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Short time scale monitoring of SiO sources

F.P. Pijpers¹, J.R. Pardo², and V. Bujarrabal²

¹ Uppsala Astronomical Observatory, Box 515, S-75120 Uppsala, Sweden

² Centro Astronómico de Yebes (OAN, IGN), Apartado 148, E-19080 Guadalajara, Spain

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Abstract. We present the results of a short time scale monitoring of SiO maser emission ($v=1$ J=1-0 transition) in four known strong sources. These sources were monitored nightly for a period of about a month. The aim of these observations is to investigate the possible presence of variations in the maser lines on time scales of a few days to weeks, due to sound waves propagating out from the central star. If sound waves are responsible for the mass loss of certain cool giants, as suggested by Pijpers & Hearn (1989) and Pijpers & Habing (1989), local variations in density and relative velocity are expected just above the stellar photosphere. These could give rise to variations in any narrow spectral line formed in this region, and therefore in particular in the SiO maser lines. Our observations indicate that variations in the line shape (leading to relative changes in the intensity of about 20%) occur in the SiO emission of Mira type stars, within short time scales of 10-20 days. The main component of the profile variability is consistent with a displacement of the velocity centroid of the dominant maser peaks, by about 1 km s^{-1} in the average. Apparent variations in the total line flux were also found, but could be partially due to calibration uncertainties.

Key words: stars: mass loss – stars: circumstellar matter – stars: oscillations – masers: SiO – radiolines: stars – stars: AGB and post-AGB

1. Introduction

Many if not all cool giants lose matter at a rate which can be as high as $10^{-4} M_{\odot} \text{ yr}^{-1}$ (e.g. Knapp 1991). Some cool giants are large amplitude radial pulsators with periods between typically 150 and 400 d, for the Miras and up to several thousands of days for OH/IR stars (Herman & Habing 1985). OH/IR stars are slightly further evolved up the asymptotic giant branch (AGB) than Miras are. Both Miras and OH/IR stars are emitters in several maser lines. In particular, SiO maser emission has been observed in a large number of objects.

Send offprint requests to: F.P. Pijpers

The precise manner in which this mass loss is driven away from these stars is still unknown. It is clear that for those stars that show a large amplitude radial pulsation, coupled with dust formation in the wind, the two stage process as envisaged by Wood (1979) and Willson & Hill (1979), and modelled in more detail by Bowen (1988), works well. However not all stars that lose mass also have large amplitude pulsations and form dust sufficiently near to the star. For these cool giants the alternative mechanism proposed by Pijpers & Hearn (1989) and applied to AGB stars by Pijpers & Habing (1989) might well be more appropriate. In this model the mass loss is driven by sound waves propagating out into the wind. These are generated either by high overtone radial or non-radial pulsation or by the convection in the surface layers of the star. As these sound waves propagate outward in the wind they transfer momentum to the wind which accelerates it outward.

At large distances from the star it is likely that the sound waves have dissipated. Near the star the amplitude of the sound waves will be large and they may well develop into shock waves before dissipating. At these distances of only a few stellar radii from the star their effects are therefore most likely to be detected.

There are a number of problems associated with the detection of sound waves in the winds of stars. Their velocity amplitude will not exceed the local sound speed by very much, if at all. This means that velocity shifts of spectral lines will be of the order of 5 km s^{-1} . The density variations associated with this will be quite large but column density variations will be much smaller, since the wave length of the sound waves is small compared to the stellar radius. Also the horizontal scales of the waves will only be large for low spherical harmonics of the pulsation and therefore their effect will average out over the stellar disc. It is therefore unlikely to find large variations for thermally excited lines (see however Bookbinder et al. 1989, 1993).

For maser lines and in particular maser lines originating near the stellar surface the situation is much more favourable. The largest amplification of the maser radiation occurs for the longest path length in the line of sight. For a spherically symmetric wind this means that an image of the maser source at a given velocity within the range spanned by the maser line will

show a ring rather than an extended disc. The spatial averaging which destroys any variation will therefore be less severe. Also, the maser intensity is expected to be in general sensitive to the column density, and the spatial scale of the masing regions is likely to be quite small (cf. Colomer et al. 1992; Bujarrabal 1994). Small variations in column density can therefore have a large effect on the intensity. Both of these effects alleviate the problems associated with the small spatial scales of sound waves. Since SiO maser emission originates near the stellar photosphere it affords a good opportunity for the detection of sound waves.

SiO masers in evolved stars are known to present strong variations in intensity (e.g. Alcolea 1993). This long-period variability essentially follows the stellar pulsating activity, but with a poor regularity and also presenting changes in the line shape. Variations in time scales of days have been reported (e.g. Balister et al. 1977), but not studied with sufficient detail and sensitivity to obtain conclusive results on their properties. The purpose of this paper is to improve our knowledge on such rapid changes, in particular making use of the much more powerful instrumentation available at present. As is shown below, the data allow inferences to be made on the possible effects of sound waves on the conditions in the maser emitting region. Four known strong maser sources were chosen to obtain the best possible S/N for the line spectra: R Leo, R Cas, Ori A, and VY CMa. Ori A is a star forming region and is considered less likely to show any variations on short time scales. VY CMa is an M5 supergiant star with intense SiO emission and irregular intensity variations. The $J=1-0 \nu=1$ ^{28}SiO line for this star is very broad and has many peaks. R Cas and R Leo are M6 and M7 Mira-type stars with periods of 431 and 312 days, respectively. During our observations, the phases of these stars advanced from 0.96 to 0.09 for R Leo and from 0.14 to 0.23 for R Cas. At this epoch emission was strong for the two stars because the SiO maximum occurs usually with a phase delay of 0.1-0.2 (see Alcolea 1993).

2. Instrumentation and observations

We have observed the $\nu=1$ $J=1-0$ SiO transition (at 43 GHz) in the well known sources R Cas, R Leo, VY CMa and Ori A. The observations were carried out in 1992 Aug.-Sept. with the 13.7-m radiotelescope of the Centro Astronómico de Yebes. The telescope was equipped with a cooled Schottky receiver with a single-sideband (SSB) system temperature of 260 K. The back-end was a 256x50kHz filter bank. A polarizer was installed in front of the receiver allowing the observation of circular polarization (the SiO masers are known to present some degree of linear polarization). The total line width of the emission from R Leo and R Cas is narrower than that of VY CMa and Ori A. For this reason the latter two stars were observed using the position switching method. For R Leo and R Cas the frequency switching mode reducing the velocity coverage to ~ 44 km/s was used. The latter method provides spectra with less noise.

The main beam antenna temperatures have been multiplied by 90 to get a maser flux in Janskys.

Our monitoring began on August 22 and concluded on September 30. Observations for each star were made typically every 24 hours, except for R Cas which was observed twice a day. In order to improve the relative calibration we observed each object at the same (average) elevation. This procedure also ensures that the projection of the polarization angle of the emission on the receiver is always the same for each source, to avoid problems with possible partial detection of linear polarization (the effects of the line polarization on the variability of maser emission are discussed in detail by Martínez et al. 1988). The observations of R Cas do not satisfy this condition. This source was in general observed twice a day, at the same elevation (50-55 degrees) but not at the same sidereal time. We integrated on each source over about 1 or 1.5 hours. The pointing was checked every 30 minutes by means of pseudocontinuum scans over the maser source itself. To guarantee that the observed variations were not due to pointing errors, most of the observations consisted in fact of several runs of small maps of five points in the sky: the expected star position and four others separated by 30° . The pointing errors were then corrected every 30 minutes by the pseudocontinuum data and, moreover, residuals were calculated from the small maps (the corresponding very small corrections to the total flux were taken into account a posteriori). In addition, the ambient temperature was measured in order to check the effect of temperature changes on the calibration. During the period of observations the sidereal time advances 3 hours and the (average) ambient temperature decreases by about 15 degrees. With these data it is possible to estimate the observational errors in the line intensity due to calibration uncertainties. This is found to be of the order of 10%.

3. Data reduction

The variation of the profiles can be expressed as a combination of a total line flux variation and as a change of the line shape. The total line flux variation is harder to detect because it is sensitive to calibration errors due to e.g. variations in the calibrator temperature. Variations in the line shape are much more straightforward to detect if the S/N ratio per channel is sufficiently large.

Various methods are used to attempt a measurement of intrinsic changes in the maser line flux. The height of the peak(s) in the line profile, the area under the line and the centre of the line profile are followed in time, as well as the ambient temperature. The correlation of the first two with the ambient temperature is checked in an attempt to determine apparent flux variations due to errors in the calibration.

In the observations of Ori A the changes in area and peak height are correlated with the ambient temperature changes, with correlation coefficients between 0.7 and 0.8. The variation observed is therefore probably for the most part due to calibration errors rather than intrinsic variation. The line centre changed from 6.4 km s^{-1} to 5.8 km s^{-1} .

In VY CMa the correlation coefficients for area/height and ambient temperature are between 0.8 and 0.9 and therefore again

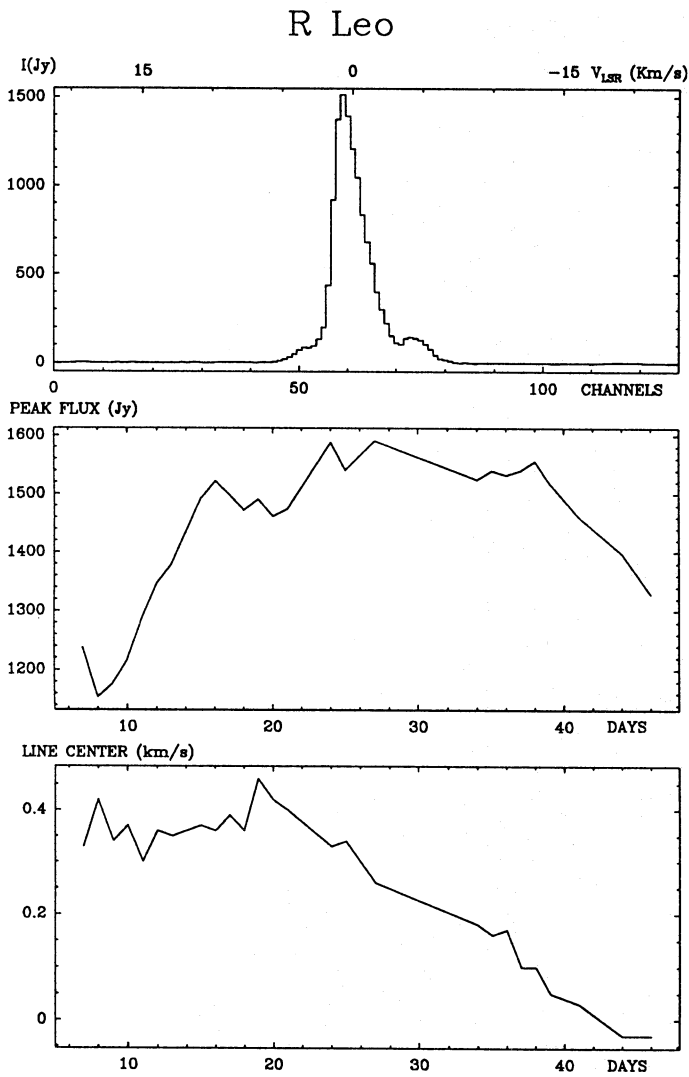


Fig. 1. Average spectrum, peak flux and line centre of the star R Leo during our short time scale monitoring. This star had a maximum of SiO maser emission ($v=1$ $J=1-0$) on September 12-15, 1992 (DAY number 1 is August 22)

intrinsic line flux variation cannot be determined. There is no change in the position of the centre of the line.

In R Leo the correlation coefficients are low, between 0.5 and 0.6. R Leo had an SiO maximum during the monitoring run, on about September 12-15 (Fig. 1). Since the curves for the peak height and the area as a function of time follow the visual light curve, it is likely that the line flux variation is mostly intrinsic. The centre of the line profile changes from 0.4 km s^{-1} to -0.1 km s^{-1} . A similar behaviour of the line centroid has been observed previously for R Leo (see Alcolea 1993) but with much less detail than in the present data.

For R Cas there was no correlation of the line maxima or the area with receiver temperature. The peak maximum and the area are both decreasing during the monitoring period (Fig. 2), which is probably intrinsic since the star was also becoming fainter visually (see the Introduction for the phase of R Cas in

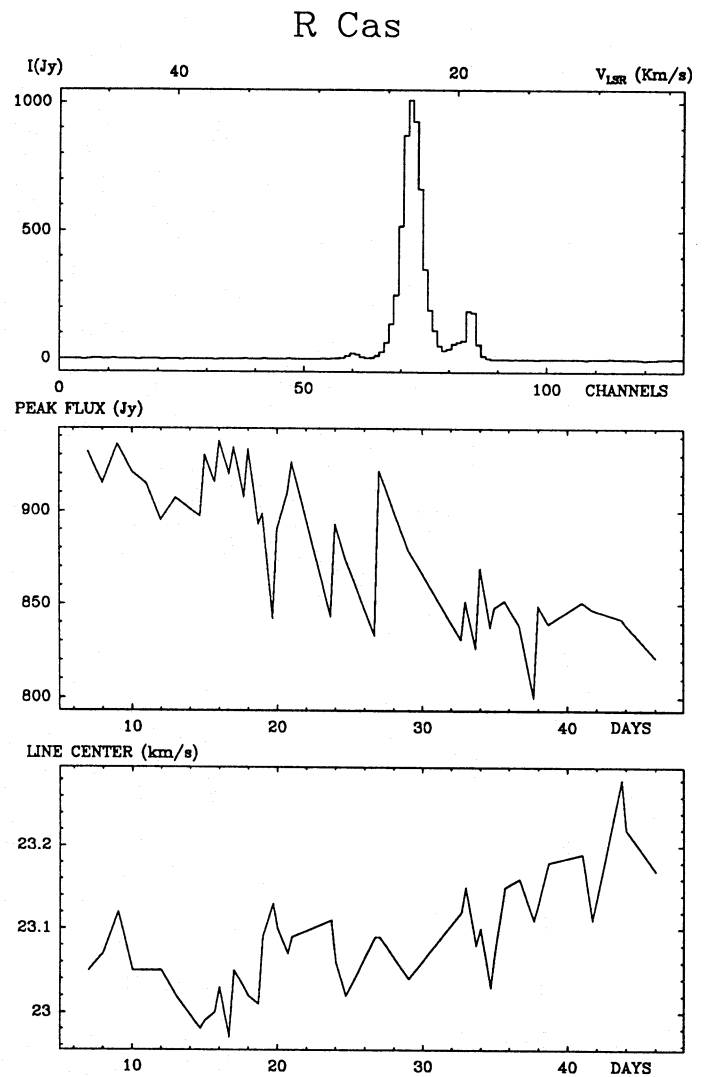


Fig. 2. Same as Fig. 1 but for the star R Cas. This source had a maximum of SiO maser emission ($v=1$ $J=1-0$) on August 10-15, 1992

the observations). The position of the line centre did not change significantly during the monitoring.

Neither in the height of the maxima of the line profile, nor in the area could any periodic change be detected for any of the sources.

To detect variations in the line profiles a mean profile is constructed by adding all the separate line profiles for the observations and then dividing by the number of separate observations. Subsequently this mean profile is subtracted from each of the individual spectra after being scaled so that it always has the same total line flux as the individual spectrum. Using channels far outside of the maser line as a reference for the noise per channel, a χ^2 test is then performed on the residual amplitude within the channels designated as containing the maser line emission. These channels are chosen by inspection of the mean profile. The χ^2 tests confirm unambiguously that the variations found with respect to the average profile are statistically significant for R Leo and R Cas. In VY CMa no line shape variations

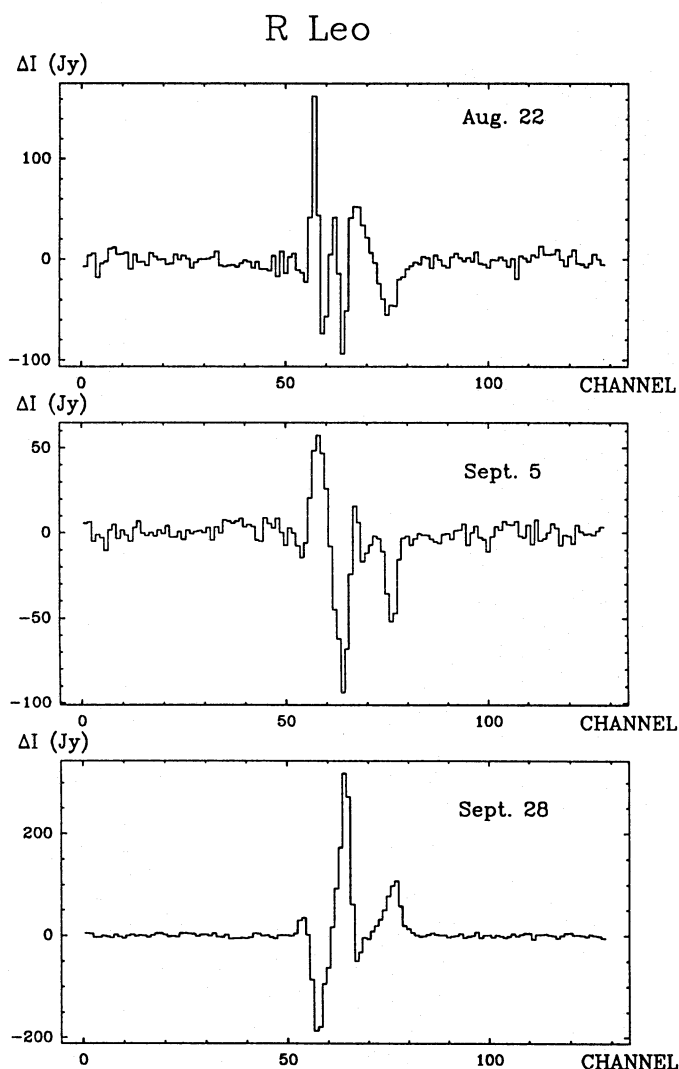


Fig. 3. Residual spectra (see text) of the star R Leo corresponding to three observations of our monitoring. Line shape variations are evident

were detected. In Ori A a significant variability was found, but will not be discussed here because of the different nature of the source. The average spectrum, and the variations found in the peak flux and profile centroid are shown in Figs. 1 and 2 for the stars R Leo and R Cas, respectively. Some selected residual profiles are shown (Fig. 3). Even without the results of the χ^2 test it is clear that the line profiles change on a time scale of days. The residual line profile after subtraction of the mean is well above the noise on many separate observations. Plots of the time variation of individual channels are shown in Fig. 4 for R Leo, the star for which the best results were obtained. In some channels a sinusoidal variation around the mean might be present, indicating a period of ≥ 30 d. However the total time span of this monitoring is too short to confirm this and any periodicity in the signal is therefore only tentative. On the other hand, it should be noted that the detection of a variability with longer periods is probably difficult (unless it is very clear), since the well known and intense long-period variations will contam-

inate the observation. The SiO maser in R Cas shows a similar behavior to that in R Leo (Fig. 5), but the effect is no clearer. The variations are smaller than in R Leo and the observational noise is relatively more important. The variations found in individual channels for these two sources range between 10 and 30 % of the peak flux, and the typical variation time is of about 10-20 d. It is remarkable in the data shown in Figs. 4 and 5 that the maximal variations are found around the two main peaks of R Leo and the main peak of R Cas, and with opposite sign from one side of the centroid to the other. Such a behaviour could be understood as due to a global displacement of the peaks, by 1 km s^{-1} in the average. Note that the relative amplitude of the detected variations is small in all cases, and the intensity and overall structure of the profiles differ only slightly from the averages shown in Figs. 1 and 2.

A very short time scale monitoring of the star R Cas (on a time scale of hours) was also performed. No clear results could be extracted from these data. Some variations of the intensity were observed but these were correlated with the ambient temperature and source elevation. Intrinsic variations on time scales of hours were not detectable because of the observational uncertainties.

Summarizing; for VY CMa no line profile variations could be detected. The channel noise is too large. Both R Cas and R Leo do show clear line profile variations on a time scale of days. If there is any period in the data it is likely to be in the range of 30-60 days. The total time span of this monitoring program was too short to make any definite conclusions.

4. Regarding the origin of the short time scale variability of SiO masers

The cause of the variations reported here could be simply the underlying light curve of the star which changes the input for the maser radiation. This however is more likely to produce a change in total maser line flux than in the line shape if the spatial structure of the masing region is really uniform and constant. Also, the period found should then be the same as the visual period of the star. Such long time scale changes in the total line flux were indeed detected in R Cas and probably in R Leo as well.

If the masing region is not uniform then differences in light travel time for different masing patches with differing radial velocities, coupled with a rapidly varying central source of maser pumping photons, could cause line profile variations. This is similar to what is seen in the broad emission lines of active galactic nuclei (AGN) If this were the case one might attempt the type of reverberation mapping that was developed for novae (Coudrec 1939) and has been used for AGN (Peterson 1993) to deduce the velocity field of the gas of which the masing patches are tracers. However for a time-scale of variation of days the difference in path length for different patches would be a few hundred stellar radii. This is far larger than the size of the SiO masing region deduced from mapping which is less than one light hour.

R Leo

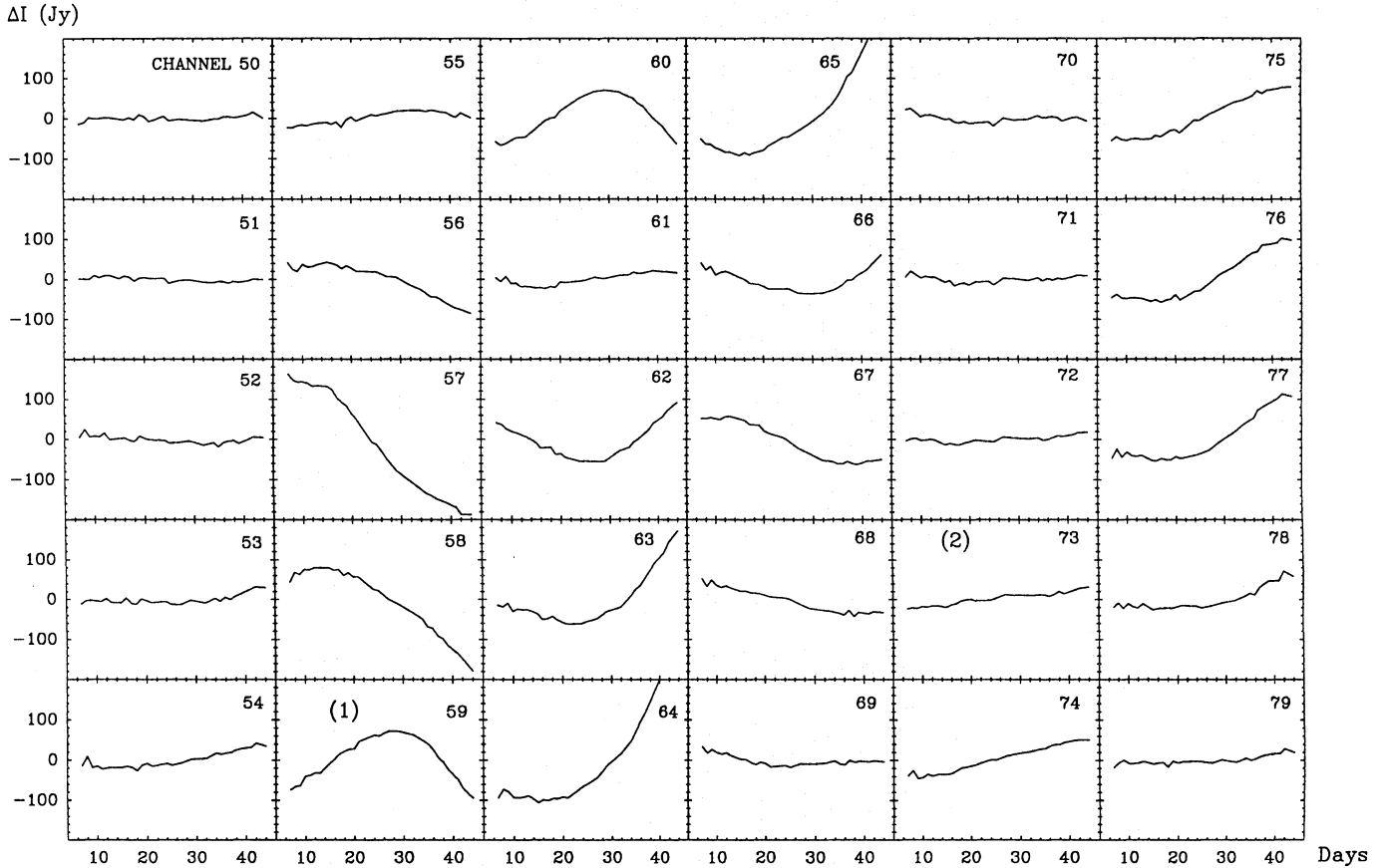


Fig. 4. Variation of the difference between individual spectra and the average for the star R Leo. The time evolution is shown of individual channels within the velocity range of the line emission. The strongest variations are found near the peaks, the main peak in channel 59 (1) and a secondary in channel 73 (2) (DAY number 1 is August 22)

If sound waves are present in the wind and if they are generated by convection they are the equivalent of the solar 5 min oscillations. The period of oscillations always scales inversely proportional to the square root of the mean density because of the period-mean density relation (see e.g. Cox 1980) :

$$\Pi \sqrt{\bar{\rho}/\bar{\rho}_{\odot}} = Q \quad (1)$$

The density $\bar{\rho}$ refers to the mean density of the star. The mass of cool giant stars is not more than a few solar masses and the radius is several hundred solar radii. Radius estimates from lunar occultation measurements are available for R Leo (Di Giacomo et al.1991), which yields $380 R_{\odot}$. Photometric analysis (e.g. Cahn & Elitzur 1979) yields a similar value for R Cas. A mass of $1.5 M_{\odot}$ does not seem unreasonable for these stars (cf. Wood 1974; Bowen 1988; Boothroyd & Sackmann 1988). The equivalents of the solar 5 min oscillations should then have a period of ~ 21 d. However the relative depth of the convective layer in cool giants is larger than it is in the sun. The current best estimates from helioseismology (cf. Guzik & Cox 1993; Roxburgh & Vorontsov 1993) give a radius of the bottom of the solar convective zone between $R = 0.71 - 0.72 R_{\odot}$. The models of Wood (1974) put the bottom of the convective layer in red giants

close to $R = 0.2 R_{*}$. As Schwarzschild (1975) argues, the peak of the energy spectrum of sound waves generated by turbulent convection may well shift to longer wave lengths because of a scaling with the total relative depth of the convective layer. This means that the most strongly excited sound waves propagating into the atmosphere and the wind may have a larger wave length and therefore a larger period by roughly a factor of 3 that such a simple scaling with the size of the convective layer suggests. The range of possible periods is therefore $\sim 21 - 63$ d.

Given an atmospheric sound speed of typically 5 km/s the wave length of sound waves with a period in this range is $\sim 9 \cdot 10^9 - 3 \cdot 10^{10}$ m. The horizontal wave length of the wave fronts can be larger than the vertical wave length. This depends on the value of the indices l and m of the spherical harmonics of the non-radial oscillations that generate the sound waves propagating into the wind. If for simplicity it is assumed that the two size scales are the same then the number of wave crests and valleys over a stellar disc is 750 – 5000. In the continuum or in a thermal line any effect due to sound waves is therefore at best only detectable at μmag -level accurate photometry (cf. Belmonte et al.1990). The advantage of using maser lines is that the masing effects tend to amplify the differences in the

R Cas

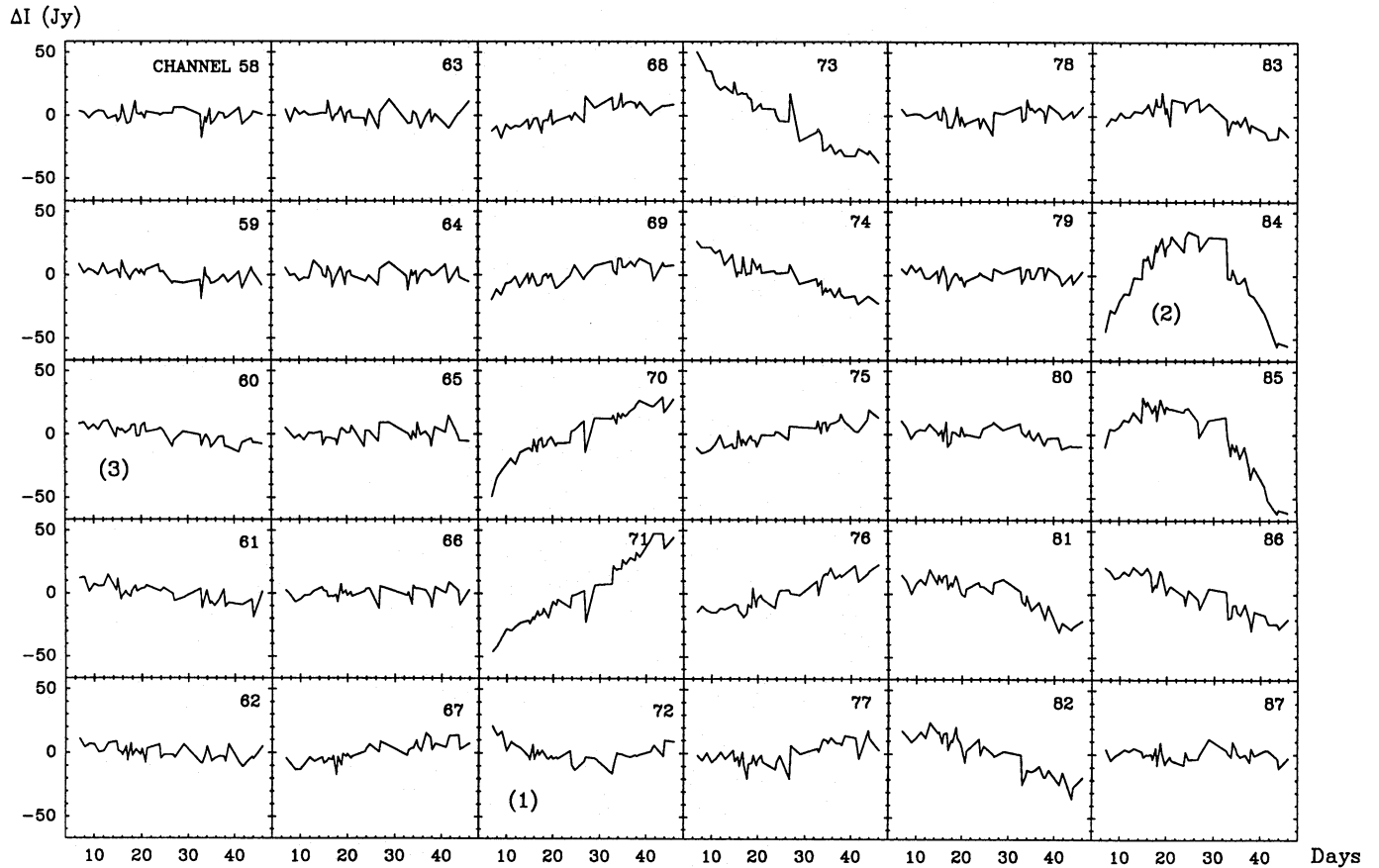


Fig. 5. Same as Fig. 4 but for the star R Cas. Note that the variations are smaller than in R Leo and there is even less evidence for periodic behavior. The three main peaks of the average spectrum are centred at channels 72, 84 and 60

path length. If one constructs a map of a source with a shell structure the overall angular distribution of brightness is a ring rather than a disc. Circumstellar SiO masers are thought to be such shell sources (Bujarrabal 1994). The spatial averaging for a sound wave front passing through this ring is over a number of wave crests and valleys (or 'elements') proportional to the ℓ of the spherical harmonic. Using the same assumption of equal horizontal and vertical size scales, the number of elements in the ring of maser emission is 60 – 200. In each of these elements the amplitude of the density and velocity variations are related according to :

$$\frac{\delta\rho}{\rho} = \frac{\delta v}{c_S} \quad (2)$$

where c_S is the sound speed. During their passage away from the star the sound waves will deform into shock waves. The limiting amplitude of these shock waves is determined by two opposing factors. One is the decrease in background density which will act to increase the wave amplitude which can be seen from arguments of conservation of energy. The other is radiative damping which tends to decrease the amplitude. The combination of these effects is likely to produce wave amplitudes of order unity (see e.g. Koninx & Pijpers 1992) in the

region of interest. Due to non-linear effects the change in density at maximum compression will be a factor somewhat larger than unity.

$$\frac{\delta\rho}{\rho} \geq \frac{\delta v}{c_S} \sim 1 \quad (3)$$

The apparent size of the emitting spots of SiO masers ranges from scales comparable to the sound wave length quoted above to about 50 times such values (Colomer et al. 1992; Bujarrabal 1994). The maser emission of the most compact spots is known to amount to about one half of the total flux and it consists of spectral spikes with typical widths of about 1 km s^{-1} . This emission is very probably saturated in which case it is strongly dependent on the contribution of its "unsaturated core" inside the amplifying path. Since the size of each of such a cores is thought not to be much larger than the sound wave length, the intensity and velocity of the spectral features emitted by these compact spots should vary significantly with the passage of sound waves. The effects of the waves on the intensity of the extended (less saturated) emission is not clear. Although many elements of density variation are expected to be present in an emitting volume, even small variations in the averaged column density can strongly affect the intensity due to the non-linear maser effect. In the unsaturated SiO masers this effect

may lead to amplification factors larger than 10^6 . In summary, sound waves are likely to produce relatively strong local variations of the density and the velocity in the inner circumstellar layers. Such phenomena should yield a significant variability in the maser features, up to percentages in intensity not much smaller than one half of the total. Significant variations in the velocity of the spectral features are also expected.

Finally it should be mentioned that the presence of sound waves in the wind does not necessarily produce periodic variations for two reasons. The first is that there is likely to be a broad-peaked spectrum for the wave lengths of the sound waves just as there is in the sun. This will produce a multiperiodic signal that cannot easily be decomposed. Furthermore, inhomogeneity of the masing region will tend to destroy any clear periodicity in the total line flux variation.

5. Conclusions

A short time scale monitoring is presented of SiO maser emission ($v=1 J=1-0$ transition) in the evolved stars R Cas, R Leo and VY CMa, and in the star forming region Ori A. Observations were typically carried out every day during 40 days. Special attention is paid to the minimization and correction of the calibration uncertainties. The measurement of (small) variations in the total intensity is always difficult, due to such uncertainties, however we believe that variations in the total emission of about 10-20% were probably detected during our observing program. On the other hand, the variations of the line shape are much more easy to determine. Significant line shape variations are found on time scales of 10-20 d close to the line peaks of R Leo and R Cas, leading to relative variations of the order of 20% in individual channels of the spectral detector. Such relative changes in intensity may be quite different inside the line spectra, in fact the sense of the variations tends to change from one side of the peaks to the other. It is argued that such relative variations are probably not due to changes in the stellar emission (i.e. in the maser pumping), but to changes of the column density and/or dynamics in the emitting region. In particular, the measured variability is consistent with global displacements of the line peak in velocity of on the average $\sim 1 \text{ km s}^{-1}$. Accordingly the conclusion must be that, at least in some cool giant stars, the gas velocity and probably also the density of the SiO emitting region changes on time scales significantly shorter than the visual period ; of the order of 10-20 days. It is suggested that this variation is due to the presence of sound waves in the inner shells of circumstellar material surrounding these stars. To better determine whether there is any periodicity present in the short time scale line profile variations, a monitoring program with a similar frequency of observations but much longer duration would be needed. However, the measurement of periods

larger than one month could be severely contaminated by the variability of the masers following the stellar cycle; a variability that is known to be often relatively irregular.

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