Structural analysis of complex ecological economic optimal control problems

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Chapter 6

Summary

This thesis is devoted to the study of parameterized families of continuous time infinite horizon optimal control problems with one dimensional non-convex state dynamics. Such models often occur in environmental economics (see Tahvonen and Salo (1996), Scheffer *et al.* (2001), Brock and Starrett (2003), Mäler *et al.* (2003), Scheffer (2009)). All the theoretical results and methods developed in the thesis are illustrated with the deterministic and stochastic lake model. They are however general and can be applied to any problem of that type.

The main contribution of the thesis is the development of the bifurcation theory of one-dimensional optimal vector fields (see Chapter 2), which allows to obtain the solution structure of parameterized non-convex optimal control problems. Such problems can exhibit multiple local optimal attractors and consequently indifference thresholds separating their basins of attraction. The theory developed in Chapter 2 allows to locate regions in the parameter space for which the controlled system exhibits thresholds. This information may suggest reconsideration of management options or reevaluation of the key system parameters.

Chapter 3 analyzes the shallow lake model using the theory developed in Chapter 2. It is a model of optimal water pollution management, which serves as a prototype of a conflict between ecologic and economic interests. Bifurcation analysis of the model shows the effects of changing the parameter values in the parameter regions where the clean and polluted steady states are globally or locally stable under the optimal dynamics. This information can affect evaluation of the parameters, such as the discount factor or relative costs of pollution, which are used by the social planner when designing the optimal policy. A slight change of the discount factor ρ can change the optimal policy radically. For instance, for some values of the pollution costs a decrease of ρ can imply that an initially clean lake will be steered to the clean equilibrium, rather than to the polluted equilibrium under the policy with a higher discount factor.

Stochastic optimal control problems with small noise intensities have been studied in Chapter 4. The solution of such a problem reduces to solution of the corresponding Hamilton-Jacobi-Bellman equation. It is a singularly perturbed second order differential equation. When the noise is set to zero, it becomes a first order dynamic programming equation of the corresponding deterministic problem. Chapter 4 develops a method of constructing approximate solutions of the Hamilton-Jacobi-Bellman equation. From these solutions, a geometric invariant - transformation invariant function - is computed. A stochastic bifurcation in the sense of Wagenmakers *et al.* (2005) is then a qualitative change of this function.

Stochastic optimal control problems exhibiting regime switching behavior are of especial interest. These are the perturbations of deterministic problems exhibiting indifference thresholds. For such stochastic problems multimodality of the transformation invariant function allows to define regime switching thresholds as its local minima. Transition between different regimes is realized by crossing these thresholds due to large shocks. Such a model can explain sudden rapid changes of a state variable, be it an ecosystem or an economy.

This thesis demonstrates the importance and effectiveness of methods of bifurcation theory applied to studying non-convex optimal control problems. It opens up a new methodological approach to investigation of parameterized economic models. While standard analytical methods are not efficient and sometimes impossible to apply to non-convex problems, the numerical geometrical methods developed in the thesis allow to solve and analyze such problems quickly. More and more deterministic and stochastic non-convex optimal control models occur in economics (Caulkins *et al.* (2001), Brock and Starrett (2003), Stachurski (2003), Crepin (2007), Heijnen and Wagener (2008), Kossioris *et al.* (2008), Heijdra and Heijnen (2009), Zeiler *et al.* (2009), Bultmann *et al.* (2010)), therefore the development of such methods is essential. In fact, some of the research work already take up the methods of the bifurcation theory of optimal vector fields: see Caulkins *et al.* (2007), Grass *et al.* (2008), Graß (2010), Hinloopen *et al.* (2010) for work on deterministic problems with continuous time, Moghayer and Wagener (2009) for deterministic one-dimensional discrete time systems, Diks and Wagener (2008) for stochastic one-dimensional discrete time systems where the authors introduce the dependence ratio reproducing the transformation invariant function.

Based on the results of this work, possible future research topics include the development of bifurcation methods for deterministic and stochastic non-convex optimal control problems with multidimensional state spaces; the generalization of these methods to dynamic games; and the development of a bifurcation methodology for multidimensional discrete time models.