Estimation methods for statistical process control

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Chapter 4

Location Estimators for $\overline{X}$ Control Charts

4.1 Introduction

This chapter studies estimation methods for the location parameter. We consider several robust location estimators as well as several estimation methods based on a Phase I analysis (recall that this is the use of a control chart to study a historical dataset retrospectively to identify disturbances). In addition, we propose a new type of Phase I analysis. The estimation methods are evaluated in terms of their MSE and their effect on the $\overline{X}$ control charts used for real-time process monitoring (Phase II). It turns out that the Phase I control chart based on the trimmed trimean far outperforms the existing estimation methods. This method has therefore proven to be very suitable for determining $\overline{X}$ Phase II control chart limits.

The remainder of the chapter is organized as follows. First, we present several Phase I sample statistics for the process location and compare their MSE. Then we describe some existing Phase I control charts and present a new algorithm for Phase I analysis. Following that, we present the design schemes for the $\overline{X}$ Phase II control chart and derive the control limits. Next, we describe the simulation procedure and present the effect of the proposed methods on Phase II performance. The final section offers some recommendations.

4.2 Proposed location estimators

To understand the behavior of the estimators it is again useful to distinguish diffuse and localized disturbances (cf. Tatum (1997)). As explained in Section 1.2, diffuse disturbances are outliers that are spread over all of the samples whereas localized disturbances affect all observations in one sample. We look at various types of estimators (both robust estimators and several estimation methods based on the principle of control charting) in Section 4.2.1 and compare their MSE in Section 4.2.2.

4.2.1 Location estimators

Recall that X_{ij}, $i = 1, 2, ..., k$ and $j = 1, 2, ..., n$, denote the Phase I data. The X_{ij}'s are assumed to be independent and largely $N(\mu, \sigma^2)$ distributed. We denote by $X_{i,(v)}$, $v = 1, 2, ..., n$, the v-th order statistic in sample i.

The first estimator that we consider is the mean of the sample means

$$\overline{\overline{X}} = \frac{1}{k}\sum_{i=1}^{k}\overline{X}_i = \frac{1}{k}\sum_{i=1}^{k}\left(\frac{1}{n}\sum_{j=1}^{n}X_{ij}\right).$$

This estimator is included to provide a basis for comparison, as it is the most efficient estimator for normally distributed data. However, it is well known that this estimator is not robust against outliers.

We also consider three robust estimators proposed earlier by Rocke (1989). They are the median of the sample means

$$M(\overline{X}) = median(\overline{X}_1, \overline{X}_2, ..., \overline{X}_k),$$

the mean of the sample medians

$$\overline{M} = \frac{1}{k}\sum_{i=1}^{k}M_i,$$

with M_i the median of sample i, and the trimmed mean of the sample means

$$\overline{\overline{X}}_\alpha = \frac{1}{k - 2\lceil k\alpha\rceil} \times \left[\sum_{v=\lceil k\alpha\rceil+1}^{k-\lceil k\alpha\rceil}\overline{X}_{(v)}\right],$$

where α denotes the percentage of samples to be trimmed, $\lceil z\rceil$ denotes the ceiling function, i.e. the smallest integer not less than z, and $\overline{X}_{(v)}$ denotes the v-th ordered value of the sample means. In our study, we consider the 20% trimmed mean, which trims the six smallest and the six largest sample means when $k = 30$. Of course other trimming percentages could have been used. In fact, we have also used 10% and 25% but the results with 20% are representative for this estimator.

Furthermore, our analysis includes the Hodges-Lehmann estimator (Hodges and Lehmann (1963)), an estimator based on the so-called Walsh averages. The h $(= n(n+1)/2)$ Walsh averages of sample i are

$$W_{i,k,l} = (X_{i,k} + X_{i,l})/2, k = 1, 2, ..., n, l = 1, 2, ..., n, k \leq l.$$

The Hodges-Lehmann estimate for sample i, denoted by HL_i, is defined as the median of the Walsh averages. Alloway and Raghavachari (1991) conducted a Monte Carlo simulation to determine whether the mean or the median of the sample Hodges-Lehmann estimates should be used to determine the final location estimate. They concluded that the mean of the sample values should be used

$$\overline{HL} = \frac{1}{k}\sum_{i=1}^{k} HL_i$$

and that the resulting estimate is unbiased.

We also include the trimean statistic. The trimean of sample i is the weighted average of the sample median and the two other quartiles

$$TM_i = (Q_{i,1} + 2Q_{i,2} + Q_{i,3})/4,$$

where $Q_{i,q}$ is the q-th quartile of sample i, $q = 1, 2, 3$ (cf. Tukey (1997) and Wang et al. (2007)). It also equals the average of the median and the midhinge $1/2\left(Q_{i,2} + \frac{Q_{i,1}+Q_{i,3}}{2}\right)$ (cf. Weisberg (1992)). We use the following definitions for the quartiles: $Q_{i,1} = X_{i,(a)}$ and $Q_{i,3} = X_{i,(b)}$ with $a = \lceil n/4 \rceil$ and $b = n - a + 1$. This means that $Q_{i,1}$ and $Q_{i,3}$ are defined as the second smallest and the second largest observations respectively for $5 \leq n \leq 8$, and as the third smallest and the third largest values respectively for $9 \leq n \leq 12$. Like the median and the midhinge, but unlike the sample mean, the trimean is a statistically resistant L-estimator (a linear combination of order statistics), with a breakdown point of 25% (see Wang et al. (2007)). According to Tukey (1977), using the trimean instead of the median gives a more useful assessment of location or centering. According to Weisberg

(1992), the "statistical resistance" benefit of the trimean as a measure of the center of a distribution is that it combines the median's emphasis on center values with the midhinge's attention to the extremes. The trimean is almost as resistant to extreme scores as the median and is less subject to sampling fluctuations than the arithmetic mean in extremely skewed distributions. Asymptotic distributional results of the trimean can be found in Wang et al. (2007). The location estimate analyzed below is the mean of the sample trimeans, i.e.

$$\overline{TM} = \frac{1}{k}\sum_{i=1}^{k} TM_i.$$

Finally, we consider a statistic that is expected to be robust against both diffuse and localized disturbances, namely the trimmed mean of the sample trimeans, defined by

$$\overline{TM}_\alpha = \frac{1}{k-2\lceil k\alpha \rceil} \times \left[\sum_{v=\lceil k\alpha \rceil+1}^{k-\lceil k\alpha \rceil} TM_{(v)} \right],$$

where $TM_{(v)}$ denotes the v-th ordered value of the sample trimeans. We consider the 20% trimmed trimean, which trims the six smallest and the six largest sample trimeans when $k = 30$.

The estimators outlined above are summarized in Table 4.1.

Estimator	Notation
Mean of sample means	$\overline{\overline{X}}$
Median of sample means	$M(\overline{X})$
Mean of sample medians	$\overline{M}$
20% trimmed mean of sample means	$\overline{\overline{X}}_{20}$
Mean of sample Hodges-Lehmann	$\overline{HL}$
Mean of sample trimeans	$\overline{TM}$
20% trimmed mean of sample trimeans	$\overline{TM}_{20}$

Table 4.1: Proposed location estimators

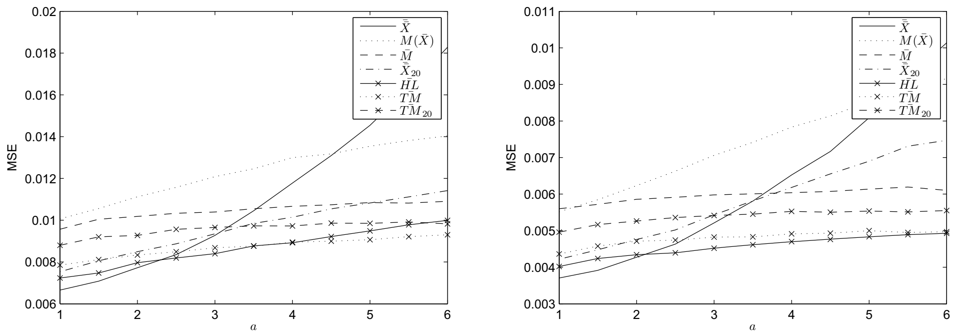
4.2.2 Efficiency of proposed estimators

As in Chapters 2 and 3, we follow Tatum (1997) and evaluate the estimators in terms of their MSE. In this case, the MSE is estimated as

$$MSE = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\hat{\mu}^i - \mu}{\sigma} \right)^2 .$$

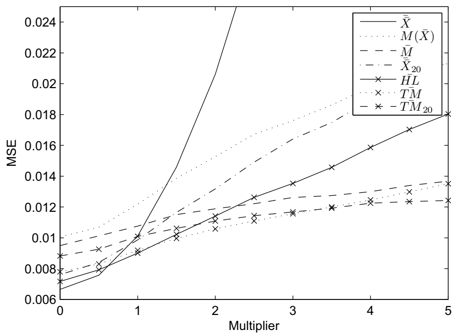
We include the uncontaminated case, i.e. the situation where all the X_{ij}'s are from the $N(0,1)$ distribution, as well as five types of disturbances. They are the four models described in Section 2.2.2 and

5. A model for localized mean disturbances in which observations in 3 out of 30 samples are drawn from the $N(a,1)$ distribution, with $a = 0.5, 1.0, ..., 5.5, 6.0$.

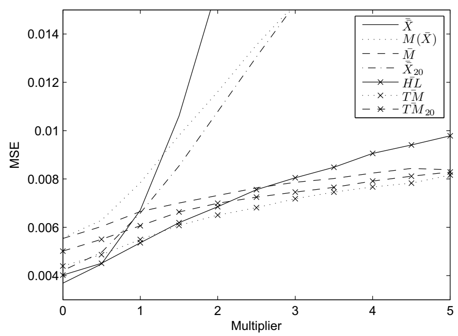


(a) (b)

Figure 4.1: MSE of estimators when symmetric diffuse variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$

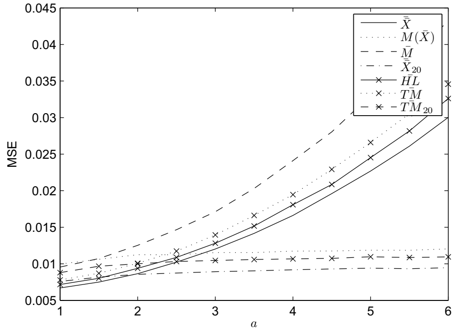


(a)

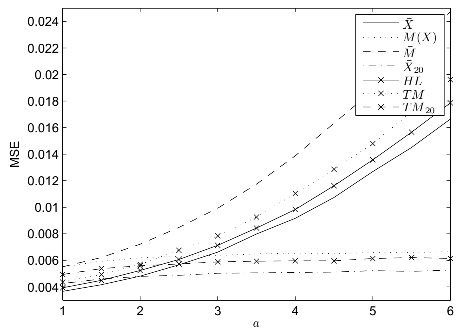


(b)

Figure 4.2: MSE of estimators when asymmetric diffuse variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$

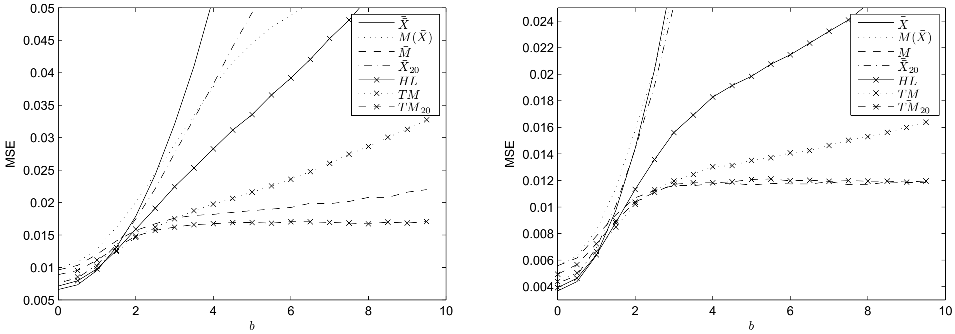


(a)



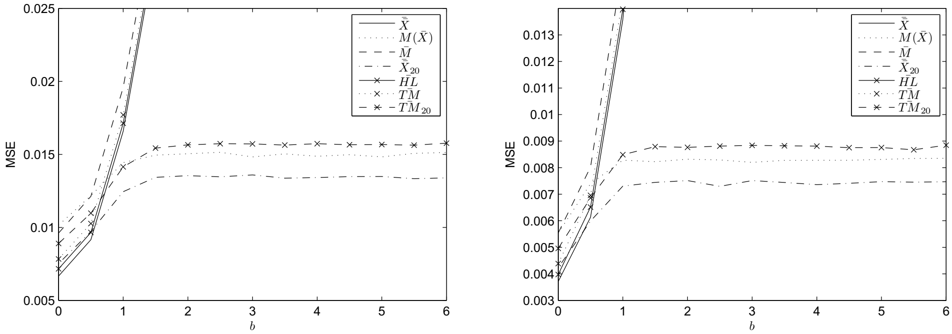
(b)

Figure 4.3: MSE of estimators when localized variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



(a) (b)

Figure 4.4: MSE of estimators when diffuse mean disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



(a) (b)

Figure 4.5: MSE of estimators when localized mean disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$

The figures show that in the uncontaminated situation the most efficient estimator is $\overline{\overline{X}}$ as was to be expected. The estimators $\overline{HL}$, $\overline{\overline{X}}_{20}$ and $\overline{TM}$ are slightly less efficient followed by $\overline{TM}_{20}$, $\overline{M}$ and $M(\overline{X})$, the reason being that they use less information.

When diffuse symmetric variance disturbances are present (Figure 4.1), the best performing estimators are $\overline{HL}$ and $\overline{TM}$. The reason why $\overline{TM}$ performs well in this situation is that it filters out the extreme high and low values in each sample. $\overline{HL}$ also performs well because it obtains the sample statistic using the median of the Walsh averages, which is not sensitive to outliers. $\overline{M}$ and $\overline{TM}_{20}$ are as efficient in the contaminated situation as in the uncontaminated situation but they are outperformed by $\overline{HL}$ and $\overline{TM}$ because the latter estimators use more information. It is worth noting that the traditional estimator $\overline{\overline{X}}$ shows relatively bad results despite the symmetric character of the outliers. $M(\overline{X})$ and $\overline{\overline{X}}_{20}$ do not perform very well because these estimators focus on extreme samples whereas in the present situation the outliers are spread over all of the samples so that the non-trimmed samples are also infected.

When asymmetric variance disturbances are present (Figure 4.2), the most efficient estimators are $\overline{TM}$, $\overline{TM}_{20}$, $\overline{HL}$ and $\overline{M}$, performing particularly well relative to the other estimators for larger sample sizes. As for the symmetric diffuse case, the estimators that include a method to trim observations within a sample perform better than the methods that focus on sample trimming.

In the case of localized variance disturbances (Figure 4.3), the estimators based on the principle of trimming sample means rather than within-sample observations - $\overline{\overline{X}}_{20}$, $\overline{TM}_{20}$ and $M(\overline{X})$ - have the lowest MSE . The estimators $\overline{\overline{X}}$, $\overline{HL}$, $\overline{TM}$ and in particular $\overline{M}$ are less successful because these statistics only perform well if no more than a few observations in a sample are infected rather than all observations, as is the case here.

When diffuse mean disturbances are present (Figure 4.4), the results are comparable to the situation where there are diffuse asymmetric variance disturbances: $\overline{M}$, $\overline{TM}_{20}$ and $\overline{TM}$ perform best, followed by $\overline{HL}$. Note that in this situation $\overline{\overline{X}}$, $M(\overline{X})$ and $\overline{\overline{X}}_{20}$ perform badly.

When localized mean disturbances are present (Figure 4.5), the results are comparable to the situation where there are localized variance disturbances: the estimators based on the principle of trimming sample means, namely $\overline{\overline{X}}_{20}$, $M(\overline{X})$ and $\overline{TM}_{20}$, perform best.

To summarize, $\overline{M}$, $\overline{TM}$ and $\overline{TM}_{20}$ have the lowest MSE when there are diffuse disturbances. $\overline{M}$ and $\overline{TM}$ lose their efficiency advantage when contaminations take the form of localized mean or variance disturbances. In such situations, $M(\overline{X})$, $\overline{\overline{X}}_{20}$ and $\overline{TM}_{20}$, which involve trimming the sample means, perform relatively well. $\overline{TM}_{20}$ has the best performance overall because it is reasonably robust against all types of contaminations.

4.3 Proposed control chart location estimators

In-control process parameters can be obtained not only via robust statistics but also via Phase I control charting. In the latter case, control charts are used retrospectively to study a historical dataset and identify samples that are deemed out of control. The process parameters are then estimated from the in-control samples. In this section, we consider several Phase I analyses which apply the principle of control charting in order to generate robust estimates of process location. We study a Phase I control chart based on the commonly used estimator $\overline{\overline{X}}$ and a Phase I control chart based on the mean rank proposed by Jones-Farmer et al. (2009). Moreover, we propose two new types of Phase I analyses. The next section presents the various Phase I control charts and the following section shows the MSE of the proposed estimation methods.

4.3.1 Phase I control charts

The standard procedure in practice is to use the estimator $\overline{\overline{X}}$ for constructing the $\overline{X}$ Phase I control chart limits. The respective upper and lower control limits of the Phase I chart are given by $\widehat{UCL}_{\bar{\bar{X}}} = \overline{\overline{X}} + 3\hat{\sigma}/\sqrt{n}$ and $\widehat{LCL}_{\bar{\bar{X}}} = \overline{\overline{X}} - 3\hat{\sigma}/\sqrt{n}$, where we estimate σ by the robust standard deviation estimator proposed by Tatum (1997), using the corrected normalizing constants presented in Chapter 2. The samples whose $\overline{X}_i$ fall above $\widehat{UCL}_{\bar{\bar{X}}}$ or below $\widehat{LCL}_{\bar{\bar{X}}}$ are eliminated from the Phase I dataset. The final location estimate is the mean of the sample means of the remaining samples

$$\overline{\overline{X}}' = \frac{1}{k'} \sum_{i \epsilon K} \overline{X}_i \times I_{\widehat{LCL}_{\bar{\bar{X}}} \leq \bar{X}_i \leq \widehat{UCL}_{\bar{\bar{X}}}}(\overline{X}_i),$$

with K the set of samples which are not excluded, k' the number of non-excluded samples and I the indicator function. This adaptive trimmed mean estimator is denoted by $ATM_{\bar{\bar{X}}}$.

We also consider a Phase I analysis that is based on the mean rank proposed by Jones-Farmer et al. (2009). It is a nonparametric estimation method which treats the observations from the k mutually exclusive samples of size n as a single sample of $N = n \times k$ observations. Let $R_{ij} = 1, 2, ..., N$ denote the integer rank of observation X_{ij} in the pooled sample of size N. Let $\bar{R}_i = (\sum_{j=1}^{n} R_{ij})/n$ be the mean of the ranks in sample i. If the process is in control, the ranks should be distributed evenly throughout the samples. For an in-control process, the mean and variance of $\bar{R}_i$ are

$$E(\bar{R}_i) = \frac{N+1}{2}$$

and

$$Var(\bar{R}_i) = \frac{(N-n)(N+1)}{12n}.$$

According to the central limit theorem, the random variable

$$Z_i = \frac{\bar{R}_i - E(\bar{R}_i)}{\sqrt{var(\bar{R}_i)}}$$

follows approximately a standard normal distribution for large values of n. A control chart for these Z_i's can be constructed with center line equal to 0, upper control limit 3 and lower control limit -3. The samples with Z_i outside the Phase I control limits are considered to be out of control and are excluded from the dataset. The location estimate is obtained from the mean of the remaining sample means

$$\overline{\overline{X}}^* = \frac{1}{k^*} \sum_{i \epsilon K^*} \overline{X}_i \times I_{-3 \le Z_i \le 3}(Z_i),$$

with K^* the set of samples which are not excluded and k^* the number of non-excluded samples. This estimation method is denoted by ATM_{MR}.

We now present two new Phase I analyses based on the principle of control charting. For the first method, we build a Phase I control chart using a robust estimator. The advantage of a robust estimator over a sensitive estimator like $\overline{\overline{X}}$ is that the Phase I control limits are not affected by any disturbances so that the correct out-of-control observations are filtered out in Phase I. An estimator shown to be very robust by the MSE study in Section 4.2.2 is $\overline{TM}_{20}$. A disadvantage is that the estimator is not very efficient under normality. To address this, we use $\overline{TM}_{20}$ to construct the Phase I limits with which we screen $\overline{X}_i$ for disturbances, but then use the efficient estimator $\overline{\overline{X}}$ to obtain the location estimate from the remaining samples. The Phase I control limits are given by $\widehat{UCL}_{\bar{TM}_{20}} = \overline{TM}_{20} + 3\hat{\sigma}/\sqrt{n}$ and $\widehat{LCL}_{\bar{TM}_{20}} = \overline{TM}_{20} - 3\hat{\sigma}/\sqrt{n}$, where we estimate σ by Tatum's estimator. We then plot the $\overline{X}_i$'s on the Phase I control chart. The samples whose $\overline{X}_i$ falls outside the limits are regarded as out of control and removed from the dataset. The remaining samples are used to determine the grand sample mean

$$\overline{\overline{X}}^{\#} = \frac{1}{k^{\#}} \sum_{i \epsilon K^{\#}} \overline{X}_i \times I_{\widehat{LCL}_{\bar{TM}_{20}} \leq \bar{X}_i \leq \widehat{UCL}_{\bar{TM}_{20}}}(\overline{X}_i),$$

with $K^{\#}$ the set of samples which are not excluded and $k^{\#}$ the number of non-excluded samples. The resulting estimator is denoted by $ATM_{\bar{TM}_{20}}$.

The fourth type of Phase I control chart resembles the chart presented above, but employs a different method to screen for disturbances. The procedure consists of two steps.

In the first step we construct the control chart with limits as we did just before. Note that, for the sake of practical applicability, we use the same factor, namely 3, to derive the $\overline{X}$ and TM charts. We then plot the TM_i's of the Phase I samples on the control chart. Charting the TM_i's instead of the $\overline{X}_i$'s ensures that localized disturbances are identified and samples that contain only one single outlier are retained. A location estimator that is expected to be robust against localized mean disturbances is the mean of the sample trimeans of the samples that fall between the control limits

$$\overline{TM}' = \frac{1}{k^{\wedge}} \sum_{i \epsilon K^{\wedge}} TM_i \times I_{\widehat{LCL}_{\bar{TM}_{20}} \leq TM_i \leq \widehat{UCL}_{\bar{TM}_{20}}}(TM_i),$$

with $K^{\wedge}$ the set of samples which are not excluded and $k^{\wedge}$ the number of non-excluded samples.

Although the remaining Phase I samples are expected to be free from localized mean disturbances, they could still contain diffuse disturbances. To eliminate such disturbances, the second step is to screen the individual observations using a Phase I individuals control chart with respective upper and lower control limits given by $\widehat{UCL}_{\bar{TM}'} = \overline{TM}' + 3\hat{\sigma}$ and $\widehat{LCL}_{\bar{TM}'} = \overline{TM}' - 3\hat{\sigma}$, where σ is estimated by Tatum's estimator. The observations X_{ij} that fall above $\widehat{UCL}_{\bar{TM}'}$ or below $\widehat{LCL}_{\bar{TM}'}$ are considered out of control and removed from the Phase I dataset. The final estimate is the mean of the sample means and is calculated from the observations deemed to be in control

$$\overline{\overline{X}}'' = \frac{1}{k''} \sum_{i \epsilon K''} \frac{1}{n'_i} \sum_{j \epsilon N'_i} X_{ij} \times I_{\widehat{LCL}_{\bar{TM}'} \leq X_{ij} \leq \widehat{UCL}_{\bar{TM}'}}(X_{ij}),$$

with K'' the set of samples which are not excluded, k'' the number of non-excluded samples, N'_i the set of non-excluded observations in sample i and n'_i the number of non-excluded observations in sample i. Note that we could also have used the double sum, divided by the sum of the n'_i. The advantage of our procedure is that, when a sample is infected by a localized disturbance, the disturbance will have a lower impact on the final location estimate when it is not detected. This estimation method is denoted by $ATM_{\bar{TM}'}$.

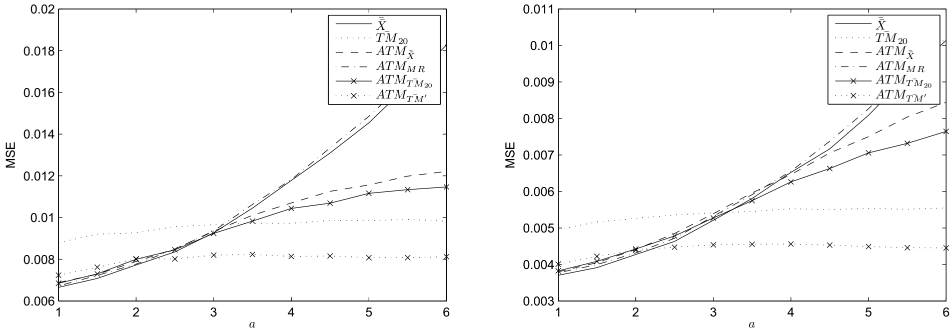
The proposed Phase I analyses are summarized in Table 4.2.

Phase I analysis	Notation
$\overline{\overline{X}}$ control chart with screening	$ATM_{\bar{\bar{X}}}$
Mean rank control chart with screening	ATM_{MR}
$\overline{TM}_{20}$ control chart with screening	$ATM_{\bar{TM}_{20}}$
$\overline{TM}'$ control chart with screening	$ATM_{\bar{TM}'}$

Table 4.2: Proposed Phase I analyses

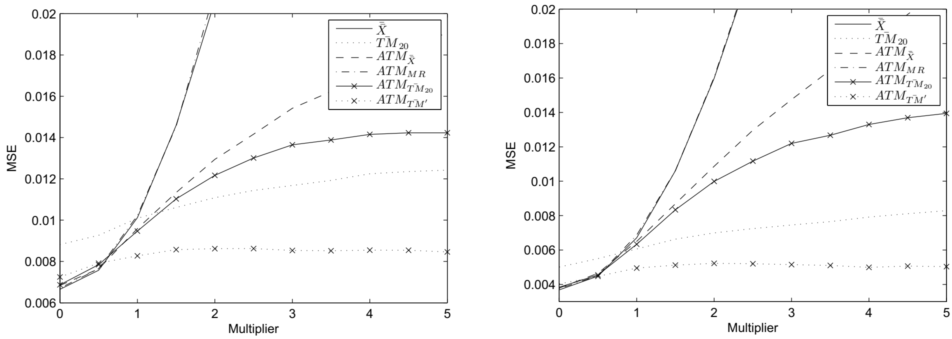
4.3.2 Efficiency of proposed Phase I control charts

To determine the efficiency of the proposed Phase I control charts, we consider the five types of contaminations defined in our MSE study of the statistics presented in Section 4.2.2. The MSE results for the Phase I control charts are given in Figures 4.6-4.10. To facilitate comparison, we have also included the MSE of the estimators $\overline{\overline{X}}$ and $\overline{TM}_{20}$.



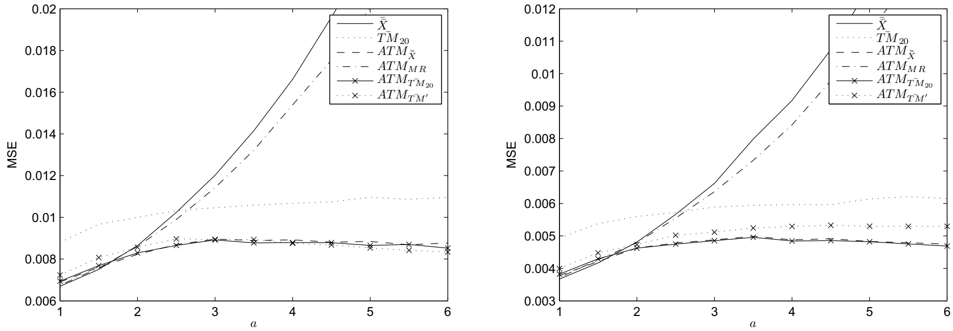
(a) (b)

Figure 4.6: MSE of estimators when symmetric diffuse variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



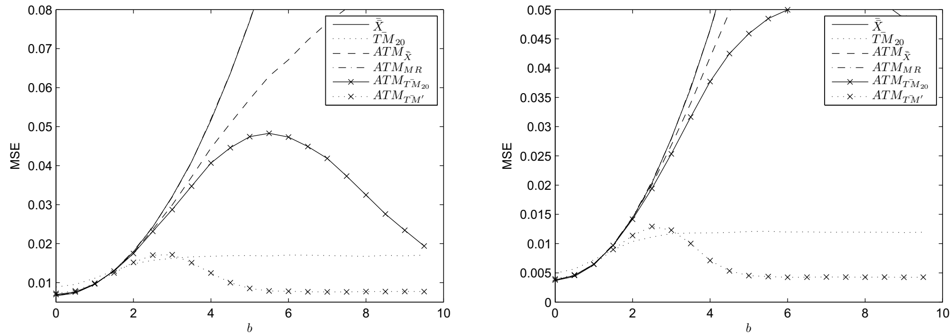
(a) (b)

Figure 4.7: MSE of estimators when asymmetric diffuse variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



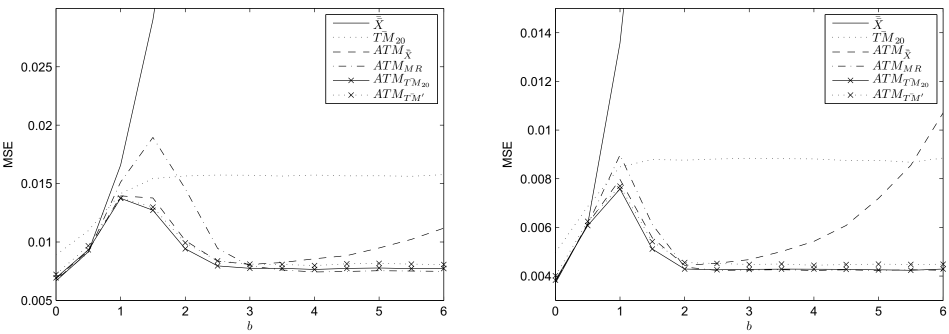
(a) (b)

Figure 4.8: MSE of estimators when localized variance disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



(a) (b)

Figure 4.9: MSE of estimators when diffuse mean disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$



(a) (b)

Figure 4.10: MSE of estimators when localized mean disturbances are present for $k = 30$ (a) $n = 5$ (b) $n = 9$

The figures show that the standard Phase I analysis method, $ATM_{\bar{\bar{X}}}$, performs almost as well as $\overline{\overline{X}}$ under normality when no contaminations are present and seems to be robust against localized variance disturbances. However, the method loses efficiency in the other situations. Since $\overline{\overline{X}}$, the initial estimate of μ, is highly sensitive to disturbances, the Phase I limits are biased and fail to identify the correct out-of-control samples.

The mean rank method, denoted by ATM_{MR}, performs well under normality and when there are localized mean disturbances. The reason is that this estimator screens for samples with a mean rank significantly higher than that of the other samples. On the other hand, ATM_{MR} performs badly when diffuse outliers are present. The mean rank is not influenced by occasional outliers so that samples containing only one outlier are not filtered out and hence are included in the calculation of the grand sample mean.

The third method, $ATM_{\bar{TM}_{20}}$, which uses the robust estimator $\overline{TM}_{20}$ to construct a Phase I control chart, seems to be more efficient under normality than $\overline{TM}_{20}$ itself. The gain in efficiency can be explained by the use of an efficient estimator to obtain the final location estimate, once screening is complete. Thus, an efficient Phase I analysis does not require the use of an efficient estimator to construct the Phase I control chart.

The final method, $ATM_{\bar{TM}'}$, which first screens for localized disturbances and then for occasional outliers, far outperforms all estimation methods. The method is particularly powerful in the presence of diffuse disturbances, because its use of an individuals control chart in Phase I to identify single outliers increases the probability that such disturbances will be detected. For example, Figure 4.9 represents the situation where diffuse mean disturbances are present. The efficiency of the estimator improves for high b values because the disturbances are more likely to fall outside the control limits and are therefore more likely to be detected.

4.4 Derivation of Phase II control limits

We now turn to the effect of the proposed location estimators on the $\overline{X}$ control chart performance in Phase II. The formulae for the $\overline{X}$ control limits with estimated parameters are given by (1.4). For the Phase II control limits, we only estimate the in-control mean μ; we treat the in-control standard deviation σ as known because we want to isolate the effect of estimating the location parameter. The factor C_n that is used to obtain accurate control limits when the process parameters are estimated is derived such that the probability of a false signal equals the desired probability of a false signal. Except for the estimator $\overline{\overline{X}}$, C_n can not be obtained easily in analytic form and is therefore obtained by means of simulation. The factors are chosen such that p is equal to 0.0027 under normality. 50,000 simulation runs are used. For $k = 30$ $n = 5$ and $n = 9$, the resulting factors are equal to 3.05 for $\overline{\overline{X}}$, $ATM_{\bar{\bar{X}}}$, $ATM_{\bar{TM}_{20}}$ and $ATM_{\bar{TM}'}$; 3.06 for $\overline{\overline{X}}_{20}$ and $\overline{TM}$ and 3.07 for $M(\overline{X})$, $\overline{M}$, $\overline{HL}$, $\overline{TM}_{20}$ and $ATM_{\bar{MR}}$.

4.5 Control chart performance

In this section we evaluate the effect on $\overline{X}$ Phase II performance of the proposed location statistics and estimation methods based on Phase I control charting. We consider the same Phase I situations as those used to assess the MSE with a, b and the multiplier equal to 4 to simulate the contaminated cases (see Section 4.2.2).

We use the unconditional run length distribution to assess performance. Specifically, we look at several characteristics of that distribution, namely the average run length (ARL) and the standard deviation of the run length (SDRL). We also report the probability that one sample gives a signal (p). We compute these characteristics in an in-control and several out-of-control situations. We consider different shifts of size $\delta\sigma$ in the mean, setting δ equal to 0, 0.5, 1 and 2. The performance characteristics are obtained by simulation. Section 4.5.1 describes the simulation method and Section 4.5.2 gives simulation results identifying which control charts perform best in the uncontaminated and various contaminated situations.

4.5.1 Simulation procedure

The performance characteristics p and ARL for estimated control limits are determined by averaging the conditional characteristics, i.e. the characteristics for a given set of estimated control limits, over all possible values of the control limits. The corresponding definitions of $p(F_i|\hat{\mu})$, $E(RL|\hat{\mu})$, $p = E(p(F_i|\hat{\mu}))$ and $ARL = E(\frac{1}{p(F_i|\hat{\mu})})$ are obtained from (1.5)-(1.8), with all variables conditioned on $\hat{\mu}$ rather than $\hat{\sigma}$ and $\hat{\sigma} = \sigma$. These expectations are obtained by simulation: numerous datasets are generated and for each dataset $p(F_i|\hat{\mu})$ and $E(RL|\hat{\mu})$ are computed. By averaging these values we obtain the unconditional values. The unconditional standard deviation is determined by (1.9).

Enough replications of the above procedure were performed to obtain sufficiently small relative estimated standard errors for p and ARL. The relative standard error of the estimates is never higher than 0.60%.

4.5.2 Simulation results

First, we consider the situation where the process follows a normal distribution and the Phase I data are not contaminated. We investigate the impact of the estimator used to estimate μ in Phase I. Table 4.3 presents p and the ARL when the process mean equals $\mu + \delta\sigma$. When the process is in control ($\delta = 0$), we want p to be as low as possible and ARL to be as high as possible. In the out-of-control situation ($\delta \neq 0$), we want to achieve the opposite. We can see that in the absence of any contamination (Table 4.3), the efficiency of the estimators is very similar. We can therefore conclude that using a more robust location estimator does not have a substantial impact on control chart performance in the uncontaminated situation.

The Phase II control charts based on the estimation methods $M(\overline{X})$, $\overline{\overline{X}}_{20}$, $\overline{TM}_{20}$, $ATM_{\overline{\overline{X}}}$ and ATM_{MR} perform relatively well when localized disturbances are present, while the charts based on $\overline{M}$, $\overline{HL}$, $\overline{TM}$ and $\overline{TM}_{20}$ perform relatively well when diffuse disturbances are present (see Tables 4.4-4.8).

The Phase II chart based on $ATM_{\bar{TM}'}$ performs best: this chart is as efficient as $\overline{\overline{X}}$ in the uncontaminated normal situation and its performance does not change much when contaminations come into play. Moreover, the chart outperforms the other methods in all situations because it successfully filters out both diffuse and localized disturbances. In the presence of asymmetric disturbances, in particular, the added value of this estimation method is substantial.

When localized mean disturbances are present, we see a strange phenomenon for the $\overline{X}$, $\overline{M}$, $\overline{HL}$ and $\overline{TM}$ charts: the in-control ARL is lower than the out-of-control ARL for $\delta = 0.5$. In other words, these charts are more likely to give a signal in the in-control situation than in the out-of-control situation for $\delta = 0.5$ and hence, in the presence of disturbances, are highly ARL-biased (cf. Jensen et al. (2006)).

n	$\hat{\mu}$	p $\delta=0$	$\delta=0.5$	$\delta=1$	$\delta=2$	*ARL* and *SDRL* $\delta=0$	$\delta=0.5$	$\delta=1$	$\delta=2$
5	$\overline{\overline{X}}$	0.0027	0.029	0.21	0.92	384 (392)	41.7 (49.4)	5.03 (4.90)	1.09 (0.32)
	$M(\overline{X})$	0.0027	0.028	0.21	0.91	390 (406)	46.2 (59.9)	5.31 (5.43)	1.10 (0.33)
	$\overline{M}$	0.0027	0.028	0.21	0.91	392 (407)	45.9 (59.0)	5.29 (5.37)	1.10 (0.33)
	$\overline{\overline{X}}_{20}$	0.0027	0.028	0.21	0.92	391 (401)	43.3 (52.4)	5.14 (5.08)	1.09 (0.32)
	$\overline{HL}$	0.0027	0.029	0.21	0.92	380 (389)	42.0 (50.4)	5.05 (4.94)	1.09 (0.32)
	$\overline{TM}$	0.0027	0.028	0.21	0.92	390 (400)	43.4 (53.0)	5.14 (5.09)	1.09 (0.32)
	$\overline{TM}_{20}$	0.0027	0.028	0.21	0.92	396 (410)	45.3 (56.9)	5.26 (5.29)	1.09 (0.33)
	$ATM_{\bar{\bar{X}}}$	0.0027	0.029	0.21	0.92	383 (392)	41.8 (49.6)	5.04 (4.92)	1.09 (0.32)
	ATM_{MR}	0.0027	0.029	0.21	0.92	383 (391)	41.5 (49.1)	5.04 (4.89)	1.09 (0.32)
	$ATM_{\bar{TM}_{20}}$	0.0027	0.029	0.21	0.92	382 (391)	41.8 (49.9)	5.04 (4.92)	1.09 (0.32)
	$ATM_{\bar{TM}'}$	0.0027	0.029	0.21	0.92	381 (390)	42.0 (50.3)	5.06 (4.96)	1.09 (0.32)
9	$\overline{\overline{X}}$	0.0027	0.064	0.48	1.00	384 (393)	17.9 (20.0)	2.13 (1.62)	1.00 (0.043)
	$M(\overline{X})$	0.0027	0.063	0.47	1.00	390 (405)	19.5 (23.5)	2.19 (1.74)	1.00 (0.046)
	$\overline{M}$	0.0027	0.063	0.47	1.00	390 (405)	19.5 (23.6)	2.19 (1.74)	1.00 (0.046)
	$\overline{\overline{X}}_{20}$	0.0027	0.063	0.48	1.00	391 (401)	18.5 (21.1)	2.15 (1.66)	1.00 (0.044)
	$\overline{HL}$	0.0027	0.064	0.48	1.00	380 (389)	18.0 (20.4)	2.13 (1.63)	1.00 (0.043)
	$\overline{TM}$	0.0027	0.063	0.48	1.00	390 (400)	18.6 (21.4)	2.16 (1.67)	1.00 (0.045)
	$\overline{TM}_{20}$	0.0027	0.062	0.47	1.00	395 (409)	19.3 (22.7)	2.18 (1.71)	1.00 (0.046)
	$ATM_{\bar{\bar{X}}}$	0.0027	0.064	0.48	1.00	382 (391)	17.9 (20.1)	2.13 (1.63)	1.00 (0.043)
	ATM_{MR}	0.0027	0.064	0.48	1.00	382 (391)	17.9 (20.1)	2.13 (1.63)	1.00 (0.043)
	$ATM_{\bar{TM}_{20}}$	0.0027	0.064	0.48	1.00	382 (391)	18.0 (20.2)	2.13 (1.63)	1.00 (0.043)
	$ATM_{\bar{TM}'}$	0.0027	0.064	0.48	1.00	380 (390)	18.0 (20.5)	2.13 (1.64)	1.00 (0.043)

Table 4.3: p, ARL and (in parentheses) SDRL of corrected limits under normality for $k = 30$

		p				ARL and $SDRL$			
n	$\hat{\mu}$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$
5	$\overline{\overline{X}}$	0.0030	0.030	0.21	0.92	358 (375)	45.0 (60.9)	5.21 (5.44)	1.09 (0.33)
	$M(\overline{X})$	0.0029	0.029	0.21	0.91	375 (395)	48.4 (67.7)	5.41 (5.77)	1.10 (0.34)
	$\overline{M}$	0.0028	0.029	0.21	0.91	387 (403)	46.5 (61.5)	5.34 (5.52)	1.10 (0.33)
	$\overline{\overline{X}}_{20}$	0.0028	0.029	0.21	0.92	376 (392)	45.1 (58.8)	5.23 (5.35)	1.09 (0.33)
	$\overline{HL}$	0.0029	0.029	0.21	0.92	370 (383)	43.2 (54.5)	5.13 (5.15)	1.09 (0.32)
	$\overline{TM}$	0.0027	0.029	0.21	0.92	384 (396)	44.2 (55.6)	5.19 (5.21)	1.09 (0.32)
	$\overline{TM}_{20}$	0.0027	0.028	0.21	0.91	390 (405)	46.0 (59.2)	5.29 (5.38)	1.10 (0.33)
	$ATM_{\overline{\overline{X}}}$	0.0030	0.030	0.21	0.92	362 (378)	44.4 (58.8)	5.18 (5.33)	1.09 (0.32)
	ATM_{MR}	0.0030	0.030	0.21	0.92	357 (374)	45.1 (61.6)	5.21 (5.44)	1.09 (0.33)
	$ATM_{\bar{TM}_{20}}$	0.0029	0.030	0.21	0.92	364 (379)	44.3 (58.3)	5.18 (5.31)	1.09 (0.32)
	$ATM_{\bar{TM}'}$	0.0028	0.029	0.21	0.92	375 (386)	42.7 (52.8)	5.09 (5.06)	1.09 (0.32)
9	$\overline{\overline{X}}$	0.0030	0.066	0.48	1.00	358 (375)	19.1 (24.0)	2.17 (1.73)	1.00 (0.045)
	$M(\overline{X})$	0.0030	0.065	0.47	1.00	371 (393)	20.6 (27.5)	2.23 (1.83)	1.00 (0.048)
	$\overline{M}$	0.0028	0.063	0.47	1.00	386 (403)	19.8 (24.5)	2.20 (1.75)	1.00 (0.047)
	$\overline{\overline{X}}_{20}$	0.0029	0.064	0.48	1.00	373 (389)	19.4 (24.1)	2.18 (1.74)	1.00 (0.046)
	$\overline{HL}$	0.0028	0.064	0.48	1.00	373 (385)	18.4 (21.5)	2.14 (1.66)	1.00 (0.044)
	$\overline{TM}$	0.0027	0.063	0.48	1.00	384 (397)	18.8 (22.1)	2.16 (1.69)	1.00 (0.045)
	$\overline{TM}_{20}$	0.0027	0.063	0.47	1.00	391 (406)	19.4 (23.3)	2.19 (1.74)	1.00 (0.046)
	$ATM_{\overline{\overline{X}}}$	0.0030	0.066	0.48	1.00	358 (375)	19.2 (24.0)	2.17 (1.73)	1.00 (0.046)
	ATM_{MR}	0.0031	0.066	0.48	1.00	356 (374)	19.1 (24.1)	2.17 (1.73)	1.00 (0.046)
	$ATM_{\bar{TM}_{20}}$	0.0030	0.066	0.48	1.00	360 (376)	19.0 (23.5)	2.16 (1.71)	1.00 (0.046)
	$ATM_{\bar{TM}'}$	0.0028	0.064	0.48	1.00	375 (386)	18.3 (21.3)	2.14 (1.65)	1.00 (0.044)

Table 4.4: p, ARL and (in parentheses) SDRL of corrected limits when symmetric variance disturbances are present for $k = 30$

n	$\hat{\mu}$	p				ARL and $SDRL$			
		$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$
5	$\overline{\overline{X}}$	0.0076	0.012	0.12	0.82	233 (295)	143 (210)	12.8 (24.0)	1.23 (0.60)
	$M(\overline{X})$	0.0034	0.019	0.16	0.88	347 (378)	77.0 (110)	7.34 (8.26)	1.14 (0.41)
	$\overline{M}$	0.0029	0.022	0.18	0.90	374 (395)	61.6 (82.4)	6.32 (6.73)	1.12 (0.37)
	$\overline{\overline{X}}_{20}$	0.0034	0.018	0.16	0.88	337 (366)	75.9 (103)	7.22 (7.91)	1.14 (0.41)
	$\overline{HL}$	0.0033	0.020	0.17	0.89	340 (363)	67.3 (90.4)	6.73 (7.36)	1.13 (0.39)
	$\overline{TM}$	0.0030	0.021	0.17	0.89	365 (384)	61.2 (69.2)	6.33 (6.61)	1.12 (0.37)
	$\overline{TM}_{20}$	0.0029	0.022	0.18	0.90	379 (398)	60.4 (78.7)	6.26 (6.56)	1.12 (0.37)
	$ATM_{\bar{\bar{X}}}$	0.0034	0.020	0.17	0.89	334 (359)	70.0 (95.0)	6.86 (7.51)	1.13 (0.39)
	ATM_{MR}	0.0076	0.012	0.12	0.82	232 (294)	143 (209)	13.0 (26.7)	1.24 (0.61)
	$ATM_{\bar{TM}_{20}}$	0.0032	0.021	0.17	0.89	347 (367)	62.9 (82.9)	6.46 (6.84)	1.12 (0.38)
	$ATM_{\bar{TM}'}$	0.0028	0.025	0.20	0.91	373 (385)	48.9 (60.2)	5.53 (5.55)	1.10 (0.34)
9	$\overline{\overline{X}}$	0.011	0.020	0.27	0.99	175 (239)	89.5 (148)	4.58 (6.42)	1.01 (0.12)
	$M(\overline{X})$	0.0044	0.035	0.36	0.99	299 (346)	42.1 (63.4)	3.04 (2.89)	1.01 (0.077)
	$\overline{M}$	0.0031	0.048	0.42	1.00	366 (389)	26.8 (34.1)	2.51 (2.12)	1.00 (0.058)
	$\overline{\overline{X}}_{20}$	0.0047	0.033	0.35	0.99	280 (324)	42.7 (61.0)	3.09 (2.89)	1.01 (0.078)
	$\overline{HL}$	0.0033	0.044	0.41	1.00	336 (358)	27.9 (33.9)	2.57 (2.15)	1.00 (0.059)
	$\overline{TM}$	0.0031	0.046	0.42	1.00	356 (378)	26.4 (31.9)	2.51 (2.08)	1.00 (0.057)
	$\overline{TM}_{20}$	0.0030	0.047	0.42	1.00	368 (391)	26.8 (33.2)	2.52 (2.11)	1.00 (0.058)
	$ATM_{\bar{\bar{X}}}$	0.0046	0.035	0.36	0.99	282 (322)	39.6 (57.4)	2.98 (2.76)	1.01 (0.074)
	ATM_{MR}	0.011	0.021	0.27	0.99	175 (240)	89.3 (148)	4.58 (6.41)	1.01 (0.12)
	$ATM_{\bar{TM}_{20}}$	0.0039	0.039	0.38	1.00	307 (339)	33.3 (44.1)	2.77 (2.44)	1.00 (0.066)
	$ATM_{\bar{TM}'}$	0.0028	0.055	0.45	1.00	370 (383)	21.4 (24.9)	2.30 (1.83)	1.00 (0.050)

Table 4.5: p, ARL and (in parentheses) SDRL of corrected limits when asymmetric variance disturbances are present for $k = 30$

		p				ARL and $SDRL$			
n	$\hat{\mu}$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$
5	$\overline{\overline{X}}$	0.0034	0.032	0.22	0.91	337 (361)	48.7 (73.0)	5.42 (6.09)	1.10 (0.34)
	$M(\overline{X})$	0.0028	0.029	0.21	0.91	382 (400)	47.4 (64.2)	5.40 (5.66)	1.10 (0.33)
	$\overline{M}$	0.0037	0.033	0.22	0.91	335 (372)	57.2 (98.5)	5.90 (7.45)	1.11 (0.36)
	$\overline{\overline{X}}_{20}$	0.0028	0.029	0.21	0.92	382 (395)	44.3 (55.8)	5.21 (5.25)	1.09 (0.32)
	$\overline{HL}$	0.0035	0.032	0.22	0.91	332 (358)	50.0 (76.8)	5.46 (6.22)	1.10 (0.34)
	$\overline{TM}$	0.0035	0.032	0.22	0.91	338 (368)	52.3 (83.4)	5.62 (6.63)	1.10 (0.35)
	$\overline{TM}_{20}$	0.0028	0.029	0.21	0.91	387 (403)	46.6 (61.7)	5.36 (5.55)	1.10 (0.33)
	$ATM_{\bar{\bar{X}}}$	0.0028	0.029	0.21	0.92	371 (382)	43.1 (53.7)	5.12 (5.13)	1.09 (0.32)
	ATM_{MR}	0.0033	0.031	0.22	0.92	342 (364)	47.7 (69.5)	5.34 (5.83)	1.10 (0.33)
	$ATM_{\bar{TM}_{20}}$	0.0028	0.029	0.21	0.92	371 (384)	42.9 (53.8)	5.12 (5.13)	1.09 (0.32)
	$ATM_{\bar{TM}'}$	0.0028	0.029	0.21	0.92	372 (384)	43.0 (53.7)	5.11 (5.10)	1.09 (0.32)
9	$\overline{\overline{X}}$	0.0034	0.068	0.48	1.00	337 (362)	20.4 (28.6)	2.21 (1.84)	1.00 (0.048)
	$M(\overline{X})$	0.0028	0.064	0.47	1.00	381 (400)	19.9 (25.0)	2.21 (1.78)	1.00 (0.047)
	$\overline{M}$	0.0038	0.069	0.47	1.00	332 (370)	24.0 (40.4)	2.32 (2.12)	1.00 (0.054)
	$\overline{\overline{X}}_{20}$	0.0028	0.064	0.48	1.00	382 (395)	18.9 (22.3)	2.17 (1.69)	1.00 (0.045)
	$\overline{HL}$	0.0035	0.069	0.48	1.00	333 (359)	20.8 (30.1)	2.22 (1.88)	1.00 (0.049)
	$\overline{TM}$	0.0035	0.068	0.48	1.00	338 (368)	21.8 (32.7)	2.25 (1.94)	1.00 (0.051)
	$\overline{TM}_{20}$	0.0028	0.063	0.47	1.00	385 (402)	19.7 (24.2)	2.20 (1.76)	1.00 (0.047)
	$ATM_{\bar{\bar{X}}}$	0.0028	0.065	0.48	1.00	372 (384)	18.4 (21.6)	2.15 (1.67)	1.00 (0.044)
	ATM_{MR}	0.0033	0.068	0.48	1.00	342 (364)	20.0 (27.2)	2.20 (1.81)	1.00 (0.048)
	$ATM_{\bar{TM}_{20}}$	0.0028	0.065	0.48	1.00	372 (384)	18.4 (21.7)	2.15 (1.67)	1.00 (0.044)
	$ATM_{\bar{TM}'}$	0.0029	0.065	0.48	1.00	368 (381)	18.6 (22.2)	2.15 (1.68)	1.00 (0.045)

Table 4.6: p, ARL and (in parentheses) SDRL of corrected limits when localized variance disturbances are present for $k = 30$

		p				ARL and $SDRL$			
n	$\hat{\mu}$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$
5	$\overline{\overline{X}}$	0.0061	0.011	0.11	0.83	224 (271)	137 (182)	10.9 (12.9)	1.22 (0.53)
	$M(\overline{X})$	0.0048	0.014	0.13	0.85	289 (340)	115 (168)	9.57 (12.2)	1.19 (0.49)
	$\overline{M}$	0.0033	0.019	0.16	0.88	351 (380)	74.6 (103)	7.12 (7.89)	1.14 (0.41)
	$\overline{\overline{X}}_{20}$	0.0049	0.013	0.13	0.85	271 (316)	115 (158)	9.52 (11.2)	1.19 (0.49)
	$\overline{HL}$	0.0042	0.015	0.14	0.86	290 (326)	93.0 (126)	8.22 (9.18)	1.16 (0.45)
	$\overline{TM}$	0.0035	0.017	0.15	0.88	333 (361)	77.8 (103)	7.33 (7.95)	1.14 (0.41)
	$\overline{TM}_{20}$	0.0032	0.019	0.16	0.88	356 (383)	72.5 (97.0)	7.00 (7.57)	1.13 (0.40)
	$ATM_{\overline{\overline{X}}}$	0.0055	0.012	0.12	0.84	245 (392)	123 (167)	10.0 (12.0)	1.20 (0.51)
	ATM_{MR}	0.0061	0.011	0.11	0.83	224 (272)	136 (182)	10.9 (12.9)	1.21 (0.53)
	$ATM_{\bar{TM}_{20}}$	0.0052	0.013	0.12	0.85	257 (302)	116 (159)	9.58 (11.2)	1.19 (0.49)
	$ATM_{\bar{TM}'}$	0.0031	0.024	0.19	0.90	356 (374)	57.0 (78.1)	6.01 (6.39)	1.11 (0.36)
9	$\overline{\overline{X}}$	0.0089	0.018	0.26	0.99	161 (208)	77.3 (107)	4.17 (4.16)	1.01 (0.11)
	$M(\overline{X})$	0.0074	0.023	0.29	0.99	212 (274)	71.7 (111)	3.96 (4.17)	1.01 (0.10)
	$\overline{M}$	0.0035	0.041	0.39	1.00	339 (371)	32.4 (42.7)	2.72 (2.39)	1.00 (0.065)
	$\overline{\overline{X}}_{20}$	0.0075	0.021	0.28	0.99	193 (246)	68.9 (98.5)	3.91 (3.90)	1.01 (0.10)
	$\overline{HL}$	0.0046	0.033	0.35	0.99	272 (310)	39.6 (51.4)	3.00 (2.69)	1.01 (0.074)
	$\overline{TM}$	0.0037	0.038	0.38	1.00	317 (349)	33.6 (42.7)	2.79 (2.43)	1.00 (0.067)
	$\overline{TM}_{20}$	0.0035	0.039	0.38	1.00	336 (368)	32.9 (42.0)	2.75 (2.40)	1.00 (0.066)
	$ATM_{\overline{\overline{X}}}$	0.0081	0.020	0.28	0.99	177 (227)	71.0 (99.9)	3.98 (3.96)	1.01 (0.10)
	ATM_{MR}	0.0089	0.019	0.26	0.99	162 (209)	77.8 (108)	4.16 (4.15)	1.01 (0.11)
	$ATM_{\bar{TM}_{20}}$	0.0074	0.022	0.29	0.99	193 (242)	64.9 (91.2)	3.80 (3.73)	1.00 (0.098)
	$ATM_{\bar{TM}'}$	0.0031	0.051	0.43	1.00	352 (371)	24.5 (30.7)	2.42 (2.00)	1.00 (0.054)

Table 4.7: p, ARL and (in parentheses) SDRL of corrected limits when diffuse mean disturbances are present for $k = 30$

		p				*ARL* and *SDRL*			
n	$\hat{\mu}$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 2$
5	$\overline{\overline{X}}$	0.017	0.0034	0.046	0.70	72.3 (87.5)	329 (351)	25.0 (28.8)	1.45 (0.83)
	$M(\overline{X})$	0.0031	0.021	0.17	0.89	366 (389)	66.0 (89.3)	6.62 (7.15)	1.13 (0.38)
	$\overline{M}$	0.017	0.0033	0.045	0.69	80.1 (105)	343 (372)	27.3 (33.6)	1.47 (0.86)
	$\overline{\overline{X}}_{20}$	0.0030	0.020	0.17	0.89	360 (379)	64.8 (81.8)	6.57 (6.81)	1.13 (0.38)
	$\overline{HL}$	0.017	0.0034	0.047	0.70	72.8 (89.3)	327 (350)	25.2 (29.2)	1.45 (0.83)
	$\overline{TM}$	0.017	0.0033	0.046	0.69	75.6 (94.4)	337 (362)	26.0 (30.7)	1.46 (0.84)
	$\overline{TM}_{20}$	0.0031	0.019	0.16	0.89	360 (385)	70.4 (92.0)	6.89 (7.32)	1.13 (0.40)
	$ATM_{\bar{\bar{X}}}$	0.0028	0.026	0.20	0.91	373 (385)	47.5 (49.1)	5.44 (5.47)	1.10 (0.33)
	ATM_{MR}	0.0028	0.029	0.21	0.92	378 (388)	42.1 (50.9)	5.08 (4.99)	1.09 (0.32)
	$ATM_{\bar{TM}_{20}}$	0.0028	0.029	0.21	0.92	378 (388)	42.8 (52.1)	5.12 (5.07)	1.09 (0.32)
	$ATM_{\bar{TM}'}$	0.0028	0.029	0.21	0.92	375 (386)	43.4 (53.4)	5.14 (5.11)	1.09 (0.32)
9	$\overline{\overline{X}}$	0.035	0.0039	0.11	0.96	34.3 (30.7)	293 (321)	10.1 (10.7)	1.04 (0.22)
	$M(\overline{X})$	0.0031	0.048	0.42	1.00	366 (389)	26.8 (34.3)	2.51 (2.12)	1.00 (0.058)
	$\overline{M}$	0.034	0.0039	0.11	0.95	37.9 (48.5)	308 (346)	10.8 (12.3)	1.05 (0.23)
	$\overline{\overline{X}}_{20}$	0.0030	0.046	0.41	1.00	359 (379)	26.3 (31.5)	2.51 (2.07)	1.00 (0.057)
	$\overline{HL}$	0.035	0.0039	0.11	0.96	34.5 (40.8)	293 (322)	10.1 (10.9)	1.05 (0.22)
	$\overline{TM}$	0.034	0.0039	0.11	0.96	35.7 (43.1)	301 (333)	10.4 (11.4)	1.05 (0.22)
	$\overline{TM}_{20}$	0.0031	0.044	0.41	1.00	361 (385)	28.4 (35.4)	2.58 (2.18)	1.00 (0.060)
	$ATM_{\bar{\bar{X}}}$	0.0029	0.055	0.45	1.00	366 (380)	21.7 (25.9)	2.30 (1.85)	1.00 (0.050)
	ATM_{MR}	0.0028	0.064	0.48	1.00	377 (387)	18.2 (20.9)	2.14 (1.64)	1.00 (0.044)
	$ATM_{\bar{TM}_{20}}$	0.0028	0.063	0.48	1.00	378 (388)	18.4 (21.1)	2.15 (1.66)	1.00 (0.044)
	$ATM_{\bar{TM}'}$	0.0028	0.063	0.48	1.00	376 (386)	18.6 (21.5)	2.16 (1.67)	1.00 (0.044)

Table 4.8: p, ARL and (in parentheses) SDRL of corrected limits when localized mean disturbances are present for $k = 30$

4.6 Concluding remarks

In this chapter we have considered several Phase I estimators of the location parameter for use in establishing $\overline{\overline{X}}$ Phase II control chart limits. The collection includes robust estimators proposed in the existing literature as well as several Phase I analyses, which apply a control chart retrospectively to study a historical dataset. The estimators have been evaluated under various circumstances: the uncontaminated situation and various situations contaminated with diffuse symmetric and asymmetric variance disturbances, localized variance disturbances, diffuse mean disturbances and localized mean disturbances.

The standard methods suffer from a number of problems. Estimators that are based on the principle of trimming individual observations (e.g. $\overline{M}$ and $\overline{TM}$) perform reasonably well when there are diffuse disturbances but not when localized disturbances are present. In the latter situation, estimators that are based on the principle of trimming samples (e.g. $M(\overline{X})$ and $\overline{\overline{X}}_{20}$) are efficient. All of these methods are biased when there are asymmetric disturbances, as the trimming principle does not take into account the asymmetry of the disturbance.

A Phase I analysis, using a control chart to study a historical dataset retrospectively and trim the data adaptively, does take into account the distribution of the disturbance and is therefore very suitable for use during the estimation of μ. However, the standard method based on the $\overline{\overline{X}}$ Phase I control chart has certain limitations. First, the initial estimate, $\overline{\overline{X}}$, is very sensitive to outliers so that the Phase I limits are biased. As a consequence, the wrong data samples are often filtered out. Second, the sample mean is usually plotted on the Phase I control chart, which makes it difficult to detect outliers in individual observations. Moreover, deleting the entire sample instead of the single outlier reduces efficiency.

To address the problems encountered in the standard Phase I analysis, we have proposed a new type of Phase I analysis. The initial estimate of μ for the Phase I control chart is based on a trimmed version of the trimean, namely $\overline{TM}_{20}$, and a subsequent procedure for both sample screening and outlier screening (resulting in $ATM_{\bar{TM}'}$). The proposed method is efficient under normality and far outperforms the existing methods when disturbances are present. Consequently, $ATM_{\bar{TM}'}$ is a very effective method for estimating μ in the limits used to construct the $\overline{X}$ Phase II chart.