Living on the Edge – Practical Information Geometry for Studying the Emergence and Propagation of Life Forms

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Living on the Edge – Practical Information Geometry for Studying the Emergence and Propagation of Life Forms

Comment on "How particular is the physics of the free-energy principle?" by Aguilera et al.

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> *"As far as the laws of mathematics refer to reality, they are not certain;*
> *and as far as they are certain, they do not refer to reality."*
> – Albert Einstein in his Address to the Prussian Academy of Sciences, Berlin 27 January 1921.

In the target article, Aguilera and colleagues present a breakdown of some of the conditions seemingly required for the application of the FEP to the emergence of complexity of life (Hesp et al., 2019; Hipólito et al. 2021). Most notably, the authors claim that their study "prevents the application of the theory to many common structures found in biological systems." (this issue, p.37).

If the FEP can indeed be leveraged for those ambitious aims, its applicability should be limited in certain ways in order to be informative for exciting frontiers of investigation such as the origins of life, both during the various stages of organismic development and over the course of evolution. Our planet Earth appears to be brimming with life when compared to the rest of our observable universe thus far, presumably because there is a limited range of circumstances under which living systems could be expected to flourish (the "Goldilocks edge" of potential habitability; e.g., Schulze-Makuch, Heller, & Guinan, 2020). At the functional level, biochemistry research suggests that life on Earth follows scaling laws in enzyme function that are very particular in a parametric sense, yet defined at a level of abstraction that could generalize to life "as we don't know it" (Gagler et al., 2022).

These examples serve to demonstrate that life is quite particular in all its variety, and any principle that might make it amenable to formal descriptions should exhibit some particularities, while also being flexible enough to capture life's diversity. Notably, while the authors of the target article argued for a limited applicability of the FEP, the opposite sentiment has also been expressed in the literature and – somewhat ironically – with overlapping contributors. For example, Baltieri, Buckley, & Bruineberg (2020) presented an active-inference account of a simple Watt Governor, and argued that near-universal applicability suggests the FEP has limited explanatory power when it comes to cognitive systems.

Now, given that the FEP has been formulated in terms of information geometry (Parr, Da Costa & Friston, 2019), it is perhaps unsurprising that its potential criticisms can be understood in terms of the differences between "purely axiomatic geometry" and "pragmatic geometry" as belabored by Albert Einstein during his lecture on "Geometry and Experience" at the Prussian Academy of Sciences. In that same address, we can find his oft-cited quotation highlighted at the top of this commentary. Under any purely axiomatic formulation, the FEP will be vacuous. Only when cast in terms of a practical information geometry can it be seen as applicable to "real" systems in a scientific sense, although by then it will inevitably have lost its flavor of universality.

In our view, axiomatic formulations of FEP can only exhibit utility indirectly, co-evolving in an intimate communion with scientific practice – thus giving rise to a practical information geometry. Such epistemic humility could be traced back at least as far as the Copernican revolution, when Osiander noted the following in his preface to Copernicus's magnum opus (1854):

> *"(...) the astronomer will take as their first choice that hypothesis which is the easiest to grasp. The philosopher will perhaps rather seek the semblance of the truth. But neither of them will understand or state anything certain unless it has been divinely revealed to them. (...) since one cannot in any way attain the true causes, one will adopt whatever suppositions enable the motions to be computed correctly from the principles of geometry."*

In a similar vein, practical information geometry is not concerned primarily with justifying the truthiness or universality of an underlying axiomatic formulation but with scientific understanding. Therefore, in line with the prescient vision that Einstein formulated so eloquently, scientists who seek to benefit from the power of information geometry will need to operate on the edge between practical constraints and axiomatic rigor, always in search of that sweet spot of informativeness between specificity and universality. In that spirit, we would now like to address more specifically some of the concerns raised in the target article. In particular, let us consider two core claims made by the authors of the target article:

Claim 1 from the target article: Achieving block diagonal structure in the solenoidal flow matrix (Q) imposes prohibitively narrow constraints on the Jacobian (J).

In the study of differential geometry, methods to mitigate the tremendous headaches caused by higher-order couplings have been an important subject of study for the past century. Most of it has been solved by tracking departures from locally linear (or pseudo-Riemannian) approximations in terms of the Ricci curvature. In general, the Ricci curvature is defined such that its higher-order moments cancel each other. In cases where the Ricci curvature has an upper bound, so-called Einstein manifolds further exploit its remarkable properties. This means the FEP applies to cases where a system's dynamics can be approximated in terms of a (temporarily) stable (pseudo-)Einstein manifold, which in turn can be tested with conditions such as the Hitchin-Thorpe inequality (Hitchin, 1974). Einstein manifolds have demonstrated wide-ranging applicability in various domains of mathematical physics, both for linear and non-linear dynamics (e.g., Gellman & Levy, 1960). Therefore, while the analyses of the target article are certainly helpful in identifying formal constraints on the applicability of the FEP, they do not appear to support the strong claim of the authors that it "prevents the application of the theory to many common structures found in biological systems." (Aguilera et al, this issue, p. 37).

Claim 2 from the target article: Correspondences between averages of flows and flows of averages are prohibitively rare.

Generally speaking, such correspondences can stabilize whenever there is a separation of temporal scales between interacting component variables (Friston, 2019; see also this recent commentary by Hesp, 2022), which also happens to dampen potential higher-order couplings among variables (speaking to Claim 1). Separating fast and slow dynamics plays a crucial role in Friston's monograph (2019) and the study of complex adaptive systems in general. To the best of our knowledge, such temporally separable dynamics are a universal characteristic of biological systems, so we would welcome counter-examples that show biological functioning without such separation. (The example systems in the appendices of the target article do not exhibit any such separation.) The question of exactly how much temporal

> separation is necessary for biological function is an important theoretical and empirical question.
>
> A related challenge has to do with Stein's example, which was tackled by Brown (1971) in his analysis of admissible estimators in spaces exceeding two dimensions, where he identified the necessary and sufficient conditions for variational minimisation based on the Laplace approximation. These conditions constrain the Ricci decomposition of the metric and correspond to local conformal symmetry, which has been exploited for optimization techniques (e.g., Franca et al., 2020).

These examples demonstrate how scientists have been operating on the edge between practical constraints and axiomatic rigor, and how that dance ends up being an interdisciplinary exercise. It shows how practical assertions require the combination of things of experience and the empty conceptual frameworks provided by axiomatic geometry:

> "it is certain that mathematics generally, and particularly geometry, owes its existence to the need which was felt of learning something about the relations of real things to one another" – Albert Einstein in his Address to the Prussian Academy of Sciences, Berlin 27 January 1921.

In closing, we note that the target article inadvertently propagates the common conflations between principles, theories, and models[1], which obfuscates the nature and scope of this contribution to the literature. Specifically, a principle is merely an abstract idea (e.g., the principle of relativity) used to formulate the foundations of theories (e.g., Einstein's theory of gravity), which constrains the formulation of models as constellations of hypotheses (e.g., about the solar system), which generates predictions about observables. Conversely, observations can be used to test models, which can be used to compare their parent theories, which can be used to evaluate underlying principles. The resulting epistemic ambiguities significantly limit the potential scope and weight of conclusions that can be drawn based on these kinds of analyses. In light of this nested (semi-Markovian) structure of the scientific method, the analyses discussed in the target article do not afford strong and wide-ranging conclusions about the applicability of the FEP as an overarching principle for the study of living systems[2], although they certainly appear to be informative for future development of models and theories inspired by the FEP.

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[1] As an example of mixing , Aguilera and colleagues state in their abstract: "These observations are critical for evaluating the generality and applicability of the FEP and indicate the existence of significant problems of the theory in its current form."

[2] The FEP, specifically, states something about how living organisms persist while engaging in adaptive exchanges with their environments. It is a principle because, while universally applicable, it also allows for particularity in life's diversity in guiding a mathematically informed direction in the investigation of cognitive life. One can develop models, theories, and hypotheses about cognitive life from this principle. Notably, however, a scientific theory does not reduce to models (Morrison, 2007). A model, guided by a principle, does not entail the whole of a theory. For example, when a model uses a Newtonian principle, it does not entail the whole of Newtonian motion mechanics. This view is captured in von Neumann's (1961) – perhaps hyperbolic – remark: 'The sciences do not try to explain, they hardly even try to interpret, they mainly make models' (p. 492). This is because a model is an abstraction of a zoomed-in part of the world one wants to understand. The model is itself inspired by a hypothesis and, as far as hypotheses are concerned, "let no one expect anything certain" (Osiander, 1854).

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