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Compositionality: its historic context

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Abstract

The aim of this chapter is to investigate the line from the 19th century, through Frege's works, to the appearance of the principle of compositionality in modern sciences. It is shown that contextuality and compositionality were discussed at the beginnings of the 19th century, but that contextuality was favoured. Also for Frege contextuality was the basic principle and he always obeyed it. Although he argued that within a given sentence, one may distinguish a compositional structure, he would never accept compositionality as we know it. Carnap was the first to give a formulation of the compositionality principle. In logic, natural language semantics, and computer science, 'compositionality' arose independently, and is a generally accepted method, but when a phenomenon is studied that has a strong influence from context, non-compositional methods are used without hesitation. The wide support for compositionality turns out to be for practical reasons, not for principled ones.
Keywords: History, Frege, Carnap, Montague, computer science, logic, natural language.

1 Introduction

The *principle of compositionality*, reads, in a formulation that is standard nowadays (Partee 1984, p. 281):

> The meaning of a compound expression is a function of the meanings of its parts and of the way they are syntactically combined.

One finds a principle like this in all fields of science that deal with language and its relation with meaning. It is found in philosophy, logic, computer science, psychology, and most prominently in the field of semantics of natural language.

If the principle is attributed to somebody, then this is, without exception, the German mathematician, logician and philosopher Gottlob Frege (1848-1925). Therefore, the principle is often called 'Frege's principle'. This attribution is illustrated below by quotes from respectively a philosopher, a logician and from linguists; for many more quotes see Pelletier (2000).

> Dummett (1981, p. 152):[In a chapter titled 'Some Theses of Frege's on Sense and Reference'. The first thesis is:] The sense of a complex is compounded out of the senses of the constituents.
>
> Cresswell (1973, p. 19): These rules reflect an important general principle which we shall discuss later under the name *Frege's Principle*, that the meaning of the whole sentence is a function of the meanings of its parts.
>
> Gazdar, Klein, Pullum & Sag (1985, p. 8): Apart from such general scientific canons as maximizing the simplicity and generality of the statements in the theory, model-theoretic semanticists have attempted to adhere to a principle of compositionality, attributed to Frege, which requires that the meaning of a complex expression should be a function of the meaning of its component parts.

The aim of this contribution is to investigate the line from the 19th century, through Frege's works, to the appearance of the principle of compositionality in modern sciences. Some questions are: who is the source of the principle, is the attribution to Frege correct, why was compositionality introduced in the different sciences, which motivation was given?

The search for the source of the principle immediately brings us into a controversy. There is another principle, viz. 'the principle of contextuality', that can directly be traced back to Frege, and that is called 'Frege's principle' as well. It appears in the introduction of his 'Foundations of arithmetic': (Frege 1884, p. x^e):

> Never ask for the meaning of a word in isolation, but only in the context of a sentence.

Pelletier (2001) describes the two (disjoined!) philosophical communities in which the name *Frege's principle* is used (for these different principles). This situation is remarkable because there is a clear tension between the two principles, and, under straightforward interpretations, they seem to exclude each other. What was Frege's position in the tension between the two principles,

did he accept both principles, did he so at the same time, or did his opinion change at some moment? Is it correct to attribute 'compositionality' to him? The answer that will be given in this paper is not the standard one, and therefore a careful argumentation and an investigation of his original writings is required.

The organization of this paper will mainly be chronological, since that will exhibit developments best. The first part of the article concerns the period before Frege. The 19th century philosophical landscape in Germany will be sketched (of course, only as far as compositionality is concerned), and special attention will be given to some authors that were influential in those days: Trendelenburg, Lotze and Wundt. In the second part, the papers of Frege which are important in the discussion concerning his position are investigated, and, again Wundt. The arguments that are used in the literature are put to test, and in some cases, rejected (this is a summary of the extensive discussion in Janssen (2001)). It will be argued that there is development in the way Frege speaks about these issues, and that his development reflects a development in the field. The third part starts with the work of Frege's student Rudolf Carnap because he played an important role in the history of compositionality. In the 20th century logic, linguistics and psychology became separated disciplines. The introduction of compositionality in the first two fields will be investigated (I have no expertise in psychology). Then authors such as Tarski, Hintikka, Ryle, Wittgenstein, Davidson and Montague will be considered. The story ends with computer science: in that field the concept of meaning and its relation to language is as important as in the other fields.

Our aim implies certain limitations:

1. Although the principle of contextuality will mentioned frequently, we do not investigate the history of contextuality. For an extended history of that principle, see Scholtz (1999).
2. We are interested in the line of history that brought compositionality to modern times, and we will therefore neglect interesting work that was not acknowledged by history. For instance, Hodges (1998) discovered compositionality in the works of Husserl (end 19th century) and formalized one of his ideas. Husserl's work will not be discussed, because his ideas about compositionality seem to have had no impact (until recently).
3. We will not provide a formalization of the principle of compositionality, but rather use a minimal interpretation. We assume that it requires

that there are expressions (we do not say how abstract they may be) and these have parts (we do not say whether the parts should be visible in the expression, nor whether it should be consecutive chunks): the syntax should somehow define what the parts of an expression are. Each expression and subexpression (part) should have a meaning (but what meanings can be, is unspecified). Furthermore, there are functions that build compound expressions from expressions, and functions that build new meanings from meanings (but what functions can do is left open). The main aspect of the interpretation is, of course, that if the meaning of the compound expression is *not* a function of the meaning of its parts (e.g. because some parts have no meaning), then the proposal does *not* obeys the principle of compositionality .

4. We are not hunting for old occurrences. The principles are so immediate and general that they can be found in the writings of several authors from many periods. The internet source Wikipedia says that the idea appears already among Indian philosophers of grammar such as Ya-ska, and in Plato's *Theaetetus*. Hodges (this handbook) discovered that a 10th century Arab author proposed compositionality, and he argues that the Arabs must have found it in 3rd century commentaries on Aristotle. Wundt (1880, p. 80) gives a connection with the founder of logic himself (Metaphysik V 17). The philosopher Dalgarno (ca.1626–1687) developed an artificial language in such a way that it might be conceived as compositional. Leibniz (1646–1716) introduced a principle that is very similar to compositionality (see Section 7). A discussion of such sources falls outside the scope of our story.

Part I
Before Frege

2 Germany, early 19th century

In the 18th and 19th century there was in Germany a growing interest in analyzing thinking and language. An important theme was the question what is primary, concept or thought? Are concepts associated with words and are thoughts formed from these concepts, or are thoughts grasped and

can concepts then be distinguished within the thoughts? One notices the ideas of compositionality and contextuality.

In the 19th century the disciplines logic, psychology and linguistics were not yet separated, and the issues under discussion could by a single author be considered from several points of view. So a book with *Logic* in its title, might for instance deal with ontology, metaphysics, investigate natural language use and speak about the human mind. In any case, it will not contain any symbolic logic, because that emerged only at the end of the 19th century; one of the pioneering works was *Begriffsschrift* (Frege 1879).

Immanuel Kant (1724-1804) taught the primacy of the judgement against the concept in his *Critique of pure reason* (Kant 1781). According to him, judgements possess an initial transcendental unity out of which we gain concept by analysis. Hence, the parts inherit their interpretation from the judgement. Sluga (1980, p. 55) informs us that the late 19th century Kant's argument became the standard argument in anti-naturalistic theories of knowledge, and that Sigwart and Lotze used it this way.

F.D.E. Schleiermacher (1768-1834) was an influential philosopher in the 19th century. In his hermeneutical writings, he makes the suggestion that the semantic value of a word is not a completely determined content, but rather something determinable, a *schema* that only gets determined in the larger context of a sentence, text etc. (for details, see Scholz (2001*a*, p. 279-280)). Modern authors transformed these ideas into the schema of a hermeneutic circle (note that our summary describes not a circle, but a mutual dependency). Using ideas from hermeneutics (especially the hermeneutic circle) recently proposals are made in which the principles of compositionality and contextuality work together in order to achieve understanding (see Rott (2000) and Prosoporov (2005)).

This sketch illustrates that both the idea of contextuality and of compositionality were discussed at the beginnings of the 19th century, but that contextuality was the favoured one. In the next section, we will consider in more detail the ideas concerning compositionality and contextuality of two predecessors of Frege: Trendelenburg and Lotze. Thereafter the first edition of Wundt's book on logic will considered, in a later section a comparison with the second edition will exhibit a development in the field.

3 1840, Trendelenburg: Logical Investigations

Trendelenburg (1802–1872), in the 19th century in Germany a well-known philosopher, wrote *Logische Untersuchungen* (Trendelenburg 1840)), a book that every German philosopher knew, and that influenced Frege's ideas (according to Sluga (1980, p. xi, p. 48–49)).

In Trendelenburg's work the two directions between thought and reality, viz. contextuality and compositionality, arise (Trendelenburg 1840, Bd. I, pp. 365):

> Since there is such a unity between thinking and being, it is not only the case that things determine a thought in such a way that the thought reconstructs them as concepts in the mind. And in case it is already realized, it recovers itself. The thought can also determine the things, in such a way that they represent it physically. In that case the thought exists before the representation, and the parts arise from the whole, and not, as in other cases where the whole comes from the parts.

Neither of the two directions seems favoured (Trendelenburg 1840, Bd. II, p. 367):

> As in reality the substance originates from the activity, and the activity on its turn from the substance, concepts originate from judgements, and judgements from concepts.

Nevertheless, he also mentions a reason why contextuality should be more basic (Trendelenburg 1840, Bd. II, p. 145), the reference he gives, is Gruppe (1834, p. 48, p. 80):

> Recently Gruppe has shown that every concept is founded in a judgement, and that therefore it is incorrect to treat the judgement after the concept, with the concept as origin.

In order to explain the relation between part and whole Trendelenburg (1840, Bd. I, p. 83) uses a metaphor that illustrates the role of contextuality:

> It is the essence of abstraction that the elements of the thought, which originally are closely grown together, are violently kept apart. What is isolated in the abstract situation, has to aim at a return from this enforced situation. Since it is torn off as part from a whole, it must have the traces of the fact that it only is a part. That is, it has to require a completion.

Frege would use the same metaphor of unsaturedness much later. An example is a quote from *Gedankengefüge* concerning negated thoughts (Frege 1923, p. 55–56):

> The whole owes its unity to the fact that the thought satisfies the unsatisfied part, or as one may say, completes the part that needs a completion.

One might consider this resemblance of the metaphors as an indication that Frege had read Trendelenburg.

The metaphor played a role in the discussion concerning the position of Frege. Resnik (1979) claims that the first appearance of this metaphor in Frege's writings dates 1891 and that it supersedes Frege's principle of contextuality from *Grundlagen* (Frege 1884). As we have seen, for Trendelenburg there was no conflict: the metaphor explains contextuality. Moreover, there was no conflict for Frege: Dummett (1981, p. 557) has refuted Resnik's argument by pointing to an occurrence of the metaphor in a letter that Frege wrote in 1882, so before he wrote *Grundlagen*. There is no conflict because the unsatisfied part is obtained (violently) from a *given* thought, so the part is considered within the context of a sentence. Another argument by Resnik's for his point will be considered in Section 8.

Concluding: forty-five years before Frege wrote about contextuality, the ideas of compositionality and contextuality were known in the field, and contextuality was considered more fundamental.

4 1874, Lotze: Logic

Lotze was philosopher at the university of Göttingen when Frege arrived in 1871 to study there as preparation for his thesis. In 1874 Lotze published his *Logik*, which Heidegger called 'the most fundamental book of modern logic' (witness Sluga (1980), p. 53). Lotze extensively discusses the contextual and the compositional method. His style gives the impression that he is attacking or defending positions taken by others, what would indicate that the methods were well known, but, unfortunately, he gives no references at all.

There is a controversy between Sluga and Dummett about Lotze's position with respect to contextuality. According to Sluga (1980, p. 55), it was due to Lotze that Frege became a proponent of contextuality, but according to Dummett (1981, p. 538) Lotze was, on the contrary, an opponent of contextuality. We will investigate the source.

Sluga refers to the following quote in which Lotze argues for the necessary priority of judgements (Lotze 1874, p. 509, in 2nd ed. p. 521):

> Previously we have used rules, i.e. sentences which express a connection between different elements, as examples which make clear what it means to

> hold in contrast with what it is to be; only with half as much clarity can this expression be transferred to independent concepts; of these it can only be said that they mean something; *but they only mean something because propositions are true of them.* [my emphasis T.J.]

Another fragment where Lotze argues for starting with judgements is Lotze (1874, p. 255):

> [...] it will essentially not be upon the analysis of our concepts and their reduction to fundamental concepts, *but upon the analysis of our judgements and their reduction to simple axioms*, that the evolving fixation of our convictions, now unsure about so much, must be based. [my emphasis T.J.]

Here Lotze seems to be a proponent of contextuality. However, Dummett argues for the opposite, and gives the quote (Lotze 1874, p. 23):

> This [...] has given rise to the statement that the theory of judgements should be placed before the treatment of concepts [...]. I consider this statement as overhasty [...].

This is indeed a negative remark about contextuality. However, I consider Dummett's conclusion that Lotze rejected contextuality as overhasty, because the quote continues as follows:

> I consider this statement as overhasty because it originates from an interchange of the aims of pure logic with applied logic [...]

This shows that Lotze is not absolutely against contextuality: it depends on the aim one has.

On the next page, Lotze makes clear that logic has to balance between the two approaches (Lotze 1874, p. 24):

> Without doubt, pure logic has to give the form of concepts preference over the form of judgements; applied logic should first learn how for the formation of certain concepts judgements can be used which consist in simpler concepts.

On the first page of his book Lotze presents an analogy for building thoughts; it expresses his appreciation for both methods (Lotze 1874, p. 14):

> It is easy to build a heap from balls only if it does not matter how they are arranged, but the building of a regular form is possible only by means of building blocks that have been given a shape with surfaces fitting for reliable connecting and stacking.

Concluding: Lotze had definite opinions on contextuality and compositionality: they are separated methods, each has its role in logic, depending on the purpose one has, but they may profit from each other's results.

5 1880, Wundt: Logic

In 1880, Wundt published a work called *Logik*, comprising two volumes, which evolved to the German standard text on logic (of course, logic in the broad 19th century sense). As a logician, he seems to be completely forgotten, but he is still honoured nowadays as one of the founding fathers of psychology. Wundt's book had four editions; we will consider here the first edition, and in Section 9 the second edition. It will be interesting to see the change in his opinion on the compositional and the contextual method.

In the first edition, Wundt considers many of the arguments concerning this issue. An example is (Wundt 1880, p. 88):

> If the concepts are the elements of our logical thinking, then it is the obvious consequence that the construction of logic has to start with them. But it is equally true that the perfect concepts of science presuppose other logical functions.

There are problems with the compositional approach and one option is:

> . . . that one reverts the customary order of investigation by putting the study of judgement function before the concepts.

and another option is:

> One could with Schleiermacher postulate a two-sided dependency in which the judgement by its nature assumes the concept, and the concept in the same way assumes the judgement . . .

Wundt considers several arguments and he concludes (p. 89):

> [. . .] therefore, it is not a contradiction if the concepts are considered on the one hand as ingredients and on the other hand as results of the process of acquiring knowledge.

Concluding: for Wundt compositionality and contextuality are separated methods; deliberations and care are required to let them cooperate.

Part II
The period of Frege

6 1884, Frege: Foundations of arithmetic

In the 19th century, there were enormous developments within calculus, and remarkable results were obtained. For instance, imaginary numbers, as the

name says, strange numbers, turned out to be very useful, the infinity was counted by Cantor, and a curve was discovered that in no point had a tangent. Many notions that previously seemed obvious now required fundamental investigations. In this situation, Frege wrote *Grundlagen der Arithmetik* (Frege 1884). It was preceded by many other publications on foundations of mathematics; he mentions 13 publications from the preceding 30 years.

As an example of Frege's argumentation, we paraphrase his discussion of Mill (1843). Mill was in those days the most discussed philosopher and politician, both in England and in Germany (witness Wundt (1880), p. VII). Frege summarizes Mill's approach as follows (Frege (1884), §7, all translations are from the English edition by Austin).

According to John Stuart Mill, all knowledge is empirical. His definitions of numbers are not definitions in the logical sense, because they do not only state the meaning of an expression, but also express empirical facts. For instance the number 3 consists, according to Mill, 'in this that collections of objects exists, which while they impress the senses thus $\star\ \star\ \star$, may be separated into two parts, thus $\star\ \star \quad \star$' (Frege (1884), p. 9^e)).

Frege comments as follows: 'What a mercy that not everything in the world is nailed down; for if it were, we should not be able to bring off this separation, and 2+1 would not be 3!'. Frege considers this approach to be ridiculous: 'On Mill's view we could actually not put $1,000,000 = 999,999+1$ unless we observed a collection of things split up in precisely this particular way [...]' (Frege (1884), p. 11^e).

The discussion of Mill is one of the sharpest discussions in *Grundlagen*. The kernel of Frege's objection (Frege 1884, § 8) is that if we call a proposition empirical on the ground that we must have made observations in order to become conscious of its content, then we are making a psychological statement, which concerns solely the content of the proposition; the question of its truth is not touched. This exhibits Frege's main motivation for his book: to save the objective truth of mathematics.

In the introduction to his book, Frege discusses psychologism (the movement in those days that tried to base the foundations of logical and mathematical knowledge on psychology). He says, 'But this account makes everything subjective. And if we follow it through to the end, it does away with truth.' (Frege (1884), p. vii^e). Therefore, mathematics should refuse all assistance from the direction of psychology (Frege (1884), p. ix^e). This shows that Frege's aim was to defend the objectivity of mathematics against the influence of psychologism. In order to reach this aim, he keeps three

principles for his investigations (Frege (1884, p. x^e)):

1. always to separate sharply the psychological from the logical, the subjective from the objective
2. never ask for the meaning of a word in isolation, but only in the context of a sentence
3. never lose sight of the distinction between concept and object.

From the previous discussion, we understand why the first point is mentioned: it is his main theme. And 'If the second point is not observed, one is almost forced to take as the meanings of words mental pictures or acts of the individual mind, and so to offend against the first principle as well. As to the third point, it is a mere illusion to suppose that a concept can be made an object without altering it' (Frege 1884, p. x^e).

Note that the only argument Frege presents in favour of the principle of contextuality, is that its consequences suit him. It is remarkable that he does not give any further argument for the correctness of the principle ((Hacker 1979, p. 223) makes the same observation); in a polemic discussion about these consequences that seems not a strong position. However, after the preceding sections, we understand this: Frege used an idea that was well known to his readers.

What was then Frege's solution? He explains it in §60 with an analogy. 'Even so concrete a thing as the Earth we are unable to imagine as we know it to be.' It is too large; there is no way to have a conception of it. Accordingly, any word, 'for which we can find no mental picture appears to have no content.' However, 'That we can form no idea of its content is [...] not a reason for excluding it from our vocabulary'. 'It is enough if the sentence as a whole has a sense; by this also its parts get content'. This is the approach that he uses to clarify the mathematical concepts. About numbers: 'The self-subsistence I claim for number, is *not* taken to mean that a number word signifies something when removed from the context of a sentence.' In §62 he says: 'Therefore the point is to explain the sense of a sentence in which a numeral occurs'. About infinity (∞) §84: 'That we cannot form any idea of an infinite number of objects is absolutely of no importance'. About infinitesimals (infinitely small units), which arise in calculus e.g. in $d\,f(x) = g(x)dx$: 'The problem is not, as might be thought, to produce a segment bounded by two distinct points whose length is dx, but rather to define the sense of an identity of the type $d\,f(x) = g(x)dx$' '[...] we ought always to keep before our eyes a complete sentence. Only therein do

the words really have a meaning.'

Conclusion: Frege presented (in 1884) contextuality as his basic principle, his solution of the foundational problems is based upon it, he meant the principle literally, and would have rejected compositionality.

7 1892, Frege: On sense and reference

In his paper 'Über Sinn und Bedeutung' (Frege 1892) the distinction between sense and reference is introduced. Frege's aim is to explain the difference between the informative sentence $b = a$ and the *a priori* true sentence $a = a$. Even if the names a and b designate the same object, they do so in a different way. For the-way-of-presentation, Frege introduced the name *sense*. Whereas the referents of a and b are the same in the first sentence, their senses are different, and therefore the first equation is informative. However, in reported speech the situation is different: then the referent of a word is what in normal speech is its sense (Frege 1892, p. 28).

Frege hypothesizes the assumption that the reference of a sentence is its truth value and investigates as follows (Frege 1892, p. 35):

> If our supposition that the reference of a sentence is its truth value is correct, the latter must remain unchanged when a part of the sentence is replaced by an expression with the same reference. And this is indeed the case.

The next step is (Frege 1892, p. 35):

> But we have not yet considered the case that the expression for which we substitute is a sentence itself. If our insight is correct, then the truth value of a sentence which has another one as part must remain unchanged if we replace the subsentence by another one which has the same truth value.

These statements are used to argue Frege now adhered to compositionality of reference. That, however, is not justified, as will be explained below.

Frege describes here what is known nowadays as the substitution principle: two expressions have the same meaning if the one can be substituted for the others without changing the truth value of the whole. This principle is known in philosophy as *Leibniz' law*, formulated by Leibniz as 'Eadem sunt quorum, quae mutuo substitui possunt, salva veritate' (Frege 1892, p. 35). It is obvious that if compositionality holds, also the substitution property holds. Vice versa, if the substitution property holds, then we may form the equivalence classes consisting of expressions that are intersubstitutable, and attribute the same meaning to all expressions in a class. A proof of the

equivalence of the two notions is given by Hodges (2001); it requires that some tidiness conditions be satisfied. So for the equivalence of substitutivity and compositionality, it is required that there are sets of expressions that are intersubstitutable.

That is, however, *not* the case in Frege's theory: two expressions may be intersubstitutable in some contexts, in other contexts not. Frege repeatedly says that there are exceptions to the substitution property. About a sentence embedded after *believes that*, Frege (1892, p. 37) says:

> In these cases, it is not allowed to replace in the subordinate clause an expression by another one that has the same customary meaning, but only by one which has the same indirect meaning, i.e. its customary sense.

Above it is shown that Frege did not accept the notion *the reference of an expression*, instead his approach is based upon the notion *the reference of an expression in a given sentence*. So Frege did not accept a notion comparable with *the meaning of an expression*. Within a given sentence a version of compositionality can be found, but the units in that analysis cannot be put into other sentences because the reference of an expression depends on the complete sentence. This is not a minor point, but an essential aspect; it will return later, especially in Sections 12 and 14.

Concluding: (in 1892) Frege adhered to contextuality, he discerned a compositional structure within a *given* sentence, but would certainly not accept the principle of compositionality.

8 1880/1893, Frege: Boole's calculating logic and the concept-script

Some authors claim that Frege abandoned the principle of contextuality because it makes no appearance in Frege's writings after *Grundlagen* (e.g. Dummett (1973, p. 192), Resnik (1967, p. 46) and Resnik (1979)), whereas other authors deny this (e.g. Currie (1982), Sluga (1971), Sluga (1975) and Sluga (1977)). In this section, we will investigate a paper that played a role in this discussion; a remarkable story comes with it.

Frege's *Posthumous writings* appeared in 1969 (Hermes, Kambartel & Kaulbach 1969), the English translation appeared ten years later (Hermes, Kambartel & Kaulbach 1979). Sluga wrote a review of the German edition (Sluga 1971). He remarks that contextuality is repeated several times therein, and points to two papers. The first one is *Booles rechnende Logik und die*

Begriffsschrift, which is considered here, and the other one is a note Frege wrote in 1919 (that is considered in (Janssen 2001)).

Frege writes (German: Hermes et al. (1969), p. 204, English: Hermes et al. (1979), p. 17):

> And so instead of putting the judgement together out of an individual as subject, and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of a possible judgement.

Here Frege describes the contextual method. In a footnote, he mentions one of the dangers of the opposite method: it would lead to unpleasant discussions about negative concepts, such as *non-triangle*. A few lines later Frege warns us that the contextual method (letting the thought fall apart) is *not* a way to obtain isolated properties or relations, and paraphrases the contextuality principle:

> But it does not follow from this that the ideas of these properties and relations are formed apart from entities [...] Hence, in the concept-script [=Begriffsschrift] their designations never occur on their own, but always in combinations that express contents of possible judgement. I could compare this with the behaviour of the atom: we suppose an atom never to be found on its own, but only combined with others, moving out of one combination only in order to enter immediately into another.

Resnik published a second paper (Resnik 1976, so after the review by Sluga) in which he repeats his opinion that Frege rejected contextuality after *Grundlagen*. He seems to accept the above quote as a formulation of contextuality, but says 'that it is clearly not relevant because it was written *before* the *Grundlagen*' (Resnik 1976, p. 47). Here he seems to have a point, *Grundlagen* is from 1884, and Frege wrote the paper under discussion in 1880. It remained unpublished; Frege submitted it several times in vain. However, this is not the whole story.

Above we quoted Frege's metaphor about atoms. Frege has a footnote attached to the last sentence of the quotation, reading 'As I have seen since, Wundt uses in his *Logik* the same image in a similar way'. The authors of the German version inform us that this picture of parts as atoms does *not* occur in the first edition of Wundt's book (dating from 1880), but in the third edition (dating from 1906). This is not quite correct: the picture of parts as atoms already occurs in the second edition from 1893 with almost the same formulations in the third edition. The dates prove that Frege cannot have added the footnote before 1893, so that he adhered to contextuality at least

until 1893. Although the *English* version of the *Posthumous writings* did not incorporate the German editorial footnote, this cannot explain Resnik's ignorance of the reference to Wundt because Resnik's paper appeared three years earlier than the English translation.

By the way, in the next section we will see that Frege in fact misunderstood Wundt's intention.

Concluding: due to the added footnote, we know that Frege adhered to contextuality both before and 10 years after *Grundlagen*.

9 1893, Wundt: Logic

The second edition of Wundt's book was extensively revised and grew to two volumes of about 650 pages each. The spread of subjects is extended considerably. It is surprising to see that contextuality and compositionality now even arise in the context of chemistry: analytic chemistry as a version of contextuality, and synthetic chemistry as a version of compositionality. However, we will restrict our discussion to the case of language and thought. The third edition (1909) grew to three huge volumes; the fragments we will discuss below occur almost unchanged in it.

It is interesting that Wundt gives many references, but none to Frege, so he considered Frege not to have made an important contribution to the issue, or he did not read him.

Wundt's statements about contextuality and compositionality are much more pronounced than in the first edition. He now explicitly mentions the context principle (Wundt 1893, p. 96):

> The real thinking consists of thoughts, and separated from a thought in which it enters a concept has no existence. The same holds for an isolated word in ordinary language when used as a sign for a concept; it has reality only in the context of a sentence.

This is a formulation of contextuality that seems even stronger than Frege's formulation ('not exist' vs. 'do not ask').

Wundt describes the late 19th century scientific landscape as follows (Wundt 1893, p. 97):

> The fact that logical concepts are not independently given, but are obtained from judgements, was a reason for many logicians to give the investigation of judgements preference over the investigation of concepts. As soon as we get rid of the still widespread opinion that real thinking consists in a linking of

> originally independently existing concepts or representations, one will hardly give this question any other value than a didactical one.

From this passage we learn that:
a) The idea of contextuality was well known at the end of the 19th century ('many logicians give the investigation of judgements preference').
b) The idea of compositionality ('the still widespread opinion') was well known in those days.
c) Although Wundt completely agrees with the contextuality principle, he nevertheless does not appreciate the approach based on it ('hardly any value than a didactical one').

Wundt prefers *not* to start with the analysis of judgements. His arguments are:

> On the other hand, it cannot be denied that the logical analysis of judgements must be based upon the investigation of properties of its concept-elements. Logic is here in the same situation as other branches of science that are compelled to similar abstractions. Although a word does not occur in isolation, grammarians study the formation of sentences from words. Although chemical elements only occur in compounds, chemists study the properties of elements. And also for logicians it will be better to follow the same order. The old methodological rule that one has to start with the simple, in order to understand the combination, still has its value, even when it is true that in real experience only the combination is given.

This expresses that the requirements of research compel logicians to give investigation of parts a primary role, so to follow the compositional method. Note that Wundt uses the metaphor of parts-as-atoms to convey the opposite of what Frege said with the picture (cf. Sect. 8): Frege used it to emphasize contextuality, whereas Wundt used it to defend the compositional approach as a necessary abstraction in science.

Concluding: due to arguments from practice, Wundt shifted from a position in-between the two principles (first edition) towards a position in favour of compositionality (second and third edition).

10 1896, Frege: Letter to Peano

We now turn to a quote from a letter that Frege wrote to Peano in reaction to his favourable review of *Grundgesetze der Arithmetik*. The quote played

an important role in the discussion whether Frege at a certain moment abandoned the principle of contextuality or not.

Frege makes some remarks about using concepts in proofs, and mentions the inscrutability of vernacular languages. Then he says (Gabriel, Hermes, Kambartel, Thiel & Veraart (1976, p. 183), and Gabriel, Hermes, Kambartel, Thiel & Veraart (1980, p. 115)) [my translation - T.J.]:

> The case is different when inferences have to be drawn: then it is essential that in two sentences the same expression occurs, and that it has exactly the same meaning in both. It must therefore have a meaning for its own that is independent of the other parts of the sentence.

The point Frege makes in the above quote would nowadays be formulated by the requirement that the expression is used 'unambiguously', i.e. that only one meaning of the word is used. Frege, however, uses a more complex formulation. He speaks about the meaning which the expression has within the two sentences ('it has the same meaning *in both*' [my emphasis T.J]), and not just about its meaning taken in isolation. His formulation takes sentences as the starting point, and thus he conforms to contextuality.

Resnik used another translation of the above quote and states that it is an explicit rejection of the context principle by Frege. The original German text reads 'Er muss also für sich eine Bedeutung haben, die unabhängig ist von den anderen Theilen des Satzes.' Resnik translates this as (Resnik 1967, p.362): 'Therefore, it must have a reference by itself which is independent of the parts of the sentence.'

On two points I disagree with this translation. The first is that 'für' is not translated into 'for' (or 'of' as in (Gabriel et al. 1980)) but into 'by', the second point is that 'anderen' ('other') is not translated. Nevertheless, this translation must be as intended: nine years later, he uses almost the same translation (Resnik 1976, p. 46). The translation 'by itself' suggests that the meaning originates from the expression itself, and thus excludes that it comes from some other source (in this case: derived from the sentence meaning). So Resnik's translation lacks an important indication of contextuality. With this indication the text is, as we have seen, in accordance with contextuality.

In addition, Dummett (Dummett 1981, p. 543) does not consider the quote as an explicit rejection of contextuality (although the formulation conforms to his opinion - based on other grounds - that Frege has rejected it). Dummett interprets it as stating that each word should have the same sense in every context (cf. section 7). That asks too much because the paragraph concerns only the two sentences involved in drawing a conclusion.

11 1923, Frege: Compound thoughts

Frege uses in his later writings several times the argument from creativity of language: how is it possible that we understand sentences we have never heard before? Several authors base their opinion concerning the relation between Frege and compositionality on those statements.

The first occurrence is in 1914 in a draft for a letter to Jourdain (German: Gabriel et al. (1976), p. 127, English: Gabriel et al. (1980), p. 79); it is the letter with mounts Ateb and Aphla. In fact, Frege rejected the draft, it was never sent, and the letter he actually sent to Jourdain has no comparable passages. Therefore it is remarkable that it is used without any hesitation in the literature: one might expect an explication why Frege did not send the letter, or at least an argumentation that his considerations for not sending had nothing to do with what he said about the subject under discussion. Below a *published* paper will be considered in which the argument from creativity is used, for a discussion of the letter to Jourdain, see Janssen (2001).

The first sentence of *Gedankengefüge* (Frege 1923) in the translation by Geach & Stoothoff (1977, p. 55) reads:

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even a thought grasped by a terrestrial being for the very first time can be put into a form of words which will be understood by someone to whom the thought is entirely new. This would be impossible, were we not able to distinguish parts in the thoughts corresponding to the parts of a sentence, so that the structure of the sentence serves as the image of the structure of the thoughts.

This fragment has been used to show that Frege abandoned contextuality and adopted compositionality (e.g. Resnik (1976), p. 47). There are several arguments why that is not a correct conclusion.

The most important argument is that the last sentence of the quote ('distinguish parts in the thought') does not speak about the formation of the sense of a sentence from the senses of its parts. It says that we start from a thought and then go to the parts of the thought. That direction is *contextuality*. The fact that within a *given* sentence a compositional structure can be conceived is not the same as *compositionality* (we have seen this in Section 7).

The second argument concerns the translation of the second sentence. It contains the phrase 'a thought [...] can be put into a form of words which

will be understood by someone ...'. One has to read this as 'the form [...] is understood', but it seems possible to parse this as 'words which will be understood' and think that first words are understood, and from that the sentence and the thought are formed. However, this option is created by the translation. The original German version reads: 'eine Einkleidung findet, in der ihn ein anderer erkennen kann ...'. A more literal translation would be: 'the thought finds an outfit in which someone else can recognize it'. So in the original the word *Wort* does not occur, and the referent of *ihn* can only be the thought. So Frege said that the thought is recognized, and *not* that (as a first step) words are understood.

Thirdly, the quote concerns only *compound thoughts*, i.e. thoughts that have a thought as part. Frege warns us that not all sentences composed from a sentence provide a serviceable example, e.g. relative clauses are not an example 'because they do not express a thought (we cannot tell what the relative pronoun is supposed to refer to)'. So the argument was not intended as a universally valid statement, let alone as a principle.

The paper has another passage from which we can conclude Frege's position with respect to compositionality. He gives a systematic discussion of all compound thoughts made up from two thoughts using negation and conjunction. When considering compound thoughts of the form *Not* [... *and* ...], Frege says (op. cit. p. 61):

> By filling the gaps with expressions of thoughts, we form the expression of a compound thought of the second kind. But we really should not talk of the compound thought as originating in this way, for it is a thought, and a thought does not originate.

So by the line of his argumentation Frege is almost forced to say that a compound thought of the form *Not[... and ...]* is formed from two other thoughts, thus following the compositional approach. That is a way of speaking which he cannot accept. Thoughts are objective (recall *Grundlagen*), we can only grasp or recognize them, but they do not originate. So Frege explicitly denies in this published paper that thoughts are formed, and therefore he would deny compositionality as well.

Concluding: Frege consequently takes the whole thought as starting point, and that within that one may distinguish a compositional structure. His research brings him close to compositionality, but that last step is, for basic reasons, not acceptable for him. Therefore, throughout his whole career, Frege advocated contextuality and never accepted compositionality.

Part III
After Frege

12 1947, Carnap: Meaning and necessity

Since the principle of compositionality cannot be traced back to Frege, the question arises: where does the attribution *Frege's principle* come from? There must have been an influential source for the attribution. Hodges (pers. comm.) found the passage, it will be given below.

Carnap was a student of Frege, and one of the three students in his class *Begriffsschrift-II* (Kreiser 2001, p. 278). In 1947 he published a book, *Meaning and Necessity* (Carnap 1947) that presents a new method for analyzing the meaning of natural language: his method of intension and extension. A substantial part (pp. 96–144) concerns the analysis of Frege's theory in *Sense and reference*. That theory had been forgotten for 50 years, until Church argued on several occasions for the sense-reference distinction and developed its formalization.

Carnap makes Frege's aims explicit, investigates carefully his arguments and gives a step-by-step reconstruction that explains Frege's statements. He shows that Frege's argument on substitutivity should be taken as basic principle and called it *Frege's principle*. A word on terminology: Carnap translates Frege's *Bedeutung* by *nominatum*, and, following Frege, he regards a sentence as a complex name (for a truth value). The relevant quote is (Carnap 1947, p. 120–121):

> In order to understand the specific sense in which Frege means his terms, we have to look not so much at his preliminary explanations as at the reasoning by which he reaches his results. When we do this, we find that Frege makes use of certain assumptions as if they were self-evident or at least familiar and plausible, without formulating them explicitly as the basic principles of his method. These assumptions can be formulated as principles of interchangeability in the following way:
>
> **Frege's Principles of Interchangeability**
>
> Let $\ldots A_j \ldots$ be a complex name containing an occurrence of the name A_j, and $\ldots A_k \ldots$ the corresponding expression with the name A_k instead of A_j.
>
> **28-6** *First principle.* If A_j and A_k have the same nominatum, then $\ldots A_j \ldots$ and $\ldots A_k \ldots$ have the same nominatum. In other words, the nominatum of

the whole expression is a function of the nominata of the names occurring in it.

28-7 *Second principle.* If A_j and A_k have the same sense, then $\ldots A_j \ldots$ and $\ldots A_k \ldots$ have the same sense. In other words, the sense of the whole expression is a function of the senses of the names occurring in it.

The heading reads *Frege's principle*, and when we substitute *meaning* for *sense* (or for *nominatum*) in the second sentence of the principle we obtain the well known formulation 'the meaning of the whole is a function of the meaning of the parts'. As Carnap explains, Frege has to regard certain cases (e.g. occurrences in indirect speech) as exceptions to these principles, and thereby Frege has to make his whole scheme rather complicated.

Carnap gives the following characterization of the differences between his and Frege's method (Carnap 1947, p. 125):

> A decisive difference between our method and Frege's consists in the fact that our concepts, in distinction to Frege's, are independent of the context. An expression in a well-constructed language system always has the same extension and the same intension: but [in Frege's theory] in some contexts it has its ordinary nominatum and its ordinary sense, in other contexts its oblique nominatum and its oblique sense.

Carnap (p. 128–129) emphasizes that there is no contradiction between the two theories, only a practical competition or conflict. He is cutting the cake in a different way.

Carnap explains the disadvantages of Frege's theory. He shows that it is a consequence of Frege's theory that an infinite hierarchy of names and senses is needed: a sentence has a sense, and in order to speak about that sense a name is needed, that name has a sense, and so on. Furthermore, with a single expression an infinite hierarchy of references has to be associated. His example is the sentence *Scott is human* occurring in the contexts such as *It is possible that Scott is human*, *John believes that it is possible that Scott is human*, *It is not necessary that John believes that it is possible that Scott is human*. Carnap mentions yet more disadvantages, discusses proposals by others (Church, Russell, Quine), and shows the advantages of his own method, in which with an expression one intension and one extension are associated.

Davidson (1968, p. 99) explains that it is this feature of Frege's language that prohibits a truth definition meeting Tarski's standards.

> What stands in the way in Frege's case is that every referring expression has an infinite number of entities it may refer to, depending on the context, and

there is no rule that gives the reference in more complex contexts based on the reference in more simpler ones.

Concluding, the relevant point is for us that in Carnap's work the principles hold without exceptions. Indeed, Montague's successful work is based upon Carnap's intension-extension distinction, and not on Frege's sense-reference distinction. Carnap was the first to give a formulation of what *he* called 'Frege's principle'. This formulation is closely related with the principle of compositionality, and if we adopt Carnap's notion of intension, then we have the principle as we know it today. Therefore the name 'Carnap's principle' would be more appropriate for the principle of compositionality.

13 Compositionality and logic

In the period before Tarski, logics were regarded as deductive theories. Tarski made an important step with his paper on 'Truth in formalized languages' (Tarski 1933, in Polish) in which he characterized the notion of a true sentence in a purely semantic way. Hodges (2008) describes extensively the road Tarski had to follow; the next paragraph is based on that paper.

At some time before 1930, Tarski became interested in metamathematics and aimed at defining concepts such as definability, entailment, and truth. It has been suggested that Tarski took the idea of induction on the complexity of formulas from quantifier elimination theory. In fact there some *other* induction was used, so he had to invent a variant of the method, a process that took at least four years. In fact Tarski:

- had no program for defining semantics,
- wouldn't have known what to try if he had such a programme,
- reached his truth definition by purely technical manipulations of other things in the Warsaw environment.

Finally the paper appeared (Tarski 1933), first in Polish, soon followed by a German translation.

For us the crucial information is that Tarski did not have the aim to design a compositional semantics. That also becomes clear from a detail of his truth definition. The clause for the existential quantifier reads (in our terminology): '$[\![\exists x\phi(x)]\!]^g$ is true if and only if there is an $h \sim_x g$ such that $[\![\phi(x)]\!]^h$'. So the definition it is not of the form $\mathcal{M}(\exists x\phi) = \mathcal{F}(\mathcal{M}(\phi))$; the meaning of the compound is not obtained by application of a function on the meaning of its parts. A prominent researcher in semantics of computer science once

said (Pratt 1979, p. 55): 'there is no such function $\mathcal{F}$ that the meaning of $\forall x\phi$ can be specified with a constraint of the form $\mathcal{M}(\forall x\phi) = \mathcal{F}(\mathcal{M}(\phi))$'. In linguistic circles, this objection has been raised against the claim that Montague grammar would be compositional, and therefore Janssen (1997, p. 423) and Dowty (2007, p. 49) had to explain the solution (see below). The philosopher Davidson (1967, p. 24) noted the fundamental difference between a recursive definition of satisfaction (that is what Tarski presents) and a compositional meaning assignment, but, as Davidson remarked, there is an obvious connection.

A reformulation of Tarski's definition is possible in which the compositional character is evident (because a closed form is used). He provided this himself, when he, around 1960, became interested in the application of methods from algebra in logic (Henkin, Monk & Tarski 1971). It requires a shift of perspective to appreciate the formulation. The meaning of a formula is defined as the set of assignments for which the formula is true. Then e.g. $\mathcal{M}(\phi \wedge \psi) = \mathcal{M}(\phi) \cap \mathcal{M}(\psi)$. The clause for the existential quantifier reads $\mathcal{M}(\exists x\phi) = \{h \mid h \sim_x g \text{ and } g \in \mathcal{M}(\phi)\}$. Graphically represented, this makes out of sets of assignments a cylinder, therefore this operation on $\mathcal{M}(\phi)$ is called a cylindrification. This definition can be found in some textbooks on logic (e.g. Monk (1976) and Kreisel & Krivine (1976)), and it is the basis of the theory of cylindric algebras (Henkin et al. 1971). However usually Tarski's definition of satisfaction is followed.

Compositional semantics is not the only way to deal with semantics of logics. We present four examples.

1. For modal logics the only way to define their semantics was for long by a proof system (until Kripke semantics was discovered), and it still is a standard way.

2. Some propositional logics are only characterized by a proof system or in another non-compositional way (e.g. relevance logics).

3. For predicate logic an alternative is the substitutional interpretation of quantifiers: $\exists x\phi(x)$ is true if and only if there is a substitution a for x such that $\phi(a)$ is true. The substitutional interpretation can be found especially in proof theory and in philosophical logic. A proponent in proof theory is Schütte (1977). According to his syntax $\forall x\phi(x)$ is formed from $\phi(a)$, where a can be an arbitrary name (the expression $\phi(x)$ does not belong to his language). Therefore, the formula $\forall x\phi(x)$ is syntactically ambiguous: there are as many derivations as there are expressions of the form $\phi(a)$. It is not

possible to define the interpretation of $\forall x\phi(x)$ from one of its derivations because $\phi(a)$ might be true for the chosen a whereas $\forall x\phi(x)$ is false. In philosophical logic, an advocate of substitutional semantics is Marcus (1962). She argues that one may believe *Pegasus is a winged horse* without believing *There exists at least one thing that is a winged horse*. At the same time she accepts that from the first sentence it follows that $\exists x\,[x$ *is a winged horse*$]$. The substitutional interpretation of the quantifier allows for this conclusion and at the same time avoids the ontological commitment.

Kripke (1976) gives a mathematical discussion of the approach. According to the syntax he presents, $\forall x\phi(x)$ is formed from $\phi(x)$, whereas the interpretation is the substitutional one. Regarding this matter, Kripke (1976, p. 330) says: 'Formulae which are not sentences will be assigned no semantic interpretation'. Therefore, the meaning of $\forall x\phi(x)$ cannot be a function of the meaning of $\phi(x)$, which according to the syntax is a part of $\forall x\phi(x)$. Therefore this substitutional interpretation is not compositional.

4. Hintikka introduced a variant of predicate logic, Independence Friendly logic (shortly: IF-logic) and defined its semantics in a non-compositional way. An example of a sentence in IF-logic is $\forall x\,\exists y_{/x}\,\phi(x,y)$. Here the second quantifier expresses that the y should be independent of the x. The interpretation is defined by means of a game between two players. The first player chooses a value for x, say c. This choice is hidden for the other player, who next chooses a value for y, say d. Finally, $\phi(c,d)$ is evaluated. This is not a compositional interpretation because the meaning of $\exists y_{/x}\,\phi(x,y)$ is not defined in terms of the meaning of $\phi(x,y)$, furthermore, the interpretation proceeds in the direction outside inside. In fact, for open formulas no interpretation is defined at all.

Hintikka claimed that it would not be possible to design a compositional semantics for IF-logic, and argued that to be a virtue. An example of his viewpoint is (Hintikka 1996, p. 110):

> Since I am in this chapter showing the very important limitations of Tarski-type definitions, I am ipso facto exposing certain serious theoretical shortcomings of the principle of compositionality.

He even published an article with the title: *Tarski's guilty secret: compositionality* (Hintikka & Sandu 1999).

Challenged by Hintikka's claim, Hodges (1997) provided a compositional semantics. The interpretation of a formula is a set of sets (of values for free variables). This semantics was not acceptable for Hintikka because of the higher order ontology (Sandu & Hintikka 2001). However, a variant of

the compositional semantics enabled Caicedo, Dechesne & Janssen (2009) to prove properties of IF-logic, e.g. a normal form theorem.

Concluding: the compositional method was not introduced in logic due to some principle, but because it was technically a good method. Although it has become a standard method, it is not the only method that is used.

14 Compositionality and natural language

Ryle (1957) gives an overview of theories of meaning in the British philosophical tradition. He lets the story begin with Mill (late 19th century), and says (p. 242) that Mill started with the idea 'that the meanings of sentences are compound of the components, which are the meanings of their ingredient words'. However, according to Ryle, 'that was a tragically false start', because (p. 294):

> Word-meanings do not stand to sentence meanings as atoms to molecules or as letters of the alphabet to the spellings of words, but more nearly as the tennis-racket stands to the strokes which are or may be made with it. [...] Their significances are their roles inside actual and possible sayings.

This describes the opinion of many British philosophers. An example is Wittgenstein. Frege's principle of contextuality appears, virtually word for word, in the Tractatus (Wittgenstein 1921, 3.3): 'Only the sentence has sense; a name has a meaning only in the context of a sentence'. It also occurs, 30 years later, in the *Philosophical Investigations*. Wittgenstein (1953, §49) considers a situation in which someone introduces names for squares occurring in a given figure, and explains then:

> [...]: naming is a preparation for description. Naming is so far not a move in the language game. We may say: *nothing* has so far been done, when a thing has been named. It has not even *got* a name except in the language game. That was what Frege meant, too, when he said that a word had meaning only as part of a sentence.

Dummett (1973, p. 193–194) gave an interpretation of Frege's contextuality principle that reconciles it with compositionality: to consider the meaning of a word as a self-contained matter and then proceed to some unrelated topic makes no sense; it only makes sense as preparation for considering a larger context. That interpretation seems to be the same as the one by Wittgenstein. However, as we have seen, Frege could not have accepted this: according to him, the meaning of a sentence should always be the point of

departure. Therefore, although Wittgenstein uses the same words as Frege, it differs from Frege's intentions. Scholz (2001*b*, p. 177-178) characterizes, with more examples, Wittgenstein's statements as a liberal reinterpretation of Frege.

Davidson's opinion on compositionality was positive. He presents a classical argument in favour of it, the argument from learnability (Davidson 1965, p. 8):

> When we regard the meaning of each sentence as a function of a finite number of features of the sentence, we have an insight not only into what there is to be learnt; we also understand how an infinite aptitude can be encompassed by finite accomplishments. For suppose that a language lacks this feature; then no matter how many sentences a would-be speaker learns to produce and understand, there will remain others whose meanings are not given by the rules already mastered.

He attributes the idea to Frege (Davidson 1967, p. 19–20):

> If we want a theory that gives the meaning (...) of each sentence, we must start with the meaning (...) of its parts. [...] Up to here we have been following Frege's footsteps; thanks to him the path is well known and even well worn.

Davidson had his own requirement for a theory: it should satisfy Tarski's *convention T* and give an account of truth. Davidson (1973, p. 68) says:

> Convention T defines a goal irrelevant to much contemporary work in semantics. Theories that characterize or define a relativized concept of truth (truth in a model, [...] valuation, or possible world) set out from the start in a direction different from that proposed by Convention T.

So although Davidson accepts a compositionality in his theory, he cannot be considered as an advocate of compositionality of semantics. But, just as in Section 13 there was a connection between Tarski's definition of satisfaction and a compositional interpretation of logic, there is a connection between a theory of truth and a theory of meaning. After a long discussion Davidson (1967, p. 23) says:

> The path to this point has been tortuous, but the conclusion may be stated simply: a theory of meaning for a language L shows 'how the meanings of sentences depend upon the meanings of words' if it contains a (recursive) definition of truth-in-L. And so far at least, we have no other idea how to turn the trick.

The first author in linguistics who presents compositionality of meaning as the leading principle of his approach seems to be Katz (1966, p. 152):

> The hypothesis on which we will base our model of the semantic component is that the process by which a speaker interprets each of the infinitely many sentences is a compositional process in which the meaning of any syntactically compound constituent of a sentence is obtained as a function of the meanings of the parts of the constituent.

Katz does not attribute the idea to Frege; his motivation is of a practical nature (Katz 1966, p. 152):

> Accordingly, we again face the task of formulating a hypothesis about the nature of a finite mechanism with an infinite output.

The issue of compositionality became prominent due to the work of Montague. He was a mathematical logician, specialized in set theory and modal logic, and he was dissatisfied with the fact that semantics was neglected by the theory of transformational grammar. In his most influential work is *The proper treatment of quantification in ordinary English* (Montague 1973) the compositional method is exemplified: for each basic expression a meaning is given, and for each syntactic rule there is a corresponding semantic rule that describes how the meanings of the parts have to be combined in order to obtain the meaning of the expression formed by the syntactic rule. Thus the semantics mirrors exactly the syntax.

An important factor that made this framework possible was that Montague, following Carnap, associated with an expression one meaning, instead of an infinity of meanings (see Section 12). However, Carnap's solution had problems that were caused by the fact that he identified possible worlds with models. To overcome these, Montague (1970, p. 233) introduced *intensional logic* in which these problems do not arise.

The principle of compositionality was for Montague not a subject of deliberation or discussion, because for him, as a mathematical logician, it was a standard way to proceed: he described his method to assign meanings to expressions in side remarks with phrases like 'following Tarski', or 'following Frege', without ever calling it a principle. His view is expressed as follows (Montague 1970, p. 222):

> There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed I consider it possible to comprehend the syntax and semantics of both kinds of languages with a single natural and mathematically precise theory.

In later years, the principle of compositionality was mentioned as the cornerstone of Montague's work in several discussions. We consider three causes.

The first was that phenomena were studied that did not easily fit in

the framework of Montague (1973) or Montague (1973). Relaxations were proposed, and that raised the question what the essential ingredients of the approach were, and which aspects could be changed without loosing the kernel.

The second cause was that a non-compositional alternative arose. In Chomsky's transformational grammar, semantics eventually received a corner. With each sentence a so-called deep structure was associated, and that structure was the input of a process that gradually transformed it into something that was called 'logical form'. So the sentence has to be completed before this semantic process could take place. Therefore the process is not compositional.

The third cause was a challenge of a principled nature. All phenomena considered by Montague (and his first successors) concern sentences in isolation. But what about discourse? Compositionality suggests that a discourse should be treated by analyzing each sentence on its own. How could one then account for discourse phenomena, such as discourse pronouns, or the tense structure of a story? The first treatment of these phenomena was introduced by Kamp (1981): Discourse Representation Theory, henceforth DRT. On the one hand, it sprung off from Montague's approach because it used model theoretic semantics; on the other hand, it was a deviation because (discourse) representations were an essential ingredient. A new semantics for binding free variables in logic had to be developed in order to treat discourse within Montague semantics (Groenendijk & Stokhof 1991), and the solution allowed dealing with more phenomena than DRT did. Nowadays there are several compositional reformulations of DRT, e.g. the one by van Eijck & Kamp (1997), Zeevat (1989) or Muskens (1989).

These discussions evoked fundamental questions. Why should we obey compositionality? What is the status of the principle of compositionality? Is it possible at all to give a compositional semantics for natural languages? What are the essential aspects of Montague's approach? Some authors considered it as a principle that could empirically be tested (for a discussion see Dowty (2007)), others authors as a methodological principle that better be obeyed because deviations yield inferior proposals (for arguments, see Janssen (1997)).

In these discussions, the mathematical results about compositionality were relevant. A mathematical description of compositionality is that syntax and semantics are algebras, and meaning assignment is a homomorphism

from syntax to semantics (see Montague (1970) and Janssen (1986)). A proof that any language can be given a compositional semantics if one is willing to accept an unnatural grammar is given in Janssen (1997). On the other hand, Zadrozny (1994) argues that if one accepts rather complex meanings (using non-wellfounded set theory), compositionality can always be achieved, even with a syntax consisting of concatenation rules. In both cases, the resulting grammar deviates from what one would like to have as result. Hodges (2001) presented another formal result. He investigated under which conditions a compositional semantics for a given fragment can be extended to a semantics for the whole language.

Concluding: the compositional method is used because it appeared, due to linguistic or mathematical arguments, to be the best method. However, when complications arose, there was not much hesitation to relax it as far as needed.

15 Compositionality and computer science

Programs are texts written to instruct computers. In the mid of the 20th century these grew so complex that the need was felt to develop tools to keep control. The earliest occurrence of the idea that one needs a form of compositionality, was, according to de Roever et al. (2001), formulated in a note by Dijkstra (1969):

> ... if we ever want to be able to compose large programs reliable, we need a discipline such that the intellectual effort E (measured in some loose sense) needed to understand a program does not grow more rapidly than proportional to the length L ....

Compositionality evidently is a method to achieve this goal. Other authors mention compositionality explicitly. Milner (1975, p. 167):

> If we accept that any abstract semantics should give a way of composing the meanings of the parts into the meaning of the whole [...].

Mazurkiewicz (1975, p. 75) says:

> One of the most natural methods of assigning meanings to programs is to define the meaning of the whole program by the meaning of its constituents.

As motivation he gives a practical argument:

> The designer of a computing system should be able to think of his system as a composite of behaviours, in order that he may factor his design problem into smaller problems [...].

Note that here is no awareness that in philosophy a *principle* of compositionality is known. The first publication in the field of semantics of programming languages that mentions this connection probably is Janssen & van Emde Boas (1977*a*).

The computers from the sixties can be regarded as calculating machines with a single processing unit. The programs were written an imperative language (FORTRAN or Algol) that instructed the machines precisely what to do. Issues in semantics were questions of correctness (does the program calculate the intended function) and termination (does the program never go into an infinite loop). Later other types of programming languages were developed, and computers are nowadays units in a large network and operate concurrently. In those situations, the semantical issues are different. The role of compositionality will be illustrated with two examples concerning the early situation; thereafter its role in the field of semantics of concurrency will be sketched.

A very simplified view of a computer program is that it consists in sequence of instructions p_1, p_2, p_3 that are performed in order, where each action changes the internal state of the computer. A prominent approach to the semantics of programming languages is the Floyd-Hoare approach. States are characterized by predicates, and the meaning of an instruction is given by describing how a predicate about the state before the action is related with a predicate about the state after the action.

The first illustration concerns the assignment statement. It is an instruction that changes the value of an identifier (something like a variable). For instance, $v := v + 1$ expresses that the value v in the next state of the computer has to be one more than the current value of v. The predicate transformer for the assignment instruction $x := t$ reads $\{[t/x]P\}\, x := t\, \{P\}$. This says that in order to guarantee that P holds after the assignment, it is necessary and sufficient that $[t/x]P$ holds before the assignment (where $[t/x]$ indicates that in P every occurrence of x has to be replaced by t). Consider the assignment $x := w + 1$, where $x > 9$ has to hold afterwards. The rule says that then initially $[w + 1/x](x > 9)$ has to hold, so $w + 1 > 9$, i.e. $w > 8$.

The predicate transformer just given is a proof rule, and not a (compositional) definition of meaning. This is a characteristic of computer science: since the aim is to prove properties, insights are formulated as proof rules. Of course, at the background there should be an abstract model of the computer (needed to define notions such as correctness and completeness of the proof system). In most cases, proof rules can easily be reformulated as defining

a meaning (only for *recursive* procedures this is not immediate). For the case of the assignment statement, the meaning is denoted by the predicate transformer $\lambda P \cdot [t/x]P$, i.e. a mapping from sets of states to sets of states. One detail requires attention. The substitution operator does not belong to the logic itself, but to the metalanguage, and in order to let the expression denote a meaning, the substitution should have a semantic interpretation as well. It can be seen as a tense operator: it shifts the state with respect to which its argument is interpreted, from the current moment to the one in which the value assigned to x equals t. This interpretation is given by Janssen & van Emde Boas (1977*b*), and, in a different context, by Müller-Olm (1997, pp. 30-32). The semantic version of the predicate transformer for the assignment statement is a very general formulation, and can be applied to pointers as well. Thus the requirement of compositionality has led towards a generalization of the rule.

The second illustration concerns the semantics of procedures with arguments. A procedure can be seen as a *name* for a series of instructions; this name can be used on several places in the program whereas the relation between the name and the instructions is defined only once. An example is *sqrt* that yields as result the positive square root of its argument, so $sqrt(4) = 2$. This procedure needs the *value* of its argument as input; therefore, this treatment of the argument bears the name *call by value*. A variant of this procedure, say *sq*, can be defined that has the effect that the resulting value is assigned to its argument, so $sq(w)$ has the same effect as $w := sqrt(w)$. In this case, the procedure needs to know *which variable* is the argument. This treatment of arguments is known as *call by name*. There is yet another mechanism for the treatment of arguments: *call by reference*.

In a compositional approach, the meaning of the entire construction (procedure with given argument) has to be obtained by combining the meaning of the procedure with the meaning of the argument. Tennent expressed this in a discussion (Neuholt 1978, p. 163) by: 'Your first two semantics are not "denotational" [...] because the meaning of the procedure call constructs is not defined in terms of the meanings of its components'. But what are the meanings of the parts of the construction? The semantics of the *call by name* construction was for long given as syntactic substitution of the argument in the description of the procedure, and that is something one would prefer not to do in a semantic analysis. A compositional analysis is given by Hung & Zucker (1991) using Montague's intensional logic. Their abstract semantic approach enabled semantics for all three kinds of treatments of arguments,

and for many types of arguments.

Finally a relativizing remark. Although compositional semantics and proof rules are important in the field of semantics of programs, it is not the only method. Alternatives are for instance model checking and (non compositional) deductive systems. And in practice the standard approach to show correctness seems to be testing, or trial and error.

Nowadays computers are part of a system with several other units (scanner, printer, storage, other computers), and each unit has computing power. These units operate in parallel, but have to communicate and work together. In this context new issues arise, such as guaranteeing fairness (each unit gets its turn for access to a resource) and avoidance of deadlock (the situation that every unit is waiting for an action by another unit, so nothing is happening anymore). The following description of the field of semantics for concurrency is based upon the introductions of de Roever et al. (2001) and de Roever, Langmaack & Pnueli (1998) (the latter bears the suggestive title *Compositionality. The significant difference*).

Concurrency is difficult to understand: several published algorithms for concurrency are incorrect. A striking example concerns a bug in the Pentium FD-IV chip: it costed Intel 475 million dollars to cover up the costs evoked by this error (since then, they have a team that verifies designs by formal methods). Methods like model checking, deductive systems, or modelling by finite automata are suitable for small systems, but for large systems their complexity explodes: if there are 80 components with 10 possible states, then the state space to be considered is 10^{80}. Using compositional methods, the parallel process can be specified using *conjunction* instead of product. 'However, the measure of success of a compositional process is its simplicity' (de Roever et al. 1998, p. 14), and there are situations where compositionality does not make the description simpler. The textbook by de Roever et al. (2001, p. 62) presents both compositional and non-compositional proof methods, and he says about them:

> Summarizing, compositional reasoning should be applied whenever it successfully solves the problems of specifier, prover and implementor alike – such is the case for instance with formalisms for synchronous constructs. However, there are many cases where the formulation of the problem at hand (developing a program preserving a certain invariant), the inherent semantic complexity of the semantics of a program construct (e.g. one requiring nine-tuples for its compositional characterization), or the tight coupling of the processes [...] prevents a practical solution.

Concluding: compositional methods are considered attractive, not for principled reasons, but for a practical reason: they reduce the complexity of controlling the whole process. However, in situations where parts are highly intertwined, non-compositional methods have to be used.

16 Conclusion

We have seen that halfway the 19th century compositionality and contextuality were regarded as two methods that each had their role. Towards the end of that, century compositionality became, due to its practical advantages, the favourite. Frege was a strong advocate of contextuality, but even he came, at the end of his carrier close to compositionality. So his development reflected the development of the field. Frege's distinction between *sense* and *reference* originally was a context dependent distinction, but in the middle of the 20th century a compositional formalization was developed by Carnap and successfully applied by Montague.

In the 19th century, the study of thought was a united science, but in the 20th century logic, linguistics and psychology became separate sciences. The history of compositionality in the first two fields and in the new field of computer science was investigated. In all those fields compositionality is a widely accepted methodology, if not the standard one. But it cannot be said that it is the only one, or the one favoured for principled reasons. Sometimes ontological considerations cause a different approach to be advocated, and, more frequently, in cases where a phenomenon is studied in which there is a strong influence from context, non-compositional methods are used without hesitation.

Generally speaking, the wide support for compositionality is not for principled, but for practical reasons. This motivation can be expressed using an old wisdom, often attributed to Julius Caesar, but probably from Philippus of Macedonia (father of Alexander the Great): compositionality implements the rule *divide et impera*.

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