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Citation for published version (APA):
Han, M. A. (2010). Short-term managerial contracts facilitate cartels. (Amsterdam Center for Law & Economics working paper; No. 2010-16). Amsterdam: Amsterdam Center for Law & Economics.
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Amsterdam Center for Law & Economics Working Paper No. 2010-16

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Short-Term Managerial Contracts Facilitate Cartels*

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December 23, 2010

Abstract

This paper shows how a series of commonly observed short-term CEO employment contracts improves cartel stability compared to a long-term contract. When a manager’s short-term appointment is renewed if and only if the firm hits a certain profit target, then (a) defection from collusion results in superior firm performance and thus reduces the chance of being fired immediately, while (b) future punishment results in inferior firm performance, thereby increasing the chance of being fired in the future. The introduction of this reemployment tradeoff intertwines with the usual monetary tradeoff and weakly improves cartel stability. Studying the impact of fixed versus variable salary components, I find that fixed components facilitate collusion with a short-term contract, while not affecting cartel stability with a long-term contract. I extend the model to argue how short-term renewable contracts are a source of cyclical collusive pricing. Finally, interpreting the results in the light of firm financing shows how debt-financed firms can form more stable cartels than equity-financed firms.

Keywords: cartels, collusion, managerial contracts, price wars

JEL codes: L10, L21, L40

*I thank Charles Angelucci, Arnoud Boot, Benno Bühler, Bruno Jullien, Henrik Lando, Evgenia Motchenkova, Erik Pot, Julien Sauvagnat, Maarten Pieter Schinkel, Randolph Sloof, Jeroen van de Ven, and Michael Yang for useful discussions, as well as seminar participants in Amsterdam (CLEEN 2010), Berlin (DIW), Düsseldorf (DICE), Istanbul (AERIE 2010), Toulouse (Brown Bag Seminar) and Utrecht (NAKE 2010). This research was initiated during a research visit at GREMAQ in Toulouse; I am grateful for their hospitality. Errors and opinions are mine.
1 Introduction

The majority of the literature on cartels and collusion treats the firm as a profit-maximising decision maker, thereby deriving key insights on cartel stability and behavior from a profit-maximising perspective. However, cartels often involve firms having separated ownership and control in such a way that the incentives of the key decision maker (CEO, manager) are not fully aligned with those of the profit-maximising shareholders. The resulting corporate governance factors can have a key impact on the operation of cartels.\footnote{See Buccirossi and Spagnolo (2007) for a research agenda on corporate governance factors and collusion.}

This paper studies how short-term renewable employment contracts affect cartel stability and behavior. Although one would expect that short-sighted CEOs are incapable of running a stable cartel, I show how ‘short-termism’ generated through such contracts can in fact improve cartel stability. Moreover, contrary to conventional wisdom, short-term contracts with fixed (as opposed to: variable) salary components weakly improve cartel stability.

From a sample of 375 CEO employment contracts of large public corporations, Schwab and Thomas (2006) find that 81% specify a length of five years or less, with a mean of 3.6 years. Similarly, Gillian, Hartzell and Parrino (2009) find that the median and mean length in a sample of 184 S&P 500 CEO contracts is 3.4 and 3.0 years, respectively. Moreover, recent work by Jenter and Lewellen (2010) indicates that CEO turnover decisions are very sensitive to stock price performance: from a sample of 2,569 publicly traded U.S. firms from 1992 to 2005, they find that 44% of CEOs left the firm within 4 years as a result of bad firm performance.\footnote{For additional evidence on the relationship of CEO turnover and firm performance, see Lausten (2002), Farrell and Whidbee (2003), Jenter and Kanaan (2010) and Kaplan and Minton (2010).}

These empirical findings show that executives typically have a short-term renewable employment contract, that is, a contract for a finite period of time which is renewed if and only if the firm performs sufficiently well.

In this paper, I take such a typical CEO contract as exogenously given and aim to understand its impact on the operation of cartels.\footnote{The model abstracts away from the underlying motivation of adopting short-term renewable contracts over long-term contracts or no contract at all. Such motivations include risk sharing or adverse selection – see the papers referenced above. In general principal-agent models with collusion, Aubert (2009) and Angelucci and Han (2010) endogenously derive optimal employment contracts.}

Assuming that managerial remuneration as well as the probability of contract renewal is positively related to firm profit, the manager can stochastically increase profit by forming a cartel with rival firms, while even further stochastically increasing profit by defecting from the collusive agreement.\footnote{To focus on the impact of short-term contracts on cartel stability, the model abstracts away from effort incentives. Aubert (2009) investigates incentives to collude and incentives to exert effort are related.} Two interrelated forces then dictate cartel stability. The manager compares the immediate expected salary gain resulting from defection, with the future expected salary losses resulting from punishment.
the monetary tradeoff. The manager also compares the immediate increase in the probability of contract renewal resulting from the higher expected profit during defection, with the future decrease in the probability of contract renewal resulting from the lower expected profits during future punishment – the reemployment tradeoff. While the monetary tradeoff parallels the familiar problem faced by profit maximising firms engaging in cartelisation, the reemployment tradeoff originates from the manager’s fear of not being rehired.

Whether short-term renewable contracts strictly enhance cartel stability compared to a long-term contract depends on two effects. First, the manager is unsure whether he will be rehired even if he colludes and, hence, the game ends with some positive probability: this is a collusion-destabilising effect. Second, noting that after defection the manager only cares about reemployment if the expected remuneration during punishment is not too small, the reemployment tradeoff can dominate the collusion-destabilising effect if the probability of reemployment is very low during punishment, while the expected remuneration during punishment is indeed not too small. This ensures that the manager misses out (in expectation) on a substantial amount of future salary as he is fired in the future with a high probability. As short-term renewable contracts can replicate a long-term contract by rehiring the manager even when a very low profit has been realised, short-term renewable contracts weakly increase cartel stability.

The model also shows how a fixed salary component may stabilise collusion when a short-term contract is in place, while not affecting cartel stability when a long-term contract is in place. The intuition is that with a long-term contract, the fixed salary component is paid out in every period independent on managerial behavior and therefore does not affect the manager’s incentives to collude. However, with a short-term contract, the fixed component is only paid out in periods in which the manager is indeed working for the firm. The manager anticipates that defection leads to a higher probability of being fired in the future, i.e. losing out on future fixed salary components, which may amount to a large expected loss when he is relatively patient. In that case, the shareholder optimally makes the fixed component an important part of the manager’s salary so as to stabilise collusion.

Moreover, I study the case in which managers may revert from punishment to collusion when one of the managers is fired and replaced by another. The rationale behind such ‘serial collusion’ is that managerial replacement restores the trust needed for a cartel to work. Interestingly, such restoration of trust destabilises collusion. The reason is that serial collusion increases the continuation value of defection, because (i) undercutting instantaneously increases the probability that the rival manager is fired, in which case collusion is immediately restored, and (ii) punishment does not last forever as the rival manager is fired at some point.

Finally, I consider the case in which managers engage in multiple sequential interactions
within the span of the employment contract, thereby showing that short-term employment contracts may be an explanation for cyclical collusive pricing. Suppose, for example, that the manager is halfway through his employment contract and has been colluding up until now, but that earlier firm profits have been extremely low due to random market conditions. In such a case, the profits during the second half of his employment contract should be extremely high for the manager’s contract to be renewed. Since extremely high profits are very unlikely to occur even with collusion, the manager anticipates that his contract is unlikely to be renewed, resulting in defection from the collusive agreement. However, rational managers fix such destabilisation by *ex-ante* agreeing that the collusive strategy entails collusion if and only if earlier firm profits during the same employment contract have not been too low. Then, the optimal collusive strategy consists of waves of competition, i.e. price wars in equilibrium.

The paper is organised as follows. Section 2 discusses related literature. Section 3 presents the model, identifies the key tradeoff, and shows that short-term contracts weakly increase cartel stability. Section 4 studies the impact of fixed salary components and serial collusion on cartel stability. Section 5 allows the employment contract to cover multiple interactions on the product market, thereby allowing for cyclical collusive pricing. Section 6 reinterprets the results in the light of firm financing, thereby showing how debt-financed firms can form more stable cartels than equity-financed firms. Section 7 concludes.

## 2 Related Literature

This paper relates to two strands of literature: (i) managerial incentives in cartels and (ii) collusion and price wars. I briefly discuss how the current paper relates to each in turn.

### Managerial incentives in cartels.

This paper departs from the traditional literature on cartels and collusion by separating the firm’s ownership and control so as to focus on the impact of short-term contracts on cartel stability and behavior. The papers discussed below deal with the impact of other common characteristics of managerial compensation on cartels.\(^5\) Spagnolo (2000) shows that stock-related compensation improves the stability of collusion: if stock markets have perfect foresight and are informed about the collusive agreement, then future punishment following a defection is immediately discounted in current stock prices, thereby instantaneously reducing the gain from defection.\(^6\) Moreover, Spagnolo (2005) argues

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\(^5\)The strategic delegation literature considers a related issue of how delegation affects the *competitive* outcome on the product market. The seminal papers are Fershtman and Judd (1987) and Sklivas (1987), which were preceded by Fershtman (1985) and Vickers (1985). Lambertini and Trombetta (2002) extend the latter to a repeated setting so as to allow for collusion.

\(^6\)In a market with stochastic, autocorrelated demand development, Neubecker (2005) confirms Spagnolo’s
that collusion is more stable when managers have a preference for ‘income smoothing’, that is, when managers are provided with low-powered incentives so that they prefer a smooth stream of profits over time. The manager is then averse to variance in profits, which disincentivises him to defect as that leads to a high profit today followed by low punishment profits in the future. Also, Spagnolo (2005) shows that well-chosen capped bonus plans allow collusion to be stable for any discount factor, since the gain from defection will be ‘capped’. In contrast to Spagnolo’s work, the current paper specifically allows contracts to be finite, thereby introducing a collusion-stabilising tradeoff based on the manager’s fear to be fired.

Since cartels are illegal, a written contract between the firm owner and the manager that explicitly induces the manager to form a cartel is not enforceable in court. Therefore, the firm owner needs to rely on a relational contract with manager to induce cartelisation. Chen (2008) shows how such a relational contract between the owner and the manager facilitates collusion as (i) it prevents the manager to cheat on the product market, because defection results in collusion breaking down in which case the owner will pay nothing to the manager and the relational contract breaks down, while (ii) the owner cannot cheat on the relational contract by inducing the manager to defect, because the owner cannot credibly commit to rewarding the manager for a defection. Thus, Chen (2008) concludes that there exists no self-enforcing relational contract that can induce the manager to cheat on the product market. This paper, however, circumvents the need for a relational contract as contracting on profits in a short-term contract introduces a new tradeoff that can be used to stabilise collusion.

The manager’s decision to form a cartel may interfere with other managerial duties. Aubert (2009) builds a model of double moral hazard in which the manager chooses whether to exert costly effort which increases profits, while either competing or colluding on the product market. In such a scenario, a high powered incentive contract aimed at inducing the manager to exert effort has the perverse effect of also inducing the manager to form a cartel. This conflict between incentives results in equilibrium contracts inducing a suboptimal effort level, leading to welfare losses even if there is no collusion in equilibrium. In a similar model of (single) moral hazard, Angelucci and Han (2010) consider a three-tier hierarchy, authority-shareholder-manager, so as to study the effects of intra-firm monitoring on fighting corporate crime, such as cartels. While Aubert (2009) and Angelucci and Han (2010) take a general approach to open up the black box of the profit-maximising firm so as to endogenously determine the optimal contract, the current paper specifically focuses on the exogenous short-termism of executive contracts.

More generally, Bull (1987) and Baker, Gibbons and Murphy (2002, 2006), for instance, study relational contracts. Such informal agreements and unwritten codes of conduct are

abundant within firms and have an important impact on the incentives of individuals. While Baker, Gibbons and Murphy (2002, 2006) show that the optimal allocation of decision rights minimises the maximum temptation to renege on relational contracts, the current paper takes the perspective of formal short-term renewable CEO contracts, while the cartel agreement is a relational contract due to its illegality.

**Cartels and price wars.** Empirical work shows that episodes of collusion often involve price wars – see for example Slade (1990), Harrington (2006) and Levenstein and Suslow (2006). As a founder of cartel theory, Stigler (1964) pointed out that prices are high during cartelisation, while *out-of-equilibrium* price wars occur after cartel breakdown. Since then, many authors have contributed to explaining patterns of elevated prices and price wars *in equilibrium.*

Such explanations include imperfect monitoring with demand uncertainty (Green and Porter, 1984; Abreu, Pierce and Stacchetti, 1986), demand cyclicity (Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991), learning about unknown demand parameters with exogenous demand shocks (Slade, 1989), renegotiation of the collusive pie (Levenstein, 1997), and inducing exit of ‘dying’ cartel members (Fershtman and Pakes, 2000).⁷ All these contributions treat the firm as profit-maximising integrated entities.

In contrast, the current paper is, to the best of my knowledge, the first to present an explanation for equilibrium price wars based on managerial issues. In particular, the fact that executives’ compensation depends on their performance during the *full* contractual employment period, induces them to optimally condition their pricing decision on profits realised earlier during their contractual employment period.

### 3 A Model of Cartel Stability With Short-Term Employment Contracts

This section introduces the model (3.1), identifies the key tradeoff associated with short-term contracts and cartel stability (3.2), shows that short-term contracts weakly improve cartel stability (3.3), performs comparative statics of the key Proposition (3.4), and discusses the robustness of the analysis (3.5).

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⁷More recent contributions include the cartel members’ concerns about detection by the antitrust authority (Harrington, 2004a), lifting prices after detection to push for an underestimate of damages in court (Harrington, 2004b), and cost variability in the presence of buyer detection (Harrington and Chen, 2006).
3.1 Set-up of the Model

Outline & players. Consider two firms $i \in \{1, 2\}$, where each firm $i$ is owned by shareholder $i$ who employs a manager $i$ to run the firm. In an infinitely repeated game, the managers interact with each other on the product market in each period $t \in \{1, \ldots, \infty\}$. I will refer to shareholders in the female form (she/her), managers in the male form (he/his) and firms in the neutral form (it/its). Figure 1 graphically summarises the players of the game.

![Figure 1. The players: shareholders and managers.](image)

Managerial action. In each period $t \in \{1, \ldots, \infty\}$, manager $i$’s action on the product market is $a_i \in \{N, C, D\}$: he either engages in Nash competition ($N$), colludes by forming a cartel ($C$), or unilaterally defects from the collusive agreement ($D$).

Managerial behavior stochastically affects the realisation of the firms’ profits. Let profit of firm $i$ in period $t$ be $\pi_t \geq 0$ with cumulative distribution function (CDF)

$$F_{a_i,a_j}(x) = \Pr[\pi_t \leq x | a_i, a_j], \forall x \geq 0,$$

which depends on action $a_i$ taken by manager $i$ and action $a_j$ taken by rival manager $j = 3-i$.

Assume that for any pair of actions $a_i$ and $a_j$, $F_{a_i,a_j}(x)$ is increasing over the entire domain $x \geq 0$, i.e. $\frac{\partial F_{a_i,a_j}(x)}{\partial x} > 0$ has full support. For notational convenience, denote firm $i$’s CDF as

1. $F_N(x) := F_{NN}(x)$ if both firms engage in Nash competition,
2. $F_C(x) := F_{CC}(x)$ if both firms engage in collusion,
3. $F_D(x) := F_{DC}(x)$ if firm $i$ defects from the collusive agreement, and
4. $F_T(x) := F_{CD}(x)$ if firm $j$ defects from the collusive agreement, that is, if firm $i$ has been tricked (‘T’).

8Gillian, Hartzell and Parrino’s (2009) empirical results show that an explicit short-term contract is more likely to be implemented when there is greater uncertainty about firm performance.
Defection is more profitable in expectation than collusion, which is in turn more profitable in expectation than Nash competition, in the sense of first order stochastic dominance, i.e.

\[ F_D(x) \leq F_C(x) \leq F_N(x), \]

for every \( x \geq 0 \) with a strict inequality for at least one value of \( x \). By definition of stochastic dominance, expected profit in period \( t \), \( E_a(\pi_t) \), thus satisfies \( E_D(\pi_t) > E_C(\pi_t) > E_N(\pi_t) \). Moreover, collusion first order stochastically dominates being tricked, i.e.

\[ F_C(x) \leq F_T(x), \]

for every \( x \geq 0 \) with a strict inequality for at least one value of \( x \). Thus, \( E_C(\pi_t) > F_T(\pi_t) \).

To stay in line with the literature and to focus on the impact of short-term contracts on cartel stability, I consider the grim-trigger collusive strategy: a manager colludes as long as his rival colluded in all previous periods, while reverting to Nash forever following a defection.

**Shareholder’s actions.** The shareholder appoints her manager for one period by offering a contract that is renewed for another period if and only if the firm’s profit \( \pi_t \) is above some threshold profit level \( \pi \in [0, \infty) \) chosen by the shareholder in period \( t = 0 \). Otherwise, the manager is fired and replaced by a new manager. This simple rule of ‘perform well or beat it’ is rationalised by (i) observations of CEOs and managers being fired after bad firm performance,\(^9\) (ii) the debate in academic journals as well as in the popular press that shareholders are focused on the short-term,\(^10\) and (iii) the fact that the stock market allows investors (shareholders) to invest in a variety of financial products, leading to the shareholder to require some minimum threshold profit level that at least ‘beats the stock market’.\(^11\)

In period \( t = 0 \), the shareholder also sets the managerial salary scheme, which consists of a fraction \( \beta \in [0, 1] \) of per-period profit \( \pi_t \), plus a fixed component \( \alpha \geq 0 \) in each period.\(^12,13\) Such a salary scheme is renegotiation proof as the shareholder has no incentive to change it at any point during any period, because the game is stationary and the salary in period \( t \) does not affect the manager’s behavior in any future period.

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\(^10\)See, for example, Bolton, Scheinkman, Xiong (2006).

\(^11\)Kaplan and Minton (2006) make the specific point that CEO replacement is related to firm performance relative to the performance of the overall stock market.

\(^12\)This paper does not consider shareholders colluding in setting managerial salary schemes, because it is likely to be difficult in practice for shareholders of different firms to coordinate. See Lambertini and Trombetta (2002) for a treatment of firm owners colluding in setting managerial incentive schemes.

\(^13\)Since the game is stationary, allowing managerial compensation to be based on profit realisations in previous periods would be irrelevant; for compensation schemes based on future profit realisations, see Spagnolo (2000) in the ‘Related Literature’ section.
Information. Managerial behavior is observable to the rival manager, but unobservable to shareholders. The motivation is that close managerial interaction on the product market reveals their actions to each other. However, as cartels are illegal, it would be virtually impossible for managers to credibly communicate to shareholders that there is a cartel without creating suspicion from the law enforcers.

Payoffs and objectives. In each period, the manager receives his salary, while the shareholder receives realised firm profit minus managerial salary. In period $t$, the shareholder’s net profit $\Pi_t(\pi_t)$ and the manager’s salary $S_t(\pi_t)$ are, respectively,

$$\Pi_t(\pi_t) = (1 - \beta) \pi_t - \alpha,$$
$$S_t(\pi_t) = \alpha + \beta \pi_t.$$

The manager has limited liability, which is taken care off by $\alpha$ and $\beta$ being non-negative by assumption. The shareholder and the manager are risk neutral, have zero outside options, discount payoffs with factor $\delta$, and maximise their discounted stream of expected payoffs.

Timing. In period $t = 0$, both shareholders independently set salary scheme $(\alpha, \beta)$, as well as the threshold profit level $\pi$ that a manager needs to realise to be reappointed. In all subsequent periods $t \in \{1, ..., \infty\}$, (i) the managers interact on the product market by taking action $a \in \{N, C, D\}$, (ii) firm profit $\pi_t$ is realised, and (iii) the manager is either fired or reappointed, depending on the realisation of profit. The timing of the game is graphically depicted in Figure 2.

![Figure 2. Timing of the game.](image)

3.2 Cartel Stability: Monetary and Reemployment Tradeoff

To focus on the main tradeoff and to keep the analysis clean, I assume for now that managerial salary entails no fixed component, i.e. $\alpha = 0$. Subsection 4.1 relaxes this assumption.
As usual in the literature on collusion, the measure of cartel stability is the lowest discount factor \( \delta \) such that the gain from defection does not exceed the expected discounted punishment: the lower is this discount factor, the more stable is the cartel.

**Benchmark Stability.** Before solving the game outlined above, consider the benchmark model in which the manager has a long-term employment contract: he is hired forever without being fired.\(^{14}\) In that case, the necessary and sufficient condition for collusion to be stable is

\[
\sum_{t=1}^{\infty} \delta^{t-1} \beta E_C(\pi_t) \geq \sum_{t=2}^{\infty} \delta^{t-1} \beta E_N(\pi_t),
\]

ifff

\[
\frac{\delta}{1-\delta} \left( E_C(\pi_t) - E_N(\pi_t) \right) \geq E_D(\pi_t) - E_C(\pi_t),
\]

which represents the familiar result that collusion is stable if and only if the immediate expected monetary gain from defection \((A)\) is not larger than the expected monetary loss from punishment in each future period \((B)\) discounted to the present \((C)\). Fraction \( \beta \) does not turn up in the stability condition as it cancels out in each term.

**Stability With Short-Term Contracts.** For notational convenience, denote by \( G_a(x) = 1 - F_a(x) = \Pr [\pi_t \geq x | a] \) the probability of realising profit \( \pi_t \geq x \) when the manager takes action \( a \). Thus, given that the shareholder requires a profit of at least \( \bar{\pi} \), the probability of reemployment for another period is \( G_a(\bar{\pi}) \) when the manager takes action \( a \). The following Lemma states stability of collusion in the model with short-term contracts.

**Lemma 1** With short-term contracts, collusion is stable if and only if

\[
\delta \left( \frac{I_a}{1 - \delta G_C(\bar{\pi})} E_C(\pi_t) - \frac{I_b}{1 - \delta G_N(\bar{\pi})} E_N(\pi_t) \right) \geq E_D(\pi_t) - E_C(\pi_t). \tag{3}
\]

\(^{14}\)In reality, even if the manager has a long-term employment contract, the shareholders may find some way to fire the manager, for example by creatively interpreting the “just cause” doctrine to terminate the contract and/or to settle on an appropriate “golden handshake” with the manager. In this paper, to focus on the impact of contract duration on cartels, I abstract away from such behavior and implicitly assume that the termination costs of a long-term contract are infinitely high.
Proof. Collusion is stable if and only if the discounted expected payoff from defection is not larger than the discounted expected payoff from collusion, i.e. if and only if

$$\sum_{t=1}^{\infty} (\delta G_C (\pi))^{t-1} \beta E_C (\pi_t) \geq \beta E_D (\pi_t) + \delta G_D (\pi) \sum_{t=1}^{\infty} (\delta G_N (\pi))^{t-1} \beta E_N (\pi_t),$$

iff.

$$\frac{E_C (\pi_t)}{1 - \delta G_C (\pi)} \geq E_D (\pi_t) + \delta G_D (\pi) \frac{E_N (\pi_t)}{1 - \delta G_N (\pi)},$$

which gives condition (3) when rearranging terms. \qed

The stability condition in Lemma 1 compares the immediate expected gain from defection (RHS) with the discounted expected future losses from punishment (LHS). Stability now depends on two interrelated tradeoffs: (i) the immediate expected monetary gain from defection compared with the discounted expected monetary loss from future punishment (monetary tradeoff), and (ii) the immediate increase in reemployment probability from defection compared to the future decrease in reemployment probability due to punishment (reemployment tradeoff). 15

The monetary tradeoff and the reemployment tradeoff are intertwined. To gain intuition, I rewrite condition (3) as

$$\frac{G_C (\pi) \delta}{1 - G_C (\pi) \delta} (E_C (\pi_t) - E_N (\pi_t)) + \left( \frac{G_C (\pi) \delta}{1 - G_C (\pi) \delta} - \frac{G_D (\pi) \delta}{1 - G_N (\pi) \delta} \right) \frac{E_N (\pi_t)}{1 - \delta G_N (\pi)} \geq \frac{E_D (\pi_t)}{1 - \delta G_D (\pi)} - \frac{E_C (\pi_t)}{1 - \delta G_C (\pi)},$$

which shows that collusion is stable if and only if

(A) the immediate expected monetary gain from defection is not larger than
(B) the future per-period expected monetary loss from punishment, taking into account that
(C) the probability of reemployment is $G_C (\pi)$ in each equilibrium period, which
(D) increases to $G_D (\pi)$ in the period of defection, while
(E) decreasing to $G_N (\pi)$ in all following punishment periods.

What is the intuition behind these effects and how do they compare to the benchmark model?

15As is the case in the benchmark stability condition, fraction $\beta$ cancels out in each term.
(A) Monetary gain. The immediate expected monetary gain from defection, \( E_D (\pi_t) - E_C (\pi_t) \), is the same as in the benchmark model. The reason is that the immediate expected monetary gain is not contingent on whether the manager will be rehired in the future.

(B) Monetary loss. Provided that a punishment period is reached, the per-period expected monetary loss from punishment, \( E_C (\pi_t) - E_N (\pi_t) \), is the same as in the benchmark model, because the expectations over profit realisations do not change by allowing managers to be fired.

(C) Continuation probability. In equilibrium, the probability that a manager is being reappointed is \( G_C (\pi) \) in each period, whereas this probability is one in the benchmark model. When discounting the expected monetary losses from punishment (B) to the present, we should therefore premultiply the discount factor by \( G_C (\pi) \). This decreases the manager’s discounted expected loss of future punishment compared to the benchmark model and thus has a collusion-destabilising effect.

(D) Reemployment gain. Defection results in a one-time increase in the probability of reemployment from \( G_C (\pi) \) to \( G_D (\pi) \). As a result, although the expected monetary payoff decreases to \( E_N (\pi_t) \) after defection, the continuation probability of the game increases from \( G_C (\pi) \) to \( G_D (\pi) \). This effectively leads to a decrease in the expected loss from punishment, which is indicated by the numerators of the factors that premultiply \( E_N (\pi_t) \) in condition (4). This decrease in the loss from punishment amounts to destabilising collusion.

(E) Reemployment loss. Punishment leads to a decrease in the probability of reappointment from \( G_C (\pi) \) to \( G_N (\pi) \) in all future periods. Therefore, not only decreases the expected monetary payoff to \( E_N (\pi) \), the continuation probability in all future periods of the game decreases from \( G_C (\pi) \) to \( G_N (\pi) \). This makes punishment fiercer through an increase in the expected loss from punishment, which is indicated by the denominators of the factors that premultiply \( E_N (\pi_t) \) in condition (4). This increase in the loss from punishment amounts to stabilising collusion.

3.3 Optimal Short-Term Contracts Weakly Increase Cartel Stability

Given profit threshold level \( \bar{\pi} \), Proposition 1 states the characteristics of the probability distributions over profits \( F_a (\cdot) \) such that collusion with short-term contracts is more stable than collusion with long-term contracts.
Proposition 1  Collusion with short-term contracts and threshold profit level $\bar{\pi}$ is more stable than collusion with long-term contracts if and only if there exists a $\delta \in [0, 1)$ such that

$$\frac{E_C(\bar{\pi}_t)}{E_N(\bar{\pi}_t)} < \frac{1 - \delta G_C(\bar{\pi})}{1 - \delta G_N(\bar{\pi})} \frac{1 - \delta G_N(\bar{\pi}) - (1 - \delta) G_D(\bar{\pi})}{1 - G_C(\bar{\pi})}.$$  \hspace{1cm} (5)

Proof. Collusion is more stable compared to the benchmark model if and only if condition (3) is satisfied for more discount factors than condition (7), i.e. if and only if there exists a $\delta \in [0, 1)$ such that

$$\delta \left( \frac{G_C(\bar{\pi})}{1 - G_C(\bar{\pi})} E_C(\bar{\pi}_t) - \frac{G_D(\bar{\pi})}{1 - G_N(\bar{\pi})} E_N(\bar{\pi}_t) \right) > \frac{\delta}{1 - \delta} (E_C(\bar{\pi}_t) - E_N(\bar{\pi}_t)),$$  \hspace{1cm} (6)

which leads to condition (5) when simplifying. ■

Short-term contracts stabilise collusion if and only if (i) the increase in reappointment probability during defection from $G_C(\bar{\pi})$ to $G_D(\bar{\pi})$ is relatively small compared to the decrease in reappointment probability during punishment $G_C(\bar{\pi})$ to $G_N(\bar{\pi})$ (RHS),\(^{16}\) while (ii) the monetary loss during punishment from $E_C(\bar{\pi}_t)$ to $E_N(\bar{\pi}_t)$ is not too large (LHS). The intuition is based on the following three forces.

First, with a short-term contract, the manager is \textit{ex-ante} unsure whether he will be rehired even if he colludes. This has a destabilising impact on collusion as it incentivises the manager to deviate so as to grab as much salary as possible in the current period, while at the same time increasing the immediate probability of reemployment.

Second, short-term contracts introduce the reemployment tradeoff, which has a stabilising impact on collusion if and only if the immediate increase in reemployment probability resulting from defection is small compared to the future decrease in reemployment probability resulting from punishment, that is, if and only if $G_D(\bar{\pi}) - G_C(\bar{\pi})$ is small relative to $G_C(\bar{\pi}) - G_N(\bar{\pi})$. If the reemployment tradeoff is sufficiently ‘positive’ for collusive stability, it can offset the first force.

Third, the higher is the reduction in expected salary resulting from punishment, the less does the manager care about being rehired in the future, that is, the lower is the impact of the reemployment tradeoff.

Altogether, the reemployment tradeoff (force 2) stabilises collusion by more than force 1 destabilises collusion if and only if (i) the increase in reappointment probability during defection is small compared to the decrease in reappointment probability during punishment, and (ii) the monetary loss from punishment is not too large.

\(^{16}\)Differently said, $G_a(\bar{\pi})$ is sufficiently concave in the action $a \in \{ N, C, D \}$.  

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The shareholder optimally sets profit threshold $\bar{\pi}$ so as to make collusion as stable as possible.

**Proposition 2** *Short-term contracts weakly stabilise cartels compared to long-term contracts.*

**Proof.** If condition (6) holds for some $\bar{\pi}$, then the shareholder chooses one of those $\bar{\pi}$ such that (3) is easiest to satisfy. If condition (6) does not hold for any $\bar{\pi}$, then the shareholder sets $\bar{\pi} = 0$, thereby effectively mimicking a long-term contract. ■

When there exists a profit level such that condition (6) is satisfied, then the optimal short-term contract stabilises collusion. When no such profit level exists, the optimal short-term contract entails the same collusive stability as is the case with a long-term contract. The reason is that a short-term renewable contract can always replicate an indefinite employment contract by setting the profit threshold level in such a way that even a manager that competes in the product market will achieve that profit level, i.e. $\bar{\pi} = 0$. A short-term contract then effectively mimics the collusive stability associated with a long-term contract.

### 3.4 Comparative Statics

The only assumptions on profit distributions are full support over the domain $\pi_t \geq 0$ and first order stochastic dominance of defection over collusion over Nash. Therefore, any change in profit distributions has an ambiguous impact on the direction as well as the size of the change of both $E_a(\pi_t)$ and $G_a(\bar{\pi})$. The ultimate impact on Proposition 1’s condition (5) then depends on the relative changes in $E_a(\pi_t)$ and $G_a(\bar{\pi})$, for every $a \in \{N, C, D\}$.

As a result, even very stylised comparative statics with respect to profit distributions leads to a large number of cases entailing tedious analyses without generating substantive insights in the model. The key point remains that changes in the parameters facilitate collusive stability if $G_a(\bar{\pi})$ becomes more concave in the action $a \in \{N, C, D\}$ and/or the monetary loss from punishment becomes smaller – see first paragraph after Proposition 1.

However, since the distribution over profit when defecting only appears once in condition (5) in the form of $G_D(\bar{\pi})$, some meaningful comparative statics are possible.

**Proposition 3** *Condition (5) is easier (more difficult) to satisfy if distribution $F_D(x)$ is*

1. shifted to the left (right),
2. more (less) dispersed, while keeping $E_D(\pi_t)$ fixed and $\bar{\pi} < E_D(\pi_t)$,
3. less (more) dispersed, while keeping $E_D(\pi_t)$ fixed and $\bar{\pi} > E_D(\pi_t)$.
Proof. See Appendix X. ■

The intuition is that changes entailing a reduction (increase) in $G_D(\pi)$, while not affecting the other parameters, make condition (5) easier (more difficult) to satisfy, because such changes make defection more attractive to the manager in the sense that the probability of reemployment increases after a defection.

3.5 Hiring and Firing Costs

This Subsection discusses the robustness of the model with respect to hiring and firing costs.

**Hiring costs.** The shareholder’s strategy to hire a new manager when profit turns out to be low is renegotiation proof, because hiring costs are assumed to be zero and all managers are of the same type. It is then costless to hire a new manager. In contrast, if hiring costs are positive,\(^{17}\) we may expect the current manager to haggle with the shareholder to renew his contract after a low profit realisation.

However, with positive hiring costs, the shareholder finds it indeed ex-post optimal to hire a new manager after a low profit realisation as long as this strategy (i) improves the stability of collusion, while (ii) the expected additional profit generated by collusion exceeds the cost of hiring a new manager. That is, the strategy is renegotiation proof as long as the cost of hiring a new manager does not exceed its expected benefit. Condition (i) holds if managers revert to Nash when a shareholder defects from her strategy of hiring a new manager after a low profit realisation. Condition (ii) is likely to hold in practice as the firm’s benefit of collusion is expected to be much higher than the costs associated with hiring a new manager.

**Firing costs.** The model treats a long-term contract as an infinite contract such that the manager is never dismissed, thereby implicitly assuming that the termination cost of a long-term contract is infinitely high. Suppose however that a long-term contract can be terminated by paying some fixed firing cost (golden parachute). In such cases, a long-term contract displays the same collusion-stabilising effect of a short-term contract, provided that the expected termination cost is smaller than the expected gain from collusion.

4 Fixed Salary Components and Serial Colluders

This section considers the impact on cartel stability of fixed salary components (4.1), as well as the possibility to revert from punishment to collusion after replacement of a manager (4.2).

\(^{17}\)E.g. due to search costs, costs associated with drafting a new contract, etc.
4.1 Fixed Salary Components Weakly Increase Cartel Stability

This subsection relaxes the assumption that contracts contain no fixed salary component, that is, the shareholders are now free to set any $\alpha \geq 0$ and $\beta \geq 0$. I thereby show that a fixed salary component affects cartel stability when short-term contracts are in place, but not when long-term contracts are in place.

**Benchmark stability.** Before deriving the stability in the model with short-term contracts, we note that the stability with long-term contracts is unchanged when allowing for a fixed salary component, because collusion is stable if and only if

$$\delta \left[ (\alpha + \beta E_C (\pi_t)) - (\alpha + \beta E_N (\pi_t)) \right] \geq \left( \alpha + \beta E_D (\pi_t) \right) - \left( \alpha + \beta E_N (\pi_t) \right) ,$$

iff.

$$\delta \left( E_C (\pi_t) - E_N (\pi_t) \right) \geq E_D (\pi_t) - E_C (\pi_t) ,$$  \hspace{1cm} (7)

which is indeed equivalent to benchmark stability condition (2). The reason is that the fixed salary component is paid out in every period and therefore cancels out.

**Stability with fixed-term renewable contracts.** What is the stability with short-term contracts and a fixed salary component?

**Lemma 2** Allowing for a fixed salary component $\alpha$, collusion is stable if and only if

$$\delta \left( \frac{G_C (\pi)}{1 - \delta G_C (\pi)} E_C (\pi_t) - \frac{G_D (\pi)}{1 - \delta G_N (\pi)} E_N (\pi_t) + K \frac{\alpha}{\beta} \right) \geq E_D (\pi_t) - E_C (\pi_t) ,$$

where $K = \frac{G_C (\pi)}{1 - \delta G_C (\pi)} - \frac{G_D (\pi)}{1 - \delta G_N (\pi)}$.

**Proof.** Collusion is stable if and only if the discounted expected payoff from defection is not larger than the discounted expected payoff from collusion, i.e. if and only if

$$\delta \left( \frac{G_C (\pi)}{1 - \delta G_C (\pi)} (\alpha + \beta E_C (\pi_t)) - \frac{G_D (\pi)}{1 - \delta G_N (\pi)} (\alpha + \beta E_N (\pi_t)) \right) \geq (\alpha + \beta E_D (\pi_t)) - (\alpha + \beta E_C (\pi_t)) ,$$

which gives condition (8) when rearranging terms. ■

For now, we note that this stability condition differs from the stability condition without the fixed salary component (see Lemma 1) by the term $\delta K \frac{\alpha}{\beta}$ on the LHS. I provide intuition for this difference below. Defining $\tilde{\delta} = \frac{G_D (\pi) - G_C (\pi)}{G_C (\pi) (G_D (\pi) - G_N (\pi))}$, the following Lemma states the optimal salary scheme from the shareholder’s perspective and the impact of the fixed salary component $\alpha$ on the stability of collusion.
Lemma 3 If $\delta > \tilde{\delta}$, collusion is stable and the optimal wage schedule entails an arbitrarily small fraction of firm profit $\beta > 0$ and a positive fixed salary component $\alpha > 0$. If $\delta \leq \tilde{\delta}$, collusion is stable if and only if condition (3) holds and the optimal wage schedule entails an arbitrarily small fraction of firm profit $\beta > 0$ and a zero fixed salary component $\alpha = 0$.

Proof. We have that $K > 0 \Leftrightarrow \delta > \tilde{\delta}$. Therefore, if $\delta > \tilde{\delta}$, the LHS of condition (8) can be made arbitrarily large by setting $\alpha$ to any positive value and $\beta$ arbitrarily small. However, if $\delta \leq \tilde{\delta}$, it is best to set $\alpha = 0$ (because $K < 0$), and collusion is stable if and only if condition (3) holds.

What is the intuition? Dividing all managerial payoffs by $\beta$, we have that with action $a \in \{N, C, D\}$ the manager’s expected payoff is $\frac{\alpha}{\beta} + E_a(\pi_t)$. The normalised fixed component $\frac{\alpha}{\beta}$ cancels out in ‘the gain from defection’ – i.e. the RHS of condition (8) –, because the fixed component is being paid in the current period independent of managerial behavior, that is, independent of whether the manager colludes or defects. However, the fixed component does not cancel out in the ‘future losses from punishment’ – i.e. the LHS of condition (8) –, because the manager is being paid the fixed component in a future period if and only if the manager is actually hired in that future period, the probability of which depends on whether the manager colludes or defects in the current period.

If the ‘future losses from punishment with regards to the fixed component’ is positive – i.e. $K > 0$ –, then it is optimal for the shareholder to set $\alpha > 0$ so as to use the fixed salary component to increase stability. The shareholder can amplify this effect by making the fixed component a relatively important part of the wage schedule compared to the variable component, that is, by setting $\beta$ sufficiently small relative to $\alpha$. Note that if $\delta > \tilde{\delta}$, stability does not depend on the size of $\pi$, because the shareholder can play around with $\alpha$ and $\beta$ to make collusion stable for every $\pi$. The next Proposition summarises these results.

Proposition 4 Allowing for a salary scheme with a fixed component weakly stabilises collusion with short-term contracts, while not affecting collusive stability with long-term contracts.

Proof. If $\tilde{\delta}$ satisfies condition (3), then we know from Lemma 3 that it is optimal to set $\alpha = 0$, resulting in condition (3) and (8) to be equivalent. If $\tilde{\delta}$ violates condition (3), then we know from Lemma 3 that it is optimal to set $\alpha > 0$, resulting in condition (8) to hold for a larger set of discount factor than condition (3). From benchmark condition (7) we know a fixed salary component does not affect collusive stability.
4.2 Serial Colluders Destabilise Cartels

Also,

\[ F_{CD}(x) \leq F_{CC}(x) \]

This subsection allows managers to revert from punishment to collusion after one of the managers has been replaced. The intuitive motivation is that mistrust is eliminated when a new manager is appointed, which allows for collusion with the newly appointed manager.

**Proposition 5** If replacement of a manager allows for switching from punishment back to collusion, then collusion is stable for less discount factors than if collusion cannot be restored.

**Proof.** Let \( V_C = \frac{E_C(\pi_t)}{1-\delta G_C(\pi)} \) be the manager’s continuation value of the collusive state. When collusion cannot (can) be restored, let \( V_D(V_D^r) \) and \( V_N(V_N^r) \) be the continuation value of the punishment state and defection state state, respectively. From 1, \( G_T(\pi) < G_C(\pi) \) is the probability of hitting at least profit level \( \pi \) when the rival manager defected from the collusive agreement.

Without restoration of collusion, collusion is stable if and only if \( V_C \geq V_D \), where

\[ V_D = E_D(\pi_t) + \delta G_D(\pi) V_N. \]

With restoration of collusion, collusion is stable if and only if \( V_C \geq V_D^r \), where

\[ V_D^r = E_D(\pi_t) + \delta G_D(\pi) [G_T(\pi) V_N^r + (1 - G_T(\pi)) V_C], \]

and \( V_N^r \) is determined by solving

\[ V_N^r = E_N(\pi_t) + \delta G_N(\pi) [G_N(\pi) V_N^r + (1 - G_N(\pi)) V_C], \]

\[ \iff V_N^r = \frac{E_N(\pi_t) + \delta G_N(\pi) (1 - G_N(\pi)) V_C}{1 - \delta (G_N(\pi))^2}. \]

Noting that \( V_N = \frac{E_N(\pi_t)}{1-\delta G_N(\pi)} \), straightforward algebra gives \( V_N^r > V_N \), and therefore we have \( V_D^r > V_D \).

If a manager can switch from punishment to collusion with a newly appointed rival manager, this effectively results in the continuation value of defection to increase for two reasons. First, defection results in an immediate decrease in the probability that the rival manager is reappointed, that is, an increase in the probability that a new rival manager is appointed and
collusion is restored instantaneously.\textsuperscript{18} Second, the continuation value of defection is higher as in each punishment period collusion will be restored with probability $G_N(\pi)$.

5 Short-Term Contracts Entail Equilibrium Price Wars

Typically, CEO contracts span several years.\textsuperscript{19} Therefore, this section studies the model when managers interact \textit{multiple} times on the product market \textit{within} the course of their contractual employment period. This allows for dynamic pricing in the sense that a manager may find it optimal to make his behavior during his employment period contingent on profit realisations occurring earlier within the same contractual period.

5.1 Set-up of the Dynamic Model

To keep the analysis clean and to focus on the impact of multiple interactions within the same employment period on cartel behavior, I assume that managers interact on the product market \textit{twice} during their employment period, and that they do \textit{not} discount their payoffs within the employment period. More than two interactions within the same contractual period and ‘within-contract’ discounting would not bring essential insights, while mathematically complicating the model substantially.

Consider the model as outlined in subsection 3.1, but now suppose that each period $t \in \{1, 2, ..., \infty\}$ consists of two sequential stages $i \in \{1, 2\}$. In both stages, the manager takes action $a_i \in \{N, C, D\}$, resulting in profit $\pi_i \geq 0$, and the manager is reappointed for the next period $t + 1$ if and only if $\pi_1 + \pi_2 \geq \pi$. The conditional probability distributions over profit are the same as outlined in subsection 3.1 and independent between periods and stages. Also, as before, managerial behavior is observed by the rival manager immediately after taking the action, thus allowing for punishment during the next interaction. The new timing of the game is depicted in Figure 3.

\textsuperscript{18}In the extreme case in which $G_F = 0$, collusion is unstable for every discount factor $\delta \in [0, 1]$ as defection automatically results in a new rival manager being appointed immediately: collusion is restored instantaneously and no punishment occurs.

\textsuperscript{19}See, for example, Schwab and Thomas (2006) and Gillian, Hartzell and Parrino (2009).

Figure 3. Timing of the dynamic game.
In terms of notation, define $G_{a_1a_2}(\pi) = \Pr[\pi_1^t + \pi_2^t \geq \pi | a_1, a_2]$ as the ex-ante probability of realising at least aggregate profit $\pi$ in period $t$, provided that the manager takes action $a_1$ in stage 1 and $a_2$ in stage 2. Noting that $E_a(\pi_1^t) = E_a(\pi_2^t)$, I leave out the argument $\pi_i^t$ in the expected value $E_a(\pi_i^t)$ for ease of notation, which then simply writes as $E_a$.

To derive more precise results, I assume the following regularity condition,

$$\frac{\partial (G_D(x) - G_C(x))}{\partial x} > 0,$$

that is, the difference in reemployment probability between defection and collusion increases in the profit threshold $\pi$. The intuitive rationale is that increasing the profit threshold $\pi$ reduces the probability of reemployment more when colluding than when defecting.

5.2 Collusion in Both Stages

The most straightforward grim-trigger collusive strategy is to collude in both stages as long as the rival manager colluded in all previous stages in all previous periods, while punishing forever otherwise. The stability of this strategy is pinned down in the following Lemma.

**Lemma 4** Collusion in both stages is a stable strategy if and only if

$$2\delta\left(\frac{G_C(\pi)}{1 - \delta G_{CC}(\pi)}E_C - \frac{G_D(\pi)}{1 - \delta G_{NN}(\pi)}E_N\right) \geq E_D - E_C,$$

(10)

**Proof.** See Appendix A. ■

For a collusive strategy to be stable in the dynamic model, the manager must (i) not defect in stage 1, and (ii) not defect in stage 2 after any possible profit realisation in stage 1, $\pi_1^t \geq 0$. Consider first the managerial incentive to defect in stage 2. The monetary tradeoff is independent of the profit realisation in stage 1: defection immediately increases the manager’s expected salary from $E_D$ to $E_C$, while decreasing it from $E_C$ to $E_N$ in each future punishment stage. However, the reemployment tradeoff does depend on the profit realisation in stage 1: defection in stage 2 immediately increases the probability of reemployment from $G_C(\pi - \pi_1^t)$ to $G_D(\pi - \pi_1^t)$, while decreasing it from $G_{NN}(\pi)$ to $G_{CC}(\pi)$ in each future punishment period. Since defection increases the immediate reemployment probability more the higher is the profit level that needs to be attained (by the regularity condition), the manager finds it most attractive to defect when the profit realisation in the first stage was low, i.e. $\pi_1^t = 0$, because then he must hit a high profit of at least $\bar{\pi} - \pi_1^t = \bar{\pi}$ in the second stage, the probability of which is substantially increased when defecting.
Consider now the managerial incentive to defect in stage 1. This is lower than the managerial incentive to defect in stage 2 after the realisation of profit $\pi_1^t = 0$. The reason is two-fold. First, in stage 1 the manager has two interactions to attain aggregate profit $\pi_1^t + \pi_2^t \geq \bar{\pi}$, while after the realisation of a zero profit in stage 1 the manager has only one interaction left to attain profit $\pi_2^t \geq \bar{\pi}$ in stage 2. Second, defection in stage 1 is immediately followed by punishment in stage 2 \emph{without} intra-period discounting, while defection in stage 2 is followed by punishment in the next period, which entails discounting.

Combining the arguments, collusion in both stages is a stable strategy if and only if the manager is patient enough not to defect in the second stage after the realisation of a zero profit in the first stage, which is represented by condition (10).

### 5.3 Collusion Conditional on Profit Realisation in Stage 1

The dynamic nature of the game allows the manager to condition his action in the second stage on the profit realisation in the first stage. By the argument based on the regularity condition in the previous subsection, we have that if the manager is patient enough to collude in stage 2 after profit realisation $\pi_1^t = \tau$, then he is also patient enough to collude in stage 2 after profit realisation $\pi_1^t > \tau$, but may not be patient enough to collude in stage 2 after any profit realisation $\pi_1^t < \tau$. The intuition is that the closer is stage 1’s profit realisation to the profit threshold $\bar{\pi}$, the less attractive is defection in terms of increasing the immediate reemployment probability.

Compared to collusion in both stages, the manager potentially increases cartel stability by adopting a strategy that entails collusion in stage 2 only after profit realisation $\pi_1^t \geq \tau$, which I define as $\tau$-\emph{conditional collusion}. The reason is that the stability of collusion in both periods is determined by the manager’s incentive to defect in stage 2 after a zero profit realisation in stage 1, while $\tau$-conditional collusion removes this concern as the manager competes as part of the collusive strategy for all $\pi_1^t < \tau$. The next two subsections study the stability of $\tau$-conditional collusion in stage 2 when the manager either colludes (Subsection 5.3.1) or competes in stage 1 (Subsection 5.3.2).

#### 5.3.1 Collusion in Stage 1 and Conditional Collusion in Stage 2

Consider the collusive strategy in which the manager colludes in stage 1, while colluding in stage 2 if and only if $\pi_1^t \geq \tau$. Define the \emph{ex-ante} reemployment probability associated with this strategy as $P_C(\bar{\pi}, \tau) = \int_\tau^\infty \Pr (\pi_1^t = x | a = C) G_C (\bar{\pi} - x) \, dx + \int_0^\tau \Pr (\pi_1^t = x | a = C) G_N (\bar{\pi} - x) \, dx$, and denote the \emph{ex-ante} expected per-period payoff as $K_C (\tau) = (1 + G_C (\tau)) E_C + (1 - G_C (\tau)) E_N$. The following Lemma pins down the stability of this strategy.
Lemma 5  Collusion in stage 1 and $\tau$-conditional collusion in stage 2 is a stable strategy if and only if

$$G_C (\tau) (E_C - E_N) + \delta \left[ \frac{P_C (\pi, \tau)}{1 - \delta P_C (\pi, \tau)} K_C (\tau) - \frac{G_{DN} (\pi)}{1 - \delta G_{NN} (\pi)} 2E_N \right] \geq E_D - E_C, \text{ and (11)}$$

$$\delta \left( \frac{G_C (\pi - \tau)}{1 - \delta P_C (\pi, \tau)} K_C (\tau) - \frac{G_D (\pi - \tau)}{1 - \delta G_{NN} (\pi)} 2E_N \right) \geq E_D - E_C, \text{ (12)}$$

and is potentially more stable than collusion in both stages.

Proof. See Appendix B. ■

By the regularity condition, the lower is stage 1’s profit realisation, the more attractive is defection in stage 2. Thus, the critical constraint for the manager’s equilibrium behavior in stage 2 to be stable is determined by the lowest possible profit realisation in stage 1 that still entails collusion in stage 2, that is, profit realisation $\pi_1^t = \tau$. This constraint is represented by condition (12): $\tau$-conditional collusion in stage 2 (i) potentially makes collusion more stable than sure collusion in stage 2 as defection entails a lower increase in reemployment probability, that is $G_D (\pi - \tau) - G_C (\pi - \tau) < G_D (\pi) - G_C (\pi)$, but (ii) may also decrease collusive stability as the equilibrium payoff is lower than collusion in both stages, that is $K_C (\tau) < 2E_C$, and its equilibrium continuation probability is lower, that is $P_C (\pi, \tau) < G_{CC} (\pi)$. The net effect depends on the precise specification of the density functions over profit and the level of $\tau$.

If the manager is patient enough to collude in stage 2 after $\pi_1^t \geq \tau$, he is not necessarily also patient enough to collude in stage 1. The intuition is that in stage 1, the manager has two interactions to achieve aggregate profit $\pi_1^t + \pi_2^t \geq \pi$, while in stage 2 the binding constraint prescribes that the manager has one interaction to achieve profit $\pi_2^t \geq \pi - \tau$. Depending on the precise specification of the density functions over profit and the level of $\tau$, the constraint in stage 1 (i.e. constraint (11)) may or may not be the binding constraint.

### 5.3.2 Competition in Stage 1 and Conditional Collusion in Stage 2

If constraint (11) is the binding constraint for collusion in stage 1 and $\tau$-conditional collusion in stage 2 to be stable, the manager may circumvent this and increase stability by constructing a collusive agreement that entails competition in stage 1 and $\tau$-conditional collusion in stage 2. Define the ex-ante reemployment probability associated with this strategy as $P_N (\bar{\pi}, \tau) = \int_{\tau}^{\infty} \text{Pr} (\pi_1^t = x | a = N) G_C (\bar{\pi} - x) \, dx + \int_{0}^{\tau} \text{Pr} (\pi_1^t = x | a = N) G_N (\bar{\pi} - x) \, dx$, and denote the ex-ante expected per-period payoff as $K_N (\tau) = G_N (\tau) E_C + (2 - G_N (\tau)) E_N$. The following Lemma pins down the stability of this strategy.
Lemma 6  Competition in stage 1 and \( \tau \)-conditional collusion in stage 2 is a stable strategy if and only if

\[
\delta \left( \frac{G_C(\pi - \tau)}{1 - \delta P_N(\pi, \tau)} K_N(\tau) - \frac{G_D(\pi - \tau)}{1 - \delta G_{NN}(\pi)} 2E_N \right) \geq E_D - E_C, \tag{13}
\]

and is potentially more stable than (i) collusion in both stages, and (ii) collusion in stage 1 and \( \tau \)-conditional collusion in stage 2.

Proof. See Appendix C. ■

Stability is not a concern in the first stage, simply because the collusive strategy prescribes competition in stage 1. Similarly to the argument presented in the previous subsection, the binding constraint for the collusive strategy to be stable in the second stage is determined by profit realisation \( \pi^1 \tau \). If constraint (11) is the binding constraint in Lemma 5, then competition in stage 1 instead of collusion in stage 1 increases cartel stability provided that constraint (13) is satisfied for a larger set of discount factors than constraint (11), which depends on the precise specification of the density functions over profit and the level of \( \tau \).

5.4 Equilibrium Price Wars, Stability and Profitability

This subsection collects the results from the previous three Lemmas and summarises the behavioral implications when the managers adopt a strategy with \( \tau \)-conditional collusion in stage 2.

Proposition 6  Short-term contracts spanning multiple managerial interactions on the product market entail price wars in equilibrium, thereby increasing cartel stability, while reducing cartel profitability.

Proof. See Appendix D. ■

Consider a manager colluding in both periods. Then, the critical constraint for collusion to be stable is whether the manager is patient enough to collude in stage 2 after a low profit realisation in stage 1, because the manager has only one opportunity left to attain its contractual profit threshold level. However, if this jeopardises cartel stability, then the managers may fix stability by adopting a collusive strategy in which they always collude in the first stage, while colluding in the second stage only if profit realisation in the first stage was relatively high. In that way, they eliminate the concern of defection after a low profit realisation, because the collusive equilibrium itself already specifies competition after a low profit realisation.
It may be the case that such a strategy is still not stable, because the managers have incentives to defect in the first stage now. Then, the managers can change their strategy and compete in stage 1 as part of the collusive equilibrium, while colluding in stage 2 only if the profit realisation in stage 1 is high enough.

Both strategies entail competition (price wars) as part of the collusive equilibrium. Although such price wars increase cartel stability, they decrease its profitability, because by stochastic dominance (i) expected profit of competition is lower than expected profit of collusion, and (ii) the continuation probability of collusion in both stages is higher than the continuation probability when the manager competes in the second (and possibly: the first) stage with some probability.

6 Debt-Financing Facilitates Cartels

The model can also be interpreted in the light of firm financing, thereby showing how firms that are financed by debt, i.e. highly leveraged firms, can form more stable cartels than firms financed by equity. This finding is consistent with Schleifer and Vishny (1992) and Khanna and Tice (2000), though through a different mechanism.

In the spirit of the model presented in this paper, consider the stylised situation of a firm fully financed by short-term debt. Assume that (i) the bank liquidates this firm immediately if it misses out on a periodical repayment, and (ii) all the profits left after the repayment are reinvested in the firm. Thus, to meet its repayment obligations, this debt-financed firm must each year make a profit of at least the amount that needs to be repaid. If such a firm colludes with its rivals, it faces the following tradeoff: defection makes it more likely to meet its current repayment obligation (due to a higher expected profit in the current period), but punishment makes it less likely to meet its future payment obligations (due to lower expected profits in future punishment periods). In other words, defection makes liquidation less likely in the current period, but more likely in future periods; this parallels the managerial tradeoff that defection makes being fired less likely in the current period, but more likely in future periods. Therefore, debt-contracts can be designed in such a way to stabilise collusion and may be a source of equilibrium price wars.

7 Concluding Remarks

This paper has presented a mechanism through which commonly observed short-term renewable employment contracts improves cartel stability, while dynamically affecting firms’ pricing behavior. Motivated by empirical observations, the model takes such short-term re-
newable contracts as exogenously given, thereby abstracting away from the question which type of employment contracts would endogenously arise when the shareholders’ main goal is to induce the CEO to either form or stay away from a cartel. This issue is at the center of the analysis in Aubert (2009) and Angelucci and Han (2010).

The model assumes that managers can observe each other’s actions. However, if managers would not be able to observe the rival’s action, Green and Porter (1984) suggest that the collusive strategy entails reversion to Nash for some periods after a low profit realisation. How such unobservability of actions interacts with the impact of short-term renewable contracts on cartel stability is left for future research.

Finally, the mechanism outlined in this paper implicitly assumes that employment contracts are observable to rival managers. Although recently an increasing number of firms publish the remuneration package and employment conditions of their executives, this may not always be the case. When rivals’ employment contracts are not observable, then correct beliefs about these contracts may still induce the cartel-stabilising mechanism. An extension of the current model with unobservable contracts can then benefit from insights of the strategic delegation literature with unobservable contracts – see REFS.
Appendix

A  Proof Lemma 4

Proof. The manager has no incentive to defect in stage 1 if and only if

$$\frac{2E_C}{1 - \delta G_{CC}(\pi)} \geq E_D + E_N + \delta G_{DN}(\pi) \frac{2E_N}{1 - \delta G_{NN}(\pi)},$$

while, given the realisation of $\pi_1^t$, he has no incentive to defect in stage 2 if and only if

$$E_C + \delta G_C(\pi - \pi_1^t) \frac{2E_C}{1 - \delta G_{CC}(\pi)} \geq E_D + \delta G_D(\pi - \pi_1^t) \frac{2E_N}{1 - \delta G_{NN}(\pi)}.$$

These two conditions rewrite as

$$\min \left\{ E_C - E_N + 2\delta \left( \frac{G_{CC}(\pi)}{1 - \delta G_{CC}(\pi)} E_C - \frac{G_{DN}(\pi)}{1 - \delta G_{NN}(\pi)} E_N \right) ; \right.$$  \hspace{1cm} (14) \\

$$\left. 2\delta \left( \frac{G_C(\pi - \pi_1^t)}{1 - \delta G_{CC}(\pi)} E_C - \frac{G_D(\pi - \pi_1^t)}{1 - \delta G_{NN}(\pi)} E_N \right) \right\} \geq E_D - E_C \hspace{1cm} (15)$$

for every profit realisation $\pi_1^t \geq 0$. Subcondition (15) can be rewritten as

$$2\delta \left[ \frac{G_C(\pi - \pi_1^t)}{1 - \delta G_{NN}(\pi)} E_N + G_C(\pi - \pi_1^t) \left( \frac{E_C}{1 - \delta G_{CC}(\pi)} - \frac{E_N}{1 - \delta G_{NN}(\pi)} \right) \right] \geq E_D - E_C,$$

where we note that (i) $G_C(\pi - \pi_1^t)$ is increasing in $\pi_1^t$, and (ii) $G_C(\pi - \pi_1^t) - G_D(\pi - \pi_1^t)$ is also increasing in $\pi_1^t$ by regularity condition (9). Thus, subcondition (15) is most difficult to satisfy if $\pi_1^t = 0$, that is, if

$$2\delta \left( \frac{G_C(\pi)}{1 - \delta G_{CC}(\pi)} E_C - \frac{G_D(\pi)}{1 - \delta G_{NN}(\pi)} E_N \right) \geq E_D - E_C. \hspace{1cm} (16)$$

Is (16) more difficult to satisfy than subcondition (14)? Subtracting the LHS of (14) from the LHS of (16) we get

$$A := E_C - E_N + 2\delta \left( \frac{G_{CC}(\pi) - G_C(\pi)}{1 - \delta G_{CC}(\pi)} E_C - \frac{G_{DN}(\pi) - G_D(\pi)}{1 - \delta G_{NN}(\pi)} E_N \right),$$

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where we rewrite

$$G_{CC}(\bar{\pi}) - G_{C}(\bar{\pi}) = \int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx - G_{C}(\bar{\pi})$$

(17)

$$= \int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx$$

(18)

$$+ \int_{\pi}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx - G_{C}(\bar{\pi})$$

(19)

$$= \int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx$$

(20)

$$+ \int_{\pi}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) dx - G_{C}(\bar{\pi})$$

$$= \int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx + G_{C}(\bar{\pi}) - G_{C}(\bar{\pi})$$

(21)

where (17) follows by definition, (18) follows by splitting up the integral, (19) follows by noting that $G_{C}(\bar{\pi} - x) = 1$ for every $x \in [\bar{\pi}, \infty)$, (20) follows by definition, yielding (21). Similarly,

$$G_{DN}(\bar{\pi}) - G_{D}(\bar{\pi}) = \int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = D) G_{N}(\bar{\pi} - x) dx - G_{D}(\bar{\pi})$$

$$= \int_{0}^{\pi} \Pr(\pi_{t}^{1} = x | a = D) G_{N}(\bar{\pi} - x) dx + G_{D}(\bar{\pi}) - G_{D}(\bar{\pi}) .$$

Therefore, $G_{CC}(\bar{\pi}) - G_{C}(\bar{\pi}) \geq G_{DN}(\bar{\pi}) - G_{D}(\bar{\pi})$, because by stochastic dominance we have (i) $\int_{0}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) dx \geq \int_{0}^{\pi} \Pr(\pi_{t}^{1} = x | a = D) dx$, and (ii) $G_{C}(\bar{\pi} - x) \geq G_{N}(\bar{\pi} - x)$ for every $x \in [0, \pi]$. Thus, noting that $E_{C} > E_{N}$ and $\frac{E_{C}}{1 - \delta G_{CC}(\bar{\pi})} > \frac{E_{N}}{1 - \delta G_{NN}(\bar{\pi})}$, we have $A > 0$ and thus (16) is the binding constraint. 

### B Proof Lemma 5

**Proof.** The manager has no incentive to defect in stage 1 if and only if

$$\frac{E_{C} + G_{C}(\tau) E_{C} + (1 - G_{C}(\tau)) E_{N}}{1 - \delta P_{C}(\bar{\pi}, \tau)} \geq E_{D} + E_{N} + \delta G_{DN}(\bar{\pi}) - \frac{2E_{N}}{1 - \delta G_{NN}(\bar{\pi})},$$

where $P_{C}(\bar{\pi}, \tau) = \int_{\tau}^{\infty} \Pr(\pi_{t}^{1} = x | a = C) G_{C}(\bar{\pi} - x) dx + \int_{0}^{\tau} \Pr(\pi_{t}^{1} = x | a = C) G_{N}(\bar{\pi} - x) dx$. 

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Given the realisation of \( \pi_t \), he has no incentive to defect in stage 2 if and only if
\[
EC + \delta GC (\pi - \pi_t) E_N + G_N (\tau) E_C + (1 - G_N (\tau)) E_N \geq E_D + \delta GD (\pi - \pi_t) \frac{2E_N}{1 - \delta G NN (\pi)},
\]
which rewrites as
\[
\delta \left( \frac{G C (\pi - \pi_t)}{1 - \delta P N (\pi, \tau) K C (\tau)} - \frac{G D (\pi - \pi_t)}{1 - \delta G NN (\pi) 2E_N} \right) \geq E_D - E_C
\]
for every profit realisation \( \pi_t \geq \tau \). By the regularity condition, this boils down to combined conditions (11) and (12) in Lemma 5.

Noting that \( P (\pi, \tau) < G CC (\pi) \) and \( E_C < K C (\tau) \), we have that (11) is more difficult to satisfy than (14). However, constraint (14) is not the binding constraint for strategy \( S_1 \) to be stable; constraint (15) with \( \pi_t = 0 \) is the binding constraint. Thus, constraint (11) does not make strategy \( S_2 \) less stable than strategy \( S_1 \) as long as \( \tau \) is such that (11) is easier to satisfy than (15) with \( \pi_t = 0 \), which depends on the precise specification of the density functions.

Fixing \( \pi_t \geq 0 \), constraint (12) is more difficult to satisfy than constraint (15), because \( P (\pi, \tau) < G CC (\pi) \) and \( E_C < K C (\tau) \). However, depending on the precise specification of the density functions, constraint (12) may be easier to satisfy than constraint (15), because (i) both constraints are more difficult to satisfy the lower is \( \pi_t \), and (ii) constraint (15) needs to be satisfied for every \( \pi_t \geq 0 \), while (iii) constraint (12) needs only to be satisfied for every \( \pi_t \geq \tau \).

Thus, depending on the precise specification of the density functions, choosing an appropriate \( \tau \) potentially results in both constraints (11) and (12) to be satisfied for a larger set of discount factors than constraint (15).

\[\Box\]

C Proof Lemma 6

Proof. Given the realisation of \( \pi_t \), the manager has no incentive to defect in stage 2 if and only if
\[
EC + \delta GC (\pi - \pi_t) E_N + G_N (\tau) E_C + (1 - G_N (\tau)) E_N \geq E_D + \delta GD (\pi - \pi_t) \frac{2E_N}{1 - \delta G NN (\pi)},
\]
which rewrites as
\[
\delta \left( \frac{G C (\pi - \pi_t)}{1 - \delta P N (\pi, \tau) K N (\tau)} - \frac{G D (\pi - \pi_t)}{1 - \delta G NN (\pi) 2E_N} \right) \geq E_D - E_C
\]
for every profit realisation $\pi^1_t \geq \tau$. By the regularity condition, we have that the above constraint is most difficult to satisfy if $\pi^1_t = \tau$; thus, stability is determined by condition (13) in Lemma 6.

D Proof Proposition 6

Proof.

First claim: price wars in equilibrium. The collusive strategies described in Lemmas 5 and 6 entail (as part of the collusive equilibrium) competition in stage 2 after the realisation of profit $\pi^1_t < \tau$ in stage 1; also, the collusive strategy described in Lemma 6 always entails competition in stage 1. Moreover, the collusive strategies described in Lemmas 5 and 6 are potentially more stable than collusion in both stages. Thus, contracts that span multiple managerial interactions potentially entail price wars in equilibrium.

Second claim: if managers adopt a strategy entailing price wars in equilibrium, then cartel stability is increased. The collusive strategies described in Lemmas 5 and 6 are potentially more stable, but entail a lower profitability (see proof below). Thus, if managers adopt such a strategy, it means that they are not patient enough to collude in both stages, and thus adopt a strategy entailing equilibrium price wars, while compromising on cartel profitability.

Third claim: price wars reduce cartel profitability. The profitability of (i) collusion in both stages, (ii) collusion in stage 1 and $\tau$-conditional collusion in stage 2, and (iii) competition in stage 1 and $\tau$-conditional collusion in stage 2 is, respectively,

$$\frac{2E_C}{1 - \delta G_{CC}(\pi)} > \frac{K_C(\tau)}{1 - \delta P_C(\pi, \tau)} > \frac{K_N(\tau)}{1 - \delta P_N(\pi, \tau)},$$

because $2E_C > K_C(\tau) > K_N(\tau)$ and $G_{CC}(\pi) > P_C(\pi, \tau) > P_N(\pi, \tau)$.
References


