD schedule in model I-2 (AD(q) for short). A sufficient condition for the AD(q) schedule to be downward sloping is $\gamma_\pi > 1$.

Equation (34) represents the (policy induced) AS schedule in model I-2 (AS(q) for short). The AS(q) schedule is upward sloping if $\left(1+\gamma_q-\beta\rho\right) - k\left(\gamma_\pi-\rho\right) > 0$ i.e. if

$$\gamma_\pi < \rho + \frac{1-\beta\rho}{k} + \frac{1}{k}\gamma_q \tag{35}$$

Notice that assumption 2 – i.e. $\gamma_\pi < \hat{\gamma}_\pi$ where $\hat{\gamma}_\pi = \rho + \dfrac{1-\beta\rho}{k}$ – is a sufficient condition for (35) to be satisfied. In other words, if we assume that the AD and AS schedules in the SIT case are well behaved, then the AD and AS schedules in the AIT case will always be well behaved.

It is important to compare the slope of the AS schedule in the SIT and AIT cases which will be labelled AS and AS(q) respectively. After some algebra we conclude that

Remark 3 *The AS(q) schedule is flatter than the AS schedule if the response of the central bank to inflation is relatively "strong" i.e.* $\gamma_\pi > \check{\gamma}_\pi$ *where* $\check{\gamma}_\pi = \rho + \dfrac{1-\beta\rho}{k + \dfrac{\lambda}{(1-\beta)(1+\delta)\rho}}$. *If, on the contrary, this response is relatively weak, i.e.*$\gamma_\pi < \check{\gamma}_\pi$*, then the AS(q) schedule is steeper than the AS schedule.*[23]

In order to understand the rationale behind this remark, recall first that, in the presence of the cost channel, the relationship between the increment of the output gap and the associated increase of inflation along the AS curve is $\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AS} = \dfrac{\lambda}{1-\beta\rho-k\left(\gamma_\pi-\rho\right)}$.

Notice now that, according to (31), the asset price is increasing with the output gap and decreasing with inflation.

- When the output gap turns positive, therefore, the asset price goes up. In the AIT case, the increase of the asset price will induce monetary

[23] The slope of the AS schedule, in fact, is $\left.\dfrac{\partial \pi_t}{\partial x_t}\right|_{AS} = \dfrac{\lambda}{(1-\beta\rho)-k\left(\gamma_\pi-\rho\right)}$ while that of the AS(q) schedule is $\left.\dfrac{\partial \pi_t}{\partial x_t}\right|_{AS(q)} = \dfrac{\lambda\left(1+\gamma_q-\beta\rho\right)+k\gamma_q(1-\beta)(1+\delta)\rho}{(1-\beta\rho)\left[\left(1+\gamma_q-\beta\rho\right)-k\left(\gamma_\pi-\rho\right)\right]}$.

tightening, i.e. an increase of the interest rate, which translates into an increase of inflation due to the cost channel. Other things being equal, this effect would account for a slope of the AS(q) schedule greater than the slope of the AS schedule.

- On the other hand, induced inflation will bring down the asset price. The contraction of the asset price associated with inflation will induce monetary easing, i.e. a reduction of the interest rate, which translates into a reduction of inflation due to the cost channel. This effect would account for a slope of the AS(q) schedule smaller than the slope of the AS schedule.

If the response of the central bank to inflation is relatively "weak" ("strong"), the first (second) effect will prevail and the AS(q) schedule will be steeper (flatter) than the AS schedule. In the following

Assumption 3: We will assume that the response of the central bank to inflation is relatively "strong": $\gamma_\pi > \check{\gamma}_\pi$.

Assumptions 2 and 3 together imply:

$$\check{\gamma}_\pi < \gamma_\pi < \hat{\gamma}_\pi$$

and

$$k < \frac{(1-\beta\rho)(1-\beta)(1+\delta)\rho}{(1-\rho)\left[(1-\beta)(1+\delta)\rho+\eta+\sigma\right]}$$

Notice moreover that when the central bank reacts to $\hat{q}_t$ also the slope of the AD curve changes with respect to the case of no reaction. In absolute value:

$$\left.\frac{\partial \pi_t}{\partial x_t}\right|_{AD} = \frac{\sigma(1-\rho)}{\gamma_\pi-\rho} < \left.\frac{\partial \pi_t}{\partial x_t}\right|_{AD(q)} = \frac{\sigma(1-\rho)\left(1+\gamma_q-\beta\rho\right)+\gamma_q(1-\beta)(1+\delta)\rho}{(\gamma_\pi-\rho)(1-\beta\rho)}$$

In words, when $\gamma_q > 0$ the AD curve is steeper – on the (x_t, π_t) plane – than in the case $\gamma_q = 0$. In order to understand why, recall that, in the case $\gamma_q = 0$, an increase of inflation brings about a contraction of output whose magnitude is $\left.\frac{\partial x_t}{\partial \pi_t}\right|_{AD} = \dfrac{\gamma_\pi-\rho}{\sigma(1-\rho)}$. This is due to the reaction of the central bank to inflation, i.e. to the increase of the interest rate. Inflation, however, leads to a fall of asset prices. In the case $\gamma_q > 0$, the central bank contrasts this tendency by "easing", i.e. reducing the interest rate with

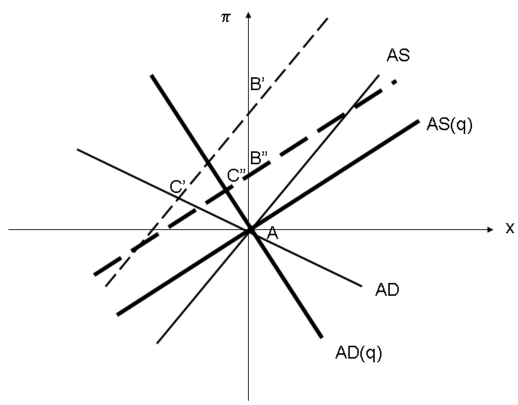

Figure 5: Effects of a supply shock in model I-2

respect to the previous interest rate hike. This will make the contractionary impact of the increase of the interest rate smaller, as shown by $\left.\frac{\partial x_t}{\partial \pi_t}\right|_{AD(q)} = \frac{(\gamma_\pi - \rho)(1 - \beta\rho)}{\sigma(1-\rho)\left(1+\gamma_q - \beta\rho\right) + \gamma_q (1-\beta)(1+\delta)\rho}$.

Solving (33) (34) gives x_t and π_t as linear functions of the shocks as shown above.

4.2.1 The effect of a supply shock

We are now ready to examine the transmission of shocks.

Suppose initially there are no shocks: $g_t = u_t = 0$. In figure 5 we represent the AD and the AS schedules in the case in which $\gamma_q > 0$ (AD(q) and AS(q) in bold). In the absence of shocks in both the SIT and AIT settings the AD and AS schedules intersect in the origin, point A.

Suppose a *supply shock* hits the economy. In the SIT case, inflation goes up by $\frac{1}{1 - \beta\rho - k(\gamma_\pi - \rho)} u_t$ (see point B' in figure 5, which corresponds to B' in figure 3). This burst of inflation incorporates the fact that the central bank reacts to the shock *raising* the interest rate, which adds to inflation on impact. The increase in inflation makes asset prices go down. When $\gamma_q > 0$, the central bank reacts to the fall of asset prices easing a bit so that the increase of the interest rate – and the additional inflation due to the cost channel – will be smaller than in the SIT case (see point B"). In other words, targeting asset prices will reduce the impact on inflation of a contractionary monetary policy in the presence of the cost channel.

The central bank then steers the economy to C". Notice that the AD curve is now steeper than in the SIT case. In the end, therefore, there will be more inflation and a milder recession than in the case in which the central bank does not react to asset prices (compare with C'). When a supply shock hits the economy, therefore, the reaction of the central bank to asset prices has a *mitigating effect* on the change in output but a *magnifying effect* on inflation.[24] In the end, the central bank adopts a more accommodating stance than in the SIT case. In fact the indirect instrument rule (32) shows that by targeting asset prices the central bank is actually concerned indirectly with the output gap.

4.2.2 The effect of a demand shock

In the case of a *demand shock*, in the AIT case the new short run equilibrium will be at the intersection B" of the AS(q) curve and the new AD(q) curve as shown in figure 6. The output gap turns positive and inflation goes up. In the AIT case, however, the expansion is stronger and inflation is higher than in the SIT case (compare with B'). When a demand shock hits the economy, therefore, the reaction of the central bank to asset prices has an effect of *amplification* on output and inflation with respect to SIT case.

What happens to the stock price? Recall that, as shown in (31) the asset price (i) falls in response to a burst of inflation and (ii) goes up in the presence of an increase of the output gap. As a consequence, $\hat{q}$ is a linear decreasing function of u because a supply shock yields an increase of inflation and a decrease of the output gap. An increase of g, on the other hand, brings about an increase of inflation – which is detrimental for the Stock market – but also an increase of the output gap, which makes dividends go up (see appendix D for details). The net effect of these two contrasting effects will depend on the strength of the response of the central bank to inflation. If the response is relatively strong – as we have assumed above (see assumption

[24]In fact, it runs out that $|a_1^q| < |a_1|$ and $b_1^q > b_1$ (see appendix D).

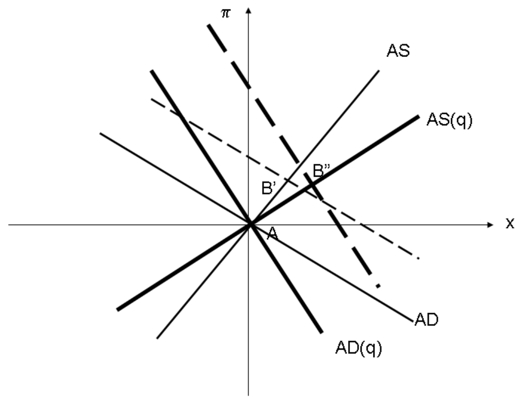

Figure 6: Effect of a demand shock in model I-2

3), the asset price will fall.

4.3 Impulse-response functions analysis

In order to provide the usual "pictorial view" of the dynamic behaviour of the model by means of impulse response functions, we have simulated the model using the following parameterization: $\sigma = 1,\ k = 0.1; \eta = 2$ (so that $\lambda = 0.3); \beta = 0.99; \rho = 0.9; \mu = 1.5$.

We borrow the calibration of the coefficient of relative risk aversion ($\sigma = 1$) and $\lambda = 0.3$ from Clarida, Galì and Gertler (2000). Having chosen $\eta = 2$, with this parameterization, $\sigma < \eta$ (as required by assumption 2).

We adopt the following parameter values for the response of monetary policy to inflation and asset prices: $\gamma_\pi = 1.1$ (solid lines) and $\gamma_q = 0.1$ (dotted lines) or $\gamma_q = 1$ (dashed lines). We use two different values for the response of monetary policy to asset prices, in particular a low value and an high value to test the robustness of our results.

Impulse-response functions: a supply shock In figure 7 the dotted (solid) (dashed) lines represent the impulse response functions when a supply shock occurs in the AIT (resp. SIT) (resp. AIT with $\gamma_q = 1$) case. A negative supply shock pushes inflation up and the output gap down. Profits and dividends follow the dynamic behaviour of the output gap. As expected (see the discussion above), targeting asset prices makes the recession milder (with respect to the SIT case) and inflation stronger. Asset prices fall because of the flight from equities and of the contraction of dividends but less than in the SIT case.

As to the interest rate, the reaction of the central bank to the supply shock makes the interest rate hike bigger in the AIT than in the SIT case. At first sight, this is strange. After all, targeting asset prices in a scenario in which they fall should lead to a monetary easing. The reason for this apparently odd result, however, is straightforward. As we noticed above, asset price targeting translates into an accommodating stance of monetary policy, leading to a big burst of inflation. The interest rate is driven up by this burst of inflation, the fall in asset prices notwithstanding. In other words the push of inflation on the interest rate prevails over the mitigating effect of the asset price bust. The stabilizing effect of the AIT rule on output, dividends and asset prices is offset by the destabilizing effect on inflation.

Impulse-response functions: a demand shock In figure 8 the dotted (solid) (dashed) lines represent the impulse response functions when a demand shock occurs in the AIT (resp. SIT) (resp. AIT with $\gamma_q = 1$) case. A demand shock pushes inflation and the output gap up. Profits and dividends follow the dynamic behaviour of the output gap. As expected (see the discussion above) targeting asset prices makes both inflation and the boom stronger (with respect to the SIT case). Asset prices fall because the flight from equities more than offset the expansion (due again to assumption 3).

As to the interest rate, the reaction of the central bank to the demand shock makes the interest rate hike bigger in the AIT than in the SIT case. The interest rate is driven up by inflation, the fall in asset prices notwithstanding. In other words the push of inflation on the interest rate prevails over the mitigating effect of the asset price fall (as in the case of a supply shock). The stabilizing effect of the AIT rule is only on asset prices while the effect is destabilizing on inflation, the output gap and dividends.

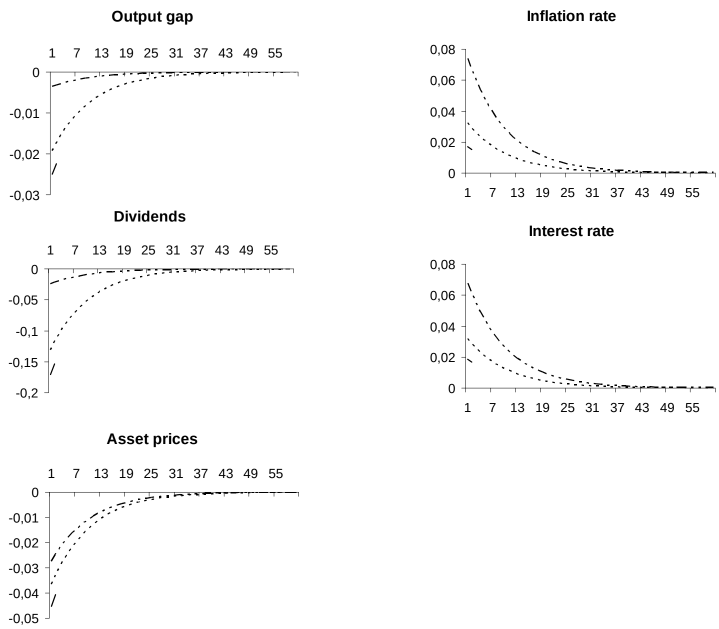

Figure 7: IRFs: a supply shock

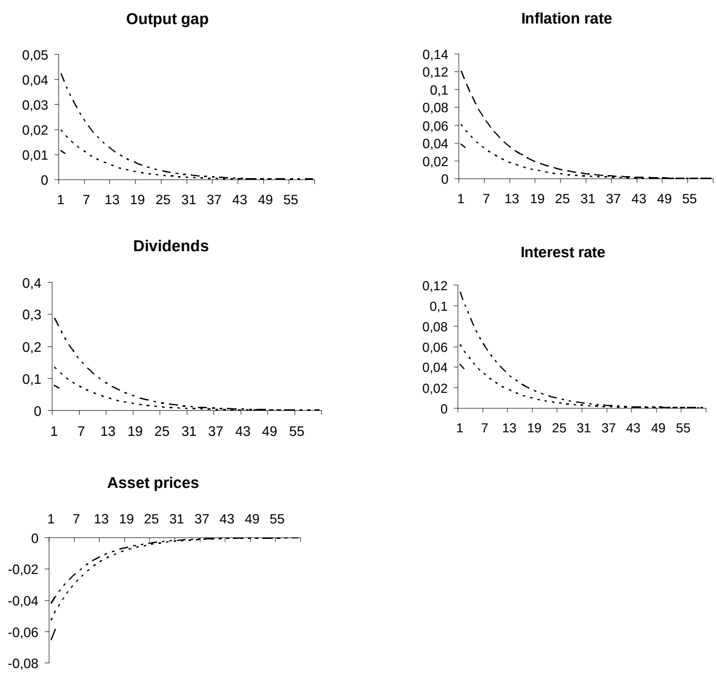

Figure 8: IRFs: a demand shock

5 Some welfare considerations

In order to sharpen our perception of the consequences of Asset Inflation Targeting on the part of the central bank, let's assume that society's preferences can be represented by a quadratic loss function whose arguments are inflation and the output gap: [25] $L = \pi_t^2 + \alpha x_t^2$ where α is a measure of aversion to output volatility. Aversion to inflation therefore can be captured by $1/\alpha$.

In the SIT case, substituting the reduced form of model M I-0 into the expression above one gets: $L = (b_1 u_t + b_2 g_t)^2 + \alpha (a_1 u_t + a_2 g_t)^2$. Rearranging and taking the expected value

$$E(L) = \left(\alpha a_1^2 + b_1^2\right)\sigma_u^2 + \left(\alpha a_2^2 + b_2^2\right)\sigma_g^2 \tag{36}$$

Analogously, in the AIT case we have:

$$E(L^q) = \left(\alpha a_1^{q2} + b_1^{q2}\right)\sigma_u^2 + \left(\alpha a_2^{q2} + b_2^{q2}\right)\sigma_g^2 \tag{37}$$

with $|a_1^q| < |a_1|, b_1^q > b_1$, $a_2^q > a_2, b_2^q > b_2$. For the sake of comparison, and with a negligible loss of generality, let's assume $\sigma_u^2 = \sigma_g^2$.

AIT is better than SIT from society's point of view if $E(L) - E(L^q) > 0$, i.e.

$$\alpha\left[\left(a_1^2 - a_1^{q2}\right) + \left(a_2^2 - a_2^{q2}\right)\right] + \left(b_1^2 - b_1^{q2}\right) + \left(b_2^2 - b_2^{q2}\right) > 0 \tag{38}$$

All the differences in parentheses are negative with the exception of the first one.

There are two cases. If $a_1^2 - a_1^{q2} > a_2^{q2} - a_2^2$, i.e.

$$\begin{aligned} a_1^{q2} &< \bar{a}_1^{q2} \\ \bar{a}_1^{q2} &:= a_1^2 + a_2^2 - a_2^{q2} \end{aligned} \tag{39}$$

– so that the expression in brackets in (38) is positive – the condition is

[25] Woodford (2003) provides a rationale for this loss function interpreting it as a second order approximation of the representative consumer's utility function.

satisfied if

$$\alpha > \bar{\alpha} \tag{40}$$
$$\bar{\alpha} := \frac{\left(b_1^{q2} - b_1^2\right) + \left(b_2^{q2} - b_2^2\right)}{\left(a_1^2 - a_1^{q2}\right) + \left(a_2^2 - a_2^{q2}\right)}$$

If, on the contrary, the inequality (39) is reversed so that the expression in brackets is negative, the condition (38) is never satisfied.

In words: AIT is preferable to SIT if:

- the reduction in output due to a supply shock in the AIT case a_1^{q2} is "small" enough, i.e. smaller than a threshold $\bar{a}_1^{q2}$ defined in (39),
- society's aversion to output volatility is "high" enough, i.e. higher than a threshold $\bar{\alpha}$ defined in (40).

This proposition can be illustrated graphically as follows. In figure 9 we represent the difference $\Delta = E(L) - E(L^q)$ as a function of α.[26] When $\Delta > 0$, AIT is preferable to SIT and viceversa.

The intercept on the y-axis is always negative. The slope is positive if condition (39) is satisfied, which is the case shown in the figure. In this case inequality (38) is satisfied if the aversion to output volativility is big enough, i.e. higher than the intercept $\bar{\alpha}$ on the x-axis. If condition (39) is not satisfied, the slope of the line becomes negative so that inequality (38) is never satisfied.

Each line is parameterized to a certain level of γ_q. In the figure we represent two lines. The solid one is parameterized to a low level, which we label γ_q^L. An increase of γ_q from γ_q^L to γ_q^H yields a higher threshold $\bar{\alpha}^H$. In other words, if the central bank becomes more reactive to asset prices, it is more likely that society becomes worse off with AIT (unless it is extremely averse to output volatility).

The increasing concave curve in figure 10 is obtained plotting the threshold $\bar{\alpha}$ which we obtain from the parameterization adopted to produce the impulse-response functions (see section 4.3) for different values of γ_q.

[26] $E(L)$ and $E(L^q)$ are defined in (36) and (37) respectively. The difference Δ is normalized by the variance of the shock which we assume to be the same for both types of shocks for simplicity.

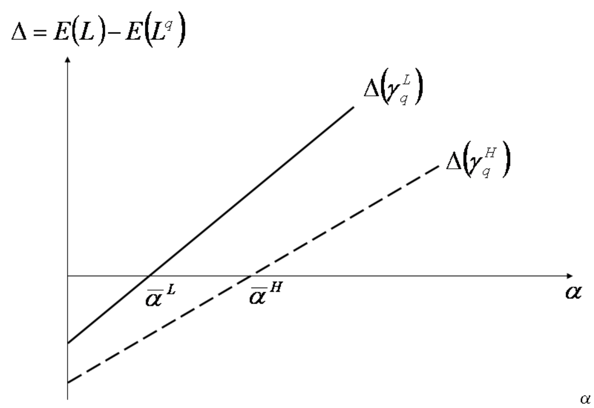

Figure 9: Difference in loss from SIT and AIT

Points above the curve represents combinations of γ_q and α such that society prefers AIT to SIT. It is clear that it takes an extremely high aversion to output volatility for society to prefer AIT even for relatively low values of γ_q.

It is clear therefore that it is highly unlikely that AIT would be preferred to SIT. Two conditions must be met: (i) the gain in output stabilization due to AIT in case of a supply shock should be non negligible and (ii) society should be very averse to output volatility.

6 Conclusions

In this paper we have presented a NK-DSGE model in which asset prices will be eventually incorporated into the NK Phillips curve. This is due to the assumption of a cost channel for monetary policy which is activated whenever monetary policy affects asset prices and therefore the return on shares. The latter in fact is the cost of external finance in our model. The novelty of the analysis consists in this peculiar treatment of financing decisions, which brings to the fore the relationship between pricing of goods and pricing of assets.

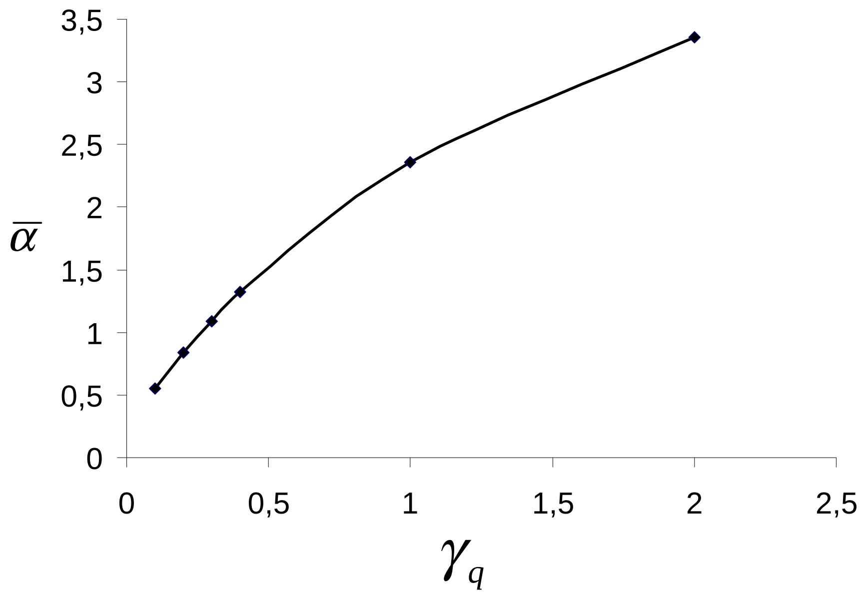

Figure 10: $\bar{\alpha}$ as a function of γ_q

We analyse two monetary policy regimes:(a) an instrument rule with no-reaction to asset prices (Strict Inflation Targeting), (b) an instrument rule with reaction to asset prices (Asset augmented Inflation Targeting).

Inflation volatility is higher in the AIT scenario irrespective of the type of shock hitting the economy. As far as output volatility is concerned, in the case of a supply shock targeting asset prices may attenuate the contractionary impact of the shock on economic activity but at the cost: Higher inflation. It turns out, therefore that targeting asset prices may not be a good idea even if the framework used to explore this issue is much different from the BG one. In the end, in the AIT scenario the central bank adopts an accommodating policy stance which results in an unsatisfactory macroeconomic performance.

In our setting, the problem with asset price targeting is, in a nutshell, the violation of Tinbergen Law. By means of an asset augmented Taylor rule, in fact, the central bank is pursuing two different objectives (asset price stabilization and inflation stabilization) with only one policy instrument, the interest rate. In order to satisfy Tinbergen Law, a two-pillar approach is necessary. An interesting proposal in this direction has been put forward by De Grauwe and Gros (2009). The same point has been forcefully made by Charlie Bean (2010).

We consider these results encouraging even if this is a very preliminary exploration of the properties of the model. We want to pursue an appropriate generalization because the model has to be enriched to explore more realistic environments. The most straightforward extension will consist in incorporating credit markets and credit market imperfections because they have a major role to play in our "story". The list of possible extensions that one can imagine, however, is quite long and will figure on top of our research agenda in the near future.

A The household's maximization problem

The representative household's problem consists in :

$$\max_{C_t, m_t, N_t, A_t, b_t} E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} (m_{t+s})^{1-\zeta} - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right]$$

subject to a sequence of budget constraints of the form:

$$C_{t+s} + m_{t+s} + b_{t+s} + A_{t+s} q_{t+s} = w_{t+s} N_{t+s} + q_{t+s} A_{t-1+s} +$$

$$+ m_{t-1+s} \frac{1}{1+\pi_{t+s}} + \frac{1+i_{t-1+s}}{1+\pi_{t+s}} b_{t-1+s} + d_{t+s} A_{t-1+s}$$

The Lagrangian therefore is:

$$\begin{aligned} L \;=\; & E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-\zeta} (m_{t+s})^{1-\zeta} - \chi \frac{N_{t+s}^{1+\eta}}{1+\eta} \right] + \\ & - E_t \sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \left[C_{t+s} + m_{t+s} + b_{t+s} + A_{t+s} q_{t+s} + \right. \\ & - w_{t+s} N_{t+s} - m_{t-1+s} \frac{1}{1+\pi_{t+s}} - \frac{1+i_{t-1+s}}{1+\pi_{t+s}} b_{t-1+s} + \\ & \left. - q_{t+s} A_{t-1+s} - d_{t+s} A_{t-1+s} \right] \end{aligned}$$

Solving the problem above we get the following $FOCs$ that hold $\forall t$:

$$\frac{\partial L}{\partial C_t} = 0 \Longrightarrow \quad C_t^{-\sigma} - \lambda_t = 0$$

$$\frac{\partial L}{\partial m_t} = 0 \Longrightarrow \gamma (m_t)^{-\zeta} - \lambda_t + \beta \lambda_{t+1} \frac{1}{1+\pi_{t+1}} = 0$$

$$\frac{\partial L}{\partial N_t} = 0 \Longrightarrow \quad -\chi N_t^{\eta} + \lambda_t w_t = 0$$

$$\frac{\partial L}{\partial A_t} = 0 \Longrightarrow \quad -\lambda_t q_t + \beta E_t \left[\lambda_{t+1} (q_{t+1} + d_{t+1}) \right] = 0$$

$$\frac{\partial L}{\partial b_t} = 0 \Longrightarrow \quad -\lambda_t + \beta E_t \left[\lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} \right] = 0$$

From the above conditions we get the Euler equations (4)(5)(6) and the asset price equation (7) as defined in section 2.

B Steady states and log-linearization

The economy consists of five markets: labor, goods, money, bonds, shares.

The equilibrium condition on the goods market is $C_t = Y_t$. Moreover, $Y_t = N_t$.

In a *symmetric* flexible price equilibrium all the firms charge the same price P_t equal to the nominal marginal cost $P_t\phi_t$ augmented by the markup μ. Therefore $\phi_t = \frac{1}{\mu}$. Recalling (15) we get the *price rule*: $w_t = \frac{1}{\mu}\frac{q_t}{E_t\left(q_{t+1}+d_{t+1}\right)}$. Plugging $C_t = Y_t = N_t$ into (6) and rearranging we get the *wage rule* $w_t = \chi Y_t^{\eta+\sigma}$. Equating these expressions and solving for output we obtain the *level* of the flexible price equilibrium: $Y_t^f = \left(\frac{1}{\chi\mu}\frac{q_t}{E_t\left(q_{t+1}+d_{t+1}\right)}\right)^{\frac{1}{\eta+\sigma}}$. In the steady state this boils down to

$$Y_s^f = \left(\frac{\beta}{\chi\mu}\right)^{\frac{1}{\eta+\sigma}} \tag{41}$$

Notice that in the canonical NK-DSGE model we have $Y_s^c = \left(\frac{1}{\chi\mu}\right)^{\frac{1}{\eta+\sigma}}$. In the present setting, therefore, the steady state flexible price equilibrium output is a fraction $\beta^{\frac{1}{\eta+\sigma}}$ of the standard one. This is, in a sense, obvious since the marginal cost is augmented, in the present, context, by the cost of esternal finance, i.e. the ROS, other things being equal.

Imposing the steady state condition in (4), it turns out that

$$\frac{1+i_t}{1+E_t\pi_{t+1}} = \beta^{-1} = 1+r \tag{42}$$

i.e. in the steady state the real interest rate is anchored to the rate of time preference r.

Using (42) and imposing the steady state condition in the asset price equation (8) we get

$$\frac{d_s}{q_s} = \beta^{-1} - 1 = r \tag{43}$$

i.e. in the steady state the dividend yield is constant and equal to the rate of time preference. From the equation above follows $q_s = d_s/r$ i.e. a pure *dividend discount* model of asset price determination: in the steady state, the asset price is the discounted sum of an infinite stream of dividends. Therefore the steady state ROS is:

$$ROS_s = \frac{q_s + d_s}{q_s} = 1 + r = \beta^{-1}$$

This is obvious: Because of the no-arbitrage condition, the real interest rate should be equal to the ROS also in the steady state.

C Model I-1

We proceed to the solution of model I-1 in two steps. First we solve model I-0, which yields the RE solutions for the output gap, inflation and the interest rate. Then we find the solution for the asset price.

C.1 Model MI-0

Model I-0 boils down to equations (20) and (21). We solve by the method of undetermined coefficients. We "guess" the following:

$$\begin{aligned} x_t &= a_1 u_t + a_2 g_t \\ \pi_t &= b_1 u_t + b_2 g_t \end{aligned}$$

So that, under assumption 1,

$$\begin{aligned} E_t x_{t+1} &= \rho \left(a_1 u_t + a_2 g_t \right) \\ E_t \pi_{t+1} &= \rho \left(b_1 u_t + b_2 g_t \right) \end{aligned}$$

After some algebra we verify that the conjecture is indeed correct and we get the following solutions:

$$\begin{aligned}
a_1 &= -\frac{\gamma_\pi - \rho}{\sigma K_0} \\
a_2 &= \frac{1 - \beta\rho - k\left(\gamma_\pi - \rho\right)}{K_0} \\
b_1 &= \frac{1-\rho}{K_0} \\
b_2 &= \frac{\lambda}{K_0}
\end{aligned}$$

where

$$K_0 := \left(1 - \beta\rho\right)\left(1 - \rho\right) + \frac{k}{\sigma}\left(\gamma_\pi - \rho\right)\left(\eta + \sigma\rho\right)$$

Under assumption 2 it turns out that $K_0 > 0$ so that $a_1 < 0, a_2 > 0, b_1 > 0, b_2 > 0$.

The coefficients for the fundamentals based interest rate rule:

$$i_t = \gamma_u u_t + \gamma_u g_t$$

can be computed as follows: $\gamma_u = \gamma_\pi b_1 > 0; \gamma_g = \gamma_\pi b_2 > 0$.

The canonical model (without the cost channel) M I-0(c) consists of equations (23) and (24). The RE solution is:

$$\begin{aligned}
a_1^c &= -\frac{\gamma_\pi - \rho}{\sigma K_1} \\
a_2^c &= \frac{1 - \beta\rho}{K_1} \\
b_1^c &= \frac{1-\rho}{K_1} \\
b_2^c &= \frac{\lambda}{K_1}
\end{aligned}$$

where

$$K_1 := \left(1 - \beta\rho\right)\left(1 - \rho\right) + \frac{k}{\sigma}\left(\gamma_\pi - \rho\right)\left(\eta + \sigma\right)$$

The coefficients have the same sign as the corresponding coefficients of the model with the cost channel. Moreover $K_1 > K_0$. Therefore $\left|a_1^c\right| < \left|a_1\right|, a_2^c >$

$a_2, b_1^c < b_1, b_2^c < b_2$.

The coefficients for the fundamentals based interest rate rule are: $\gamma_u = \gamma_\pi b_1^c > 0; \gamma_g = \gamma_\pi b_2^c > 0$.

C.2 Asset prices

From (27) follows that the solution for $\hat{q}_t$ is $\hat{q}_t = c_1 u_t + c_2 g_t$ where

$$\begin{aligned} c_1 &= -\frac{\gamma_\pi - \rho}{1 - \beta\rho} b_1 + \frac{1 - \beta}{1 - \beta\rho} (1 + \delta) \rho a_1 \\ c_2 &= -\frac{\gamma_\pi - \rho}{1 - \beta\rho} b_2 + \frac{1 - \beta}{1 - \beta\rho} (1 + \delta) \rho a_2 \end{aligned}$$

Therefore $c_1 < 0$. c_2 has uncertain sign. It turns out that $c_2 < 0$ if

$$\gamma_\pi > \rho + \frac{(1 - \beta\rho)}{k + \dfrac{\lambda}{(1 - \beta)(1 + \delta)\rho}}$$

This completes the solution of model I-1.

C.3 Determinacy

Substituting the monetary policy rule (18) into the IS schedule and the NK Phillips curve of model I-0 and rearranging, we can write the model in matrix format:

$$Z_t = A E_t Z_{t+1} + B W_t$$

where Z_t is the column vector of endogenous variables, $E_t Z_{t+1}$ is the column vector of the expectations taken in t of the endogenous variables in t+1, W_t is the column vector of the shocks:

$$
\begin{aligned}
Z_t &= \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}; E_t Z_{t+1} = \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} \\
A &= \frac{1}{1-\gamma_\pi \left(k - \frac{\lambda}{\sigma}\right)} \begin{bmatrix} \beta - k + \frac{\lambda}{\sigma} & \lambda \\ \frac{1-\gamma_\pi \beta}{\sigma} & 1 - k\gamma_\pi \end{bmatrix} \\
B &= \begin{bmatrix} 1 & \lambda \\ -\frac{\gamma_\pi}{\sigma} & 1 - \frac{\lambda \gamma_\pi}{\sigma} \end{bmatrix} \\
W_t &= \begin{bmatrix} u_t \\ g_t \end{bmatrix}
\end{aligned}
$$

The RE solution is determinate if the Blanchard-Kahn conditions are satisfied.

Determinacy requires all the eigenvalues of A to lie inside the unit circle. Necessary and sufficient conditions for this to happen are

$$
\begin{aligned}
D &< 1 \\
T - D &< 1 \\
T + D &> -1
\end{aligned}
$$

where D and T denote respectively the determinant and the trace of matrix A.

After some tedious algebra we reach the following conclusions:

- If $\sigma < \eta$,determinacy requires $\gamma_\pi > 1$
- If $\sigma > \eta$,determinacy occurs if and only if $1 < \gamma_\pi < \dfrac{2\sigma(1+\beta)}{k(\sigma-\eta)} - 1 :=$ $\bar{\gamma}_\pi$.

Let's assume $\sigma < \eta$. From section 4.1 we know that a well behaved AD curve requires requires $\gamma_\pi > \rho$. Hence in order to have both determinacy and well behaved solutions we have to impose

$$
1 < \gamma_\pi < \rho + \frac{1-\beta\rho}{k} := \hat{\gamma}_\pi \tag{44}
$$

and assume that $k < \dfrac{1-\beta\rho}{1-\rho}$.

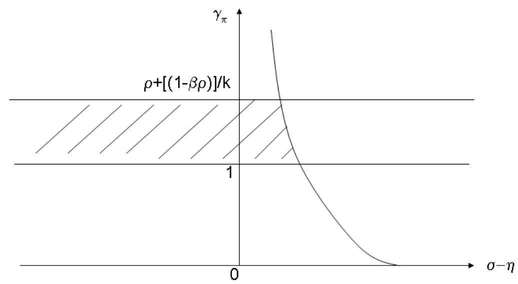

Figure 11: Determinacy of RE solution in M I-0.

In case $\sigma > \eta$, in order to have both determinacy and well behaved solutions we have to impose

$$1 < \gamma_\pi < \min\left(\hat{\gamma}_\pi, \bar{\gamma}_\pi\right) \tag{45}$$

In figure 11 the downward sloping curve represents the threshold $\bar{\gamma}_\pi$ as a function of the difference $\sigma - \eta$ (when $\sigma > \eta$). The dashed area represents the locus of $(\gamma_\pi, \sigma - \eta)$ points such that both determinacy and well behaved curves are guaranteed.

D Model I-2

In order to find the RE solution of model I-2 we "guess" the following:

$$\begin{aligned} x_t &= a_1^q u_t + a_2^q g_t \\ \pi_t &= b_1^q u_t + b_2^q g_t \\ \hat{q}_t &= c_1^q u_t + c_2^q g_t \end{aligned}$$

So that, under assumption 1,

$$\begin{aligned} E_t x_{t+1} &= \rho \left(a_1^q u_t + a_2^q g_t\right) \\ E_t \pi_{t+1} &= \rho \left(b_1^q u_t + b_2^q g_t\right) \\ E_t \hat{q}_{t+1} &= \rho \left(c_1^q u_t + c_2^q g_t\right) \end{aligned}$$

After some algebra we verify that the conjecture is indeed correct and we get the following solutions:

$$\begin{aligned} a_1^q &= -\frac{\gamma_\pi - \rho}{\sigma K_2} \\ a_2^q &= \frac{\left(1 + \gamma_q - \beta\rho\right) - k\left(\gamma_\pi - \rho\right)}{K_2} \\ b_1^q &= \frac{\sigma\left(1-\rho\right)\left(1 + \gamma_q - \beta\rho\right) + \gamma_q\left(1-\theta\right)\left(1+\delta\right)\rho}{\sigma\left(1-\beta\rho\right)K_2} \\ b_2^q &= \frac{\lambda\left(1 + \gamma_q - \beta\rho\right) + k\gamma_q\left(1-\beta\right)\left(1+\delta\right)\rho}{\left(1-\beta\rho\right)K_2} \\ c_1^q &= \frac{\left(1-\beta\right)\left(1+\delta\right)\rho a_1^q - \left(\gamma_\pi - \rho\right)b_1^q}{1 + \gamma_q - \beta\rho} \\ c_2^q &= \frac{\left(1-\beta\right)\left(1+\delta\right)\rho a_2^q - \left(\gamma_\pi - \rho\right)b_2^q}{1 + \gamma_q - \beta\rho} \end{aligned}$$

where

$$\begin{aligned} K_2 &: = \left(1-\rho\right)\left[\left(1 + \gamma_q - \beta\rho\right) - k\left(\gamma_\pi - \rho\right)\right] + \\ & + \frac{1}{\sigma}\left[\lambda\left(\gamma_\pi - \rho\right) + \gamma_q\left(1-\theta\right)\left(1+\delta\right)\rho\right] \end{aligned}$$

Under assumption 2 it turns out that $K_2 > 0$. Notice moreover that $\frac{\lambda}{\sigma} - k\left(1-\rho\right) = k\left(\frac{\eta}{\sigma} + \rho\right) > 0$.Therefore $a_1^q < 0, a_2^q > 0, b_1^q > 0, b_2^q > 0, c_1^q < 0$. The sign of c_2^q is uncertain.

We can determine the coefficients for the interest rate in the fundamentals based rule:

$$i_t = \gamma_u u_t + \gamma_u g_t$$

as follows: $\gamma_u = \gamma_\pi b_1^q + \gamma_q c_1^q; \gamma_g = \gamma_\pi b_2^q + \gamma_q c_2^q$.

This completes the solution of model I-2.

Comparing the Re solutions in the SIT and AIT case, it turns out that: $|a_1^q| < |a_1|, b_1^q > b_1$.If $\gamma_\pi > \rho + \dfrac{(1-\beta\rho)}{k + \dfrac{\lambda}{(1-\beta)(1+\delta)\rho}}$ then $a_2^q > a_2, b_2^q > b_2$.

D.1 Determinacy

Substituting the monetary policy rule (29) into the Asset price equation, the NK Phillips curve and the IS schedule of model I-2 – i.e. equations (26), (14), (9) – and rearranging, we can write the model in matrix format:

$$Z_t = AE_t Z_{t+1} + BW_t$$

where Z_t is the column vector of endogenous variables (π_t, x_t, q_t), $E_t Z_{t+1}$ is the column vector of the expectations taken in t of the endogenous variables in t+1, W_t is the column vector of the shocks (u_t, g_t).Matrix A reads as follows: $A = \Lambda\ \bar{A}$ where $\Lambda = \dfrac{1}{1 - \gamma_\pi\left(k - \frac{\lambda}{\sigma}\right) + \gamma_q - \lambda\gamma_q\gamma_\pi\left(1 - \frac{1}{\sigma}\right)}$ and the entries of matrix $\bar{A}$ are

$$\begin{aligned}
\bar{a}_{11} &= \beta\left(1+\gamma_q\right) - \left(k - \tfrac{\lambda}{\sigma}\right) - \lambda\gamma_q\left(1 - \tfrac{1}{\sigma}\right)\\
\bar{a}_{12} &= \lambda\left(1+\gamma_q\right) + (1+\delta)(1-\beta)(k-\lambda)\gamma_q\\
\bar{a}_{13} &= \beta(k-\lambda)\gamma_q\\
\bar{a}_{21} &= (1-\beta\gamma_\pi)\left[\tfrac{1}{\sigma} - \left(1 - \tfrac{1}{\sigma}\right)\gamma_q\right]\\
\bar{a}_{22} &= 1 - k\gamma_\pi + \gamma_q - (1+\delta)(1-\beta)\left[\gamma_q(1-k\gamma_\pi) + \tfrac{1}{\sigma}k\gamma_q\gamma_\pi\right]\\
\bar{a}_{23} &= -\beta\left[\gamma_q(1-k\gamma_\pi) + k\tfrac{1}{\sigma}\gamma_q\gamma_\pi\right]\\
\bar{a}_{31} &= 1 - \beta\gamma_\pi\\
\bar{a}_{32} &= -\lambda\gamma_\pi + (1+\delta)(1-\beta)\left[1 - \gamma_\pi\left(k - \tfrac{\lambda}{\sigma}\right)\right]\\
\bar{a}_{33} &= \beta\left[1 - \gamma_\pi\left(k - \tfrac{\lambda}{\sigma}\right)\right]
\end{aligned}$$

As shown by Brooks (2004) determinacy of the RE solution requires all the eigenvalues of A to lie inside the unit circle. Necessary and sufficient

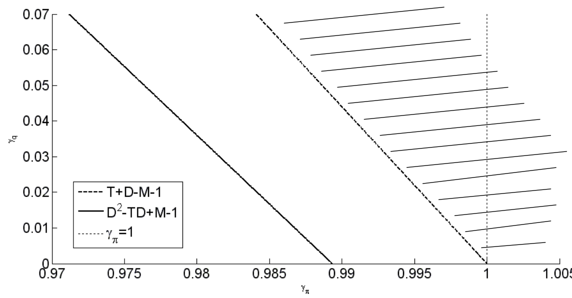

Figure 12: Determinacy of Re solutions in M I-2

conditions for the eigenvalues of A to lie inside the unit circle are:

$$\begin{aligned} D - 1 &< 0 \\ T + D - M - 1 &< 0 \\ D^2 - TD + M - 1 &< 0 \end{aligned}$$

where D, T and M denote respectively the determinant, the trace and the sum of leading minors of order two of matrix A.

We assume the following parameter values (see section 4.3): $\sigma = 1,\ k = 0.1; \eta = 2$ (so that $\lambda = 0.3$); $\beta = 0.99, \mu = 1.5$. With this parameterization, the LHS of each of the three conditions above becomes a function of γ_q and γ_π only. Therefore the determinacy region can be represented on the $\left(\gamma_q, \gamma_\pi\right)$ plane (see figure 12)

It turns out that:

- the first condition $(D - 1 < 0)$ is always satisfied for positive γ_q and γ_π;
- the second condition $(T + D - M - 1 < 0)$ is satisfied for points lying above the steep downward sloping curve;

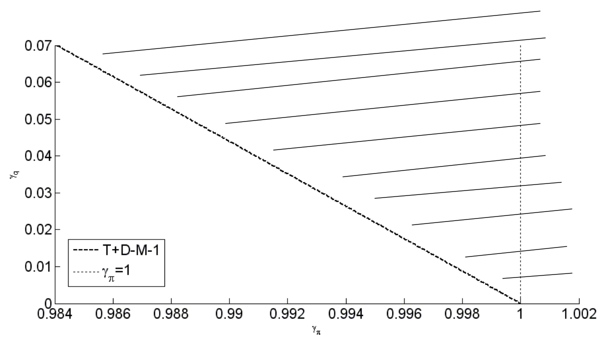

Figure 13: Magnification of the determinacy area

- the third condition $(D^2 - TD + M - 1 < 0)$ is satisfied for points lying above the flat downward sloping line.

When the second condition is satisfied, therefore, also the third one is satisfied. We can conclude that a sufficient condition for determinacy is for the combination (γ_q, γ_π) to lie in the dashed region of the plane.

Figure 13 is a magnification of a portion of figure 12.

It is clear that the Taylor principle is a sufficient condition for determinacy whatever the value of γ_q. Notice, however, that one can have determinacy in this model also when the reaction of the central bank to inflation is not as aggressive as the Taylor principle would require.

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