Dynamic logic of questions
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Chapter 1

Introduction and Motivation

The logical study of questions is important for various reasons, it has an interesting history, and significant potential for further development and applications.

The interest in questions as an intellectual tool is manifest already in Antiquity, in the emergence of philosophical thinking. The ‘Socratic method’ makes questions an essential element of argumentation and inquiry. Many subsequent advances in science can be understood as refinements of best methods for raising questions, and finding better instruments for giving answers, leading to progressive theory improvement.

The study of questions in a formal setting coincides with the emergence of modern logic in the beginning of the 20th century. Already Kazimierz Ajdukiewicz suggested in the 1930s that asking questions is a logical act, analogous in importance to drawing conclusions. While this did not become mainstream logic at the time, there has been a constant flow of interest in formal modeling of questions, in (epistemic) logic, epistemology, philosophy of science, artificial intelligence, information theory, game theory, and natural language semantics.

Asking and answering questions has been considered the decisive criterion for defining and recognizing intelligence [87]. Contemporary practical applications of questioning theory include query languages in database theory [61, 36], searching the web, guiding rovers exploring Mars towards the best way of performing inquiry in a new environment [58], or computers able to beat human champions in open-questions contests like Jeopardy, as Watson did recently [29].

There are many traditions in the logical study of questions, which started in a serious manner in the 1960s: cf. the discussion in [13, 48, 44, 117]. It would be tedious to discuss all of these, but here are a few major approaches:

- answer sets and question logic analogous to logics of assertions [13],
- partition representation for question semantics to natural language questioning acts [40],
- questions as epistemic devices exploring truth in the world [48],
- questions related to inferential strategies in erotetic scenarios [117],
- algorithmic modeling of questions inside a process of inquiry [56].

These approaches range from quite concrete to highly abstract. There have even been meta-theoretical results on incompleteness [43] and undecidability [81] inherited from the underlying first order logic. But decidability of more constrained reasoning tasks with questions has also been established [15] as well as efficiency of model checking in suitable modal logics.

In this thesis, we want to add something to this existing literature. We will focus on questions as dynamic actions that change some current issue and guide investigation. We will then formalize this view in terms of current ‘dynamic-epistemic logics’, asking ourselves: what, precisely, is the natural repertoire of questions and question-related actions, and what are the natural valid principles of reasoning about them?

But before getting to this point, we discuss a number of major existing approaches to set the scene, identify some major insights that are available, and then state what further things we want to achieve.

1.1 Brief History of Approaches to Questions

We start by a brief overview of major previous approaches to questions that will be most relevant for later developments in this thesis.

- The Logic of Interrogation (LoI), close to natural language semantics, [40, 37] starts from considering a set of possibilities. Then a context is defined as an equivalence relation on these. A language containing first order sentences and formulae is used to talk about such structures.

1. The semantics of both indicative and interrogative sentences is given in terms of ‘context change potential’. Indicative sentences change the context by eliminating possibilities. In contrast, interrogative sentences change the context by disconnecting possibilities, so not by providing new data but by ‘raising new issues’.¹

¹Without delving in all formal details here is how the difference between indicative and interrogative context change is presented in [37]. Indicative sentences provide new data, hence an indicative sentence will change the context by eliminating possible worlds: \( C[\&\phi] = \{(w,v) \in C \mid w = \phi \land v = \phi\} \). Interrogative sentences also change the context, not by giving new information but by raising an issue, hence the context change potential of an interrogative sentence is to disconnect certain possible worlds: \( C[?\phi] = \{(w,v) \in C \mid \|\phi\|_w = \|\phi\|_v\} \). Where \( C \) is the context equivalence relation on \( W \), the set of possibilities, and \( \|\phi\|_w = \{g \in D^{PV}(\phi) \mid w, g = \phi\} \) the extension of \( \phi \) in a triple \( (W, D, I) \) where \( W \) is a set of possible worlds, \( D \) a set of individuals, and \( I \) an interpretation function [37].
2. Next, a notion of semantic entailment is defined also in terms of context change potential. This describes relations between informative and interrogative sentences, both as answerhood (either partial or complete) of an indicative sentence, or as compliance (or licensing) between a context resulting from a sequence of assertions and questions. Using licensing and entailment notions of optimality and informativity for answers can also be defined in this framework.

3. This framework leads to a complete axiomatization in [84] and to a syntactic characterization for the answerhood relation [83] under the assumption of rigid designation. The notion of questions entailment is reduced to the notion of entailment between indicative first order formulae by means of a development of a set of formulae and definability and interpolation for first order logic [84].

4. Indicative sentences as primary information carriers are studied in close connection to questions. Later approaches consider hybrid combination of indicative and inquisitive content of sentences [38, 39] and [18].

- **Interrogative Epistemic Logic** and the resulting Interrogative Model of Inquiry, close to epistemology [49, 48, 78], starts by specifying standard tableau decomposition rules for indicative sentences. Then the resulting tableaux are enriched with an Oracle and rules for questioning moves are added. These rules establish the interaction with the available sources of information: only questions with established presuppositions can be asked and only available information will trigger answers from the Oracle.

1. Knowledge operators can be added to this setting and patterns of dependence and independence between quantifiers and the implicit epistemic quantification are used to specify new decomposition rules. Questions to and answers from the Oracle have also their tableau rules.

2. IMI establishes a connection between a first order questioning model using wh-questions [which object(s) have some property?] and a basic propositional questioning model using the simplest yes-no questions [is the case?] via the Yes-No Theorem, cf. [49].

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2Skipping the formal details for the rules describing standard logical operations, here is how an (in)dependence pattern involving the epistemic modality $K$ looks like as tableau rules [49]:

\[
\begin{align*}
\forall^K, \frac{\Pi, S_0[S_1(\forall/K)S_2] \rightarrow \Sigma}{\Pi, S_0[(S_1 \land f(\overrightarrow{y})) = 0] \lor (S_2 \land f(\overrightarrow{y}) \neq 0)] \land \forall y(\exists x/K)(f(\overrightarrow{y}) = x)) \rightarrow \Sigma} \\
\exists^K, \frac{\Pi, S_0[\exists x/K]S_1[x] \rightarrow \Sigma}{\Pi, S_0[\exists x/K]S_1[x] \rightarrow \Sigma} \\
\end{align*}
\]

for $S_1, S_2$ occurring inside $S_0$ within the scope of universal quantifiers $\forall y = (\forall y_0)\ldots(\forall y_n)$ and where $\overrightarrow{y} = y_0, \ldots, y_n$ and $f$ is a new function symbol.
3. IMI establishes a first link between strategic aspects in questioning and the choice of a decomposition rule in an interrogative tableau via the Strategy Theorem.

A more detailed comparison and a comprehensive merge between IMI and DEL can be also found in [41].

– Dynamic Epistemic Logic based approaches. Inside the broader DEL paradigm questions also have a more recent but fruitful history which is already very close to our own desiderata. A first account of questions as ‘communication acts’ was introduced in [8]:

1. Questions are a special case in a more general setting of ‘abstract dialogues’ or ‘communication sequences’. Interrogatives, or queries are in this setting a particular kind of communication acts.\(^3\)

2. Communication acts come with a ‘timestamp’ determining the legal or illegal ‘(communication) moves’ in a ‘dialogue game’.

3. Various kinds of questioning moves are captured by complex preconditions for execution and/or the publicity of the action, the security of the communication channel, responsiveness, sincerity, etc.

Questions as processes. Another recent approach inside the DEL paradigm is [88, 105, 106]. Here questions are understood as processes combining PDL tests with sequential composition and choice to model changes in the structure of an epistemic model.

1. An update by a question is a complex program that changes a focus equivalence relation representing the content of the asked formula.\(^4\)

2. Standard notions of answerhood and interrogative entailment can be captured in this setting in a natural way, and a large variety of questioning actions and their presuppositions projections can be modeled such as embedded questions, constituent questions, etc.

3. The questioning moves interact with standard informative actions such as public announcements to describe a complete epistemic dynamics via model restrictions and the focus relation induced by questions.

\(^3\)See also Section 4.1 later in the thesis, where we will further discuss the relevance of this approach for questioning games.

\(^4\)We cannot present all the formal details here, but here is the main idea from [88] in a nutshell: the effect of asking the question whether \(\phi\) is a complex program constructed by composing several simple ones as follows: \(F\phi =_{df} (\text{Test}\phi; G; \text{Test}\phi) \cup (\text{Test}\neg\phi; G; \text{Test}\neg\phi)\), where \(G = W \times W\) is the total binary relation and \(; \cup \), and \(\text{Test}\) have their standard meaning from PDL. The extensional result of executing a question is then given by refining the focus relation: \([F\phi]^{++} = \{ (w, w') \mid M, w \models \phi \text{ iff } M, w' \models \phi \} \).
4. Another relevant contribution in this approach is the fruitful implementation of the DEL theory in a computational framework provided by functional programming in Haskell [106, 109, 25].

There are also related approaches and neighboring fields in which the questioning research agenda is reinforced by similar results and research interests:

- **Inferential Erotetic Logic** [119, 118, 117], starts from an account of inferences with questions as premises and builds an approach to problem solving via ‘erotetic search scenarios’. This setting leads to results that link erotetic derivation with a systematic search for an answer to one main wh-question using a ‘golden path’ of propositional yes/no questions which are the simplest in a logical and epistemological hierarchy.

- **Other Inquiry Calculi** There are other traditions aiming to develop abstract calculi for inquiry using various formal frameworks. The approach in [72] and [26] adds a research agenda to the AGM framework for belief revision and studies resulting logics. The approach from [56, 57] provides an algorithmic model for the role of questions inside a process of inquiry. The approach in [20, 59] derives questioning inference rules from algebraic properties of questions in an analogous way in which Boolean algebra is connected with assertions and Bayesian inference [58, 60]. Finally, the approach in [14] studies questions and answers in an orthoalgebraic framework using a ‘decorated’ partition theory and a formal theory of ‘observables’.

- **Game Theory** Approaches that establish a connection in both directions between logical and game theoretical aspects are not new [50], [80, 77], [90]. But even in standard game theoretic approaches questions and experiments have played an important part as ancillary elements in probabilistic belief dynamics [7]. Approaches to awareness and unawareness in imperfect information games have also many common points with the logic of issue management we are going to develop here. A exhaustive presentation is beyond the scope of this brief history, we also refer to Chapters 4, 5, 6 and 7 where we add a game theoretic twist to dynamic epistemic logic with questions (henceforth DELQ), and we also mention [23] which provides a comprehensive link between a DEL approach with binary experiments and agreement theorems. Moreover, there are results inside game theory showing how belief dynamics based on questioning actions gives rise to specific, useful and irreducible properties in contrast to the dynamics induced by informative actions [46].

A complete presentation of all these fields is beyond the scope of this brief introduction, nevertheless it suggest a coherent research agenda that includes topics that resonate in many other research projects.
1.2 Main Topics in the DELQ Approach

In this broad theoretical background boundaries between alternative approaches aimed at explaining the same underlying phenomena of questioning are blurred and clear distinctions might be only partial and sometimes elusive. One way to distinguish the specific of one approach among many others is by finding pivotal criteria for what it aims to achieve and aspects considered important.

We already listed the achievements of previous approaches, we will focus now on some general desiderata that remain unfulfilled.

Even though the connection between knowledge and questions is of interest in many previous approaches, this is not approached in an setting that makes this connection explicit and with a language that can describe both aspects and their intersection with adequate modalities. The issue-knowledge resolution will be studied at both a static level, expressed by an intersection modality, and a dynamic one, by performing the intersection between the two relations. The intersection will be an essential tool in the formal repertoire needed to understand the interdependence between questions and information.

A next natural desideratum left unfulfilled by previous approaches concerns an interactive framework. We will use a genuine multi-agent setting that allows for higher order questioning and information dynamics, private communication and group notions of knowledge and issues.

Another unfulfilled desideratum concerns an exploration of strategic abilities in questioning scenarios and the connection between a questioning theory and designing efficient querying strategies. We will also provide a bridge between a theory of questions and known search heuristics using backtrack oracles.

Many of the topics already discussed, issues acknowledged and problems raised in previous approaches will constitute the main points to be addressed in our approach. Some of the most relevant open problems received from previous traditions will define the profile of our current approach and will guide the solutions we aim at and the general results that emerge in the DELQ approach.

There are at least three general goals that will characterize our approach inside a more general framework of related approaches, and we state them in retrospect from the very beginning. First, we will be interested in developing a setting in which questions are analyzed and understood in their intricate conceptual, logical, and practical interdependence and essential connection with knowledge and information dynamics. Second, a genuine and fruitful guiding interest for a setting in which questions arise in an interactive multi-agent environment doubled by a study of the strategic aspects that emerge in such a setting. Finally, a third defining criterion for our approach will be a constant interest to find the right balance between expressive power and computational complexity. This will be substantiated in a parallel connection with implementing theoretical aspects and in developing logics that will provide at least decidability of reasoning task and practical efficiency for model checking of realistic questioning scenarios.
1.2. Main Topics in the DELQ Approach

The discussion so far and the list of further desiderata already provides the main points on the research agenda to be pursued throughout this thesis. We list them here once more in a systematic way:

- make actions explicit, and determine the repertoire systematically,
- multi-agent character of questions,
- role of strategic interaction,
- role of protocols and global temporal perspective,
- need for a ‘reality check’ in implementation.

All these constitute relevant components in the general theoretical background on which the approach presented in the following chapters will emerge. There are also many topics for further research and connections with similar or alternative approaches that will only be partially resolved in this thesis. We only mention here one of them and reserve a more exhaustive discussion to Section 1.3 after the details of our approach and the induced results have been given.

Developing an adequate theory of questioning actions for first order logic and the corresponding wh-questions using our framework will remain an important desideratum for further research. In comparison to most of previous approaches this is an important missing bit in our current approach. However, the current questioning theory can already handle much of the relevant and interesting questioning scenarios involving first order definable properties by using oracles. The main idea here is that once FO entailment between FO properties is independently proved, a DELq theory can be applied to oracles that use such a FOL implication. And designing efficient questioning strategies for such oracles does not require higher order logics. We illustrate this with a concrete example in Chapter 6 using an oracle of first order definable properties.

The rationale behind the implementations we include in our approach are the following: They can be understood in connection with previous implementations for PAL and DEL from [108, 107, 109, 106]. We will provide a significant extension of previous implementations which focus on informative actions with analogous functionality for questioning actions. Moreover, we will also extend the standard DEL functionality with strategic aspects that emerge in games with questions. In addition to previous Haskell implementations for DEL functionality from DEMO, we will also use Alloy Analyzer [52] to capture entailment between oracles of local properties in characterizing solution concepts, and the Haskell module for probabilistic functional programming from [27]. The implementations are used to provide concrete illustrations for many theoretical and algorithmic aspects developed in the text. However, their role goes beyond merely providing illustrations and often they will give rise to interesting conceptualizations and provide additional useful results.
Overview of the Thesis  Before going into the details of each chapter, we give a bird’s view perspective on the succession of the main topics that will converge into the guiding storyline of the entire thesis.

There are two entangled strains contributing to the guiding direction of this thesis. The first storyline will be theoretical and will provide a formal analysis of questioning actions in epistemic and interactive contexts. This storyline will be intertwined and doubled by the storyline of implementing the formal aspects and applying them for analyzing realistic questioning scenarios.

The two story-lines are closely related but the succession of chapters allows them to be followed independently. The even chapters will only contain theoretical aspects while the odd chapters complement these with implementation details and further concrete illustrations of relevant example applications.

The first chapter is an introductory chapter in which we also review some of the previous approaches to questions that are most relevant for this thesis.

We start in Chapter 2 with setting the ground for an analysis of questioning by introducing a basic logical framework. We extend the standard epistemic models with equivalence relations for questions. We introduce a static logical language to describe such structures by means of corresponding modalities, most important being the resolution modality which uses the intersection of the two equivalence relations. We then go on to define actions changing both epistemic and questioning structure in such models. Most important are the actions of asking and resolution which operate by refining partitions. Then we add dynamic modalities to this language and provide a completeness result by means of reduction axioms. We conclude the chapter with two appendices containing some theoretical background and definitions respectively the proofs of the main results.

We continue in Chapter 3 by presenting and discussing the implementation behind our logic of questions. This will be a literate Haskell program that extends the previous implementation for epistemic model checking from DEMO [107, 108] with questioning specific functionality.

The main new additions are a richer, more expressive language that includes formulas with intersection modalities, model checking for resolution, questioning and epistemic formulae and a general and extensible implementation for complex questioning and resolution dynamic actions that emerge in this framework. We will also prove the implementation useful by analyzing some paradigmatic examples of questioning scenarios in epistemic settings.

We return to a theoretical approach in Chapter 4 by defining and investigating games with questioning moves. We first look at strategic games with two players and question-answer moves. We then extend this basic approach to settings with more players, sequential moves, and oracles encoding interactions between imperfect agents or limitations in external information sources or experimental procedures. We define solution concepts for such games and consider some illustrative examples. We continue by basic results about questioning games and
compute outcomes in some illustrative interactions. We then investigate strategic abilities in epistemic games with extensive questioning moves and show how and why the diffraction property is an important property of questioning games that is relevant for a general framework for long term questioning actions in inquiry. Two final appendices contain background definitions and proofs of main results.

In Chapter 5 we return to implementation by presenting and discussing the Haskell scripts behind the questioning games introduced in the previous chapter. These also extend basic epistemic functionality from [109] to include strategic aspects specific for a game theoretic approach of questioning actions. Some of the most noticeable new functionalities include the use of nominals to define the execution value of questioning strategies by linking the semantic level based on partitions of the domain with a corresponding syntactic level in expressive harmony. This gives us a notion of strategy equivalence based on the execution value for a question in an issue-epistemic model.

Using this notion of strategy equivalence we build the space of strategy profiles for players in the issue-epistemic model at a state. Then using this local perspective we construct the induced game that represents the questioning interaction at a global level of the model. Finally, we build the game matrix in the induced global game by performing model checking for the goal formulae of the agents and then by aggregating the results over the entire domain. We prove the implementation useful by analyzing in detail the process of counting strategies and goals and computing the outcomes in a representative example of an interactive and competitive questioning scenario and possible variations.

We also give an algorithm for minimizing issue-epistemic structures while preserving the truth value of formulae using the intersection modalities. This is based on a refinement algorithm that solves the birelational coarsest partition problem. To match this fixedpoint refinement process we also define a notion of behavioral equivalence between issue-epistemic models and we show that this is the adequate invariance notion for our questioning language.

Chapter 6 approaches the topic of designing questioning strategies in problem solving from a theoretical perspective. We take again solving games as our point of departure and a rich test case representative for a more general theory. In this context we investigate the problem of solving the location game played on a line. Our solution concept of choice will be Nash equilibrium with pure strategies.

We give a characterization of NE by means of local properties in the game. Then we use an approach based on querying an oracle of local properties to design questioning strategies that solve the game in an efficient way. We end by a discussion of the general relevance of this result for designing querying strategies in problem solving by using oracles of operational properties to solve a principal problem using efficiently available sources of information. As in the previous chapters, two appendix sections conclude the study by presenting some useful background definitions and the proofs for the main results.
Section 6.4 approaches the dynamics of questioning in a setting enriched with probabilities. We start by considering previous approaches to DEL in a probabilistic setting in general, and, in particular approaches that add questions into the mix. Then we give an algorithm for minimizing probabilistic issue-epistemic models while preserving the truth value of formulae using the intersection modalities. This is based on a refinement algorithm that solves the birelational coarsest partition problem. Based on this we also give a notion of behavioral equivalence between probabilistic issue-epistemic models and we show that this is the adequate invariance notion for our language.

Next, as we did until now, in Chapter 7, we present and discuss the implementation behind the results in the previous chapter. In this case we will use two software tools. The first is, as in the previous chapters Haskell. We will show how queries of local properties in the game can be used to search for equilibrium strategy profiles in an implementation using list comprehension. This approach assumes the existence of oracles of local properties and uses this to search NE.

Because both NE and the local properties are expressible by formulae of first order logic we will use a second implementation, in Alloy Analyzer [52], to build a model for the location game and check assertions about logical entailment within a predetermined scope between formulae expressing local properties and NE. Once more, we will illustrate the use of both implementation tools by considering in detail representative examples of concrete strategic interactions in LG.

Chapter 8 is dedicated to draw the general conclusions and to discuss how our approach gives rise to a coherent research agenda and many directions for further study and broader connections with other relevant topics both inside the DEL paradigm and in comparisons with alternative approaches.

The Sources of Some Material The content in Chapter 2 is based on material which has has been previously presented in preliminary versions at the LIRa seminar in Amsterdam and subsequently published in the LIRa Yearbook 2009. It was also presented at the Second International Workshop on Logic, Rationality and Interaction (LORI-II) in Chongqing, China, October 6-11, 2009, and subsequently published in the conference proceedings as [100].

Section 4.4 has been presented in preliminary version at the 9th Conference on Logic and the Foundations of Game and Decision Theory (LOFT 9), in Toulouse, France, 5-7 July, 2010, as [1]. Section 4.4 has been presented and subsequently published in the proceedings of the second ILCLI International Workshop on Logic and Philosophy of Knowledge, Communication and Action, (LogKCA-10), Donostia - San Sebastian, Spain, 3-5 November 2010 as [70].

A preliminary version of implementing questioning dynamics was presented and subsequently published in the proceedings of the Second International Conference on Computer Supported Education (CSEDU 2), in Valencia, Spain, 7-10
April, 2010 as [71]. Section 5.1.1 is the implementation corresponding to the theoretical framework from [1] and was also used in [70]. The content in Chapter 6 originated from a homework exercise while the author was grading assignments for the Strategic Games [4] course at ILLC in 2010 and it has been presented before in the ComSoc seminar in Amsterdam and in the Student Session of Sino-European Winter School in Logic, Language and Computation (SELLC) Guangzhou, China, December 3-18, 2010, subsequently included in the electronic proceedings as [69].

1.3 Comparisons with Alternative Approaches

There are many classical traditions [42, 5, 65, 13] that provide useful insights for an approach to questions and many of these are reinforced when considered in the light of our current dynamic logics of questioning actions. It is beyond the scope of this brief introduction to have comprehensive comparison to all of them, we will only focus to some of them that offer the closest connections.

One which is directly connected to our approach is the active program of Interrogative Model of Inquiry (IMI) [48, 49] and [50, 78, 32]. As already mentioned, questions are treated here as requests for new information, which function intertwined with deductive indicative moves in ‘interrogative tableaux’. There is an extensive theory of answerhood, as well as an analysis of various types of question in a predicate-logical setting, beyond what we have done here. The framework has a number of nice theoretical results, including meta-theorems about the scope of questioning in inquiry and discovery. A systematic comparative study of the relations between the two approaches is still needed in order to have a complete picture of the connections. There are previous studies that already started such an investigation and proposed meaningful and fruitful merges between the frameworks. A bridge in this direction has been given by [41], and comparison points can be also found in [76]. Much of what we did in this thesis can also be seen as a further development of some of the themes put forward in IMI.

Such themes also emerge in Inferential Erotetic Logic (IEL) [119, 118, 117] which provides an account of inferences with questions as premises and, starting from this notion, an approach to problem solving via ‘erotetic search scenarios’ linking erotetic derivation with a systematic search for an answer to one main wh-question using a ‘golden path’ of propositional yes/no questions which are the simplest in a logical and epistemological hierarchy.

Our approach in Chapter 6 establishes a fruitful link between a questioning theory and known search heuristics using backtrack oracles. This shows that a questioning theory is relevant and can be useful in designing efficient querying strategies by using oracles of first order properties. It also shows that designing such query strategies can be done with yes/no questions alone as long as the higher first order entailment between properties encoded in the available oracles has been established. This comparison is in no way complete but it provides a first
step towards a more general theory for efficient problem solving via questioning mechanism design and issue management.

Another close comparison to our approach stemming from the LoI tradition is the recent inquisitive semantics ([39, 38, 18]). Inquisitive semantics gives propositions an “interrogative meaning” defined in a universe of information states over propositional valuations, with sets of valuations expressing issues. This supports a compositional semantics for the language of propositional logic, where, for instance, a conditional is true in an informational sense if every subset (stronger information state) supporting the antecedent also supports the consequent. Interrogative meanings are then defined in terms of generalized partitions of the set of worlds, where partition cells may now also overlap. This is a significant extension of the traditional issue picture. Based on this semantics, a propositional logic arises that describes valid consequence and other important relations between questions, and for questions and answers. The program has found a variety of applications to natural language semantics and pragmatics. Of course, we cannot do full justice to this framework here: for recent updates see [86]. At some level of abstraction, the ideas in this system sound very close to ours: there is information dynamics, questions also change current possibilities, and so on.

However, a systematic study of the formal relationship between the two does not yet exist in the literature. Neither does the scope and purpose of this thesis provide the proper setting for such a comparison. The best we can do at this stage is to give a starting point for a methodological comparison, while acknowledging the fact that further investigation is needed in order to have a complete picture of the existing connections bridging the two approaches in both directions.

We start from a basic observation that seems the key point of difference between the two approaches. Inquisitive semantics puts the dynamic information about questions in a new account of the meaning of interrogative sentences in a propositional language. This is not classical declarative meaning, and hence some deviant propositional logic emerges.

By contrast, the DELQ approach wants to give an explicit account of questions and other actions of issue management, but it does so by means of dynamic modalities on top of a classical logical language. In particular, there is no meaning shift: but rather an expansion of the domain of study of classical logic.

The distinction is similar to one in logic itself (cf. [89, 92]). Intuitionistic logic studies knowledge and information implicitly by changing the meaning of the classical logical constants, and then picking a fight with classical logic in the set of ‘validities’. By contrast, epistemic logic analyzes knowledge explicitly as an additional operator on top of classical propositional logic: there is no meaning shift, but agenda expansion. In our view, DELQ stands in exactly the same relationship to inquisitive logic: it makes the dynamics explicit, and steers away from foundational issues of meaning and validity. Comparisons between the two approaches can be quite delicate (cf. [92]), and the same may also be true here.
1.3. Comparisons with Alternative Approaches

Much of what we did in chapters 3 and 5 is an investigation related to themes put forward in the inquisitive approach. We have considered many examples of natural language conversational and epistemic scenarios in which various gradients of mixing information flow with raising issues emerge. We have also given reduction axioms for some of these combinations, and, perhaps most importantly, we have given an unifying method, a way to treat all such gradients in an uniform way by using a unique product update rule. We have also shown how adequate models for this ‘art of pragmatic modeling’ can be generated using implementation tools. We have also shown how the partition model can be extended to deal with questions inducing a cover representation using the product update rule.

A complete comparison should proceed by giving characterization results by explicit reduction axioms that will bridge the two approaches, this is a very interesting enterprise for future research. Our approach that deals with birelational structures might also bare some relevance to more recent inquisitive approaches that also consider the interaction between issues and attention or awareness.

Finally, our multi-agent approach to question-answer games revealed the fact that the interactive desideratum is not just a collateral extension. We have shown that multi-agent interaction gives rise to concepts about questioning that generalize traditional notions. The questioning games analyzed in Chapters 4 and 5 showed that received standard notions of relevance and informativity for questions turn out to be one particular case in a more general conceptualization that also takes the interaction with epistemic aspects into account.

But the closest comparison is inside DEL itself. There are previous DEL approaches that inspired and guided what we have done. The first DEL approach to questions as communicative actions [8] was already using product update and gave reduction axioms for legal ‘timestamped’ moves in an abstract dialogue with questions and answers. Our approach shows how this methodology can be lifted from epistemic events to sets of epistemic events representing the answers to questions and shows that this works essentially in the same way as it did for logics focusing on informative actions. The same issues of privacy and publicity that were treated for informative actions emerge for questioning actions.

Another close comparison inside DEL is the approach using a PDL tests and regular operations to refine the focus relation [106, 105, 88]. This is already very close to our approach both in desiderata and in formal details. The main further contribution provided by our approach is to add intersection and show how to handle it in a formal framework that captures the interaction between questions and knowledge. Another relevant comparison point that emerges here consists in the fruitful connection with implementations. The preexisting functionality for dynamic epistemic modeling from DEMo [107, 109] was the starting point for our extension to questions. We add functionality for questioning actions to this framework, we also give algorithms for minimizing birelational models and we link this to an notion of structural equivalence adequate for questions. We also show
how the Haskell implementation can be used for extensions modeling strategic aspects in question-answer games and also in basic probabilistic scenarios.

Last but not least we mention the close connection with game theory. This connection is a major theme of the thesis and bridges its topics in both directions.

First we will show how the standard DELQ approach can lead to interesting results when enriched with game theoretic concepts and we will add a more general game theoretic twist to questioning actions in chapters 4 and 5 by considering and analyzing games with questions.

Second, in chapters 6 and 7, we will start from a game theoretical analysis of a concrete interactive situation in the location game on a line and use queries of local properties together with backtrack oracle heuristics to design query strategies for solving the game.