Estimation methods for statistical process control

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Chapter 5

A Robust $\overline{X}$ Control Chart

5.1 Introduction

This chapter studies alternative standard deviation estimators that serve as a basis to determine the $\overline{X}$ control chart limits used for real-time process monitoring (Phase II). Several existing (robust) estimation methods are considered. In addition, we propose a new estimation method based on a Phase I analysis, whereby a control chart is used to identify disturbances in a dataset retrospectively. The method constructs a Phase I control chart derived from the trimmed mean of the sample interquartile ranges in order to identify out-of-control data. An efficient estimator, namely the mean of the sample standard deviations, is used to obtain the final standard deviation estimate from the remaining data. The estimation methods are evaluated in terms of their MSE and their effects on the performance of the $\overline{X}$ Phase II control chart. It is shown that the newly proposed estimation method is efficient under normality and performs substantially better than standard methods when disturbances are present in Phase I.

The chapter is structured as follows. We first present the estimation methods for the standard deviation and assess the MSE of the estimators. In the following sections, we present the design schemes for the $\overline{X}$ Phase II control chart and derive the control limits. Next, we describe the simulation procedure and present the effect of the proposed methods on Phase II performance. The final section offers some recommendations.

5.2 Proposed Phase I estimators

We analyze various types of standard deviation estimators and compare them under various types of disturbances. The next two sections introduce the standard deviation and location estimators respectively, while the third section presents the MSE of the standard deviation estimators.

5.2.1 Standard deviation estimators

We again denote the Phase I data by X_{ij}, $i = 1, 2, ..., k$ and $j = 1, 2, ..., n$. The X_{ij}'s are assumed to be independent and $N(\mu, \sigma^2)$ distributed. We denote by $X_{i,(v)}$, $v = 1, 2, ..., n$, the v-th order statistic in sample i. We look at several robust estimators proposed in the existing literature and introduce a new method incorporating a Phase I control chart.

We consider the traditional estimators of the standard deviation, namely the mean of the sample standard deviations $\overline{S}$ (see (2.2)), the mean of the sample ranges $\overline{R}$ (see (2.5)) and the mean of the sample interquartile ranges (see (2.6)). We saw in Chapter 2 that $\overline{S}$ is slightly less efficient under normality than the pooled sample standard deviation $\tilde{S}$.

Rocke (1989) proposed the trimmed mean of the sample interquartile ranges

$$\overline{IQR}_{\alpha} = \frac{1}{k - 2\lceil k\alpha \rceil} \times \left[\sum_{v=\lceil k\alpha \rceil + 1}^{k-\lceil k\alpha \rceil} IQR_{(v)} \right],$$

where $IQR_{(v)}$ denotes the v-th ordered value of the sample interquartile ranges. We consider the 20% trimmed mean of the sample interquartile ranges, which trims the ten smallest and the ten largest sample interquartile ranges when $k = 50$ and the twenty smallest and the twenty largest sample interquartile ranges when $k = 100$. An unbiased estimator of σ is given by $\overline{IQR}_{20}/d_{I\bar{Q}R_{20}}$, where $d_{I\bar{Q}R_{20}}$ is a normalizing constant. The value of this constant is 0.925 for $n = 5$ and 1.108 for $n = 9$.

We also evaluate a robust estimator proposed by Tatum (1997). This estimator is defined in (2.11).

We now present a new estimation method based on the principle of Phase I control charting. We build a Phase I control chart using a robust estimator for the standard deviation, namely $\overline{IQR}_{20}$. A disadvantage of this estimator is that it is not very efficient under normality. To address this, we use $\overline{IQR}_{20}$ to construct the Phase I limits with which we screen the estimation data for disturbances, but then use the efficient estimator $\bar{S}$ to obtain a standard deviation estimate from the remaining data. The Phase I standard deviation control chart limits are given by $\widehat{UCL}_{I\bar{Q}R_{20}} = U_n \overline{IQR}_{20}/d_{I\bar{Q}R_{20}}$ and $\widehat{LCL}_{I\bar{Q}R_{20}} = L_n \overline{IQR}_{20}/d_{I\bar{Q}R_{20}}$. For simplicity, we derive U_n and L_n from the 0.99865 and 0.00135 quantiles of the distribution of IQR/d_{IQR}. These quantiles are obtained from 1,000,000 simulation runs. The respective values of U_n and L_n are 3.220 and 0.035 for $n = 5$ and 2.487 and 0.145 for $n = 9$. We then plot the IQR_i/d_{IQR}'s of the Phase I samples on the Phase I control chart. Charting the IQR instead of the sample standard deviation or the sample range ensures that localized variance disturbances are identified and samples that contain only one single outlier are retained. A standard deviation estimator that is expected to be robust against localized variance disturbances is based on the mean of the sample interquartile ranges of the samples that fall between the control limits

$$\overline{IQR}' = \frac{1}{k'} \sum_{i \epsilon K} IQR_i \times 1_{\widehat{LCL}_{I\bar{Q}R_{20}} \leq IQR_i/d_{IQR} \leq \widehat{UCL}_{I\bar{Q}R_{20}}}(IQR_i),$$

with K the set of samples which are not excluded and k' the number of non-excluded samples. The resulting estimate $\overline{IQR}'/d_{IQR}$ is unbiased.

Although the remaining Phase I samples are expected to be free from localized variance disturbances, they could still contain diffuse disturbances. To eliminate such disturbances, the next step is to screen the individual observations using a Phase I individuals control chart. To screen the individual observations, we determine the residuals in each sample by subtracting the trimean value from each observation in the corresponding sample: $resid_{ij} = X_{ij} - TM_i$ with

$$TM_i = (Q_{i,1} + 2Q_{i,2} + Q_{i,3})/4.$$

Note that $Q_{i,2}$ is the median of sample i. Substracting the sample trimeans ensures that the variability is measured within samples and not between samples. The control limits of the individuals chart are given by $\widehat{UCL}_{I\bar{Q}R'} = 3\overline{IQR}'/d_{IQR}$ and $\widehat{LCL}_{I\bar{Q}R'} = -3\overline{IQR}'/d_{IQR}$. The residuals $resid_{ij}$ that fall above $\widehat{UCL}_{I\bar{Q}R'}$ or below $\widehat{LCL}_{I\bar{Q}R'}$ are considered out of control and their corresponding observations are removed from the Phase I dataset. The final estimate is the mean of the sample standard deviations S_i and is calculated from the observations deemed to be in control

$$\bar{S}' = \frac{1}{k^{\wedge}} \sum_{i \epsilon K^{\wedge}} S_i(\{X_{ij} \times 1_{\widehat{LCL}_{I\bar{Q}R'} \leq resid_{ij} \leq \widehat{UCL}_{I\bar{Q}R'}}\})(X_{ij}),$$

with $K^{\wedge}$ the set of samples which are not excluded and $k^{\wedge}$ the number of non-excluded samples. The normalizing constant is 0.980 for $n = 5$ and 0.984 for $n = 9$. This adaptively trimmed standard deviation is denoted by ATS.

The proposed standard deviation estimators are summarized in Table 5.1.

Estimator	Notation
Mean of sample standard deviations	$\bar{S}$
Mean of sample ranges	$\bar{R}$
Mean of sample interquartile ranges	$\overline{IQR}$
20% trimmed mean of sample interquartile ranges	$\overline{IQR}_{20}$
Tatum's estimator	$D7$
$\overline{IQR}'$ control chart with screening	ATS

Table 5.1: Proposed standard deviation estimators

5.2.2 Location estimator

The above standard deviation estimators are used to construct the $\overline{X}$ Phase II control limits. To ensure a fair comparison, we use the same location estimator in each case. The location estimation method uses a procedure similar to ATS above. Chapter 4 showed that this procedure performs better than the standard procedures based on estimators such as the mean, median, trimmed mean and Hodges-Lehmann. Again, the procedure consists of two steps.

In the first step we determine a location estimator that is robust against both localized and diffuse mean disturbances, namely the 20% trimmed mean of the sample trimeans

$$\overline{TM}_{\alpha} = \frac{1}{k - 2\lceil k\alpha \rceil} \times \left[\sum_{v=\lceil k\alpha \rceil + 1}^{k - \lceil k\alpha \rceil} TM_{(v)} \right].$$

Note that we start with the entire dataset. The respective upper and lower control limits for the sample location are given by $\widehat{UCL}_{\bar{TM}_{20}} = \overline{TM}_{20} + 3\hat{\sigma}/\sqrt{n}$ and $\widehat{LCL}_{\bar{TM}_{20}} = \overline{TM}_{20} - 3\hat{\sigma}/\sqrt{n}$, where σ is estimated by the corresponding standard deviation estimator from Table 5.1. We then plot the TM_i's of the Phase I samples on the control chart. Charting the TM_i's instead of the $\overline{X}_i$'s ensures that localized disturbances are identified and samples that contain only one single outlier are retained. A location estimator that is expected to be robust against localized mean disturbances is the mean of the sample trimeans of the samples that fall between the control limits

$$\overline{TM}' = \frac{1}{k^*} \sum_{i \epsilon K^*} TM_i \times 1_{\widehat{LCL}_{\bar{TM}_{20}} \leq TM_i \leq \widehat{UCL}_{\bar{TM}_{20}}}(TM_i),$$

with K^* the set of samples which are not excluded and k^* the number of non-excluded samples.

Although the remaining Phase I samples are expected to be free from localized mean disturbances, they could still contain diffuse disturbances. To eliminate such disturbances, the next step is to screen the individual observations using a Phase I individuals control chart with respective upper and lower control limits given by $\widehat{UCL}_{\bar{TM}'} = \overline{TM}' + 3\hat{\sigma}$ and $\widehat{LCL}_{\bar{TM}'} = \overline{TM}' - 3\hat{\sigma}$, where σ is estimated by the corresponding standard deviation estimator from Table 5.1. The observations X_{ij} that fall above $\widehat{UCL}_{\bar{TM}'}$ or below $\widehat{LCL}_{\bar{TM}'}$ are considered out of control and removed from the Phase I dataset. The final estimate is based on the mean of the sample means and is calculated from the observations deemed to be in control

$$\overline{\overline{X}}' = \frac{1}{k''} \sum_{i \epsilon K''} \frac{1}{n_i'} \sum_{j \epsilon N_i'} X_{ij} \times 1_{\widehat{LCL}_{\bar{TM}'} \leq X_{ij} \leq \widehat{UCL}_{\bar{TM}'}}(X_{ij}),$$

with K'' the samples which are not excluded, k'' the number of non-excluded samples, N_i' the non-excluded observations in sample i and n_i' the number of non-excluded observations in sample i.

5.2.3 Efficiency of proposed standard deviation estimators

We again use the MSE of the proposed standard deviation estimators to evaluate their efficiency. The MSE is estimated as

$$MSE = \frac{1}{N} \sum_{i=1}^{N} \left(\hat{\sigma}^i - \sigma \right)^2,$$

where $\hat{\sigma}^i$ is the value of the unbiased estimate in the i-th simulation run and N is the number of simulation runs. We consider the uncontaminated case, i.e. the situation where all the X_{ij}'s are from the $N(0, 1)$ distribution, as well as four types of disturbances (see Section 2.2.2).

The MSE is obtained for $k = 50, 100$ subgroups of sizes $n = 5, 9$. The number of simulation runs N is equal to 50,000.

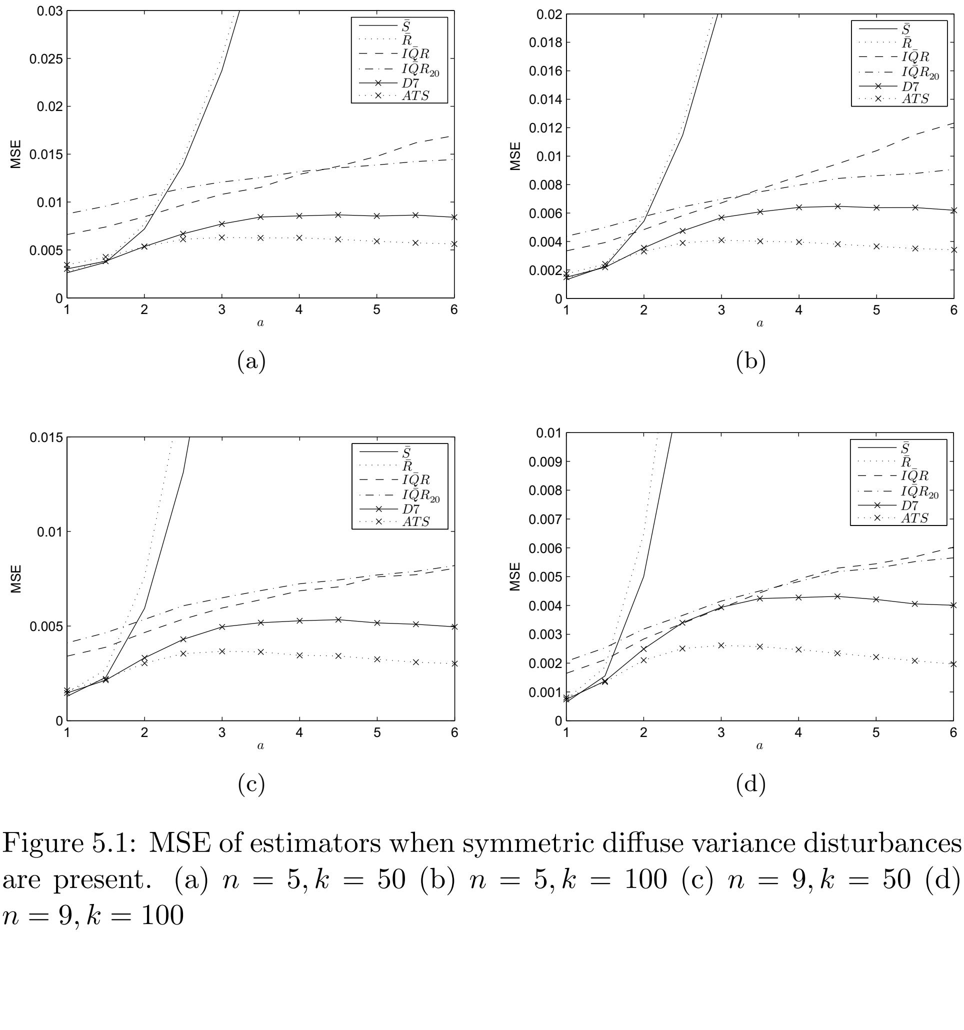

Figure 5.1: MSE of estimators when symmetric diffuse variance disturbances are present. (a) $n = 5, k = 50$ (b) $n = 5, k = 100$ (c) $n = 9, k = 50$ (d) $n = 9, k = 100$

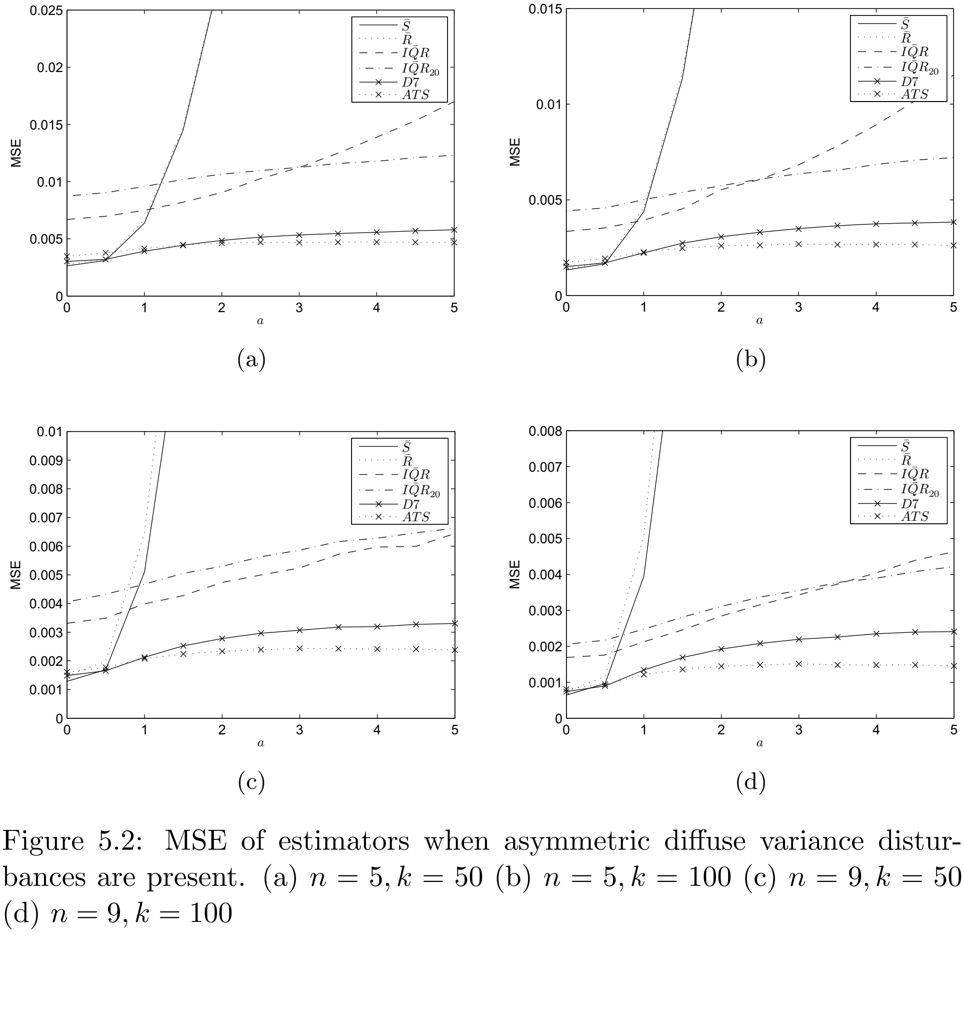

Figure 5.2: MSE of estimators when asymmetric diffuse variance disturbances are present. (a) $n = 5, k = 50$ (b) $n = 5, k = 100$ (c) $n = 9, k = 50$ (d) $n = 9, k = 100$

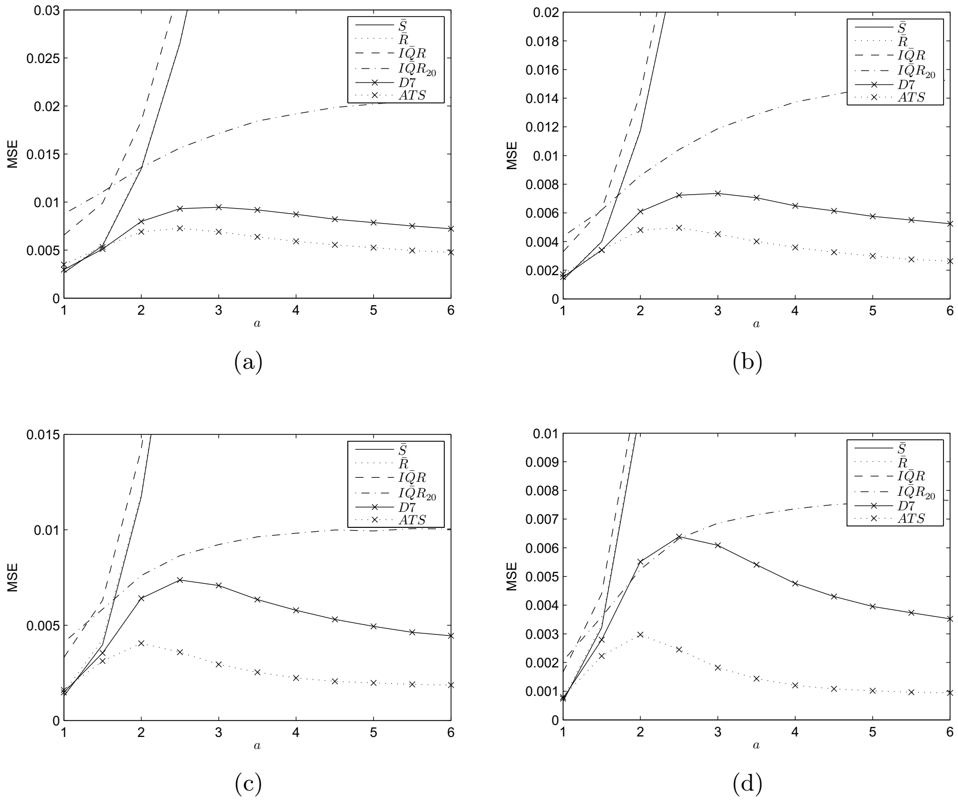

Figure 5.3: MSE of estimators when localized variance disturbances are present. (a) $n = 5, k = 50$ (b) $n = 5, k = 100$ (c) $n = 9, k = 50$ (d) $n = 9, k = 100$

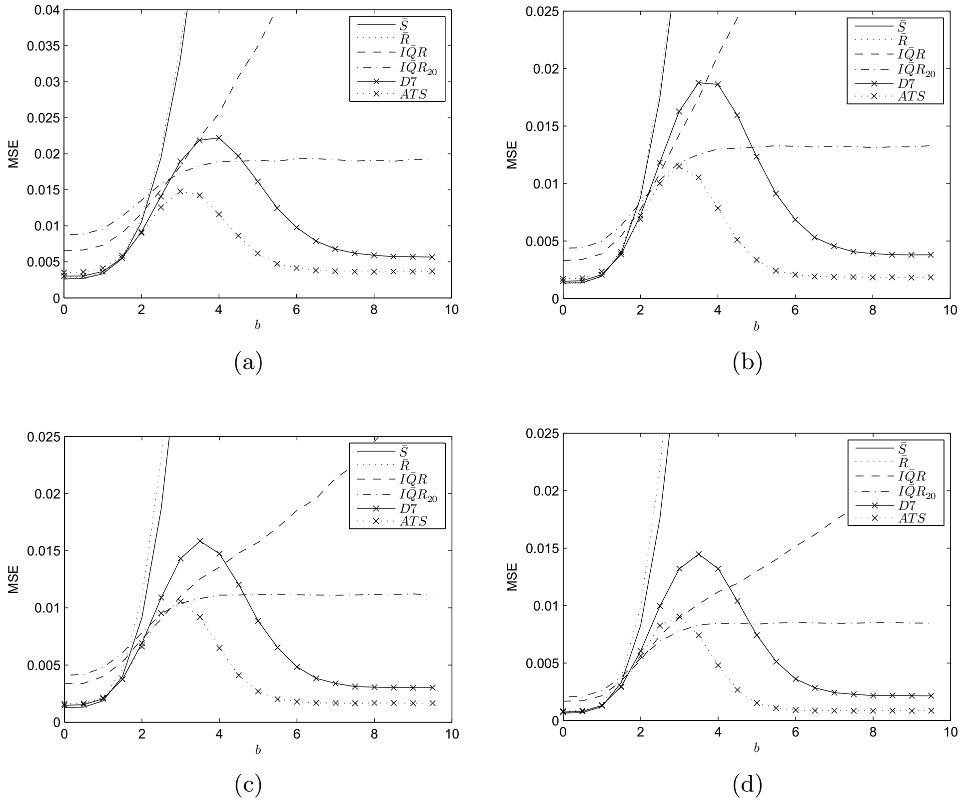

Figure 5.4: MSE of estimators when diffuse mean disturbances are present. (a) $n = 5, k = 50$ (b) $n = 5, k = 100$ (c) $n = 9, k = 50$ (d) $n = 9, k = 100$

Figures 5.1-5.4 show the MSE of the proposed estimators. The following results can be observed. The standard estimators $\bar{S}$ and $\bar{R}$ are not robust against either localized or diffuse disturbances. The $\overline{IQR}$ is less efficient under normality when there are no contaminations, but performs reasonably well when there are diffuse disturbances. The reason why $\overline{IQR}$ performs so well in these situations is that it trims the highest and lowest observations in each sample. However, this estimator remains biased when there are asymmetric diffuse disturbances because the trimming method does not take the distribution of the contaminations into account. Furthermore, this estimator is not efficient when there are localized variance disturbances as it trims only the observations within the sample instead of the sample interquartile ranges.

An estimator that combines within-sample and between-sample trimming, namely $\overline{IQR}_{20}$, performs reasonably well for all types of contaminations. However, its efficiency is relatively low under normality. $D7$ is efficient under normality as well as for contaminated data but relatively less so when the contamination consists of localized variance disturbances.

The estimator ATS is slightly less efficient under normality than the standard estimators, but much more robust than $\overline{IQR}$ and $\overline{IQR}_{20}$. Moreover, it shows outstanding performance when contaminations are present. We can therefore conclude that this estimator effectively filters out extreme observations.

5.3 Derivation of Phase II control limits

We now turn to the effect of the proposed estimators on the performance of the $\overline{X}$ Phase II control chart. The formulae for the $\overline{X}$ control limits with estimated parameters are given by (1.4). The factor C_n that is used to obtain accurate control limits when the process parameters are estimated is derived such that the probability of a false signal equals the chosen type I error probability p. The factors can not be obtained easily in analytic form. Therefore, they are obtained by means of simulation. The chosen type I error probability p is 0.0027. 50,000 simulation runs are used. The resulting factors are presented in Table 5.2.

	Factors for control limits			
	$n=5$		$n=9$	
$\hat{\sigma}$	$k=50$	$k=100$	$k=50$	$k=100$
$\bar{S}$	3.065	3.030	3.050	3.025
$\overline{R}$	3.070	3.035	3.055	3.025
$\overline{IQR}$	3.125	3.060	3.080	3.040
$\overline{IQR}_{20}$	3.155	3.080	3.090	3.045
$D7$	3.070	3.035	3.050	3.025
ATS	3.085	3.040	3.055	3.025

Table 5.2: Factors C_n to determine Phase II control limits

5.4 Control chart performance

In this section we evaluate the effect on $\overline{X}$ Phase II performance of the proposed standard deviation estimators. We consider the same Phase I situations as those used to assess the MSE with a, b and the multiplier equal to 4 to simulate the contaminated cases (see Section 5.2.3).

The performance of the Phase II control charts is assessed in terms of the unconditional p and ARL as well as the conditional ARL. The conditional ARL values express the ARL values for the control limits associated with the 2.5% and 95.7% quantiles of p in the in-control situation. We consider different shifts of size $\delta\sigma$ in the mean in Phase II, namely δ equal to 0, 0.25, 0.5 and 1. The performance characteristics are obtained by simulation. Sections 5.4.1 and 5.4.2 describe the simulation procedure and simulation results, enabling a comparison of control charts across the uncontaminated and contaminated situations.

5.4.1 Simulation procedure

The performance characteristics p and ARL for estimated control limits are determined by averaging the conditional characteristics, i.e. the characteristics for a given set of estimated control limits, over all possible values of the control limits. The corresponding definitions of $p(F_i|\hat{\mu},\hat{\sigma})$, $E(RL|\hat{\mu},\hat{\sigma})$, $p = E(p(F_i|\hat{\mu},\hat{\sigma}))$ and $ARL = E(\frac{1}{p(F_i|\hat{\mu},\hat{\sigma})})$ are obtained from (1.5)-(1.8), where the variables are conditioned on $\hat{\mu}$ and $\hat{\sigma}$. These expectations are obtained by simulation: numerous datasets are generated and for each dataset $p(F_i|\hat{\mu}, \hat{\sigma})$ and $E(RL|\hat{\mu}, \hat{\sigma})$ are computed. By averaging these values we obtain the unconditional values.

Enough replications of the above procedure were performed to obtain sufficiently small relative estimated standard errors for p and ARL. The relative standard error of the estimates is never higher than 0.80%.

5.4.2 Simulation results

The performance metrics are obtained in the in-control situation as well as in the out-of-control situation. When the process is in control ($\delta = 0$), we want p to be as low as possible and ARL to be as high as possible. In the out-of-control situation ($\delta \neq 0$), we want to achieve the opposite.

Tables 5.3 and 5.4 show the results for the $\overline{X}$ Phase II charts under normality. In this case we have estimated both the in-control μ and σ in Phase I. Unlike the $\overline{X}$ Phase II performance presented in Chapter 4, where only the mean was estimated to isolate the effect of location estimation, the ARL values are much higher than the desired 370. Thus, estimating the process standard deviation as well as the mean has substantially more impact on the $\overline{X}$ Phase II control chart than only estimating the process mean.

The conditional ARL values are presented in parentheses. The first value represents the ARL for the control limits associated with the 97.5% quantile of the simulated p in the in-control situation, while the second value represents the ARL for the control limits associated with the 2.5% quantile of the simulated p in the in-control situation. The results show that the conditional ARL values vary quite strongly, even when k equals 100.

In the absence of any contamination, the charts based on $\bar{S}$, $\bar{R}$, $D7$ and ATS show comparable performance. The charts based on $\overline{IQR}$ and $\overline{IQR}_{20}$ are less powerful under normality.

The analysis shows that, when there are disturbances in the Phase I data, the performance of all charts changes considerably: there is a sizeable decrease in p and increase in ARL. In other words, when the Phase I data are contaminated, shifts in the process mean are less quickly detected. When symmetric disturbances are present (Tables 5.5 and 5.6), their impact is the smallest for the charts based on $\overline{IQR}$, $\overline{IQR}_{20}$, $D7$ and ATS. These charts are also least affected when there are asymmetric disturbances (Tables 5.7 and 5.8). Both tables show that the chart based on ATS outperforms the others.

When there are localized disturbances (Tables 5.9 and 5.10), the charts based on the estimators $D7$ and ATS perform best, the reason being that these charts trim extreme samples. Finally, in the case of diffuse mean disturbances (Tables 5.11 and 5.12), the charts based on $\overline{IQR}_{20}$, $D7$ and ATS perform better than the other charts.

Overall, the ATS chart performs best. Under normality, the chart essentially matches the performance of the standard charts based on $\bar{S}$ and $\bar{R}$ and, in the presence of any contamination, the chart outperforms the alternatives.

5.5 Concluding remarks

We have analyzed several estimation methods for the standard deviation parameter and compared the MSE of the estimators under a range of circumstances: the uncontaminated situation and various situations contaminated with diffuse symmetric and asymmetric variance disturbances, localized variance disturbances and diffuse mean disturbances. We have also investigated the effect of estimating the standard deviation on $\overline{X}$ Phase II control chart performance when the methods are used to determine the Phase II limits.

One of the proposed methods has allowed us to address certain problems exhibited by the standard methods. Estimators that trim observations (e.g. $\overline{IQR}$) perform reasonably well when there are diffuse disturbances but not when there are localized disturbances. In the latter case, estimators that include a method to trim sample statistics (e.g. $\overline{IQR}_{20}$) are efficient. But all such methods are biased when disturbances are asymmetric, as the trimming does not take asymmetry into account.

A Phase I analysis - using a control chart to study a historical dataset retrospectively and trim the data adaptively - does consider the distribution of the disturbance, making it well-suited to the estimation of σ. In this chapter we have proposed a new type of Phase I analysis. The initial estimate of σ for the Phase I control chart is given by an estimator that is robust against both diffuse and localized disturbances, namely $\overline{IQR}_{20}$. We have shown that this estimator is not very efficient under normality. However, when $\overline{IQR}_{20}$ is only used to construct the Phase I control chart limits, and when the standard estimation method $\bar{S}$ is used to determine the final estimate of σ after screening, the resulting estimator (ATS) is efficient under normality. Moreover, ATS outperforms the other estimation methods when there are contaminations. It is therefore a suitable method for determining the value of σ in the $\overline{X}$ Phase II control chart limits.

Note that in Chapter 3 we evaluated the standard deviation estimator $\overline{MD}^{i,s}$, whose performance essentially matches that of ATS. The difference is that, in the case of ATS, the estimators used in Phase I and Phase II are not the same. The estimator used in Phase I is very robust so that the Phase I control limits better represent the in-control state of the process. As a result, only one iteration of the screening step is required. We think that, on the whole, ATS is of more practical use.

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	0.0027	0.0073	0.029	0.21
	$\bar{R}$	0.0027	0.0073	0.028	0.21
	$\overline{IQR}$	0.0027	0.0071	0.027	0.20
	$\overline{IQR}_{20}$	0.0027	0.0069	0.026	0.19
	$D7$	0.0027	0.0073	0.028	0.21
	ATS	0.0027	0.0072	0.028	0.20
100	$\bar{S}$	0.0027	0.0075	0.029	0.22
	$\bar{R}$	0.0027	0.0074	0.029	0.22
	$\overline{IQR}$	0.0027	0.0074	0.029	0.21
	$\overline{IQR}_{20}$	0.0027	0.0072	0.028	0.21
	$D7$	0.0027	0.0074	0.029	0.21
	ATS	0.0027	0.0073	0.029	0.21
		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	489	193	44.9	5.24
		(155; 1256)	(64.1; 418)	(18.6; 85.7)	(3.21; 7.94)
	$\bar{R}$	500	196	46.2	5.30
		163; 1318)	(63.3; 489)	(18.4; 98.1)	(3.19; 8.58)
	$\overline{IQR}$	770	289	60.7	6.09
		(110; 3127)	(75.7; 1838)	(22.6; 307)	(3.48; 17.4)
	$\overline{IQR}_{20}$	1066	375	73.2	6.74
		(96.4; 5089)	(42.8; 1332)	(13.5; 227)	(2.69; 14.9)
	$D7$	507	201	46.2	5.30
		145; 1374)	(81.2; 493)	(23.1; 98.6)	(3.56; 8.63)
	ATS	543	211	48.4	5.45
		(135; 1536)	(57.8; 451)	(17.1; 91.2)	(3.06; 8.32)
100	$\bar{S}$	419	159	38.5	4.83
		(194; 818)	(77.1; 398)	(21.5; 83.7)	(3.48; 7.66)
	$\bar{R}$	428	161	38.9	4.85
		(193; 857)	(76.7; 282)	(21.4; 61.8)	(3.48; 6.50)
	$\overline{IQR}$	521	189	43.9	5.18
		(146; 1487)	(61.1; 522)	(17.9; 103)	(3.14; 8.89)
	$\overline{IQR}_{20}$	598	213	47.9	5.41
		(133; 1929)	(56.6; 553)	(16.8; 108)	(3.03; 9.26)
	$D7$	431	163	39.1	4.88
		(189; 877)	(81.6; 290)	(22.6; 63.2)	(3.57; 6.59)
	ATS	446	168	40.1	4.94
		(179; 940)	(80.0; 296)	(22.3; 64.3)	(3.53; 6.68)

Table 5.3: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values under normality for $n = 5$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	0.0027	0.012	0.063	0.48
	$\bar{R}$	0.0027	0.012	0.063	0.48
	$\overline{IQR}$	0.0027	0.012	0.062	0.47
	$\overline{IQR}_{20}$	0.0027	0.012	0.061	0.46
	$D7$	0.0027	0.012	0.064	0.48
	ATS	0.0027	0.012	0.063	0.48
100	$\bar{S}$	0.0027	0.012	0.065	0.49
	$\bar{R}$	0.0027	0.012	0.063	0.48
	$\overline{IQR}$	0.0027	0.012	0.064	0.49
	$\overline{IQR}_{20}$	0.0027	0.012	0.064	0.48
	$D7$	0.0027	0.012	0.065	0.49
	ATS	0.0027	0.012	0.065	0.49

		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	427	106	18.0	2.13
		189; 846)	(148; 157)	(23.8; 24.6)	(2.31; 2.48)
	$\bar{R}$	440	107	18.2	2.14
		(186; 923)	(53.1; 171)	(10.9; 26.2)	(1.75; 2.54)
	$\overline{IQR}$	537	124	20.1	2.22
		142; 1512)	(38.2; 257)	(8.61; 35.8)	(1.61; 2.94)
	$\overline{IQR}_{20}$	588	135	21.1	2.26
		(135; 1858)	(36.7; 388)	(8.37; 48.7)	(1.60; 3.36)
	$D7$	432	106	18.0	2.14
		(182; 885)	(65.8; 184)	(12.7; 27.6)	(1.85; 2.59)
	ATS	441	107	18.2	2.15
		(177; 931)	(127; 171)	(21.1; 26.3)	(2.21; 2.55)
100	$\bar{S}$	397	95.5	16.4	2.06
		(232; 643)	(56.4; 132)	(11.4; 21.6)	(1.79; 2.33)
	$\bar{R}$	398	92.8	16.4	2.06
		(223; 668)	(61.2; 135)	(12.1; 21.9)	(1.83; 2.35)
	$\overline{IQR}$	440	99.8	17.3	2.10
		(182; 941)	(47.2; 226)	(10.0; 32.1)	(1.71; 2.75)
	$\overline{IQR}_{20}$	462	103	17.6	2.12
		(174; 1066)	(46.6; 193)	(9.94; 28.7)	(1.70; 2.65)
	$D7$	403	93.1	16.4	2.07
		(225; 670)	(61.1; 130)	(12.1; 21.4)	(1.83; 2.33)
	ATS	401	92.9	16.4	2.07
		(219; 681)	(75.8; 143)	(14.1; 22.7)	(1.92; 2.38)

Table 5.4: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values under normality for $n = 9$

			p		
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	4.3×10^{-4}	0.0014	0.0070	0.081
	$\bar{R}$	4.1×10^{-4}	0.0013	0.0067	0.079
	$\overline{IQR}$	0.0015	0.0042	0.018	0.15
	$\overline{IQR}_{20}$	0.0016	0.0046	0.018	0.15
	$D7$	0.0015	0.0044	0.019	0.16
	ATS	0.0019	0.0053	0.022	0.17
100	$\bar{S}$	3.5×10^{-4}	0.0012	0.0065	0.082
	$\bar{R}$	3.3×10^{-4}	0.0012	0.0062	0.079
	$\overline{IQR}$	0.0014	0.0042	0.018	0.16
	$\overline{IQR}_{20}$	0.0016	0.0046	0.019	0.16
	$D7$	0.0015	0.0044	0.019	0.16
	ATS	0.0019	0.0053	0.022	0.18

			ARL		
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	4.3×10^{4}	3609	441	20.2
		(343; 1.4×10^{4})	(76.1; 1675)	(7.12; 58.7)	
	$\bar{R}$	1.4×10^{4}	364	475	21.1
		562; 7.8×10^{4})	(275; 1.5×10^{4})	(61.7; 1758)	(6.35; 61.1)
	$\overline{IQR}$	2041	657	120	9.08
		(166; 9948)	(109; 2979)	(30.1; 449)	(4.10; 33.3)
	$\overline{IQR}_{20}$	2288	746	128	9.28
		144; 1.2×10^{4})	(70.4; 2700)	(20.3; 413)	(3.34; 22.4)
	$D7$	1107	401	82.6	7.38
		(221; 3820)	(102; 1209)	(27.1; 209)	(3.94; 14.0)
	ATS	898	335	70.8	6.73
		(170; 3064)	(69.9; 860)	(19.9; 157)	(3.33; 11.7)
100	$\bar{S}$	5922	1599	255	15.2
		880; 2.4×10^{4})	(309; 5101)	(66.9; 713)	(6.80; 32.5)
	$\bar{R}$	6506	174	271	15.9
		(912; 2.7×10^{4})	(302; 6155)	(65.4; 837)	(6.73; 36.1)
	$\overline{IQR}$	1116	375	77.1	7.17
		240; 3632)	(99.8; 952)	(26.6; 171)	(3.91; 12.4)
	$\overline{IQR}_{20}$	1131	370	76.5	7.10
		204; 4062)	(83.7; 1046)	(23.0; 185)	(3.61; 13.1)
	$D7$	874	304	64.8	6.57
		(303; 2126)	(126; 785)	(32.1; 146)	(4.36; 11.0)
	ATS	693	247	55.1	5.96
		235; 1696)	(190; 507)	(49.3; 101)	(5.40; 8.83)

Table 5.5: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when symmetric variance disturbances are present in Phase I for $n = 5$

				p	
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.9×10^{-4}	0.0019	0.015	0.23
	$\bar{R}$	2.0×10^{-4}	0.0014	0.011	0.20
	$\overline{IQR}$	0.0016	0.0078	0.045	0.40
	$\overline{IQR}_{20}$	0.0017	0.0079	0.046	0.41
	$D7$	0.0016	0.0078	0.046	0.41
	ATS	0.0019	0.0091	0.051	0.43
100	$\bar{S}$	2.5×10^{-4}	0.0017	0.014	0.23
	$\bar{R}$	1.7×10^{-4}	0.0012	0.011	0.20
	$\overline{IQR}$	0.0016	0.0079	0.047	0.42
	$\overline{IQR}_{20}$	0.0017	0.0081	0.048	0.42
	$D7$	0.0016	0.0077	0.046	0.42
	ATS	0.0019	0.0091	0.053	0.45
				ARL	
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.0×10^{4}	1445	127	5.14
		(934; 4.8×10^{4})	(185; 1.2×10^{4})	(67.7; 732)	(2.60; 14.0)
	$\bar{R}$	2.2×10^{4}	2702	206	6.42
		(1162; 1.2×10^{5})	(230; 2.2×10^{4})	(32.7; 1246)	(2.80; 19.3)
	$\overline{IQR}$	983	209	29.4	2.62
		(218; 3166)	(62.1; 459)	(12.2; 56.3)	(1.83; 3.66)
	$\overline{IQR}_{20}$	1047	219	30.3	2.64
		(196; 3690)	(109; 533)	(18.5; 63.2)	(2.11; 3.87)
	$D7$	799	180	26.7	2.52
		(271; 1945)	(85.7; 348)	(15.4; 45.0)	(2.00; 3.26)
	ATS	667	154	23.7	2.39
		(225; 1618)	(64.3; 274)	(12.5; 37.6)	(1.85; 3.00)
100	$\bar{S}$	6623	934	95.2	4.61
		(1446; 2.2×10^{4})	(244; 2308)	(34.4; 203)	(2.89; 7.15)
	$\bar{R}$	1.2×10^{4}	1492	135	5.49
		(1851; 4.6×10^{4})	(305; 5238)	(40.9; 390)	(3.13; 10.2)
	$\overline{IQR}$	785	160	24.6	2.44
		287; 1831)	(67.6; 416)	(13.1; 51.2)	(1.89; 3.42)
	$\overline{IQR}_{20}$	785	160	24.4	2.44
		(263; 1953)	(61.9; 317)	(12.2; 42.1)	(1.84; 3.17)
	$D7$	724	152	23.7	2.41
		(354; 1367)	(91.8; 240)	(16.3; 33.9)	(2.06; 2.86)
	ATS	591	129	20.9	2.28
		(287; 1106)	(71.4; 195)	(13.6; 29.0)	(1.92; 2.67)

Table 5.6: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when symmetric variance disturbances are present in Phase I for $n = 9$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.9×10^{-4}	9.2×10^{-4}	0.0045	0.054
	$\bar{R}$	2.8×10^{-4}	9.2×10^{-4}	0.0046	0.055
	$\overline{IQR}$	0.0016	0.0045	0.018	0.15
	$\overline{IQR}_{20}$	0.0018	0.0050	0.020	0.16
	$D7$	0.0019	0.0053	0.022	0.17
	ATS	0.0022	0.0061	0.024	0.19
100	$\bar{S}$	1.7×10^{-4}	6.07×10^{-4}	0.0033	0.048
	$\bar{R}$	1.7×10^{-4}	6.3×10^{-4}	0.0034	0.049
	$\overline{IQR}$	0.0016	0.0045	0.019	0.16
	$\overline{IQR}_{20}$	0.0018	0.0051	0.021	0.17
	$D7$	0.0018	0.0053	0.022	0.18
	ATS	0.0022	0.0061	0.025	0.19

		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.4×10^{10}	4.1×10^{14}	1.5×10^{8}	6.2×10^{4}
		549; 2.2×10^{7})	(277; 2.1×10^{7})	(62.3; 1.1×10^{6})	(6.38; 7196)
	$\bar{R}$	4.2×10^{9}	2.9×10^{7}	2.5×10^{7}	3532
		(575; 2.2×10^{7})	(194; 1.4×10^{6})	(95.4; 9.6×10^{4})	(5.40; 1172)
	$\overline{IQR}$	4219	1376	158	9.88
		(158; 1324)	(72.3; 2951)	(20.6; 446)	(3.38; 23.6)
	$\overline{IQR}_{20}$	1944	670	118	8.74
		(132; 1.0×10^{4})	(62.8; 8338)	(18.4; 1152)	(3.17; 42.0)
	$D7$	820	318	67.7	6.62
		(189; 2565)	(76.4; 702)	(21.4; 132)	(3.47; 10.5)
	ATS	723	283	60.7	6.23
		(155; 2292)	(77.7; 647)	(22.0; 123)	(3.49; 10.1)
100	$\bar{S}$ 8.7×10^{5}	6.7×10^{4}	1.6×10^{4}	123	
		(1107; 1.5×10^{6})	(467; 2.9×10^{5})	(94.7; 2.4×10^{4})	(8.34; 402)
	$\bar{R}$	6.8×10^{5}	7.0×10^{6}	5843	69.0
		1097; 1.0×10^{6})	(402; 1.4×10^{5})	(83.1; 1.3×10^{4})	(7.75; 260)
	$\overline{IQR}$	1204	400	78.8	7.23
		(223; 4523)	(109; 1148)	(29.0; 200)	(4.07; 13.8)
	$\overline{IQR}_{20}$	988	341	69.9	6.75
		188; 3510)	(80.2; 1006)	(22.3; 179)	(3.54; 12.7)
	$D7$	669	246	55.1	5.94
		(254; 1525)	(99.4; 450)	(26.4; 91.1)	(3.91; 8.30)
	ATS	571	213	49.0	5.54
		(209; 1312)	(83.8; 397)	(23.0; 82.0)	(3.62; 7.77)

Table 5.7: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when asymmetric variance disturbances are present in Phase I for $n = 5$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.3×10^{-4}	8.5×10^{-4}	0.0071	0.14
	$\bar{R}$	1.0×10^{-4}	6.7×10^{-4}	0.0058	0.12
	$\overline{IQR}$	0.0018	0.0084	0.048	0.42
	$\overline{IQR}_{20}$	0.0019	0.0087	0.049	0.42
	$D7$	0.0020	0.0092	0.052	0.44
	ATS	0.0023	0.010	0.056	0.45
100	$\bar{S}$	7.4×10^{-5}	5.7×10^{-4}	0.0054	0.13
	$\bar{R}$	5.5×10^{-5}	4.4×10^{-4}	0.0043	0.11
	$\overline{IQR}$	0.0018	0.0086	0.050	0.43
	$\overline{IQR}_{20}$	0.0019	0.0089	0.051	0.43
	$D7$	0.0019	0.0092	0.053	0.45
	ATS	0.0022	0.010	0.058	0.46

		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.4×10^{8}	1.1×10^{7}	1.9×10^{5}	190
		(1173; 1.7×10^{7})	(208; 1.1×10^{6})	(30.5; 3.5×10^{4})	(2.72; 166)
	$\bar{R}$	2.1×10^{9}	2.1×10^{8}	9.0×10^{10}	322
		(1408; 4.1×10^{7})	(260; 3.1×10^{6})	(36.0; 9.2×10^{4})	(2.92; 314)
	$\overline{IQR}$	880	200	28.5	2.56
		(194; 2921)	(49.7; 444)	(10.4; 54.7)	(1.73; 4.60)
	$\overline{IQR}_{20}$	905	203	28.9	2.57
		(177; 3084)	(61.5; 768)	(12.1; 82.3)	(1.81; 4.29)
	$D7$	628	154	23.6	2.38
		(232; 1423)	(131; 243)	(21.3; 34.4)	(2.23; 2.88)
	ATS	551	137	21.8	2.30
		(198; 1248)	(63.0; 216)	(12.3; 31.3)	(1.83; 2.76)
100	$\bar{S}$	4.8×10^{5}	3.6×10^{4}	1405	15.1
		(2429; 1.9×10^{6})	(577; 1.0×10^{5})	(66.0; 4594)	(3.86; 45.8)
	$\bar{R}$	1.2×10^{6}	1.0×10^{5}	2.5×10^{3}	19.9
		(3124; 3.6×10^{6})	(532; 2.9×10^{5})	(62.6; 1.1×10^{4})	(3.82; 77.3)
	$\overline{IQR}$	699	152	23.6	2.40
		(254; 1592)	(60.7; 443)	(12.1; 52.6)	(1.83; 3.44)
	$\overline{IQR}_{20}$	685	150	23.4	2.38
		(235; 1674)	(58.4; 354)	(11.7; 45.4)	(1.81; 3.24)
	$D7$	572	131	21.2	2.29
		(293; 1026)	(123; 192)	(20.1; 28.6)	(2.21; 2.64)
	ATS	496	116	19.4	2.20
		(251; 894)	(81.4; 178)	(14.9; 27.0)	(1.97; 2.57)

Table 5.8: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when asymmetric variance disturbances are present in Phase I for $n = 9$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.3×10^{-4}	5.2×10^{-4}	0.0030	0.047
	$\bar{R}$	1.3×10^{-4}	5.1×10^{-4}	0.0030	0.046
	$\overline{IQR}$	1.8×10^{-4}	6.3×10^{-4}	0.0033	0.047
	$\overline{IQR}_{20}$	0.0011	0.0033	0.014	0.13
	$D7$	0.0015	0.0044	0.018	0.16
	ATS	0.0021	0.0057	0.023	0.18
100	$\bar{S}$	1.2×10^{-4}	4.7×10^{-4}	0.0029	0.048
	$\bar{R}$	1.1×10^{-4}	4.6×10^{-4}	0.0029	0.047
	$\overline{IQR}$	1.4×10^{-4}	5.3×10^{-4}	0.0031	0.048
	$\overline{IQR}_{20}$	0.0011	0.0033	0.015	0.14
	$D7$	0.0014	0.0043	0.019	0.16
	ATS	0.0020	0.0056	0.023	0.19
		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.3×10^{4}	6410	828	32.8
		(1843; 1.1×10^{5})	(1054; 2.1×10^{4})	(192; 2442)	(12.8; 76.9)
	$\bar{R}$	2.5×10^{4}	6946	868	33.7
		(1805; 1.3×10^{5})	(767; 2.3×10^{4})	(144; 2606)	(10.8; 80.7)
	$\overline{IQR}$	1.5×10^{5}	3.0×10^{4}	2.7×10^{3}	60.7
		(918; 7.2×10^{5})	(743; 1.2×10^{5})	(150; 1.1×10^{5})	(10.8; 231)
	$\overline{IQR}_{20}$	3703	1171	186	11.8
		(193; 2.1×10^{4})	(76.8; 4467)	(41.4; 636)	(3.48; 30.0)
	$D7$	1112	408	83.5	7.48
		(224; 3717)	(90.4; 1536)	(24.5; 257)	(3.74; 15.8)
	ATS	843	321	68.0	6.58
		(155; 2899)	(216; 780)	(70.2; 144)	(6.50; 11.2)
100	$\bar{S}$	1.5×10^{4}	3766	534	25.4
		(2865; 5.0×10^{4})	(780; 9525)	(144; 1222)	(11.1; 47.3)
	$\bar{R}$	1.6×10^{4}	3904	549	26.1
		(2870; 5.3×10^{4})	(1034; 1.2×10^{4})	(184; 1456)	(12.8; 53.0)
	$\overline{IQR}$	2.9×10^{4}	6682	827	32.4
		(1630; 1.6×10^{5})	(264; 2.7×10^{4})	(144; 3022)	(10.8; 90.0)
	$\overline{IQR}_{20}$	1775	554	105	8.68
		(285; 6672)	(107; 1654)	(28.1; 272)	(4.06; 16.9)
	$D7$	886	309	66.0	6.65
		(309; 2132)	(114; 629)	(29.6; 120)	(4.18; 9.89)
	ATS	652	236	52.9	5.80
		218; 1603)	(89.9; 468)	(24.4; 94.4)	(3.73; 8.47)

Table 5.9: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when localized variance disturbances are present in Phase I for $n = 5$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.2×10^{-4}	8.9×10^{-4}	0.0083	0.17
	$\bar{R}$	1.2×10^{-4}	9.0×10^{-4}	0.0083	0.17
	$\overline{IQR}$	1.4×10^{-4}	9.9×10^{-4}	0.0086	0.17
	$\overline{IQR}_{20}$	0.0014	0.0067	0.040	0.38
	$D7$	0.0016	0.0076	0.045	0.41
	ATS	0.0025	0.011	0.060	0.46
100	$\bar{S}$	1.1×10^{-4}	8.6×10^{-4}	0.0082	0.18
	$\bar{R}$	1.1×10^{-4}	8.6×10^{-4}	0.0083	0.18
	$\overline{IQR}$	1.2×10^{-4}	9.0×10^{-4}	0.0084	0.18
	$\overline{IQR}_{20}$	0.0014	0.0069	0.042	0.40
	$D7$	0.0015	0.0076	0.045	0.42
	ATS	0.0025	0.011	0.061	0.47
			ARL		
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.5×10^{4}	2.3×10^{3}	193	6.54
		(2724; 5.1×10^{4})	(505; 6959)	(59.9; 487)	(3.72; 11.4)
	$\bar{R}$	1.6×10^{4}	2432	201	6.69
		(2608; 6.0×10^{4})	(773; 1.6×10^{4})	(82.4; 930)	(4.26; 16.0)
	$\overline{IQR}$	3.0×10^{4}	4082	292	7.66
		(1671; 1.7×10^{5})	(277; 1.4×10^{4})	(38.0; 899)	(3.02; 16.7)
	$\overline{IQR}_{20}$	1281	273	35.7	2.82
		(229; 4581)	(87.2; 621)	(15.6; 71.4)	(2.00; 4.12)
	$D7$	823	190	27.6	2.55
		(273; 2013)	(118; 348)	(19.6; 45.1)	(2.18; 3.26)
	ATS	509	126	20.4	2.24
		(176; 1211)	(96.9; 225)	(17.0; 32.2)	(2.04; 2.79)
100	$\bar{S}$	1.2×10^{4}	1650	152	5.95
		(3975; 3.0×10^{4})	(690; 2965)	(56.6; 249)	(4.21; 8.00)
	$\bar{R}$	1.3×10^{4}	1691	154	5.98
		(3776; 3.3×10^{4})	(1534; 3841)	(142; 303)	(5.53; 8.84)
	$\overline{IQR}$	1686	2130	180	6.38
		(2599; 6.5×10^{4})	(404; 6258)	(50.8; 453)	(3.47; 11.2)
	$\overline{IQR}_{20}$	971	194	28.3	2.60
		(310; 2447)	(86.6l 387)	(45.6; 49.1)	(2.02; 3.41)
	$D7$	748	158	24.5	2.44
		(359; 1422)	(79.6; 249)	(14.8; 34.9)	(1.99; 2.90)
	ATS	455	105	18.0	2.14
		(226; 842)	(57.4; 184)	(11.6; 27.5)	(1.80; 2.58)

Table 5.10: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when localized variance disturbances are present in Phase I for $n = 9$

		p			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.5×10^{-4}	8.8×10^{-4}	0.0046	0.062
	$\bar{R}$	2.3×10^{-4}	8.3×10^{-4}	0.0045	0.060
	$\overline{IQR}$	9.4×10^{-4}	0.0028	0.012	0.12
	$\overline{IQR}_{20}$	0.0013	0.0036	0.015	0.13
	$D7$	9.6×10^{-4}	0.0029	0.013	0.12
	ATS	0.0017	0.0047	0.019	0.16
100	$\bar{S}$	2.1×10^{-4}	8.1×10^{-4}	0.0045	0.063
	$\bar{R}$	2.0×10^{-4}	7.6×10^{-4}	0.0042	0.061
	$\overline{IQR}$	8.7×10^{-4}	0.0027	0.012	0.12
	$\overline{IQR}_{20}$	0.0012	0.0036	0.016	0.14
	$D7$	8.8×10^{-4}	0.0028	0.013	0.13
	ATS	0.0016	0.0045	0.019	0.16
		ARL			
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	1.2×10^{4}	6444	976	32.3
		(991; 5.4×10^{4})	(341; 1.5×10^{4})	(72.5; 1754)	(7.15; 60.0)
	$\bar{R}$	1.3×10^{4}	7379	1117	35.3
		(1030; 6.1×10^{4})	(1068; 4.4×10^{4})	(216; 4737)	(13.4; 115)
	$\overline{IQR}$	4141	2120	306	15.4
		(245; 2.2×10^{4})	(108; 6419)	(28.4; 866)	(4.05; 36.4)
	$\overline{IQR}_{20}$	3177	1530	264	12.4
		(176; 1.7×10^{4})	(83.2; 9626)	(23.1; 1253)	(3.60; 45.4)
	$D7$	2269	1006	182	11.5
		(299; 9368)	(146; 3666)	(36.7; 359)	(4.67; 25.9)
	ATS	1343	598	115	8.57
		(175; 5726)	(108; 1457)	(29.6; 244)	(4.07; 15.7)
100	$\bar{S}$	7771	3188	484	22.4
		1554; 2.5×10^{4})	(823; 2.2×10^{3})	(155; 2638)	(11.2; 75.0)
	$\bar{R}$	8521	3520	521	23.6
		(1651; 2.8×10^{4})	(951; 1.5×10^{4})	(176; 1756)	(12.1; 57.6)
	$\overline{IQR}$	2173	790	143	10.2
		(366; 8109)	(142; 1924)	(35.4; 310)	(4.63; 18.4)
	$\overline{IQR}_{20}$	1564	568	106	8.65
		(262; 5767)	(99.9; 1570)	(26.5; 260)	(3.93; 16.2)
	$D7$	1662	618	117	9.27
		(433; 4800)	(172; 2467)	(421.3; 389)	(5.07; 20.5)
	ATS	934	350	73.2	6.94
		(247; 2762)	(94.9; 1218)	(25.4; 213)	(3.83; 13.9)

Table 5.11: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when diffuse mean disturbances are present in Phase I for $n = 5$

			p		
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	2.2×10^{-4}	0.0014	0.011	0.20
	$\bar{R}$	1.4×10^{-4}	0.0010	0.0089	0.17
	$\overline{IQR}$	0.0012	0.0060	0.037	0.36
	$\overline{IQR}_{20}$	0.0014	0.0066	0.040	0.38
	$D7$	0.0011	0.0055	0.035	0.36
	ATS	0.0017	0.0081	0.047	0.41
100	$\bar{S}$	1.8×10^{-4}	0.0013	0.011	0.20
	$\bar{R}$	1.3×10^{-4}	9.7×10^{-4}	0.0087	0.17
	$\overline{IQR}$	0.0011	0.0059	0.037	0.38
	$\overline{IQR}_{20}$	0.0013	0.0068	0.041	0.397
	$D7$	0.0010	0.0054	0.035	0.37
	ATS	0.0017	0.0082	0.048	0.42
			ARL		
k	Chart	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 1$
50	$\bar{S}$	9178	3394	272	7.00
		(1508; 3.2×10^{4})	(2351; 5964)	(216; 425)	(6.50; 10.4)
	$\bar{R}$	1.5×10^{4}	6628	462	9.05
		(1923; 5.8×10^{4})	(512; 7.8×10^{4})	(60.0; 3614)	(3.67; 33.9)
	$\overline{IQR}$	1479	373	44.8	3.10
		(278; 5152)	(64.8; 693)	(12.7; 77.7)	(1.87; 4.29)
	$\overline{IQR}_{20}$	1305	332	41.0	2.95
		(235; 4557)	(391; 964)	(56.5; 98.8)	(3.29; 4.74)
	$D7$	1336	350	43.5	3.06
		(361; 3751)	(100; 533)	(17.4; 63.2)	(2.11; 3.87)
	ATS	819	215	30.0	2.59
		(228; 2339)	(159; 496)	(24.8; 58.8)	(2.37; 3.66)
100	$\bar{S}$	7369	2127	178	6.07
		(2201; 1.9×10^{4})	(450; 1.5×10^{4})	(54.6; 893)	(3.55; 14.9)
	$\bar{R}$	1.1×10^{4}	3640	273	7.51
		2834; 3.1×10^{4})	(999; 7126)	(101; 488)	(4.67; 11.1)
	$\overline{IQR}$	1137	259	34.2	2.82
		(376; 2822)	(82.2; 592)	(15.1; 67.5)	(2.01; 3.92)
	$\overline{IQR}_{20}$	979	220	30.8	2.67
		(316; 2457)	(71.6; 373)	(13.6; 47.9)	(1.92; 3.38)
	$D7$	1166	265	35.8	2.87
		(484; 2483)	(101; 521)	(17.6; 61.1)	(2.14; 3.74)
	ATS	702	163	24.7	2.44
		303; 1479)	(93.3; 259)	(16.4; 36.0)	(2.05; 2.94)

Table 5.12: Unconditional p and ARL and (in parentheses) the upper and lower conditional ARL values when diffuse mean disturbances are present in Phase I for $n = 9$