Behavioural models of technological change

Zeppini, P.

Chapter 4

A behavioural model of endogenous technological change

4.1 Introduction

Innovation is an important strategy in firm competition. Two alternative ways to innovate are *R&D* investment and imitation of others' innovation (Nelson and Winter, 1982). Imitation can be perceived not just as free riding, but also as exploitation of external sources. This can involve public knowledge such as published research but also spillovers and leakages from private knowledge (Spence, 1984). Considering the taxonomy of Malerba (1992), innovation and imitation refer to *learning by searching* and *learning from spillovers*. In the latter we may include all different kinds of information flows, from knowledge leakages to pure product copying activity. Here we will consider imitation in a similar way as Conlisk (1980) model of costly optimizers versus cheap imitators, being interested in the interplay between innovation and imitation as strategies and its effect on innovation dynamics. Technology is a non-rival partially excludable good (Romer, 1990), which makes direct imitation possible, although with different levels of difficulty from case to case. In the end, it will always be possible to copy a design of a new product once it is in production. Benoit (1985) addresses non-patentable innovations and studies the interplay of innovators and imitators in the strategic setting of a duopoly. We consider a population of firms instead, where innovation and imitation are two alternative strategies.

The empirical evidence shows a substantial unexplained inter-firm and intra-sectoral variability of innovation proxies ($R\&D$ expenditure, innovative output, patenting activity, etc.) after allowing for the firms' size effect (Dosi, 1988). This suggests that other factors affect the innovation process, such as heterogeneity of strategies and firms interactions. Innovative and imitative behaviour are not static, but rather dynamic phenomena in market competition, with the possibility that firms switch strategies.

We propose a market model of innovation dynamics based on adaptive heterogenous strategies of firms. Heterogeneity is central to the process of innovation, since it gives rise to a differentiated cost structure or to differentiated products. Our aim is to examine the impact of heterogeneity and adaptive behaviour on innovation dynamics, understanding what endogenous forces determine the prevalence of innovation or imitation. Moreover, a dynamic market model based on demand and supply allows the study of the mutual interaction between technological innovation and market dynamics.

We consider a perfectly competitive market where one technology underlies the production of a homogeneous good. Firms can innovate their production technology and increase total factor productivity (TFP) by reducing the unit cost of production. If they imitate they rely on the state-of-the-art technology. Cost reduction is not only process innovation but can also arise from product innovations. An example are photovoltaic cells, where innovative products cost less or have a better performance for the same cost.

In a competitive market firms need to catch up either via innovation or imitation in order to recover in terms of profitability and market competitiveness (Dosi, 1988). In our model innovation and imitation are seen as two strategies in line with Schumpeter's hypothesis of *routinization* of innovation (Schumpeter, 1942), and more generally with Simon's view about bounded rationality (Simon, 1957): information gathering and processing costs are an obstacle to the optimal strategy, whether to innovate or not. In order to study this behavioural diversity we adopt the discrete choice strategy switching framework of Brock and Hommes (1997). Our model addresses interacting firms that make a choice about whether or not to invest in innovation in order to be more productive. The idea of imitation as a cheap heuristic opposed to a costly sophisticated innovative strategy is similar in spirit to Grossman and Stiglitz (1976)'s model of informed and uninformed agents in a competitive asset market. In our model this idea can be expressed by saying that it may be more efficient for some firms to exploit other firms than to invest in innovation themselves. Because of these different elements, our model of innovation combines the approach of neoclassical economics with the evolutionary-economic approach of dynamic heterogeneous populations.

In our model a negative feedback makes it profitable to individual agents to change their strategy in an environment where the strategy becomes dominant, as in Conlisk (1980). Differently from Conlisk, in our model the economic environment evolves under the endogenous interplay of market and firms' choices, instead of an exogenous stochastic process. Another endogenous model of interacting sophisticated and naive agents is Sethi and Franke (1995). However, this model as well as Conlisk's model is globally stable: if it was not for exogenous random shocks the economy in these models would converge to a homogeneous equilibrium where all agents use the cheap strategy. In our model the equilibrium may be unstable, and long run endogenous cycles are possible without any exogenous shocks. Iwai (1984) proposes a model where firms are described by a distribution of production cost. Our evolutionary selection dynamics of innovation as cost reduction, brings together the Iwai (1984) and Conlisk (1980) models.

The negative feedback of a dominant strategy in our market environment stems from the effect of innovation on the price. Innovators lower the market price, because innovation in our model means cost reduction. A lower price translates into lower profits, which hurt innovators more than imitators due to the investment cost of innovation. Hence there is a negative feedback: when innovators (imitators) dominate the market it is better to imitate (innovate). Because of bounded rationality, firms adopt the best strategy with some probability smaller than one. In the simplest model, we consider synchronous updating of strategies, which means that in each time interval all agents evaluate utilities and possibly change strategy. Asynchronous updating corresponds to the more realistic case where agents gradually change strategies and only a fraction update their strategy in each period. In case of synchronous updating, the model has either a stable equilibrium where the two strategies coexist, or period 2 cycles, where firms switch between innovation and imitation every period. Although asynchronous strategy updating turns out to dampen the amplitude of oscillations in cases where the equilibrium is unstable, it may qualitatively destabilize price behaviour and lead to irregular chaotic dynamics. Asynchronous strategy updating thus provides an endogenous behavioural explanation of the variability of empirically observed innovation proxies.

A further extension of the model consists of the introduction of technological progress. This amounts to relax the hypothesis that price and fractions dynamics do not interact with technology dynamics. We instead let the production technology grow endogenously depending on the number of innovators in past periods. Doing so we build a behavioural model of endogenous technological change, where we study how agents choices shape technology through innovation, and how technology on its turn drive agents' choices. Numerical simulations of this model are compared to empirically observed data on industry price indices. The model is able to replicate a number of stylised facts related to technological innovation, such as learning curves, Schumpeterian rents and market breakdown.

The chapter is organized as follows. Section 4.2 introduces the general framework and presents a basic model. Section 4.3 studies the effect of asynchronous updating. Section 4.4 extends the model with technological progress, and finally Section 4.5 concludes.

4.2 Costly innovators versus cheap imitators

4.2.1 The model

Consider an industry with N firms producing the same good in a perfectly competitive market. *Innovation* means to reduce the production cost, while *imitation* means to adopt the currently available technology. Firms are either *innovators*, with fraction n, or *imitators*, with fraction $1-n$. Choosing the strategy (innovation or imitation) sets the production technology and the cost structure or total factor productivity (TFP) of a firm. The quantity $S^h(p_t)$ supplied in period t by a firm choosing strategy h is a function of price and depends on the cost structure of strategy h. In each period the market clears in a Walrasian equilibrium:

$$D(p_t) = n_t S_t^{INN}(p_t) + (1-n_t) S_t^{IM}(p_t), \tag{4.1}$$

where $h = INN$ stands for innovation, and $h = IM$ for imitation. Eq. (4.1) results from the aggregation of demand over consumers and supply over firms, and then dividing by the total number of firms N.[1] The supply is a convex combination of innovators' and imitators' production, with n_t and $1-n_t$ the fractions of innovators and imitators, respectively. Profits of an individual firm of type h in period t are $\pi_t^h = p_t q_t^h - c^h(q_t^h)$, with $q_t^h \equiv S_t^h(p_t)$. We choose a quadratic cost function as in Jovanovic and MacDonald (1994): the cost of producing quantity q for a firm adopting strategy h is $c^h(q) = \frac{q^2}{2s^h} + C^h$, where C^h represents the fixed costs of the strategy. This choice keeps the model as simple as possible: maximization of profits with respect to quantity q gives a linear supply:

$$S_t^{INN}(p_t) = s_t^{INN} p_t, \qquad S_t^{IM}(p_t) = s_t^{IM} p_t. \tag{4.2}$$

[1] Aggregation of supply gives $S_t = \sum_{i=1}^{N_t^{INN}} S_{i,t}^{INN} + \sum_{j=1}^{N_t^{IM}} S_{j,t}^{IM}$. Subgroups of innovators (imitators) are homogeneous i.e. $S_{i,t}^{INN} = S_t^{INN}$ ($S_{j,t}^{IM} = S_t^{IM}$) for all i (j). Hence $S_t = N_t^{INN} S_t^{INN} + N_t^{IM} S_t^{IM}$. Dividing by the number of firms N one gets the right-hand side of (4.1).

The parameters s_t^{INN} and s_t^{IM} are proportional to TFP, and consequently depend on the production technology of the firm.[2] Consider a linearly decreasing demand $D(p_t) = a - dp_t$ ($d > 0$). The market equilibrium equation (4.1) becomes

$$a - dp_t = n_t s_t^{INN} p_t + (1 - n_t) s_t^{IM} p_t. \tag{4.3}$$

An innovator invests $C^{INN} = C > 0$ and increases TFP, expressed by $s^{INN} > s^{IM}$, cutting down the production cost $c(q)$ (see Jovanovic and MacDonald (1994)). Cost reduction is larger for larger values of output: $\Delta c = -\frac{q^2}{2s^2}\Delta s$. This means that larger firms profit more from innovation. Imitation is free ($C^{IM} = 0$) and it amounts to using the state-of-the-art technology, a sort of publicly available technological frontier.[3] This setting is similar to Iwai (1984), the difference being that here we have two types of firms instead of a continuous distribution. If we focus on TFP, our model resembles the model of competition driven by *R&D* in Spence (1984), provided that time is discrete and firms are homogeneous but for their choice about innovation, as in Llerena and Oltra (2002). In order to close the model we need to specify how the fractions of innovators and imitators are determined in each period. We assume that firms switch between innovation and imitation based on the difference of profits between the two strategies $\Delta\pi_t = \frac{1}{2}\Delta s_t p_t^2 - C$. For a quadratic cost function, profits are:

$$\pi_t^{INN} = \frac{1}{2} s_t^{INN} p_t^2 - C, \qquad \pi_t^{IM} = \frac{1}{2} s_t^{IM} p_t^2. \tag{4.4}$$

In particular $\Delta\pi = 0$ for $p = \overline{p} \equiv \sqrt{2C/\Delta s}$. Average costs are $\gamma^{INN} \equiv \frac{c^{INN}(q)}{q} = \frac{p}{2} + \frac{C}{s^{INN}p}$ and $\gamma^{IM} = \frac{p}{2}$, with $\gamma^{INN} \geq \gamma^{IM}$ and $\gamma^{INN} = \gamma^{IM}$ in the limit of infinite price. This is

[2] If we think in terms of a production function like $q = A\phi(K, L)$, where ϕ is a function of capital and labour, the parameter s is positively related to the production technology factor A.

[3] In principle imitators have the advantage of not replicating an unsuccessful innovation. Here we assume that innovation is always successful. One can also interpret the model in a slightly different way, thinking that innovation is an uncertain event, and that innovators improve their productivity with a given (exogenous) probability. Say that S^{INN} is the expected value of productivity from this innovation process. With a large number of identical innovating agents, everything goes as if all innovating agents are given the improved productivity S^{INN}

an indication that innovators benefit from a high price, although their aggregate effect is exactly in the opposite direction, i.e. more innovators lower the price.

We adopt the discrete choice framework of Brock and Hommes (1997) (BH henceforth) to model the endogenous evolutionary selection between costly innovation and cheap imitation. An agent i enjoys the random utility $\tilde{u}_{i,t} = u_{i,t} + \epsilon_{i,t}$. The deterministic term is given by market profits: for an agent adopting strategy h, $u_{i,t} = \pi_t^h$. The noise $\epsilon_{i,t}$ (*iid* across agents) represents imperfect knowledge of strategy h utility. This means that agents choose the best performing strategy with some probability. In the context of technological innovation of this model, the noise represents the uncertainty of the innovation process. If the noise $\epsilon_{i,t}$ has a double exponential distribution, in the limit of an infinite number N of agents, this probability has a logit (or 'Gibbs') distribution (Hommes, 2006). Consequently, innovators at time t are distributed as:

$$n_t = \frac{e^{\beta \pi_{t-1}^{INN}}}{e^{\beta \pi_{t-1}^{INN}} + e^{\beta \pi_{t-1}^{IM}}} = \frac{1}{1 + e^{-\beta \Delta \pi_{t-1}}}, \tag{4.5}$$

with $\Delta \pi_t \equiv \pi_t^{INN} - \pi_t^{IM}$. The larger the difference of profits, the more firms become innovators. The intensity of choice β is inversely proportional to the variance of the utility noise, and measures the ability of firms to choose the best strategy. For $\beta = 0$ agents split equally among the different types. On the other hand, $\beta = \infty$ represents the rational limit where all agents choose the optimal strategy. This setup recalls the quantal response game of McKelvey and Palfrey (1995). The difference is that choices here are based on past experience, without anticipating other agents' action.

In a first specification of the model we ignore the mutual effects of technological advance and agents decisions, focusing on the relative effects of innovators and imitators interplay. We assume that innovation is like buying a shortcut which results in lower production costs in one period. A similar assumption is in Aghion et al. (2005), where profits depend only on the gap between leading and laggard firms, and not on the absolute level of technology. Section 4.4 relaxes this hypothesis, and considers technological progress.

The next step is to model the competitive advantage of innovators, which boils down to specifying the production costs of innovators and imitators. Assume TFP's of innovators and imitators do not depend on time, and $R\&D$ expenditure enhances the TFP of innovators by an exponential factor (Nelson and Winter, 1982; Dosi et al., 2005): $s_t^{INN} = se^{bC}$ and $s_t^{IM} = s$, where $b > 0$ represents the benefits of the innovation investment. Marginal production costs are: $c'(q) = \frac{q}{s}$ for imitators and $c'(q) = \frac{q}{se^{bC}}$ for innovators. Solving the market equilibrium (4.3) for p_t we get

$$p_t = \frac{a}{d + se^{bC} n_t + s(1 - n_t)} \equiv \hat{f}(n_t), \tag{4.6}$$

where fractions depend on last period price and are given by (4.5). The function $\hat{f}$ is decreasing in n, because $e^{bC} > 1$, which means that *an increase in the number of innovators drives down the price*. When everybody innovates the price reaches its minimum value $p^*_{INN} = a/(d + se^{bC})$. On the other hand, the maximum market price is $p^*_{IM} = a/(d+s)$, when there are only imitators,[4] as illustrated in Fig. 4.1. The more innovators, the steeper

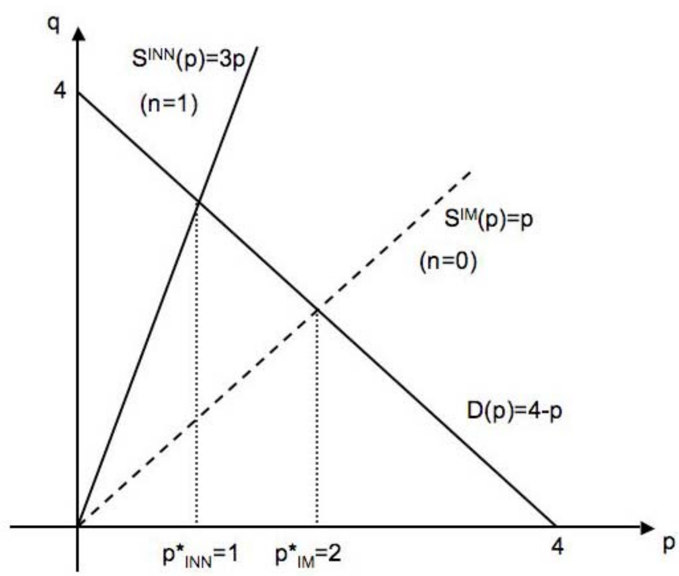

Figure 4.1: Demand and supply curves with $D(p) = 4 - p$, $s^{INN} = 3$ and $s^{IM} = 1$. This gives $S(p) = n^{INN} 3p + n^{IM} p$ $p = 2/(1 + n^{INN})$, with $p^*_{INN} = 1$ and $p^*_{IM} = 2$.

[4] We can think of this limit as a situation with only one innovator: If $N \gg 1$ we have $n \simeq 0$.

is the aggregate supply curve and the lower is the price. Using (4.5) and (4.4) we can reason the other way around:

$$n_t = \frac{1}{1 + e^{-\beta\left[\frac{1}{2}s(e^{bC}-1)p_{t-1}^2 - C\right]}}. \tag{4.7}$$

Eq. (4.7) tells that lower prices bring more imitators: when the price is too low, it is difficult to profit from the innovation advantage, because of the fixed costs of innovation. The opposite is true when the price is too high.

Eq. (4.3) expresses the Walrasian equilibrium of demand and supply. This is a dynamic equilibrium. There may be conditions for a stable equilibrium, where fractions and price remain unchanged through time. In order to find the equilibria of a dynamic system, one studies the flow map of its state variables. Our system in one-dimensional, and either the price p_t or the innovators fraction n_t can be used. By substituting Eq. (4.7) into (4.6) we obtain the price map:

$$p_t = \frac{a}{d + s\left\{1 + \frac{(e^{bC}-1)}{1+e^{-\beta\left[\frac{1}{2}s(e^{bC}-1)p_{t-1}^2 - C\right]}}\right\}} \equiv f(p_{t-1}). \tag{4.8}$$

If instead we substitute (4.6) into (4.7), we obtain a map for the fraction of innovators:

$$n_t = \frac{1}{1 + e^{-\beta\left\{\frac{s(e^{bC}-1)a^2}{2\left[d+se^{bC}n_{t-1}+s(1-n_{t-1})\right]^2} - C\right\}}} \equiv g(n_{t-1}). \tag{4.9}$$

The one-dimensionality of the system means that either p_t or n_t is sufficient to determine the state of the system at each time step.

4.2.2 Steady states and stability

An equilibrium is expressed by a value of the price p^* that satisfies Eq. (4.8), i.e. a fixed point $p^* = f(p^*)$. Equivalently, an equilibrium is a fixed point n^* of the map g.

Proposition 4.2.1. *There exists one and only one steady state p^* (and only one n^*).*

The proof of this proposition follows from the fact that the map f is monotonically decreasing. The stability of the equilibrium p^* depends on the intensity of choice β and on the difference of profits between innovators and imitators, $\Delta\pi^* = \frac{1}{2}s(e^{bC}-1)(p^*)^2 - C$:

Proposition 4.2.2. *In the limit $\beta \to 0$, p^* is stable.*

The proof is given in Appendix 4.A. In the opposite case $\beta = \infty$, the stability of the equilibrium depends on the other parameters of the model, namely the fixed costs C, the innovation benefits b, and the actual technology level s. Consider the price $\overline{p}$ where imitators and innovators make the same profit, $\overline{p} = \sqrt{2C/[s(e^{bC}-1)]}$. When β is infinite

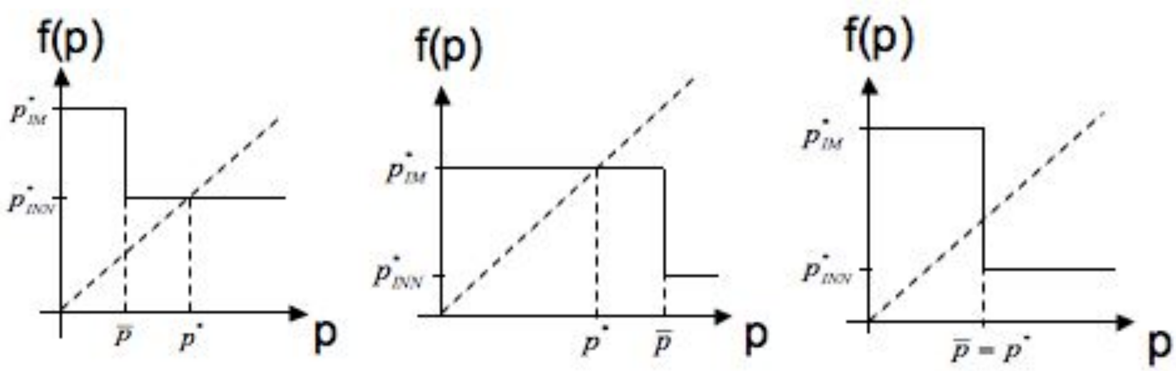

Figure 4.2: Price map $f(p)$ in the case $\beta = \infty$. Left: $p^* > \overline{p}$. Centre: $p^* < \overline{p}$. Right: $p^* = \overline{p}$.

the price map (4.8) is a step function, with a discontinuity at $p = \overline{p}$ (Fig. 4.2):

$$f(p) = \begin{cases} p^*_{IM} = \frac{a}{d+s} & \text{if } p < \overline{p} \\ p^*_{INN} = \frac{a}{d+se^{bC}} & \text{if } p > \overline{p}. \end{cases} \tag{4.10}$$

The following proposition summarizes the stability properties of the steady state for $\beta = +\infty$ in the different cases:

Proposition 4.2.3. *In the limit $\beta = +\infty$, three cases are possible:*

- *$p^* > \overline{p}$ ($\Delta\pi^* > 0$), then $p^* = p^*_{INN}$ is stable and all firms are innovators.*

- *$p^* < \overline{p}$ ($\Delta\pi^* < 0$), then $p^* = p^*_{IM}$ is stable and all firms are imitators.*

- *$p^* = \overline{p}$ ($\Delta\pi^* = 0$), then p^* is unstable, and we have a stable 2-cycle, with oscillations between all firms being innovators and all firms being imitators.*

For finite values of β different situations may occur. In general the equilibrium is locally stable whenever the map (4.8) is such that $-1 < f'(p^*) < 0$. The first derivative f' at the equilibrium price can be expressed as follows (Appendix 4.A):

$$f'(p^*) = -\frac{1}{a}\beta s^2(e^{bC} - 1)^2(p^*)^3 n^*(1 - n^*). \tag{4.11}$$

A large value of p^* and a balanced population ($n^* \sim 1 - n^*$) tend to be associated with an unstable equilibrium. Some sufficient conditions for stability are summarized in the following proposition:

Proposition 4.2.4. *When β is finite (bounded rationality), the equilibrium p^* is stable if at least one of the following is true:*

- *$\beta \simeq 0$ (low intensity of choice)*

- *$C \simeq 0$ (cheap innovation)*

- *$b \simeq 0$ (low benefits from innovation)*

- *$s \simeq 0$ (inelastic supply)*

Since $0 \leq p^* \leq \frac{a}{d}$ is bounded, these stability results follow directly from (4.11).

4.2.3 Period doubling and period halving bifurcations

Now we look at cases where the equilibrium is not stable. If $|f'(p^*)| > 1$ the market cannot attain a stable equilibrium. Moreover, Since the map f is decreasing and bounded, when the equilibrium is unstable a (stable) 2-cycle occurs. Hence a *period doubling bifurcation* may occur when increasing one of the parameters β, b, C or s.

The intuition for cyclical dynamics is as follows. Innovation drives down the price, and as a consequence at some point the profits from innovation become too low (even negative) due to the fixed costs C, so that imitation becomes preferable. Agents start switching to imitative behaviour then, and the price goes up. An increasing price boosts innovators' profits more than imitators', because of larger TFP. When innovators profits become largest, agents switch back to innovation again, and the story repeats. This cyclical behaviour reflects a "minority game" dynamics: a strategy adopted by the minority is more appealing. Stated differently: innovation works better in a market dominated by imitators, while imitation is more profitable in an environment dominated by innovators. Hence, there is a negative feedback from strategy adoption. Such a negative feedback mechanism resembles the dynamic counterpart of the inverted-U relationship between competition and innovation studied in Aghion et al. (2005): a fall of the price means stronger competition and it is associated with a surge in innovation, but at the same time it creates incentives for imitation, and innovation slows down.

In general innovators and imitators coexist in this model, due to the finite value of β. Whether one or the other type prevails, depends on innovation benefits compared to its cost. Nevertheless, innovation benefits are always enough to compensate some innovator for investing in $R\&D$. This case is similar to Grossman and Stiglitz (1976) heterogeneous equilibrium of informed and uninformed agents. In our model the role of information is replaced by the degree of rationality of agents, measured by the intensity of choice β. In Fig. 4.3 we report two examples of time series of the innovators fraction n. On the left we have a case where the market converges to a stable equilibrium $n^* \simeq 0.57$. On the right we have convergence to a stable 2-cycle. These two different dynamics are obtained by slightly increasing the intensity of choice, from $\beta = 2.5$ to $\beta = 3$.

A *bifurcation* is a qualitative change of the dynamics when a parameter varies. This model has a number of different bifurcation scenarios for its parameters β, a, d, s, b and C, involving the transition from stable equilibrium to a stable 2-cycle and viceversa.

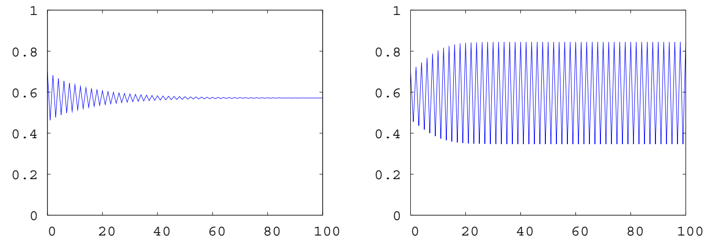

Figure 4.3: Two examples of time series of innovators fraction n. Left: $\beta = 2.5$. Right: $\beta = 3$. In both cases $b = C = d = 1$, $s = 2$, $a = 4$.

There may be a period doubling bifurcation, possibly followed by a period halving bifurcation. An analytic computation of bifurcation values is not feasible. Therefore we turn to numerical simulations of the model, in order to show qualitatively the dynamics. The long run behaviour of the system for different values of a parameter is well represented by a bifurcation diagram[5] In the case of β we obtain the diagram in the left part of Fig.

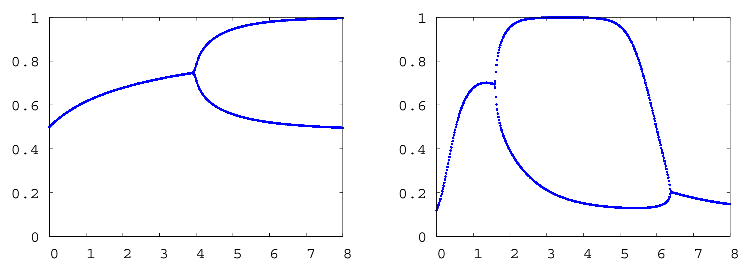

Figure 4.4: Long run dynamics of n. Left: bifurcation diagram of the intensity of choice β (horizontal axis) with $b = 1$. Right: bifurcation diagram of the innovation benefit b (horizontal axis) with $\beta = 2$. Here $b = s = C = d = 1$, $a = 4$.

4.4. For $\beta \simeq 2.7$ the steady state loses stability and a stable 2-cycle is created. As β gets larger, the 2-cycle approaches $\{0, 1\}$, meaning that in the case of synchronous updating, when all agents switch immediately to the best performing strategy, the market switches between a state where all firms innovate and a state where all firms imitate.

The right panel of Fig. 4.4 reports a bifurcation diagram of b with a period doubling at $b \simeq 1.7$ and a period halving at $b \simeq 6.3$; only for intermediate b-values a stable 2-cycle arises. This diagram shows a trade-off in setting higher innovation benefits: a larger value of b is not necessarily good for innovators. The effect of b on n is positive for small

[5]The bifurcation diagrams have been obtained with a transient time of 1000 periods, using the software package *E&F Chaos* (Diks et al., 2008).

values of b, but negative for large values. This is due to a double effect of innovation on innovators' profits: a positive direct effect comes from the exponential factor of the improved profitability $s^{INN} = se^{bC}$. A negative indirect effect comes through the price: since innovation drives down the price, this effect is stronger the larger is b. The price effect hampers innovators' profits more than imitators', as we have seen. If the price effect is prevailing, innovators become less frequent as b gets larger.

The joint effect of any two parameters can be analysed by looking at a basin of attraction, such as the ones reported in Fig. 4.5. The dark regions indicate pairs of

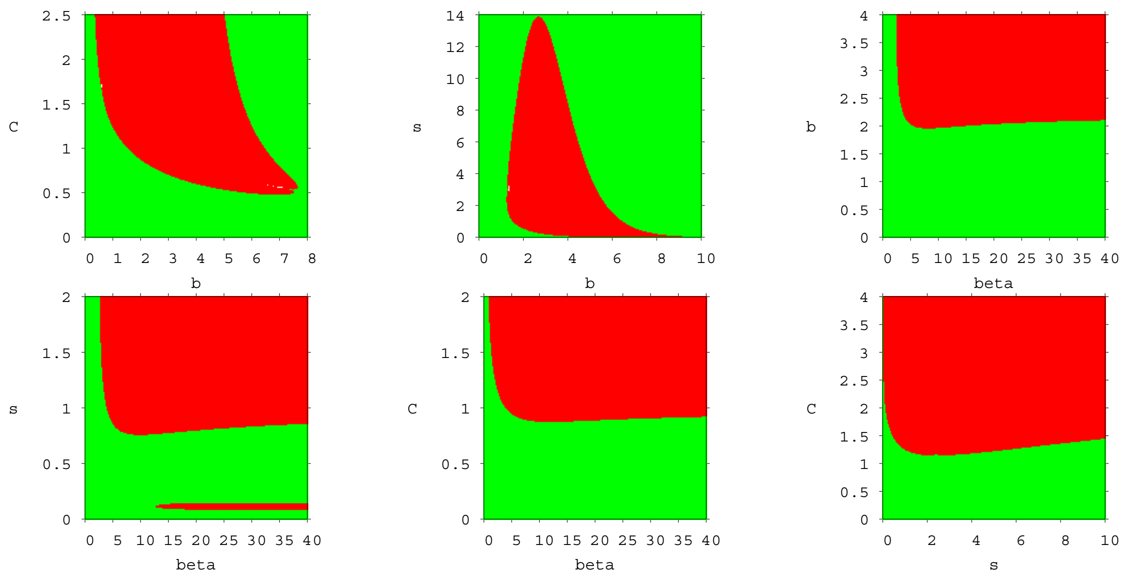

Figure 4.5: Basins of attraction. Light grey colour corresponds to stable equilibrium, dark colour to stable 2-cycle. Top-left: b and C ($\beta = 2$, $a = 4$, $s = d = 1$). Top-centre: b and s ($\beta = 2$, $a = 4$ $C = d = 1$). Top-right: b and β ($s = C = a = d = 1$). Bottom-left: β and s ($a = 4$, $C = b = d = 1$). Bottom-centre: β and C ($a = 4$, $b = s = d = 1$). Bottom-right: s and C ($\beta = 2$, $a = 4$, $b = d = 1$).

parameter values that give rise to a stable 2-cycle, while the light colour refers to stable equilibrium. Boundary curves between different basins of attractions are period doubling (or period halving) bifurcation curves. The basin of β and s, for instance (bottom-left panel), presents three bifurcation values of s for β larger than 13: as s increases, a period doubling bifurcation followed by period halving, followed by another period doubling bifurcation arises. The basins of attraction in Fig. 4.5 show that depending on the value of one parameter, the system may undergo different qualitative changes of its dynamics.

These plots may not tell the whole story though, and one must search the entire range of one parameter in order to know all its bifurcations. For instance Fig. 4.6 shows four bifurcation values for productivity s, if one extends the range up to $s = 15$.

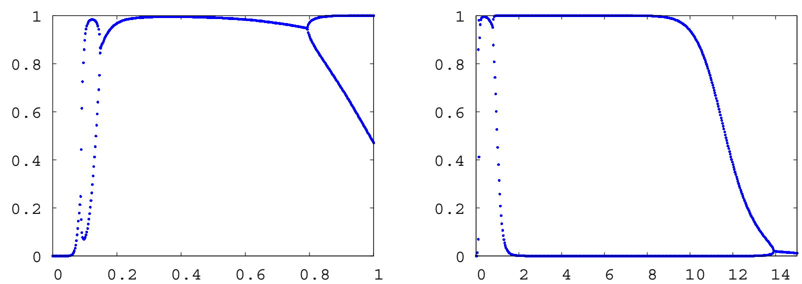

Figure 4.6: Bifurcation diagrams of s (horizontal axis) for n (vertical axis). Left: range $s \in [0, 1]$. Right: range $s \in [0, 15]$. Here $\beta = 20$, $a = 4$, $b = C = d = 1$.

Beside the supply side of the market, the equilibrium and its stability are affected also by the demand. In particular, the parameter a shifts the demand curve, while d rotates it around the intercept with the vertical axis (Fig. 4.1). Relatively speaking, the effect of a is larger. Both Fig. 4.1 and Eq. (4.8) indicate that a positive demand shock that lifts a increases the price. In cases of unstable equilibrium, a positive demand shock leads to a larger average price over cycles.

The intuitive pictures for the case $\beta = \infty$ of Fig. 4.2 help to understand how the price map is affected by a demand shock: If a is increased, the price map shifts up and the gap $p^*_{IM} - p^*_{INN}$ enlarges. The discontinuity point $\overline{p} = \sqrt{\frac{2C}{s(e^{bC}-1)}}$ is unaffected though, because it does not depend on demand parameters. The consequence is that an increase of a shifts the graph from a stable fixed point (Fig. 4.2, middle panel) to an unstable fixed point (Fig. 4.2, right panel), and to stable fixed point again (Fig. 4.2, left panel). Fig. 4.7 reports two bifurcation diagrams of the fraction of innovators with respect to shifts in demand, i.e. the parameter a. In the left panel ($\beta = 2$) we have only stable equilibrium, with a fraction of innovators which increases as a gets larger, in accordance with the fact that the equilibrium price goes down. In the case $\beta = 4$ (right panel), an instability region appears: there is a period doubling bifurcation for $a = a_1 \simeq 2.5$ and

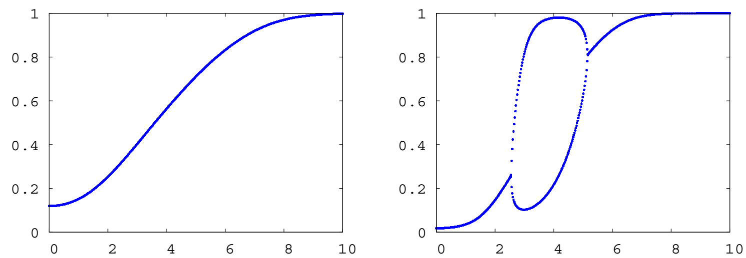

Figure 4.7: Bifurcation diagrams of a (horizontal axis) for the fraction of innovators n (vertical axis). Left: $\beta = 2$. Right: $\beta = 4$. $d = 1$, $b = 1$, $C = 1$, $s = 2$.

a period halving bifurcation for $a = a_2 \simeq 5$. Consequently, the dynamics changes from stable equilibrium to a stable period 2 cycle, and then back to a stable equilibrium.

4.3 Asynchronous updating of strategies

We now consider the more realistic case of asynchronous strategy updating. Synchronous updating assumes that in each time period all firms simultaneously make a decision about which strategy to adopt. In reality firms' strategies show a good degree of persistence (Dosi, 1988). The empirical evidence of persistence in firms' propensity to innovate or not-innovate holds across countries and industrial sectors (Cefis and Orsenigo, 2001). It is therefore more realistic to assume that agents stick to their technology to some extent, and only a fraction $1 - \alpha$ with $\alpha \in [0, 1]$ update their strategy in a given period. The discrete choice model with *asynchronous updating* is given by (Diks and van der Weide, 2005; Hommes et al., 2005):

$$\begin{aligned} n_t &= \alpha n_{t-1} + (1-\alpha)\frac{e^{\beta\pi_{t-1}^{INN}}}{e^{\beta\pi_{t-1}^{INN}} + e^{\beta\pi_{t-1}^{IM}}} \\ &= \alpha n_{t-1} + (1-\alpha) g(n_{t-1}) \equiv \hat{g}(n_{t-1}), \end{aligned} \tag{4.12}$$

where the function g is the map (4.9) of the basic model with synchronous updating. Notice that for $\alpha = 0$ we are back to that model. This system is still one-dimensional. The map $\hat{g}$ in (4.12) is a convex combination of an increasing function, n_{t-1}, and a decreasing function, $g(n_{t-1})$, and therefore can be non-monotonic depending on the value of α (Fig. 4.8). In particular, $\hat{g}$ is decreasing for $\alpha = 0$, it becomes non-monotonic for

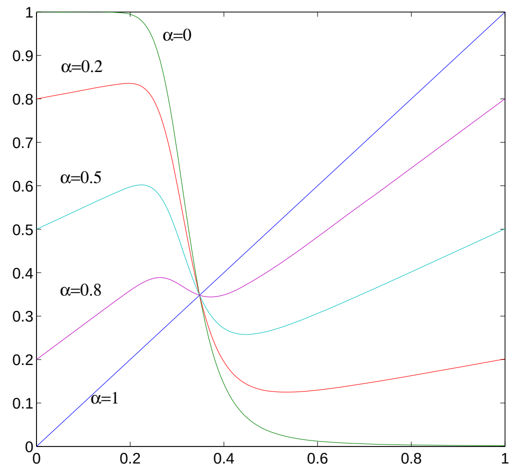

Figure 4.8: Graph of the map of innovators fraction $n_t = \hat{g}(n_{t-1})$ with asynchronous updating for different values of the weight α ($\beta = 10$, $C = 2$, $b = 1$, $s = 2$, $a = 4$ $d = 1$).

intermediate values of α and it is increasing for α close to 1. The non-monotonicity of the map $\hat{g}$ can lead to more complicated dynamics when the steady state is unstable. Indeed, chaotic dynamics can arise, as illustrated in the bifurcation diagram of Fig. 4.9. When β and C are relatively small (left panel), the system converges either to a 2-cycle or to a stable equilibrium. A higher cost of innovation C destabilizes the dynamics, with

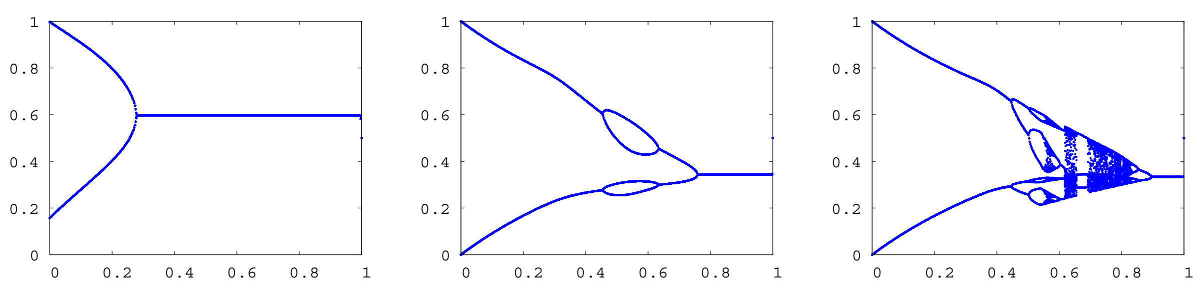

Figure 4.9: Bifurcation diagrams of the updating fraction α (horizontal axis) for n (vertical axis). Left: $\beta = 5$ and $C = 1$. Centre: $\beta = 5$ and $C = 2$. Right: $\beta = 12$ and $C = 2$. ($b = 1$, $s = 2$, $a = 4$, $d = 1$).

cycles of period 4 (middle panel). By increasing also β we obtain irregular dynamics for intermediate values of α (right panel). These examples indicate that in general, when most agents stick to their strategy (large α), the industry converges to a stable equilibrium.

When a small fraction of agents update their strategy instead (low α), the dynamics converge to a period 2-cycle. Intermediate values of the updating fraction α may lead to a period doubling bifurcation route to irregular chaotic dynamics. Beside irregular dynamics, we also observe how the variability of n decreases with larger α. This means that asynchronous updating is quantitatively stabilizing, but qualitatively destabilizing: it dampens the amplitude of the cycles, but at the same time the cycles become unstable and chaos occurs. This global dynamics is similar to the cobweb model with adaptive expectations of Hommes (1994), with the asynchronous updating fraction α playing the role of the adaptive expectations weight factor. The following proposition shows the occurrence of chaos in the case of asynchronous strategy updating.

Proposition 4.3.1. *Let $\hat{g}$ be the map (4.12). If β and C are sufficiently large, there exist values α_1, α_2 and α_3 with $0 < \alpha_1 < \alpha_2 < \alpha_3 < 1$ such that the following holds true:*

- *(A1) $\hat{g}$ has a stable period 2 orbit for $\alpha \in [0, \alpha_1)$,*
- *(A2) the map $\hat{g}$ is chaotic in some interval $[\alpha_2 - \epsilon, \alpha_2 + \epsilon]$,*
- *(A3) $\hat{g}$ has a stable equilibrium for $\alpha \in (\alpha_3, 1]$.*

A proof is given in Appendix 4.B. The value of β is critical in dictating the effect of asynchronous updating. In Fig. 4.10, we compare the bifurcation diagrams of innovation benefits b with low β (left) and large β (right), for $\alpha = 0.5$ i.e. when half of the agents update every period. If β is low, there is only a stable equilibrium in the range of b considered. If β is large, 3-cycles and chaotic dynamics appear. Notice that dealing with a one-dimensional system (4.12), the presence of orbits of period 3 is a sufficient condition for chaotic behaviour (Li and Yorke, 1975).

Another aspect of the irregular behaviour introduced by asynchronous updating is the persistence of strategies. The time series of Fig. 4.11 is an example where oscillations of the fraction of innovators is strongly reduced in several periods of time, in a very irregular

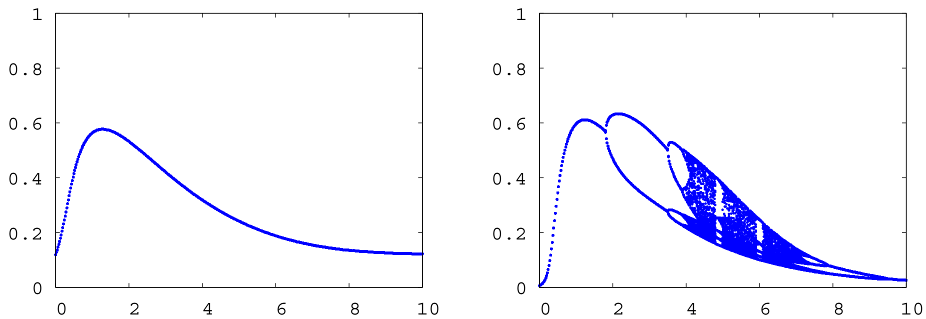

Figure 4.10: Bifurcation diagrams of the innovation benefits rate b (horizontal axis) for n (vertical axis) with asynchronous updating ($\alpha = 0.5$). Right: $\beta = 2$. Left: $\beta = 5$. Here $C = 1.1$, $s = 2$, $a = 4$, $d = 1$.

fashion. It is noteworthy that such variability is obtained with only 25% of firms updating strategy in one period ($\alpha = 0.75$).

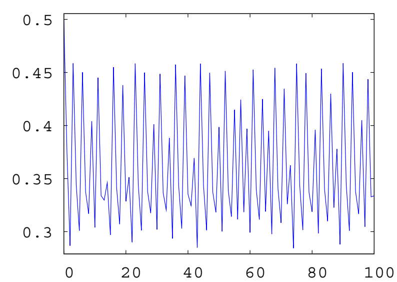

Figure 4.11: Time series of innovators fraction with asynchronous updating $\alpha = 0.75$ ($\beta = 2$, $a = 4$, $b = d = 1$, $s = C = 2$).

This model is very sensitive to the demand, especially when β is large. Consider Fig. 4.2, and assume a is small (middle panel). By increasing a we get to the condition of the right panel, and enter an instability region. With asynchronous updating such instability may be characterized by chaotic dynamics. Increasing a further, we pass to the situation of the left panel eventually, with stable equilibrium again. Fig. 4.12 reports a bifurcation diagram of a. The market converges to a stable equilibrium for small values of a and for large values, as in the case with synchronous updating. In between, we have two regions of values of a where the markets exhibits chaotic dynamics. There is also a region where the market converges to a stable cycle of period 2, and a region where we have a stable period 3 cycle. In order to obtain such a rich dynamic spectrum, a relatively large value of the intensity of choice is required ($\beta = 12$ in this example).

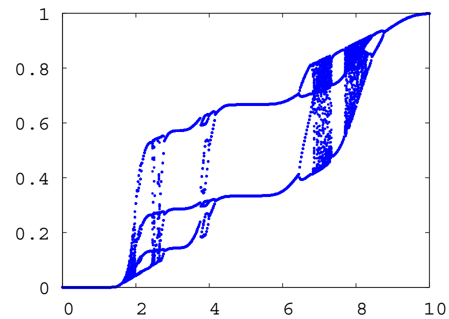

Figure 4.12: Bifurcation diagram of a (horizontal axis) for the innovators fraction n (vertical axis) with asynchronous updating $\alpha = 0.5$. Here $\beta = 12$, $b = 1$, $C = 2$, $s = 2$, $d = 1$.

We conclude this section with a bifurcation diagram of the productivity parameter s (Fig. 4.13). This figure should be compared to Fig. 4.6. As s increases, a period doubling bifurcation route to chaos arises (e.g. for $8 \leq s \leq 10$), followed by a period halving bifurcation route to a stable steady state.

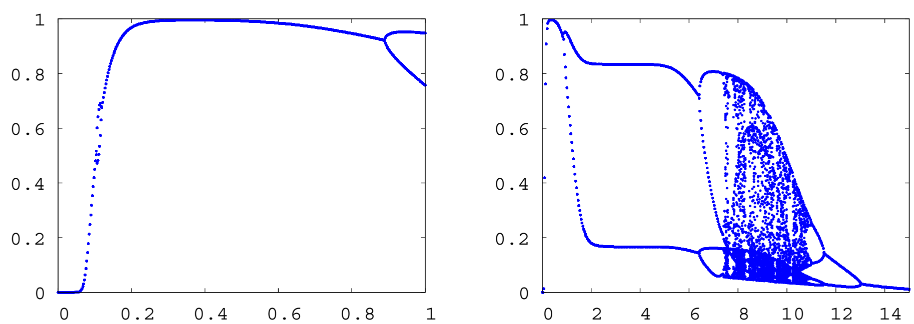

Figure 4.13: Bifurcation diagrams of s (horizontal axis) for n (vertical axis) with asynchronous updating ($\alpha = 0.2$). Left: range $s \in [0, 1]$. Right: range $s \in [0, 15]$. Here $\beta = 20$, $a = 4$, $b = C = d = 1$.

4.4 Technological progress

In the previous sections we have studied the dynamics of the interplay between innovation and imitation assuming that this does not interfere with the underlying technological progress. In some sense, this means that firms start from scratch in each period and then decide to innovate or not. In this section we study the mutual effects of *technological progress* and strategy switching between innovation and imitation.

4.4.1 The model

The fundamental assumption of this extension of the model is that innovation cumulates: in each period the achievements of innovators contribute to a technological frontier. The frontier consists of all past innovations, and has the connotation of a learning curve.[6] Imitators have access to the technological frontier, while innovators expand it, obtaining a better production technology due to their innovation investment. We introduce a cumulation rate γ for innovations, and also a depreciation rate δ. Based on this we define the technological frontier:

$$s(t) = se^{\sum_{i=1}^{t-1}[\gamma n_i - \delta]}. \tag{4.13}$$

This technological frontier grows over time exponentially by a time-varying factor $\gamma n_i - \delta$, where n_i is the fraction of innovators in period i. Imitators exploit the frontier technology, while innovators build on it, getting a competitive advantage. Consequently the two productivity levels are as follows:

$$s_t^{INN} = s(t)e^{bC}, \qquad s_t^{IM} = s(t). \tag{4.14}$$

This model is related to the "Schumpeterian" version of the innovation-based endogenous growth theory of Grossman and Helpman (1991) and Aghion and Howitt (1992). Beside the fact that we focus on the market dynamics of supply and demand, while they focus on the production function, two other differences of our model must be stressed. The first is the heterogeneity of TFP, which stems from the heterogeneity of firms. The second is that we propose a behavioural explanation of endogenous technological growth.

Formally nothing changes with respect to the basic model: innovators increase TFP by the factor e^{bC}, after investing C in innovation. This advantage lasts one period, because

[6] Learning curves are usually proposed in two versions, namely a relationship between marginal cost and output quantity (Argote and Epple, 1990), or a relationship between marginal cost and time like Moore's law (Koh and Magee, 2006). The latter version is basically the technological frontier of our model, since the price reflects marginal costs.

it becomes publicly available afterwards. The difference with the basic model is that *innovation now exhibits endogenous growth* and cumulates at a rate γ, resulting in an advancing technological frontier. An agent can innovate today and imitate tomorrow, without loosing the benefits from its previous innovation, although everybody else can use it as well. It has to be noted that a dynamic technological frontier $s(t)$ makes the technological gap $\Delta s(t) = s(t)(e^{bC} - 1)$ change over time. In particular, technological progress causes Δs to enlarge.

The rate γ measures two effects, namely cumulativeness of knowledge and spillovers of technological innovations. The implicit assumptions here are that innovation always cumulates and spills over at the same rates, in line with the assumption of our model that innovation benefits b and costs C are the same in every period.

At first it is instructive to consider synchronous updating ($\alpha = 0$), with the fraction of innovators given by the logistic distribution (4.7) of the basic model. Once we substitute the static parameter s by the technological frontier $s(t)$ we have:

$$n_t = \frac{1}{1 + e^{-\beta\left[\frac{1}{2}s(t)(e^{bC}-1)p_{t-1}^2 - C\right]}}. \tag{4.15}$$

Similarly we obtain the new expression of the price (cfr. Eq. 4.6):

$$p_t = \frac{a}{d + s(t)e^{bC}n_t + s(t)(1 - n_t)}. \tag{4.16}$$

By substituting (4.15) into (4.16), or the other way around, we obtain the price map $p_t = F(p_{t-1}; s(t))$, and the fraction map $n_t = G(n_{t-1}; s(t))$, respectively. These maps are identical to (4.8) and (4.9), after exchanging the static parameter s with the technological frontier $s(t)$. The technological frontier works as a slowly changing parameter that spans the technology dimension of the model. Fig. 4.14 illustrates how the graph of the map G evolves because of technological change $s(t)$. For small values $0 \leq s(t) \leq 0.5$ the map has a stable steady state whose value increases as $s(t)$ increases. For intermediate values

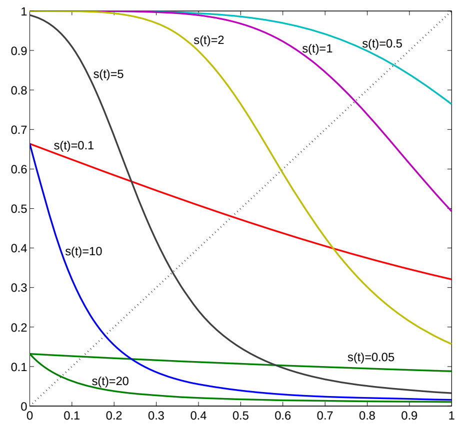

Figure 4.14: Graph of the map of the innovators fraction $G(x; s(t))$ for different values of the technological frontier $s(t)$. Here $\beta = 5$, $C = 1$, $b = 1$, $a = 4$ and $d = 1$.

$1 \leq s(t) \leq 5$ the steady state value decreases and it becomes unstable, leading to stable 2-cycle. For larger values $s(t) > 15$ the steady state becomes stable again. Hence, because of technological growth, the dynamics may go through different phases, from stable steady state to 2-cycle to stable steady state again.

Before turning to numerical simulations of the model there are some theoretical results that one can anticipate. Let us write the technological frontier as follows:

$$s(t) = se^{-\delta(t-1)} e^{\gamma \sum_{i=1}^{t-1} n_i}. \tag{4.17}$$

The first factor is an exponential depreciation at rate δ, while the second factor is a positive exponential endogenous growth due to technological innovation, where the argument depends on the amount of past innovations, that is the sum of the fraction of innovators in previous periods. One does not know these fractions *a priori*, and one does not know whether the series converges or diverges, because the technological frontier feeds back into agents choice. The growth rate of $s(t)$ has an upper and a lower bound:

$$-\delta \leq \gamma n_i - \delta \leq \gamma - \delta.$$

Depending on the value of the upper bound we have the following proposition:

Proposition 4.4.1. *The following cases are possible for the long run dynamics:*

1. *for* $\gamma < \delta$*:* $s(t) \to 0$, $p_t \to \frac{a}{d}$ *and* $q_t = a - dp_t \to 0$ *(market breakdown).*

2. *for* $\gamma = \delta$*:* $s(t) = se^{-\gamma \sum_1^{t-1}(1-n_i)}$ *and we have two subcases:*

   (a) *if* $\sum_1^{\infty}(1-n_i) \to \infty$*, then* $s(t) \to 0$, $p_t \to \frac{a}{d}$ *and* $q_t \to 0$ *(market breakdown).*

   (b) *if* $\sum_1^{\infty}(1-n_i) \to \Sigma < \infty$*, then* $s(t) \to se^{-\gamma\Sigma}$ *and* $p \to p^*$ *stable or unstable, with* $0 < p^* < \frac{a}{d}$ *(balanced technological change).*

3. *for* $\gamma > \delta$ *we have three subcases:*

   (a) *if* $\gamma \sum_{i=1}^{t-1} n_i < \delta t$*, then* $s(t) \to 0$, $p_t \to \frac{a}{d}$ *and* $q_t \to 0$ *(market breakdown).*

   (b) *if* $\gamma \sum_{i=1}^{t-1} n_i \sim \delta t$*, then there is* $0 < \Sigma < +\infty$ *such that* $s(t) \to se^{-\gamma\Sigma}$ *and* $p \to p^*$ *stable or unstable, with* $0 < p^* < \frac{a}{d}$ *(balanced technological change).*

   (c) *if* $\gamma \sum_{i=1}^{t-1} n_i > \delta t$*, then* $s(t) \to \infty$, $p_t \to 0$ *and* $q_t \to a$ *(technological progress).*

In case (1) the technological frontier $s(t) \to 0$, the price reaches its maximum value a/d (Eq. 4.16), with zero supplied quantity (see Fig. 4.1). Case (2) depends on whether we have enough imitators during the preliminary phase (case 2a) or whether innovators dominate the market (case 2b). In case (2a) a market breakdown occurs as in case (1). In case (2b) the sequence of innovators n_t converges to 1, which is a necessary condition for the series $\sum(1-n_t)$ to converge to a finite value Σ, and consequently also the technological frontier settles to a finite positive value $s_0 e^{-\gamma\Sigma}$. This value determines whether the limiting map (4.16) has a stable or unstable equilibrium. The intuition is that in this case technological progress is exactly enough to compensate for depreciation. Case (3), with the technology growth rate γ larger than the depreciation rate δ, is the most realistic, but also the most uncertain, because anything can happen. If the process of knowledge accumulation is not strong enough to compensate technological depreciation, a market breakdown occurs (case $3a$). This is the case if γ is only slightly larger than δ. If instead knowledge accumulation goes at a rate similar to δt (case $3b$), we are in a situation similar to case ($2b$), where depreciation and technological progress offset each other. In case ($3c$) technological accumulation is stronger than depreciation, and price and marginal cost $c'(q) = p/s$ fall down to zero. This case occurs when $\gamma \gg \delta$, for instance, and on average there are enough innovators in the history of the market. Notice that the three different cases ($3a$), ($3b$) and ($3c$) may all occur with a divergent series $\sum_{i=1}^{t-1} n_i$ of innovators: what matters is the relative value of accumulated innovation compared to the linear depreciation δt.

The cases of balanced technological change ($2b$ and $3b$) can only present stable equilibrium or 2-cycles, because also in the limit the map (4.16) is monotonic decreasing. In all cases of stable equilibrium we can simplify Proposition 4.4.1 in the following way:

Proposition 4.4.2. *Assume that the model converges to a stable equilibrium, with $n_i \to n^*$. Consider the quantity $\nu^* \equiv \gamma n^* - \delta$. Three cases are possible:*

(i) $\nu^* < 0$, then $s(t) \sim se^{\nu^*(t-1)} \to 0$, $p_t \to \frac{a}{d}$ and $q_t \to 0$ (market breakdown).

(ii) $\nu^* = 0$, then $s(t) \to se^{-\Sigma}$, $p_t \to p^*$, with $0 < p^* < \frac{a}{d}$ (balanced technological change).

(iii) $\nu^* > 0$, then $s(t) \sim se^{\nu^*(t-1)} \to \infty$, $p_t \to 0$ (technological progress).

Case (i) can occur in all three cases of Proposition 4.4.1, and in particular it coincides with cases (1), ($2a$) and ($3a$). Case (ii) implies that the equilibrium value of the innovators fraction is $n^* = \frac{\delta}{\gamma} \leq 1$, so it may occur in cases (2) and (3) of Proposition 4.4.1. Actually case (ii) falls in (but does not coincide with) cases ($2b$) and ($3b$) of Proposition 4.4.1. Finally, case (iii) implies $\gamma > \delta$ and implies case ($3c$) of Proposition 4.4.1.

Market breakdown concerns industries that have ended their activities, because technological progress did not compensate for depreciation. Balanced technological change explains cases where real technological progress is missing and innovation sounds more like re-novation. This case resembles the situation of Schumpeterian rents, where innovation is just enough to award to a firm its presence in the market, but competition is absent and the price misses to fall beyond a certain value. Technological progress extinguishes entrepreneurial rents with a falling price, that follows after the unlimited reduction of production cost. This is the case of *learning curves*, that will be studied with particular attention in the last part of this section.

The relationship between n_t and $s(t)$ has an economic interpretation: it is the relationship between $R\&D$ intensity and innovation in an industry (Nelson, 1988). In our model such relationship is bi-directional, because of the endogeneity of technological progress.

4.4.2 Numerical simulations

We run some simulations of the model in order to illustrate the cases described above. First we consider $\gamma \leq \delta$, with $\gamma = \delta = 0.1$ as a typical example (Fig. 4.15). After some

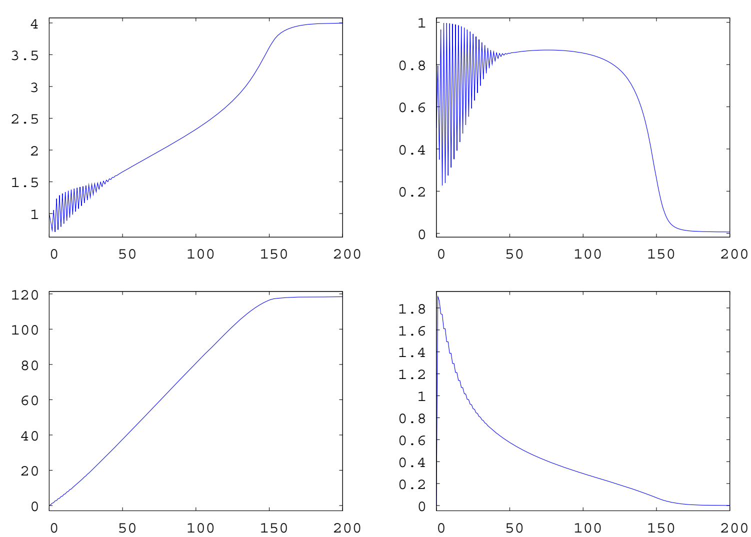

Figure 4.15: Model with technological progress, with $\gamma = \delta = 0.1$. Top-left: price p_t. Top-right: fraction of innovators n_t. Bottom-left: series $\sum_{i=1}^{t-1} n_i$. Bottom-right: technological frontier $s(t)$. Here $\beta = 5$, $b = C = d = 1$, $a = 4$, $s = 2$.

initial fluctuations, the price goes up to the highest level $\frac{a}{d}$. Innovators disappear after the transitory oscillatory phase, $n^* \to 0$. As a consequence, the series $\sum_{i=1}^{t-1} n_i$ converges, while the technological frontier goes down to zero. This is an example of market breakdown, that is case (1) or $(2a)$ of Proposition 4.4.1 and case (i) of Proposition 4.4.2. A cumulation rate $\gamma \leq \delta$ is not sufficient to counter technological depreciation. By rising γ and δ simultaneously one gets a faster convergence to the market breakdown ($p \to \frac{a}{d}$).

With $\gamma > \delta$ the spectrum of possible dynamics enlarges considerably. In the example of Fig. 4.16 we set $\gamma = 0.2$ and $\delta = 0.1$. The market converges to a stable 2-cycle, where

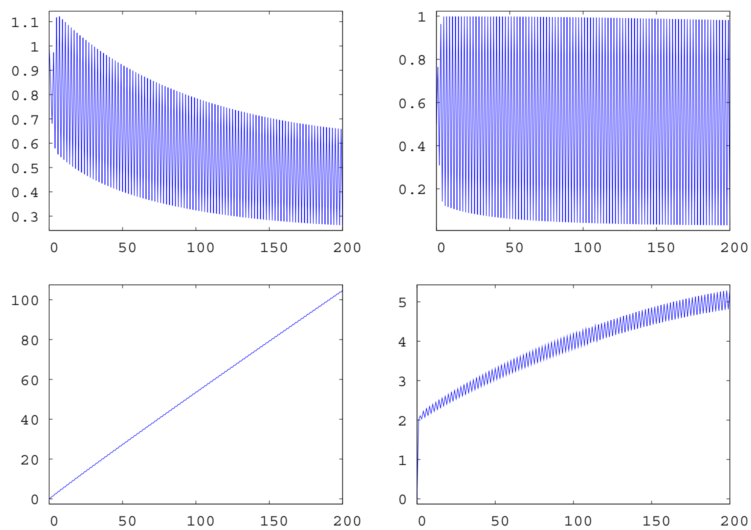

Figure 4.16: Model with technological progress, with $\gamma = 0.2$, $\delta = 0.1$. Top-left: price p_t. Top-right: fraction of innovators n_t. Bottom-left: series $\sum_{i=1}^{t-1} n_i$. Bottom-right: technological frontier $s(t)$. Here $\beta = 5$, $b = C = d = 1$, $a = 4$, $s = 2$.

the price oscillates between a high and a low value. Innovation and imitation alternate as the dominant strategy. The series $\sum n_i$ diverges in a step-wise fashion. The technological frontier goes up steadily, but it exhibits up and down oscillation along the growth path. This example fits into case 3b of Proposition 4.4.1 and in particular it presents an unstable equilibrium with cyclical dynamics.

The dynamics in the long run strongly depends on the intensity of choice β. If we lower the intensity of choice in the example of Fig. 4.16 to $\beta = 2$, for instance, after initial oscillations, the price converges to an equilibrium value $p^* \simeq 0.6$ (Fig. 4.17). We are still in case (ii) of Proposition 4.4.2, but this time p^* is stable. Agents fractions converge to $n^* = \frac{\delta}{\gamma} = 0.5$, and $\nu^* = \gamma n^* - \delta = 0$ as predicted by Proposition 4.4.2. Accordingly, the technological frontier converges to a finite positive value $se^{-\Sigma}$. This is due to the fact that the argument of the series is converging to $\nu^* = \gamma n^* - \delta = 0$, since $n^* = \frac{\delta}{\gamma}$, as predicted by Proposition 4.4.2.

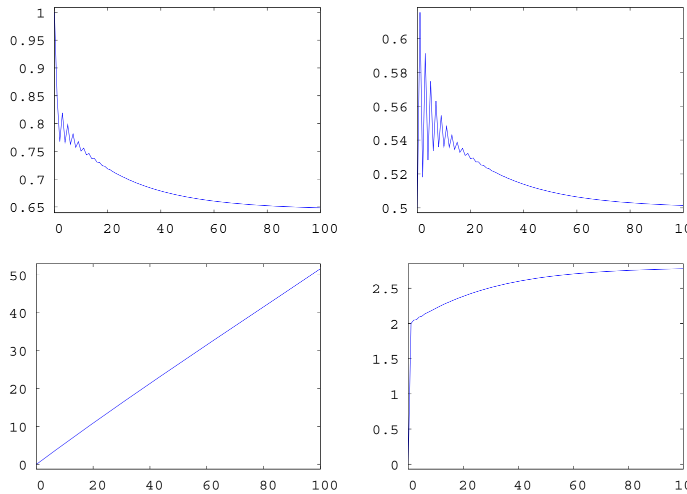

Figure 4.17: Model with technological progress. $\gamma = 0.2$, $\delta = 0.1$ and $\beta = 2$. Top-left: price p_t. Top-right: innovators fraction n_t. Bottom-left: series $\sum_{i=1}^{t-1} n_i$. Bottom-right: technological frontier $s(t)$. Here $b = C = d = 1$, $a = 4$, $s = 2$.

In the example of Fig. 4.18 we have $\gamma = 0.5$ and $\delta = 0.05$. The very low depreciation rate allows the technological frontier to grow exponentially, sending the price down to zero. This is a case of technological progress, i.e. case (iii) of Proposition 4.4.2. The fraction of innovators converges to $n^* \simeq 0.12$, which means that $\nu^* \simeq 0.5 \cdot 0.12 - 0.05 = 0.01$. Although the market converges to a scenario with more imitators, depreciation is so

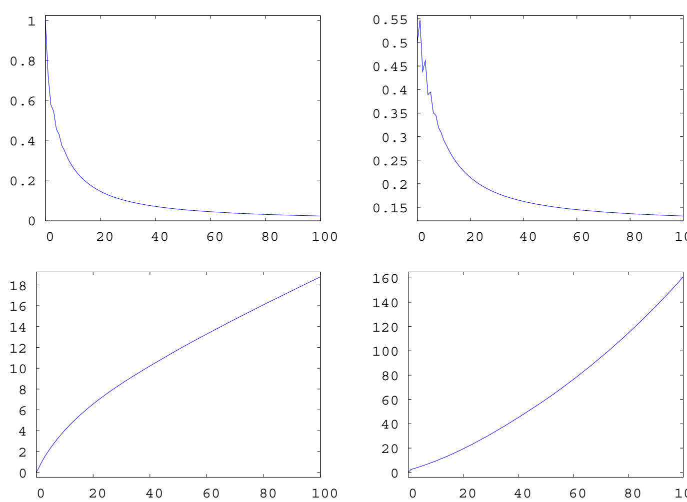

Figure 4.18: Model with technological progress. $\gamma = 0.5$, $\delta = 0.05$. Top-left: price p_t. Top-right: innovators fraction n_t. Bottom-left: series $\sum_{i=1}^{t-1} n_i$. Bottom-right: technological frontier $s(t)$. $\beta = 2$, $b = C = d = 1$, $a = 4$, $s = 2$.

low that only a few innovators are enough to keep technological progress at a positive net rate, with a consequent technological advance. This example is relevant for hi-tech sectors like microelectronics, which present a learning curve with exponential progress and are characterized by a few innovators and many imitators.

The last two examples (Fig. 4.17 and Fig. 4.18) introduce the question whether more competition is good or bad for innovation (cf. Aghion et al. (2005)). If one measures competition by the number of innovating firms (remember that the total number of firms is fixed and large, by assumption), than competition seems to go along with less innovation, because the technological frontier advances at a slower pace in the example of Fig. 4.17 than in the example of Fig. 4.18. But if the price level is taken as a measure of competition instead, meaning that a lower price characterizes more competitive industries, then the opposite is true, with more competition associated to more innovation as in the example of Fig. 4.18. The point is that when cumulativeness of innovation is high and spillovers are strong ($\gamma \gg \delta$) the strong fall in price lowers the number of innovating firms, because a lower price reduces profits from innovation, and this effect overcomes the direct effect of a larger productivity (larger $s(t)$). This is the double effect of innovation that we have encountered already in the basic model of Section 4.2. We observe that higher cumulativeness (larger γ) rewards more innovators, but it leads to more concentrated industries, because selection is tougher, in line with Dosi (1988).

4.4.3 Path-dependence and learning curves

A stylised fact of technological change is path-dependence. Our model with technological progress presents this feature. Cumulative innovation is fundamental in this respect: the technological frontier advances depending on firms choices in every period, and its level drives firms' choices on its turn. This mutual relationship determines the value of productivity $s(t)$ in the long run. The long run dynamics of the model depends on the limit map $G(x; s^{\infty})$, where $s^{\infty} = \lim_{t\to\infty} s(t)$. Fig. 4.14 illustrates how the map G

depends on $s(t)$. If $G(x; s^{\infty})$ is steep enough at the steady state, the long run dynamics will be a 2-cycle, as it is the case with the example of Fig. 4.16. Otherwise the model will converge to a stable equilibrium (as in both Fig. 4.15 and Fig. 4.18). In Fig. 4.19 we show path-dependence with respect to the initial value of productivity s, with the same initial condition ($p_0 = 1$). In two cases (low s) the model converges to $p = a = 4$, which means

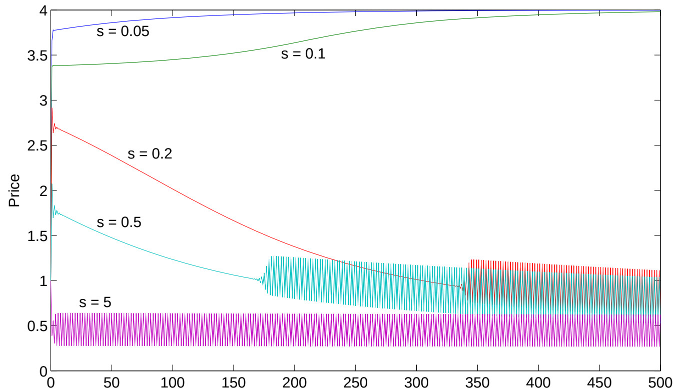

Figure 4.19: Simulated time series of p_t with five different values of the productivity s (technological frontier at time $t = 0$). Here $\beta = 5$, $a = 4$, $d = b = C = 1$, $\alpha = 0$, $\gamma = 0.02$, $\delta = 0.01$.

market breakdown. In three cases (larger s) we end up with a 2-cycle. The critical value of s that separates the two different dynamics lies between $s = 0.1$ and $s = 0.2$. With the example of Fig. 4.20 we set the model in this critical condition with $s = 0.103$, and simulate the time series of innovators fraction with two different initial values: $n_0 = 0.1$ and $n_0 = 0.9$. In the first case ($n_0 = 0.1$) the price converges to the market breakdown

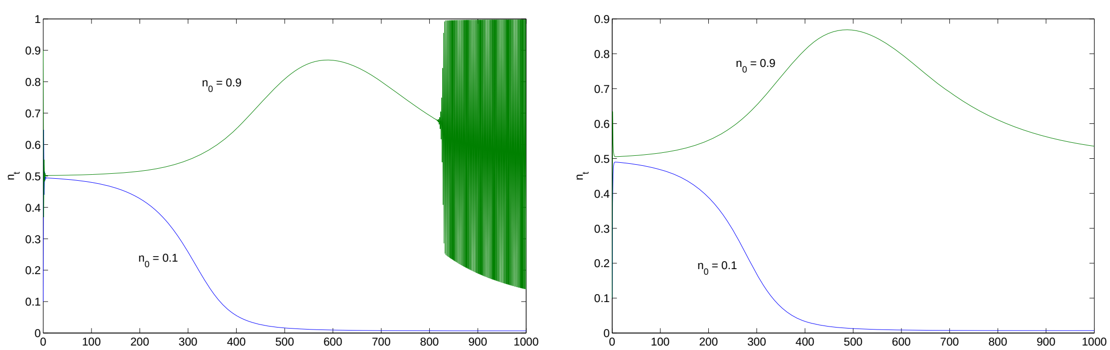

Figure 4.20: Simulated time series n_t with $s = 0.103$ and with two different initial conditions. Left: $\alpha = 0$. Right: $\alpha = 0.5$. Here $\beta = 5$, $a = 4$, $d = b = C = 1$, $\gamma = 0.02$, $\delta = 0.01$.

value $p = a = 4$. In the second case ($n_0 = 0.9$) we end up with a 2-cycle. The fact that in the same setting different initial conditions lead to completely different trajectories and even to different dynamics qualitatively, is an illustration of path-dependence.

We now combine asynchronous strategy updating to technological progress. The right panel of Fig. 4.20 reports the same example of the left panel but with $\alpha = 0.5$: instead of a 2-cycle for $n_0 = 0.9$ the price converges to a different stable steady state, $p \simeq 0.65$, which means asynchronous updating has a stabilizing effect here. This is by no means always the case. In the example of Fig. 4.21 the same fraction of updating firms ($\alpha = 0.5$) produces an irregular dynamics which persists in the long run, as the right panel illustrates: in

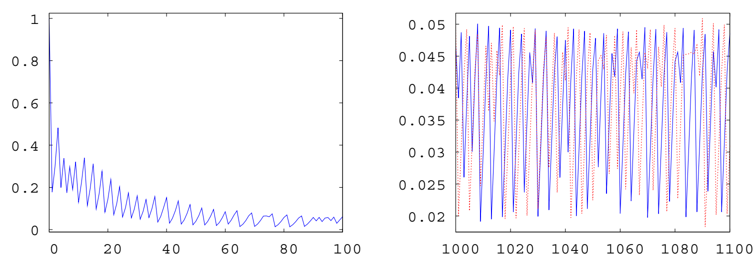

Figure 4.21: Simulated time series of p_t with asynchronous updating: Left: first 100 periods. Right: after 1000 periods with two different initial conditions: $n_0 = 0.1$ and $n_0 = 0.9$ ($\alpha = 0.5$, $\gamma = 0.2$, $\delta = 0.01$, $\beta = 10$, $b = 2$, $C = 1.5$, $a = 4$, $d = 1$, $s = 1$).

this case, different initial conditions do not lead to different long run outcomes. However, different initial conditions give different values of the state variable at a given time t, which is an indication of chaos.

In most cases G converges to a map with regular dynamics, either 2-cycles or stable equilibrium. Nevertheless asynchronous updating is responsible of irregular behaviour in the short run in many cases. Fig. 4.22 reports the time series of the price for three different values of the cumulation rate γ: innovation drives down the price p_t in irregular fashion. The time series of price with asynchronous strategy updating resembles empirical learning curves. Price is the same as marginal production cost in our model, and it falls as a consequence of technological progress. Fig. 4.23 reports the empirical time series of the price index for the automobile tire industry in the US, from Jovanovic and MacDonald

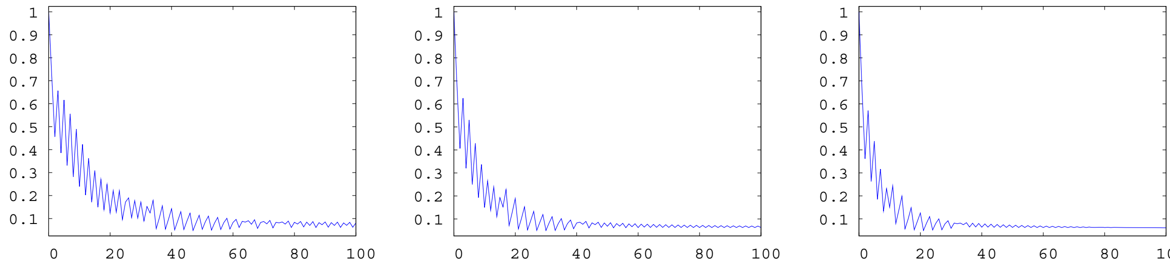

Figure 4.22: Simulated time series of p_t with asynchronous updating ($\alpha = 0.4$). Left: $\gamma = 0.2$. Centre: $\gamma = 0.3$. Right: $\gamma = 0.4$ ($\delta = 0.01$, $\beta = 10$, $b = 2$, $C = 1$, $a = 4$, $d = 1$, $s = 1$).

(1994): the price gradually decreases exhibiting short run fluctuations. The simulated

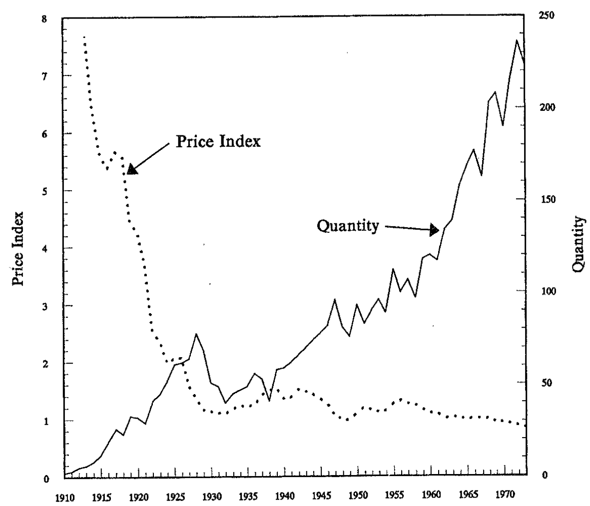

Figure 4.23: Empirical time series of the price index in the US automobile tire industry (Jovanovic and MacDonald, 1994)

time series of our model in Fig. 4.22 present a similar pattern.

Learning curves are another stylised fact of technological change that our model is able to reproduce. In Fig. 4.24 we report the simulated time series of price and aggregate quantity together, in the same conditions of the left panel of Fig. 4.22. In our model there is a limit to aggregate production, due to the fixed number of firms on the one hand, and to the fact that firms cannot scale up production: the quantity they produce increases only if their productivity increases. Another limitation of our model in reproducing the empirical time series of quantity is the linear demand (Eq. 4.3). Notice that we obtained learning curves, fluctuations in prices and production growth without adding any exogenous shocks,

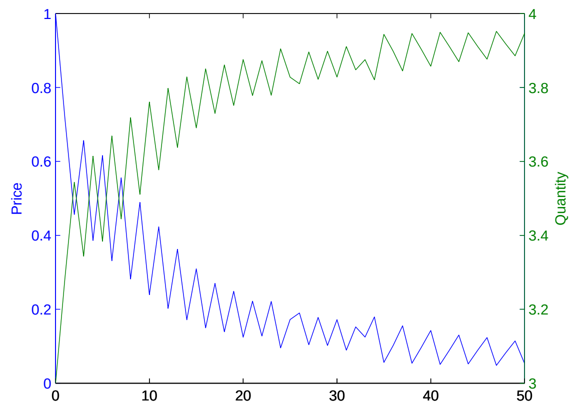

Figure 4.24: Simulated time series of price and aggregate quantity, with $\gamma = 0.2$, $\delta = 0.01$, $\beta = 10$, $b = 2$, $C = 1$, $a = 4$, $d = 1$, $s = 1$.

but just as endogenous dynamics.

4.5 Conclusion

We investigate the dynamics of innovation and imitation as two market strategies that affect total factor productivity in a perfectly competitive market, using a discrete choice mechanism. By modelling the interaction between innovators and imitators and its effect on supply and demand, we describe the market forces that lead a firm to innovate or to imitate. The general focus of the model are the market conditions and the behavioural characteristics of agents that cause a prevalence of innovation or imitation, and in particular what factors are important for more or less innovation in an industry.

The core of the model is an evolutionary mechanism of agents' choices that affect endogenously the production technology. This evolutionary environment exhibits negative feedback, because one strategy (innovation or imitation) is more profitable when the opponent strategy is dominant. Innovators drive down the market price because of cost reduction, but on the other hand they profit more from a high price. Such opposite incentives may end up offsetting each other in a stable equilibrium where both strategies coexist in some proportion. Alternatively, this minority game may lead to cyclical dynamics.

Two extensions of this basic model are studied, which relax some of its main assumptions, namely asynchronous updating of strategies and technological progress. The main result for asynchronous updating is that 2-cycles may turn into chaotic dynamics of agents' choices and market price. Although qualitatively destabilizing, asynchronous updating is quantitatively stabilizing, because it reduces the amplitude of possibly chaotic market oscillations.

Technological progress is modelled endogenously with a technological frontier which builds up as the cumulation of innovators' actions in each period. Doing so, we relax one of the main assumption of the basic model, and study how agents choice between innovation and imitation shapes dynamically the technological environment, and how technological change feeds back into agents choice. This extension of the model reproduces a number of different stylised facts of industrial dynamics, such as learning curves and path-dependence. When asynchronous updating is added to technological progress the model is able to reproduce the observed time patterns of the price index in an industry.

The final message is that a simple one-dimensional model of technological change with linear supply and demand and no lags in price expectation is able to produce a rich dynamics where market price and strategy distribution can have very different outcomes in the long run, from globally stable equilibrium to chaotic dynamics. The interplay of factors that are introduced step-by-step in the model allows to address a number of interrelated issues as firms strategic heterogeneity, stickiness of strategies and technological progress as knowledge cumulation. All outcomes in this model stem endogenously from the interplay of agents' choices and market price, and provide a simple micro-founded behavioural explanation of market and technological innovation dynamics.

Appendix

4.A Proofs

The first derivative of the price map (4.8) is:

$$f'(p) = -s^2(e^{bC}-1)^2\beta \frac{pe^{-\beta\Delta\pi(p)}}{\left[1+e^{-\beta\Delta\pi(p)}\right]^2} \frac{a}{\left\{d+s\left[1+\frac{(e^{bC}-1)}{1+e^{-\beta\Delta\pi(p)}}\right]\right\}^2}, \tag{4.18}$$

where $\Delta\pi(p) = \frac{1}{2}s(e^{bC}-1)p^2 - C$ is the difference in profits between innovators and imitators. We always have $f'(p) < 0$ for $0 < p < \infty$, which means that there exists a unique $p = p^*$ such that $f(p^*) = p^*$. This proves Proposition 4.2.1. By making use of (4.7) the exponential factor can be expressed with the product of agents' fractions $n(1-n)$:

$$f'(p) = -s^2(e^{bC}-1)^2\beta pn(1-n)\frac{a}{\left\{d+s\left[1+\frac{(e^{bC}-1)}{1+e^{-\beta\Delta\pi(p)}}\right]\right\}^2}. \tag{4.19}$$

In the steady state p^* we can use the equilibrium condition (4.8), $f(p^*) = p^*$:

$$f'(p^*) = -\frac{\beta s^2(e^{bC}-1)^2}{a}n^*(1-n^*)(p^*)^3. \tag{4.20}$$

The steady state p^* is bounded between $p^*_{INN} = \frac{a}{d+se^{bC}}$ and $p^*_{IM} = \frac{a}{d+s}$ (see Eq. 4.6), while $0 \le n^* \le 1$. If $\beta \to 0$ then $f'(p^*) \to 0$ and p^* is stable. This proves Proposition 4.2.2.

4.B Conditions for chaotic dynamics

The model with asynchronous updating is specified by the map $\hat{g}$ of Eq. (4.12):

$$\hat{g}(n) = \alpha x + (1-\alpha)g(n), \tag{4.21}$$

where g is the map (4.9) of the basic model with synchronous updating:

$$g(n) = \frac{1}{1 + e^{-\beta\left\{\frac{s(e^{bC}-1)a^2}{2\left[d+se^{bC}n+s(1-n)\right]^2}-C\right\}}}. \tag{4.22}$$

Consider property $(A3)$ of Proposition 4.3.1, first. The stability condition of the steady state n^* is $-1 < \alpha + (1-\alpha)g'(n^*)$. Since $\hat{g}'$ is bounded for $\beta < \infty$, there will always be a value of α close to 1 so that the stability condition holds true. Regarding property $(A1)$, the lower α, the closer the map $\hat{g}$ is to the map g of the basic model with synchronous updating. This means that in all situations where g has a stable 2-cycle, $\hat{g}$ has the same type of dynamics whenever α is close enough to 0. Finally, to prove $(A2)$ we follow Hommes (1994) p. 370. The map $\hat{g}$ of Eq. (4.12) is in the same class of functions of Eq. (12) in Hommes (1994), because it is obtained as a convex combination of a linear map (the diagonal) and a decreasing S-shaped map. Such functions have two critical points, c_1 and c_2, such that $\hat{g}'$ is decreasing in $[c_1, c_2]$ when β and C are large, and increasing outside with $0 < \hat{g}' < 1$. For intermediate values of α and for β and C large, the map $\hat{g}$ has a 3-cycle (see Hommes (1994)) and chaotic behaviour then follows by applying the Li-Yorke "Period 3 implies chaos" theorem (Li and Yorke, 1975). Shifting the graph of such a map leads to bifurcations from a stable 2-cycle to chaos, and back to stable steady state (se e.g. Fig. 4.12).