# Behavioural models of technological change

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# Chapter 5

# Competing technologies: a discrete choice model

## 5.1 Introduction

In this chapter we study the interaction of three factors affecting technological competition: decisions externalities, technological progress and environmental policy. Regarding externalities, the case of technological competition is peculiar, because on top of generic social interactions, there are other sources of feedback, called "network externalities", stemming from technological standards and infrastructure. Cases where social interactions and network externalities are important include, for instance, information and communication technologies, and power generation. Fig. 5.1 reports time series data for computer servers operating systems. In this figure we see how Linux entered the market in 1999 and managed to overcome Unix as the dominant operating system in about 5 years. Fig. 5.2 contains the time series of different sources of energy production in the United States. In this case there are little changes from 1972 until 2008, and for instance renewable energy is not able to gain momentum. In both these two examples of technology competition network externalities give rise to barriers which are strong to be broken.

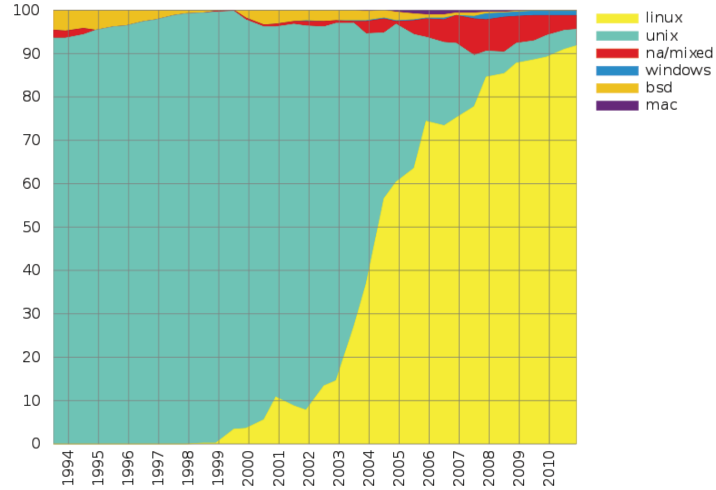


Figure 5.1: Computer servers: time series of operating systems shares (source: TOP500 Supercomputer).

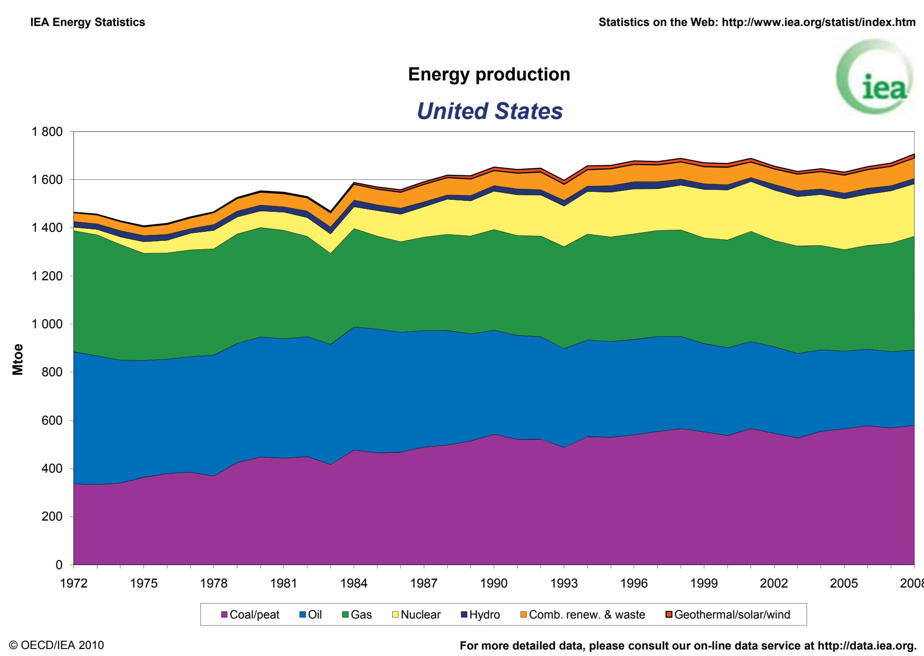


Figure 5.2: Energy production in US: time series of different sources (source: OECD/IEA).

This scenario translates into multiple equilibria, and once the economy is stuck in one of those, with one technology dominating the market (technological lock-in), it is hard for alternative technologies to gain market shares, let alone to overcome the dominant technology. This happened in the case of computer servers operating systems, but not for home computers operating systems, where Windows is still the unrivalled leader. In the case of energy production, a shift from the equilibrium represented by fossil fuels may be even harder to happen, due to the importance of energy infrastructures. In order to tackle these issues we propose a discrete choice model of interacting non-strategic decision makers who can adopt different technological solutions. The theoretical framework of the model is the discrete choice model with social interaction of Brock and Durlauf (2001).

A behavioural model of competing technologies that greatly influenced the literature on technology diffusion is Arthur (1989). With a sequential decision model Arthur focuses in particular on the role of increasing returns. His model can explain path-dependence of technological trajectories and lock-in, where one technology conquers the whole market. The technical version of this model (Arthur et al., 1987) is based on urn schemes, also known as a Polya process. These processes are a powerful tool in explaining positive feedback and path-dependence. A shortcoming of the model is that it employs a non-autonomous difference equation, which makes analytical study very difficult. One of our objectives is to reproduce Arthur's results with the simpler mathematics of a discrete choice model. Then we build upon this model by introducing technological progress and environmental policy, first separately and then in combination. Chapter 3 of this thesis proposes a different model of competing technologies with environmental policy. The main differences in the model of the present chapter are first, that here agents' decisions are modelled with a 'mean-field' approach, while in the model of Chapter 3 they are sequential. Second, fluctuations of technology shares here may result as endogenous chaotic dynamics of a deterministic model, and not as the outcome of a stochastic process.

Two popular models of externalities in collective decision making are Banerjee (1992) and Bikhchandani et al. (1992). These are sequential decision models which do not consider increasing returns on adoption, and consequently do not address technological choices, but financial markets and fashion dynamics, instead. Kirman (1993) proposes a model of recruitment based on social ants behaviour, where he explains how casual interactions may cause symmetry breaking in a symmetric system, with a large majority of agents choosing one out of two identical options, and how regime shifts may occur. Also this model does not consider any forward looking behaviour and the externality does not stem from increasing returns to adoption, but from recruitment, that is a sort of contagion effect, instead. In our model the way to understand the effect of network externalities and social interactions is the dynamics of technological shares. Our model is quite close to the discrete choice model proposed by Brock and Durlauf (2010), who study the adoption curve of one technology. The present chapter considers two technologies available for adoption, focusing on technology competition and on the dynamics of technology shares. Moreover we also extend the discrete choice mechanism to the case of an environmental policy and to technological progress.

Technological competition is characterized by several sources of positive feedback from decision externalities. Besides contagion effects, imitation and learning through social interactions, technology choices are also affected by network externalities and technological infrastructures. Because of this, positive feedback is common in technology adoption decisions, leading to path-dependence and lock-in into a dominant technology Arthur (1989). Here we propose a discrete choice model of technological competition which reproduces the main results of Arthur (1989). The simpler structure of the discrete choice framework has the advantage that it allows for a partly analytical study of the equilibrium and stability of the system, and of the qualitative changes in dynamics due to changes in parameters values (bifurcations).

The discrete choice framework further allows for several useful extensions of the model. A first extension is the case of competition between technologies that have an effect on the environment due to their generation of pollution. This in turn makes it possible to study the impact of environmental policy. Here we consider a simple scenario with the competition between a “clean” and a “dirty” technology, and a policy which attempts to reduce the market share of the latter. Together with network externalities and social interactions, the environmental policy affects the dynamics of the system and its equilibria.

We show that a policy whose effort is aimed at directly altering technology market shares may miss that goal, giving place to cyclical behaviour.

A second extension of the model is the cumulative effect of technological progress, which adds an element of irreversibility to technology competition. This works in two directions: technological competition driven by network externalities and social interactions affects and is affected by technological progress (innovation). This specification of the model offers a tool for studying innovation policy, as it deals with the trade-off between the advantages of technological variety and the advantages of the rate of progress of a single technology: intuitively, technological variety counters the rate of development of each technology by reducing the amount of resources allocated to each of them.

A final extension brings together technological progress and environmental policy. This model is particularly relevant for power generation, where the competition of the different energy resources is heavily affected by technological progress. Here we also consider two types of environmental policy, one linked to technology shares, and one that tries to close the profitability gap between "clean" and "dirty" technologies. The interplay of environmental policy and decision externalities shapes the catching-up of "clean" technologies, and consequently it affects the outcome of the environmental policy in its attempt to unlock the market from the "dirty" technology. In particular we observe that policy alone hardly succeeds in this attempt, and a stronger effort directed to technological innovation of the "clean" solution is needed.

The structure of the remainder of this chapter is as follows. Section 5.2 presents a basic version of the model. Section 5.3 proposes an application to environmental economics with the competition of "dirty" and "clean" technologies. Section 5.4 introduces technological progress. Section 5.5 brings together environmental policy and technological progress. Section 5.6 concludes.

## 5.2 Social interactions and network externalities

In this section we present the basic version of the model, modelling the effect of social interactions and network externalities in technology competition. Network externalities are a source of self-reinforcement due to increasing returns to adoption: the utility from one technology increases with the number of fellow adopters (Arthur, 1989), because of technological standards and infrastructure (have you ever tried to use Linux in a department where everybody uses Microsoft Windows? Or go around with your fuel cell car and run out of hydrogen?). Social interactions instead convey all positive externalities that occur as a contagion effect, "word of mouth" learning or recruitment (Kirman, 1993), or as conformity effects and habit formation (Alessie and Kapteyn, 1991). The main difference is that network externalities stem from measurable contributions to utility with the diffusion of one technology (a network of users), while social interactions imply social contacts and do not cause any tangible benefit in terms of performance of the technology adopted.

Social interactions and network externalities are important to a different extent in different technology sectors. For instance, computer software packages present strong network externalities, due to standards and compatibility barriers. On the other hand, web browsers are perfectly compatible, and their competition is likely to be characterized only by social interactions. There may also be cases where social interactions give place to a negative feedback, as with conspicuous consumption aiming at social status. We discard this possibility, and we model social interactions and network externalities together as a unique source of self-reinforcement in technology decisions.

Consider $M$ technologies competing in the market for adoption or for $R\&D$ investment by $N$ agents ($N \gg M$). The utility from choosing technology $a$ in period $t$ is

$$u_{a,t} = \lambda_a + \rho_a x_{a,t}, \tag{5.1}$$

where $\lambda_a$ is the profitability of technology $a$, and $x_{a,t}$ is the fraction of agents that choose technology $a$ in period $t$. For the moment we assume $\lambda_a$ to be constant, that is we discard technological progress. In Section 5.4 we relax this assumption. The parameter $\rho_a > 0$ expresses the intensity of positive externalities in agents' decisions. The term $\rho_a x_{a,t}$ describes the self-reinforcing effect of decision externalities.

We model agents' choices about technology by the discrete choice framework of Brock and Durlauf (2001). The general case with $M$ choice options is addressed in Brock and Durlauf (2002) and in Brock and Durlauf (2006). According to this model, each agent experiences a random utility $\tilde{u}_{i,t} = u_{i,t} + \epsilon_{i,t}$, where the noise $\epsilon_{i,t}$ *iid* is across agents, and it is known to an agent at the decision time $t$. What the agent does not know with infinite precision is the decision of other agents, that is the social term $\rho_a x_{a,t}$ of Eq. (5.1). In the limit of an infinite number of agents, when the noise $\epsilon_{i,t}$ has a double exponential distribution, the probability of adoption of technology $a$ converges to the Gibbs probability of the multinomial logit model:

$$x_{a,t} = \frac{e^{\beta u_{a,t-1}}}{\sum_{j=1}^{M} e^{\beta u_{j,t-1}}}. \tag{5.2}$$

The parameter $\beta$ is the *intensity of choice* and it is inversely related to the variance of the noise $\epsilon_{i,t}$ (Hommes, 2006). In the limit $\beta \to 0$ the different technologies tend to an equal share $1/M$. The limit $\beta \to \infty$ represents the "neoclassical-economic" or "rational agent" limit, where everybody chooses the optimal technology. The main difference of our model with respect to Brock and Durlauf (2001) is in the timing of utility computation entering Eq. (5.2): their model is based on rational expectations, so as to have the utility of time $t$ dictating the agents' fraction $x_{a,t}$. In our model instead agents' decision is based on *past experience*, either involving technological network externalities or social interactions. Our focus is on technology competition dynamics, which calls for modelling learning and decision dynamics as in Brock and Hommes (1997), instead of the decision rule (5.2) being a condition for equilibrium consistency as in Brock and Durlauf (2001).

Of all the possible learning heuristics we adopt the most simple one, naive expectations, according to which agents decide today based on the last experienced utility. In the Industrial Organization literature the model by Smallwood and Conlisk (1979) considers a similar switching mechanism where consumers take into account the market share of products, beside their quality. The main difference of our model is the strong accent on dynamics of choices.

Consider the simplest scenario with two competing technologies, labelled $a$ and $b$. This model is one-dimensional: one state variable, the fraction of technology $a$, $x_a \equiv x$, is enough for knowing the state of the system at a given time ($x_b = 1 - x$). Assume for simplicity an equal increasing return on adoption $\rho_a = \rho_b \equiv \rho$ for the two technologies. The difference of utilities is central in this model:

$$u_{b,t} - u_{a,t} = \lambda + \rho(1 - 2x_t), \tag{5.3}$$

where $\lambda \equiv \lambda_b - \lambda_a$ is the difference in profitability between the two technologies. The probability of adoption (and the market share) of technology $a$ in period $t$ is:

$$x_t = \frac{e^{\beta(\lambda_a + \rho x_{t-1})}}{e^{\beta(\lambda_a + \rho x_{t-1})} + e^{\beta[\lambda_b + \rho(1 - x_{t-1})]}} = \frac{1}{1 + e^{\beta[\lambda + \rho(1 - 2x_{t-1})]}} \equiv f(x_{t-1}). \tag{5.4}$$

Analytical results regarding the dynamics of the system (5.4) are in line with Brock and Durlauf (2001). The fixed points of the map $f$ give the equilibrium values for $x_t$.

**Proposition 5.2.1.** *The system (5.4) has either one stable steady state or an unstable steady state $x^*$ and two stable steady states $x_1^*$ and $x_2^*$ such that $x_1^* \leq x^* \leq x_2^*$.*

A first observation is that $x = 0$ and $x = 1$ (technological monopoly) are equilibria only for $\beta = \infty$. For finite $\beta$ the less adopted technology never disappears. Fig. 5.3 shows some examples with different values of $\beta$ for $\lambda = 0$ and $\lambda = 0.2$ (with $\rho = 1$). In the symmetric case $\lambda = 0$ (left panel) the steady state $x = 1/2$ is stable if $f'(1/2) \leq 1$, which is true if $\beta \leq 2$. Whenever the intensity of choice is smaller than 2, the adoption

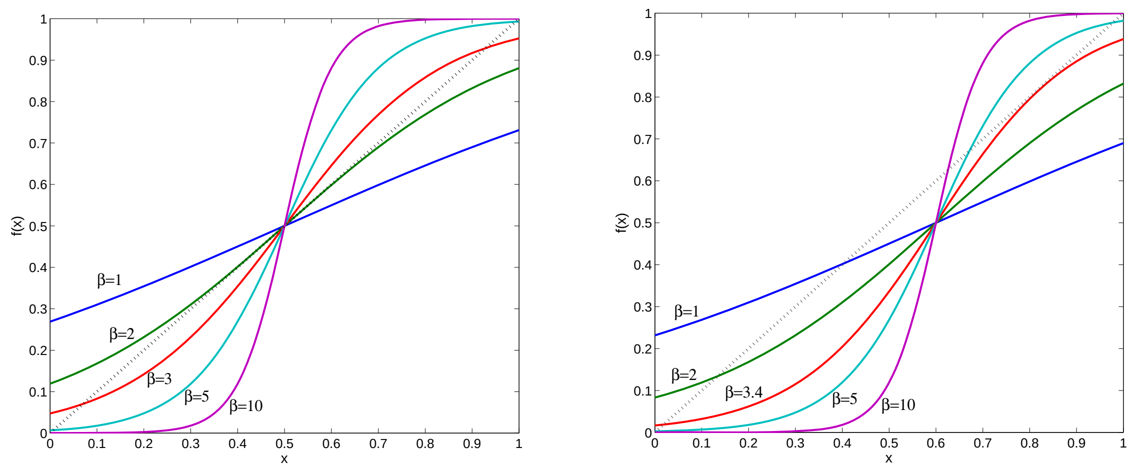


Figure 5.3: Map $f$ for different values of $\beta$ ($\rho = 1$). Left: $\lambda = 0$. Right: $\lambda = 0.2$.

process will converge to equal shares of technologies $a$ and $b$. Conversely, for $\beta > 2$ the system converges to one of two alternative steady states, where one technology is dominant. Such qualitative change in the dynamics of a model due to a change in one parameter is called a *bifurcation*. The critical value ($\beta = 2$) is the bifurcation value. Symmetry of the two technologies ($\lambda = 0$) gives place to a "pitchfork bifurcation" for $\beta = 2$, where the steady state $x = \frac{1}{2}$ loses stability and two new stable steady states are created. When one technology is more profitable than the other one ($\lambda \neq 0$) additional steady states are created by a "tangent bifurcation". The right panel of Fig. 5.3 shows a tangent bifurcation for $\beta \simeq 3.4$, in which two steady states are created, one stable and one unstable. The role of $\rho$ is somewhat similar to the role of $\beta$, as illustrated in Fig. 5.4. In this case a larger $\lambda$ also lowers the value of the map in the flex point, $f(\hat{x})$. Fig. 5.5 describes the occurrence of a tangent bifurcation for $\lambda$. Here two tangent bifurcations occur for $\lambda \simeq -0.27$ and $\lambda \simeq 0.27$. When $\lambda \neq 0$, the worse technology may still attain a larger share in equilibrium. This is due to the positive externality in (5.1), which renders the initial condition very important. A positive value of $\lambda$ (technology $b$ better than $a$), for instance, shifts the map to the right, with an unstable steady state $x^* > 1/2$. If the initial condition $x_0 > x^* > \frac{1}{2}$, the system converges to $x_2^*$, with a larger share of technology

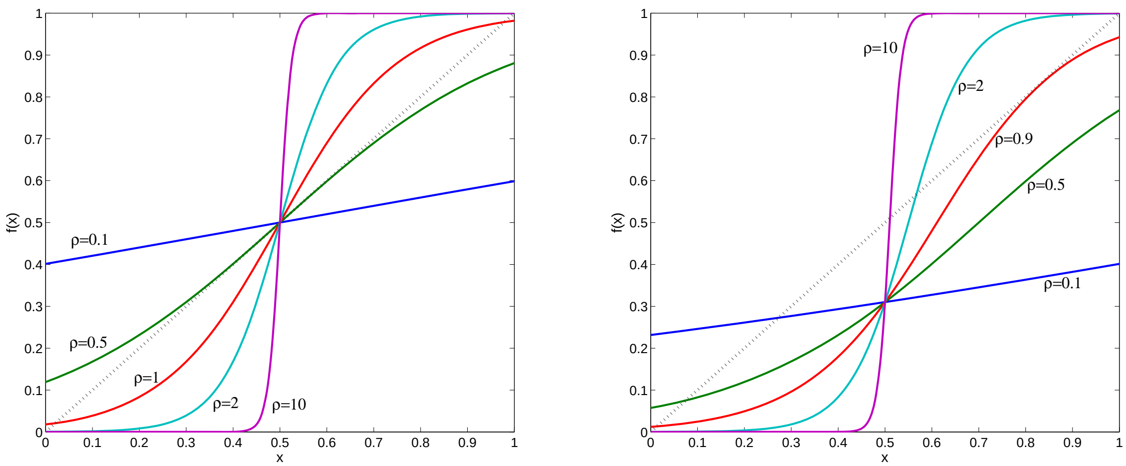


Figure 5.4: Graph of map $f$ for different values of $\rho$ ($\beta = 4$). Left: $\lambda = 0$. Right: $\lambda = 0.2$.

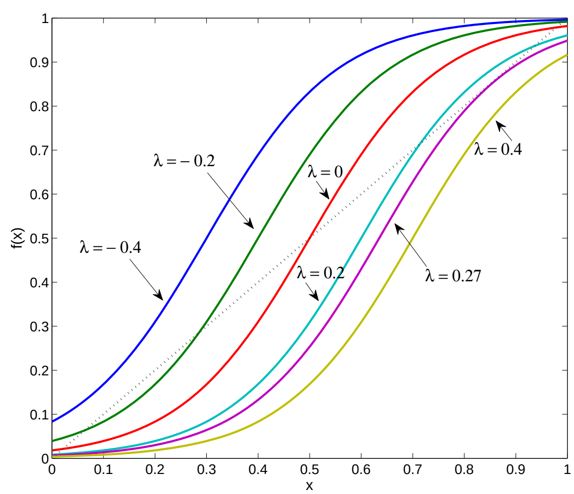


Figure 5.5: Map $f$ for different values of $\lambda$ ($\beta = 4$, $\rho = 1$).

$a$, despite this one being worse than technology $b$.

A bifurcation diagram gives a broader picture of the qualitative dynamics of this system, consisting of the long run value(s) of the state variable $x_t$ from many different initial conditions in a given range of parameter values. Fig. 5.6 reports four bifurcation diagrams of $\lambda$ for four different values of $\beta$. In two cases ($\beta = 1$ and $\beta = 2$) there is a smooth change in the equilibrium value of the share of technology $a$ following a change in $\lambda$, namely the share $x$ decreases continuously as the profitability gap with technology $b$ increases. For higher values of the intensity of choice ($\beta = 3$ and $\beta = 4$) an increase of $\lambda$

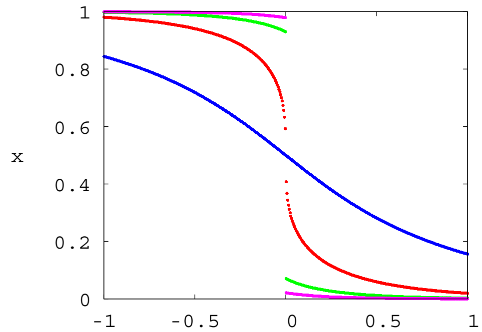


Figure 5.6: Bifurcation diagram of $\lambda$ (horizontal axis) for the share $x_t$ of technology $a$ (vertical axis) with $\beta = 1$, 2, 3 and 4, where increasing $\beta$ the diagram gets closer to a step. Here $\rho = 1$ and $x_0 = 0.5$.

near the bifurcation value $\lambda = 0$ may cause an abrupt change from $a$ to $b$ as the dominant technology. The values of $\beta$, $\rho$ and $\lambda$ together determine whether or not multiple equilibria exist. The following two necessary conditions hold true:

**Proposition 5.2.2.** *$\rho\beta > 2$ and $-\rho < \lambda < \rho$ are necessary conditions for multiple equilibria.*

The proof is based on the position of the inflection point $\hat{x} = \frac{\lambda+\rho}{2\rho}$ of the map $f$ and on the maximum derivative $f'(\hat{x}) = \frac{\beta\rho}{2}$ (see Appendix 5.A).

The intensity of choice regulates the shape of the map (5.4): the larger is $\beta$, the more $f$ is similar to a step function, with a discontinuity in $\hat{x} = \frac{\lambda+\rho}{2\rho}$. The following holds true:

**Proposition 5.2.3.** *Consider map (5.4):*

- *when $\beta \approx 0$, there is a unique equilibrium, and it is stable.*
- *when $\beta \approx \infty$, there may be three cases:*
  1. *if $\lambda < -\rho$ the equilibrium $x_2^* = 1$ is unique and stable,*
  2. *if $\lambda > \rho$ the equilibrium $x_1^* = 0$ is unique and stable,*
  3. *if $-\rho < \lambda < \rho$, $x^* = \hat{x} = \frac{\lambda+\rho}{2\rho}$ is unstable, while $x_1^* = 0$ and $x_2^* = 1$ are stable.*

The proof of Proposition 5.2.3 relies on the fact that when $\beta = \infty$, the two conditions of Proposition 5.2.2 are also sufficient for multiple equilibria, because the system depends only on the position of the inflection point $\hat{x}$; in the third case, $\hat{x}$ falls inside the interval $[0, 1]$, and both $x = 0$ and $x = 1$ are stable equilibria. In this case the market will be completely taken by one or the other technology, depending on the initial condition.

We conclude this section with a numerical implementation of this model which aims to reproduce the main result of Arthur (1989). Consider the case of two equally profitable technologies ($\lambda = 0$) and set $\beta = 3$ and $\rho = 1$. These settings meet all requirements of Proposition 5.2.2, that is two stable equilibria. Fig. 5.3 indicates that one equilibrium, $x_1^*$, is located between 0.05 and 0.1, and the other, $x_2^*$, between 0.9 and 0.95. Assume that technologies $a$ and $b$ start with equal shares ($x_0 = 0.5$). This initial condition coincides with the unstable equilibrium. By adding an arbitrarily small noise term to the state variable $x$, one escapes this unstable equilibrium. The point is that sometimes the system converges to $x_1^*$, where technology $b$ is dominant, and at other times to $x_2^*$, where $a$ is dominant. Fig. 5.7 reports five simulated time series of the share $x$ produced by five different runs of the model with a noise component $\epsilon_t \sim N(0, 0.01)$ added to Eq. (5.4). The fact that no one can tell which technology will be the winner is one of the

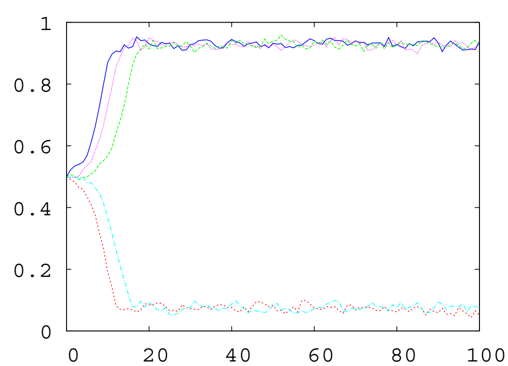


Figure 5.7: Five time series of the share of technology $a$ obtained by running the model with noise for two equally performant technologies ($\lambda = 0$) starting with equal shares ($x_0 = 0.5$), with $\beta = 3$ and $\rho = 1$.

main insights from Arthur (1989). Path-dependence and the lock-in effect are stronger in Arthur's model, because the probability distribution of the states of the system changes with time. This is not the case with probability distribution (5.4), instead. By adding memory to the utility in our model we also can obtain a stronger lock-in effect.

## 5.3 Competing technologies with an environmental policy

An application of this model in environmental economics pertains to the case of technologies with different degrees of pollution. Power generation is an example, and it will be our reference story in this section. Say $d$ is a "dirty" technology, fossil fuels for instance, with a high pollution intensity, and $c$ is a "clean" technology, for example renewable energy, with low pollution intensity. We assume the clean technology has higher costs and/or lower performance compared to the dirty technology, which translates into a profitability gap $\lambda = \lambda_d - \lambda_c > 0$. In principle one can unlock the economy from the dirty technology by making $\lambda$ low enough, so as to eliminate the socially less desirable equilibrium (Fig. 5.5). This is the goal of an environmental policy. The way such a policy is enforced has strong consequences for the dynamics of technology competition, and a tougher policy does not necessarily lead to an equilibrium with a larger share of clean technology, due to non-linearities in the system, as we will show. In this section we study the changes in the dynamics of competing technologies due to the introduction of an environmental policy.

In the case of power generation, environmental policies aim at the "grid parity", where clean energy reaches the profitability of traditional dirty energy. To realize this, a number of different policies have been implemented in different countries (Fischer and Newell, 2008). Such policies can be grouped in various ways, but in essence they either impose taxes on pollution or provide subsidies for the clean(er) technology. Taxes make the dirty technology more expensive, by internalizing the pollution externality. Subsidies make the clean technology less expensive. Both measures result in an attempt to lower the profitability gap $\lambda$. Here we consider the case of subsidies, which are implemented by the so-called "Feed-in-Tariffs" in Germany and Denmark, for instance (Lipp, 2007).

An energy policy tends to be endogenous to technology competition, because its effort usually decreases as the share of the clean technology $x_c \equiv x$ increases. If $\sigma$ is a sort of fixed price (tariff) for the unit of clean energy produced, the profitability of the clean technology is augmented by the subsidy $\sigma_t = \sigma(1 - x_t)$ in each period. Consequently the profitability gap after the introduction of subsidies changes as follows:

$$\lambda^\sigma(x) = \lambda_0 - \sigma(1 - x), \tag{5.5}$$

where $\lambda_0 = \lambda_d - \lambda_c$ is the profitability gap without policy. After substituting $\lambda$ with $\lambda^\sigma(x)$ in the utility (5.1), the difference in utility between the two technologies becomes:

$$u_d - u_c = \lambda_0 - \sigma(1 - x) + \rho(1 - 2x) \tag{5.6}$$

and the new map of the system is:

$$f_\sigma(x) = \frac{1}{1 + e^{\beta[\lambda_0 + \rho(1-2x) - \sigma(1-x)]}}. \tag{5.7}$$

The dynamics of the share of clean technology is given by $x_t = f_\sigma(x_{t-1})$. In Appendix 5.B we show that a pollution tax leads to the same dynamic model. Without policy ($\sigma = 0$) one is back to the basic model (5.4). The main difference to the basic model is that here the map can be decreasing, depending on the policy effort $\sigma$:

**Proposition 5.3.1.** *$f^\sigma$ is downward sloping for $\sigma > 2\rho$ and upward sloping otherwise.*

The case $\sigma = 2\rho$ gives a flat map, with one steady state which is stable. The proof of Proposition 5.3.1 is in Appendix 5.C. This proposition says that beyond a threshold value of policy effort the steady state becomes unstable and period 2 cycles of technology shares occur. The intuition for a cyclical dynamics of the technology market is the following. An environmental policy that reduces the profitability gap as indicated by Eq. (5.5) is shut down as soon as the clean technology reaches a certain market share (here this threshold is equal to one, for simplicity). But without policy the profitability gap widens in the next period, calling for the policy to be enforced again, and so the story repeats. Such cyclical behaviour is easier to attain the lower is the intensity of externalities $\rho$:

**Proposition 5.3.2.** *There are six cases:*

1. *the map $f^\sigma$ is upward sloping ($\sigma < 2\rho$):*

   (a) $\lambda_0 < \rho$*: raising $\sigma$ leads to a tangent bifurcation. With both one or three steady states, raising $\sigma$ increases the equilibrium(a) share. The flex point is $\hat{x} < 1$.*

   (b) $\lambda_0 = \rho$*: there is only one steady state, which is stable. The flex point is $\hat{x} = 1$.*

   (c) $\lambda_0 > \rho$*: there is only one steady state, which is stable. The flex point is $\hat{x} > 1$.*

2. *the map $f^\sigma$ is downward sloping ($\sigma > 2\rho$):*

   (a) $\lambda_0 < \rho$*: there is only one steady state, which is stable. Increasing $\sigma$ increases the equilibrium share. The flex point is $\hat{x} > 1$.*

   (b) $\lambda_0 = \rho$*: there is only one steady state, which becomes unstable for $\sigma$ sufficiently large, giving place to a stable period 2 cycle. The flex point is $\hat{x} = 1$.*

   (c) $\lambda_0 > \rho$*: there is only one steady state, which becomes unstable for $\sigma$ sufficiently large, giving place to a stable period 2 cycle. The flex point is $\hat{x} < 1$.*

The proof is in Appendix 5.C. Fig. 5.8 illustrates the different cases of Proposition 5.3.2 with a number of examples. A tougher policy (larger $\sigma$) generally leads to a larger

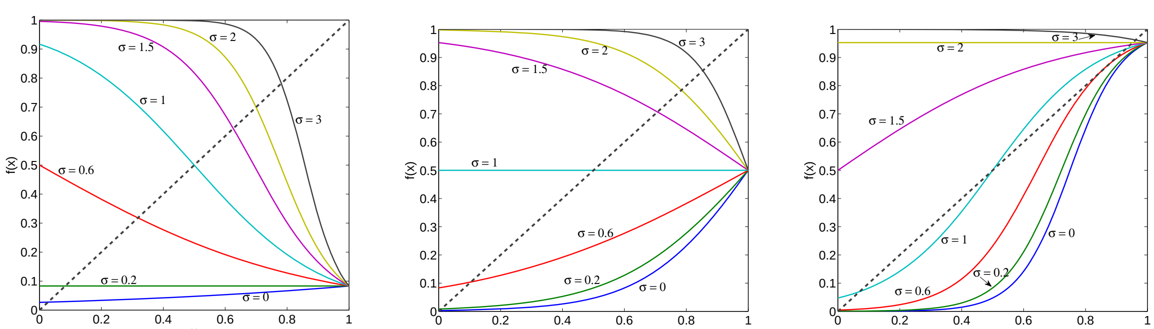


Figure 5.8: Model with environmental policy. Map $f_\sigma$ for seven different values of subsidy level $\sigma$ (with $\lambda_0 = 0.5$ and $\beta = 6$). Left: $\rho = 0.1$. Centre: $\rho = 0.5$. Right: $\rho = 1$.

share of clean technology, as one may expect. More surprising, beyond a threshold value of $\sigma$ cyclical dynamics occur. Both effects are clear in the left and middle panels of Fig. 5.8. In the left panel ($\lambda_0 > \rho$, cases (1c) and (2c) of Proposition 5.3.2) there is always a unique steady state, and an increasing effort shifts the flex point $\hat{x}$ to the right. In the middle panel ($\lambda = \rho$, cases (1b) and (2b)) there is still only one steady state, but the flex point position $\hat{x} = 1$ is unaffected. In the right panel ($\lambda_0 < \rho$, cases (1a) and (2a)), rising $\sigma$ leads to a tangent bifurcation for $\sigma \simeq 0.6$, with the appearance of two additional steady states, one of which stable. Another tangent bifurcation above $\sigma = 1$ reduces the number of steady states again to only one. We can resume the effect of the environmental policy in the condition of the right panel as follows: for low effort values the marginal effect of the policy on the market share of the clean technology is very small. For middle values of the effort, the environmental policy creates an alternative equilibrium, which is socially desirable. Higher efforts lead to a sudden shift, eliminating the suboptimal equilibrium. If the economy is locked-in into a dirty technology, this event tips the market towards the clean technology. Concluding the remarks on the examples of Fig. 5.8, stronger network externalities and/or social interactions are responsible for the occurrence of multiple equilibria, which are the fundamental condition for lock-in into one technology. When such externalities are relatively weak, it is easier for the environmental policy to increase the share of the clean technology. But if the policy effort is too strong it destabilizes the market with cyclical dynamics.

Fig. 5.9 on the left reports a simulated time series of the share $x_t$ that converges to a period 2 cycle. The middle panel of Fig. 5.9 is a bifurcation diagram of the subsidy level $\sigma$. By comparing the left and right panels of Fig. 5.8 we see that lower network externalities and social interactions make it easier for environmental policy to trigger cycles. This is evident also from the bifurcation diagram of $\rho$ in the right panel of Fig. 5.9.

The switching behaviour of the discrete choice framework may be unrealistic in cases where large sunk costs cause stickiness in the decision process. The power generation

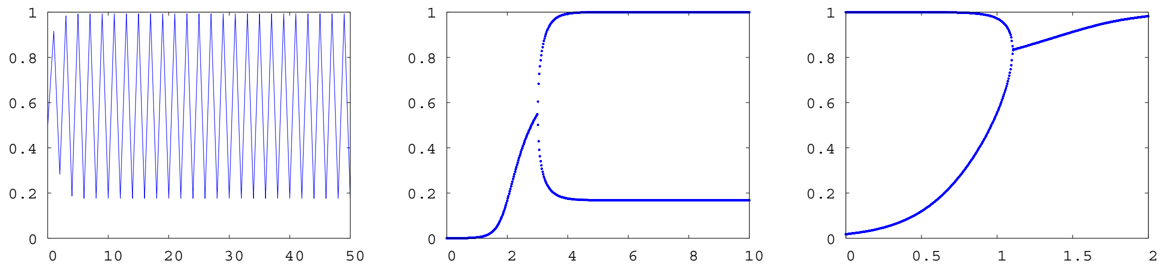


Figure 5.9: Model with environmental policy. Left: time series of the share of clean technology $x$. Centre: bifurcation diagram of $\sigma$ (horizontal axis) for the share $x$ (vertical axis). Right: bifurcation diagram of $\rho$ (horizontal axis) for the share $x$ (vertical axis). $\lambda_d = 2.4$, $\lambda_c = 1$, $\rho = 1$, $\beta = 4$ and $\sigma = 4$.

sector is an example, although utilities can switch energy source to some extent, by switching on and off different power plants. Nevertheless we can improve the realism of the model by capturing persistence of behaviours through asynchronous updating. This extension of the model responds to the idea that not all agents update their strategy in every period. The discrete choice model with asynchronous updating is given by

$$x_{i,t} = \alpha x_{i,t-1} + (1-\alpha)\frac{e^{\beta u_{i,t-1}}}{\sum_{j=1}^{M} e^{\beta u_{j,t-1}}}, \tag{5.8}$$

where $\alpha$ is the portion of agents that stick to their previous strategy, while a fraction $1-\alpha$ chooses a strategy based on the discrete choice mechanism (5.2). A larger $\alpha$ means more persistence of strategies. Fig. 5.10 reports a bifurcation diagram of $\alpha$. Whenever $\alpha$

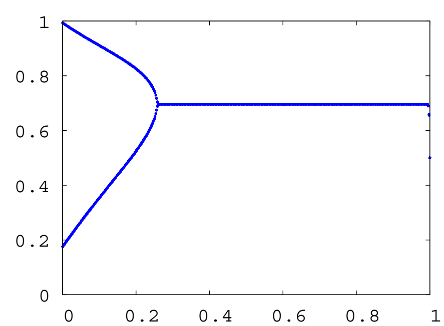


Figure 5.10: Model with environmental policy and asynchronous updating. Bifurcation diagram of $\alpha$ (horizontal axis) for the share of clean technology $x$ (vertical axis). $\lambda_d = 2.4$, $\lambda_c = 1$, $\rho = 1$, $\sigma = 4$, $\beta = 4$.

is larger (more stickiness) the amplitude of the fluctuations is smaller. Above a threshold value of $\alpha$ there is a unique stable equilibrium.

Although asynchronous updating has a stabilizing effect in general, it may lead to chaotic dynamics. If the map $f_\sigma$ is downward sloping (see Fig. 5.8), the map with asynchronous updating is a convex combination of an upward and a downward non-linear function, which may result in a non-monotonic map. For the case of two competing technologies we have:

$$f_{\sigma,\alpha}(x) = \alpha x + (1-\alpha)\frac{1}{1+e^{\beta[\lambda_0+\rho(1-2x)-\sigma(1-x)]}}. \tag{5.9}$$

A non-monotonic map may generate chaotic dynamics. The left part of Fig. 5.11 reports an example of a bifurcation diagram in a setting with chaotic dynamics. Here the dynamics

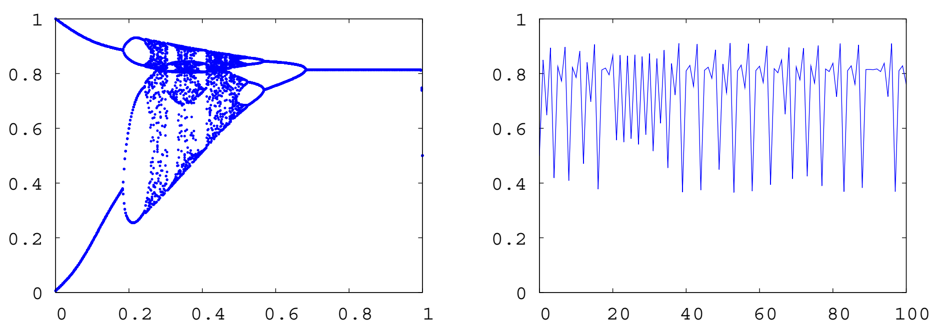

Figure 5.11: Model with environmental policy and asynchronous updating. Left: bifurcation diagram of $\alpha$ (horizontal axis) for the share of clean technology $x$ (vertical axis). Right: time series of $x$, with $\lambda_d = 3$, $\lambda_c = 1$, $\rho = 1$, $\sigma = 9$, $\alpha = 0.3$ (for the time series) and $\beta = 5$.

of $x_t$ is chaotic for $\alpha$ between 0.25 and 0.5, where a cascade of period doubling and period halving bifurcation occur, respectively. Large values of $\sigma$ are responsible for this occurrence, because they give a map with a steep negative slope (Fig. 5.8). An example of a time series $x_{c,t}$ showing chaotic dynamics is reported in the right panel of Fig. 5.11.

## 5.4 Competing technologies and technological progress

In this section we extend the discrete choice model of technology competition of Section 5.2 in a different direction, by introducing an endogenous mechanism of technological progress. The stream of research that goes under the name of "endogenous growth theory" addresses the mutual relationship between economic growth and technological progress (Romer, 1990; Aghion and Howitt, 1998). The main feature of this approach to economic growth is the recognition of mutual effects between the economy and technological change, going beyond the traditional one-way relationship from science to technology to the economy. Although these models are generally claimed to have micro-foundations, relatively little attention is given to the decision of agents concerning which technology to adopt. Here we propose a behavioural approach, and study the decision process that underlies the interplay of technological competition and technological progress. Building on the discrete choice mechanism of the basic model, we can study in particular how network externalities and social interactions shape technological progress.

In general, technological progress may take the form of costs reduction or performance enhancement of one technology (vertical product differentiation), and it can give rise to an increase in product variety (horizontal product differentiation). Here we consider the former case, and we model it through changes in the profitability of competing technologies. We focus on gradual innovation and do not consider here the occurrence of radical innovation represented by the advent of new technological regimes.

Consider again the competition of two technologies $a$ and $b$ with utility given by (5.1). The profitabilities $\lambda_a$ and $\lambda_b$ increase because of technological progress. Assume that for each technology the progress depends on the cumulative investment through history, and in every period the technology investments $I_{a,t}$ and $I_{b,t}$ are proportional to the technology market share: $I_{a,t} = h_a x_t$ and $I_{b,t} = h_b(1-x_t)$. Growth is concave in investments (Barlevy, 2004), and a concave function is used also to describe endogenous technological progress (Aghion and Howitt, 1998). We model technological progress with the following learning curves:

$$\lambda_{a,t} = \lambda_{a0} + \psi_a \left( h_a \sum_{j=1}^{t} x_j \right)^{\zeta}, \quad \lambda_{b,t} = \lambda_{b0} + \psi_b \left( h_b \sum_{j=1}^{t} (1 - x_j) \right)^{\zeta}, \tag{5.10}$$

with $\lambda_{a0}$ and $\lambda_{b0}$ the profitabilities without technological progress. According to this specification, the difference in profitabilities (the *technological gap*) between the two technologies becomes:

$$\lambda_t = \lambda_{b,t} - \lambda_{a,t} = \lambda_0 + \psi_b \left( h_b \sum_{j=1}^{t} (1 - x_j) \right)^{\zeta} - \psi_a \left( h_a \sum_{j=1}^{t} x_j \right)^{\zeta}, \tag{5.11}$$

where $\lambda_0$ is the technological gap without progress. $\psi_a, \psi_b$ measure how investments translate into technological progress, and together with $h_a$ and $h_b$ describe $R\&D$ in each technology. $\zeta \in [0, 1]$ is a sector specific parameter which dictates the curvature of the investment function. In general, an old established technology $a$ is likely to present low values of $h_a$ and $\psi_a$, while the opposite is true for a young innovative technology.

The difference in utility between $b$ and $a$ with technological progress becomes:

$$u_{b,t} - u_{a,t} = \lambda_t + \rho(1 - 2x_t), \tag{5.12}$$

Here technology competition is driven by externalities (the second term of the right hand side) as well as technological progress (the first term): the share of technology $a$ according to (5.2) is now given by:

$$x_t = \frac{1}{1 + e^{\beta[\lambda_t + \rho(1 - 2x_{t-1})]}} \equiv f_t(x_{t-1}). \tag{5.13}$$

The map $f_t$ depends on time. It is identical to the map $f$ of the basic model (5.4) after substituting the static parameter $\lambda$ with the time varying technological gap of Eq. (5.11). Endogenous technological progress as expressed by the dynamic technological difference $\lambda_t$ works as a slowly changing parameter that modifies the flow map of the competing technology system as shown in Fig. 5.5. The long run equilibrium, or long run dynamic of this system is known only by looking at the limit map given by the limit value $\lim_{t \to \infty} \lambda_t \equiv \lambda_\infty$. The technology gap $\lambda$ does not modify the curvature of the map, but it may affect the number of stable equilibria by shifting it. There can be two cases: 1) if the steady state $x^*$ of Proposition 5.2.1 is stable, a change in $\lambda$ changes gradually the equilibrium market shares as one technology slowly catches up, or 2) if $x^*$ is unstable, a change in $\lambda$ can cause a change from one to two stable equilibria (or the other way around) through a tangent bifurcation (Fig. 5.5). In this second case, a less adopted technology may suddenly overcome the other, unlocking the economy from the previous dominant technology.

The convergence of the series $\lambda_t$ plays a key role in determining the long run equilibrium of the technology market. Whenever one technology has a faster pace (due to larger values of $h$ and/or $\psi$), we have $\lambda_t \to \pm\infty$, that is lock-in into technology $a$ ($\lambda_t \to -\infty$) or $b$ ($\lambda_t \to +\infty$). The linear case $\zeta = 1$ allows to derive some analytical results on the dynamics of $\lambda_t$. Eq. (5.11) in this case becomes

$$\begin{aligned} \lambda_t &= \lambda_0 + \psi_b h_b \sum_{j=1}^{t} (1 - x_j) - \psi_a h_a \sum_{j=1}^{t} x_j \\ &= \lambda_0 + h_b \psi_b t - (h_a \psi_a + h_b \psi_b) \sum_{j=1}^{t} x_j. \end{aligned} \tag{5.14}$$

The following proposition lists the possible outcomes in the linear case:

**Proposition 5.4.1.** *In the long run ($t \to \infty$) the technological gap $\lambda_t$ for $\zeta = 1$ (Eq. 5.14) has the following limit behaviour:*

1. *$\lambda_t$ converges if and only if $\exists\ p, q$ constants such that $\sum_{j=1}^{t} x_j \sim g(t) = p + qt$, with $q = \frac{h_b \psi_b}{h_a \psi_a + h_b \psi_b}$ and $p = -\frac{\lambda_\infty}{h_a \psi_a + h_b \psi_b}$.*

2. *If $\sum_{j=1}^{t} x_j$ is slower than $g(t)$, then $\lambda_t$ diverges to $+\infty$ (lock-in into $b$).*

3. *If $\sum_{j=1}^{t} x_j$ is faster than $g(t)$, then $\lambda_t$ diverges to $-\infty$ (lock-in into $a$).*

Condition 1 implies that $x_t = \frac{h_b \psi_b}{h_a \psi_a + h_b \psi_b}$, on average. Conditions 2 and 3 represent situations where one technology systematically grows faster than the other, and eventually it conquers the entire market (eq. 5.4). The only possibility for technological market segmentation is condition 1.

One important question that this model can address is how social interactions and network externalities affect technological progress through technological competition. To answer that question, one needs a measure of technological progress. A rough measure is the sum of profitabilities, $\Lambda = \lambda_a + \lambda_b$:

$$\Lambda_t = \lambda_{a0} + \lambda_{b0} + \psi_a \Big( h_a \sum_{j=1}^{t} x_j \Big)^{\zeta} + \psi_b \Big( h_b \sum_{j=1}^{t} (1 - x_j) \Big)^{\zeta}. \tag{5.15}$$

$\Lambda_t$ gives the technological frontier of the market. In the linear case $\zeta = 1$ it becomes

$$\Lambda_t = \lambda_{a0} + \lambda_{b0} + h_b \psi_b t + (h_a \psi_a - h_b \psi_b) \sum_{j=1}^{t} x_j. \tag{5.16}$$

Consider Proposition 5.4.1: if the technology gap $\lambda_t$ converges (condition 1), we have:

$$\Lambda_t = \lambda_{a0} + \lambda_{b0} + 2 \frac{h_a h_b \psi_a \psi_b}{h_a \psi_a + h_b \psi_b} t. \tag{5.17}$$

The rate rate of change of $\Lambda_t$ is the rate of technological progress $r$:

$$r = 2 \frac{h_a h_b \psi_a \psi_b}{h_a \psi_a + h_b \psi_b} = \frac{\eta_a \eta_b}{\eta_a + \eta_b}. \tag{5.18}$$

As long as $\zeta = 1$, all the quantities above depend on the products $h_i \psi_i \equiv \eta_i$. If one can make either $\eta_a$ or $\eta_b$ as large as desired, the rate $r$ is unbounded. But if there are limits to cumulation and/or to investments, there are also conditions that maximize $r$. With a linear constraint $\eta_a + \eta_b = E$, we have for the rate of technological progress

$$r = 2 \eta_a \Big( 1 - \frac{\eta_a}{E} \Big), \tag{5.19}$$

which is maximum in the symmetric case $\eta_a = \eta_b = E/2$. Since convergence of $\lambda$ (case 1 of Proposition 5.4.1) implies $x_t \sim \frac{\eta_b}{\eta_a + \eta_b}$ in the long run, technologies $a$ and $b$ converge to equal shares in this condition. Summarizing, under the assumption of linear cumulation of investments ($\zeta = 1$), if market segmentation persists, with both technologies growing so that $\lambda_t \to \lambda_\infty < \infty$, the rate of total technological progress $\lambda_t$ is higher for a maximally diversified technological market.

In all cases when $\lambda_t \to \pm\infty$, the rate of growth of the total technological level $\Lambda_t$ (rate of technological progress) is obtained with Eq. (5.16). If we are in case 2 of Proposition 5.4.1, the rate of growth of $\lambda_t$ is higher for $\eta_a < \eta_b$. If we are in case 3 of Proposition 5.4.1, the other way around is true, with a higher rate of technological progress for $\eta_a > \eta_b$.

Whenever $\zeta < 1$, the rate of technological progress is lower, but the results obtained above do not change as long as concavity is the same for both technologies. At this point it is left to understand what determines the competing technologies system to fall into one or the other of the three cases of Proposition 5.4.1, which amounts to understand the effect of externalities $\rho$ on technological progress. In order to do that, we rely on numerical observations with a simulation of the model (5.13). Consider the following example: cumulative technological investments are set with $h_a = 1, h_b = 0.8, \psi_a = 0.5, \psi_b = 1.2$, with concavity parameter $\zeta = 0.5$. Initial qualities are $\lambda_{a0} = 2$ and $\lambda_{b0} = 1$, and $\beta = 1$ for the intensity of choice. Fig. 5.12 reports the simulated time series of the share $x_t$, the technology gap $\lambda_t$ and the total technological level $\Lambda_t$, for three different externalities conditions, $\rho = 0.1$ (weak) $\rho = 1$ (medium) and $\rho = 10$ (strong). This example gives two main messages: first, network externalities strongly affect the long run technology shares (compare top and middle panels in the second column of Fig. 5.12 to the bottom panel), and second, strong externalities can lower the rate of technological progress (right column of Fig. 5.12). Initial events are important for the long run values: although the two technology start with equal shares, initially technology $a$ performs better (left column of Fig. 5.12). If network externalities and social interactions are weak, this pattern is halted thanks to a more effective *R&D* for technology $b$, that catches up first and outpaces technology $a$ (see the reversal of $x_t$ in the top and middle panels of the left column of Fig. 5.12). If such externalities are strong instead, the initial advantage of technology $a$

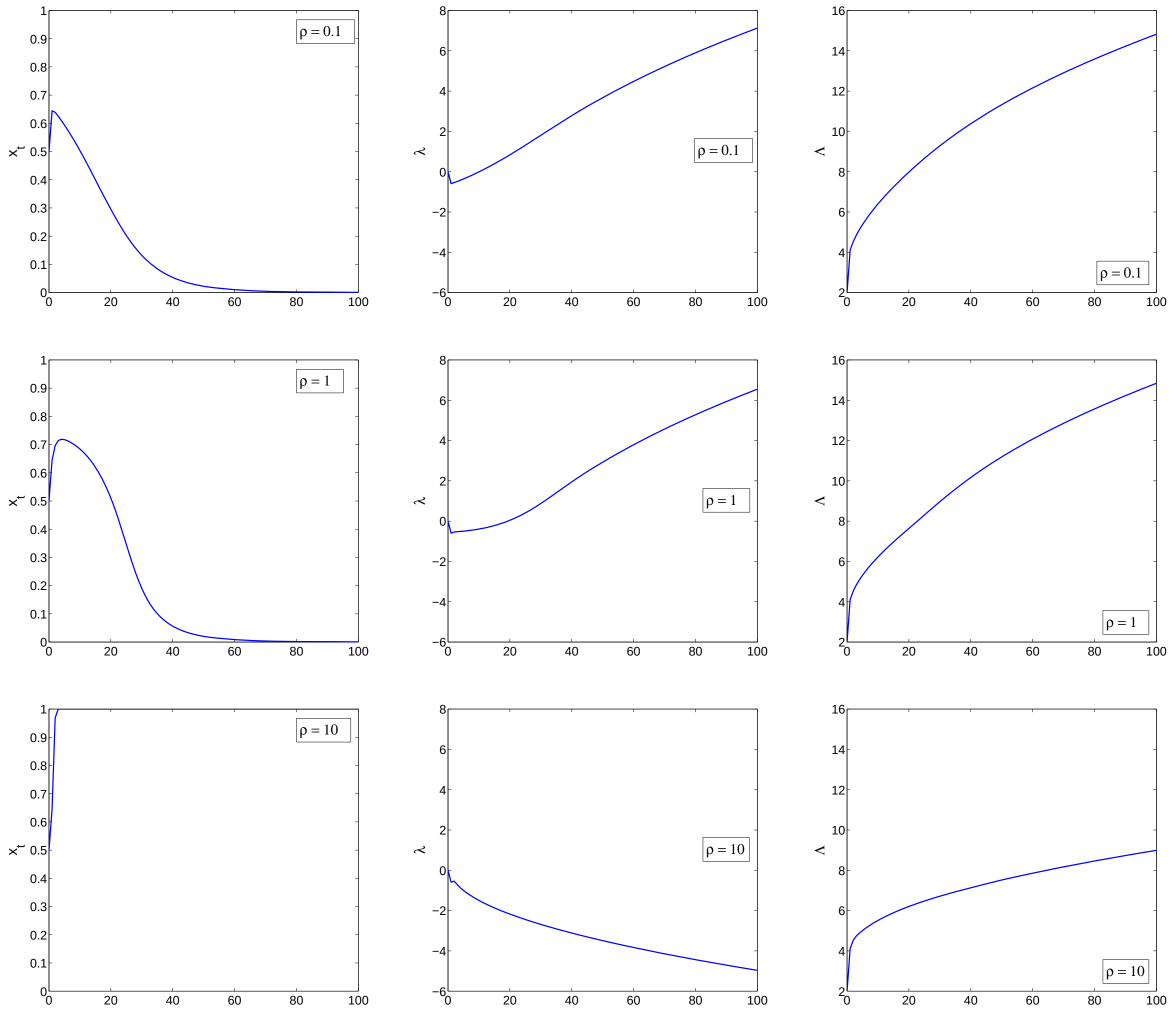

Figure 5.12: Time series of technology share $x_t$ (left), technological gap $\lambda_t$ (centre) and total technological level $\Lambda_t$ (right), with low externalities, $\rho = 0.1$ (top), medium externalities, $\rho = 1$ (centre) and strong externalities, $\rho = 10$ (bottom). $\beta = 1$, $\lambda_{a0} = 2$, $\lambda_{b0} = 1$, $h_a = 1, h_b = 0.8, \psi_a = 0.5, \psi_b = 1.2$, $\zeta = 0.5$.

weights more, and the better $R\&D$ process of $b$ is not enough to outpace technology $a$.

The examples of Fig. 5.12 show that network externalities and social interactions amplify technological advantages. Initial values of profitability ($\lambda_0$) play their role early, while $R\&D$ ($h_i, \psi_i$) needs time. $R\&D$ can reverse an initial technology gap, but if network externalities or social interactions are too strong, this may never happen. On the other hand, these externalities may help $R\&D$ in a technology transition, after $R\&D$ has covered the initial technological gap. This message is relevant to innovation policy, indicating that effort can diminish, as the desired technology acquires market shares .

## 5.5 Technological progress and environmental policy

In Section 5.3 we analyze the impact of an environmental policy on technological competition, assuming a constant profitability for the competing technologies. Now we introduce technological progress, bringing together the models of Section 5.3 and Section 5.4. Consider again two competing technologies, a clean and a dirty one, labeled with $c$ and $d$ respectively. Because of technological progress, the profitabilities $\lambda_{c,t}$ and $\lambda_{d,t}$ follow the learning curve (5.10), and the profitability gap $\lambda_t = \lambda_{d,t} - \lambda_{c,t}$ evolves according to Eq. (5.11). We assume that without intervention, the clean technology has a lower profitability, which means $\lambda_0 > 0$ at time $t = 0$. Let alone, the market would converge to a complete dominance of the dirty technology. A government steps in, enforcing an environmental policy to foster the market share of the clean technology, by reducing the profitability gap $\lambda$ through subsidies. We consider two options: first, a subsidy proportional to the market share of the dirty technology, $\sigma_t = \sigma(1 - x_t)$. This is the policy studied in Section 5.3, to which we refer as policy $I$. Second, a subsidy linked to the technology gap $\lambda_t$, i.e. to the technology learning curves, shaped by the endogenous technological progress. This feature is commonly present in Feed-in-Tariffs (Lipp, 2007). Germany is a paradigmatic example, where the tariff payed by utilities to renewable energy producers is adjusted to production costs (ResAct, 2000). The idea is that subsidies decrease as the production costs of clean energy go down. This is intended to foster permanently the clean technology, through scale effects as well as technological progress. We model this policy with a subsidy proportional to the previous period profitability gap, $\sigma_t = \sigma\lambda_{t-1}$, and we refer to this as policy $II$.

Environmental policy $I$ modifies the profitability gap $\lambda_t$ as follows:

$$\lambda_t^{\sigma I} = \lambda_t - \sigma(1 - x_t), \tag{5.20}$$

with $\lambda_t$ the gap without policy, given by Eq. (5.11). The profitability gap $\lambda_t^{\sigma I}$ changes due to technological progress and to the environmental policy (Eq. 5.5). Equipped with the profitability gap of Eq. (5.20), the discrete choice mechanism works exactly as before: the differential utility (5.3) becomes

$$\begin{aligned} u_{b,t} - u_{a,t} &= \lambda_t^{\sigma I} + \rho(1 - 2x_t) \\ &= \lambda_t - \sigma(1 - x_t) + \rho(1 - 2x_t), \end{aligned} \tag{5.21}$$

and the map for the share of clean technology $x_t$ (Eq. 5.4) is

$$x_t = \frac{1}{1 + e^{\beta[\lambda_t^{\sigma I} + \rho(1-2x_{t-1})]}} \equiv f_t^{\sigma I}(x_{t-1}). \tag{5.22}$$

These two equations are to be compared to Eq. (5.3) and Eq. (5.4) of Section 5.2 (basic model), to Eq. (5.6) and Eq. (5.7) of Section 5.3 (environmental policy) and to Eq. (5.12) and Eq. (5.13) of Section 5.4 (technological progress).

We simulate the model under different conditions. We consider a setting in which the profitability of the clean technology without policy is half the profitability of the dirty technology, with an initial condition $\lambda_{c0} = 1$ and $\lambda_{d0} = 2$. Now the model contains three factors: the positive feedback of social interactions and/or network externalities, an environmental policy and technological progress. With this model we address the following questions: first, how much the rate of technological progress matters in the effort of unlocking the market from the lock-in into the dirty technology. Second, how strict an environmental policy must be to achieve this target. Third, what is the role of social interactions and network externalities in this dynamics. Far from attempting an exhaustive study of the model under all possible conditions, we restrict our analysis to two scenarios, one where the two technologies have the same rate of technological progress, and one where the clean technology grows faster. Let us start with equal technological progress for the clean and dirty technologies (scenario $A$). This means to set $h_c = h_d$ and

$\psi_c = \psi_d$ in Eq. (5.11), that we rewrite:

$$\lambda_t = \lambda_{d,t} - \lambda_{c,t} = \lambda_0 + \psi_d \left( h_d \sum_{j=1}^{t} (1 - x_j) \right)^{\zeta} - \psi_c \left( h_c \sum_{j=1}^{t} x_j \right)^{\zeta}. \tag{5.23}$$

We consider three levels of policy efforts, with subsidies $\sigma = 0, 0.3, 0.9$, and two different intensities of positive feedback from social interactions and network externalities, $\rho = 0.1$ (weak externalities) and $\rho = 1$ (strong externalities). Fig. 5.13 reports the time series of the share of the clean technology under these different conditions. This simulation tells

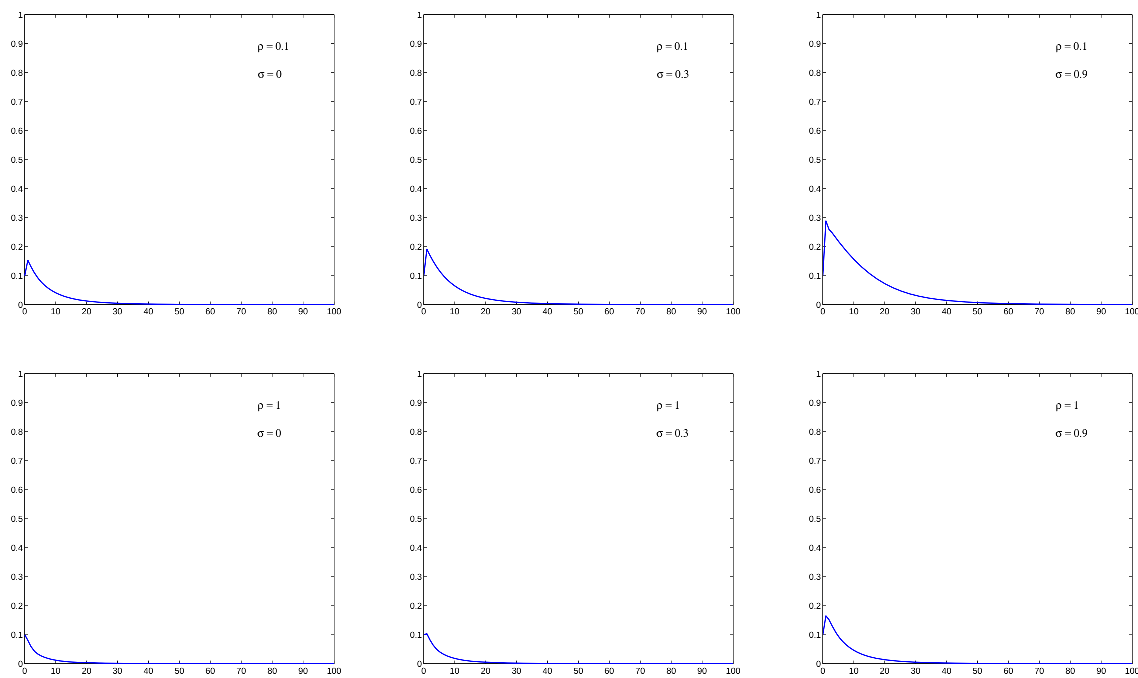


Figure 5.13: Time series of $x_t$ (share of clean technology) with policy $I$. Scenario A: $h_c = h_d = 1$, $\psi_c = \psi_d = 1$. Top: $\rho = 0.1$ (weak externalities). Bottom: $\rho = 1$ (strong externalities). Left: $\sigma = 0$. Centre: $\sigma = 0.3$. Right: $\sigma = 0.9$. Other parameters are $\beta = 1$, $\lambda_{c0} = 1$, $\lambda_{d0} = 2$, $\zeta = 0.5$.

us two things: first, the environmental policy is not able to unlock the market from the lock-in into the dirty technology, which conquers all the market with $x_t$ converging to $x = 0$ ("bad" equilibrium). Second, strong externalities make this process faster.

We consider a second scenario (scenario $B$) where the clean technology has a higher rate of progress, with $h_c > h_d$ (larger investments in innovation per firm) and $\psi_c > \psi_d$ (higher return to investment in terms of profitability). Fig. 5.14 shows the simulation results under the same conditions considered for the scenario $A$. Now for three out of

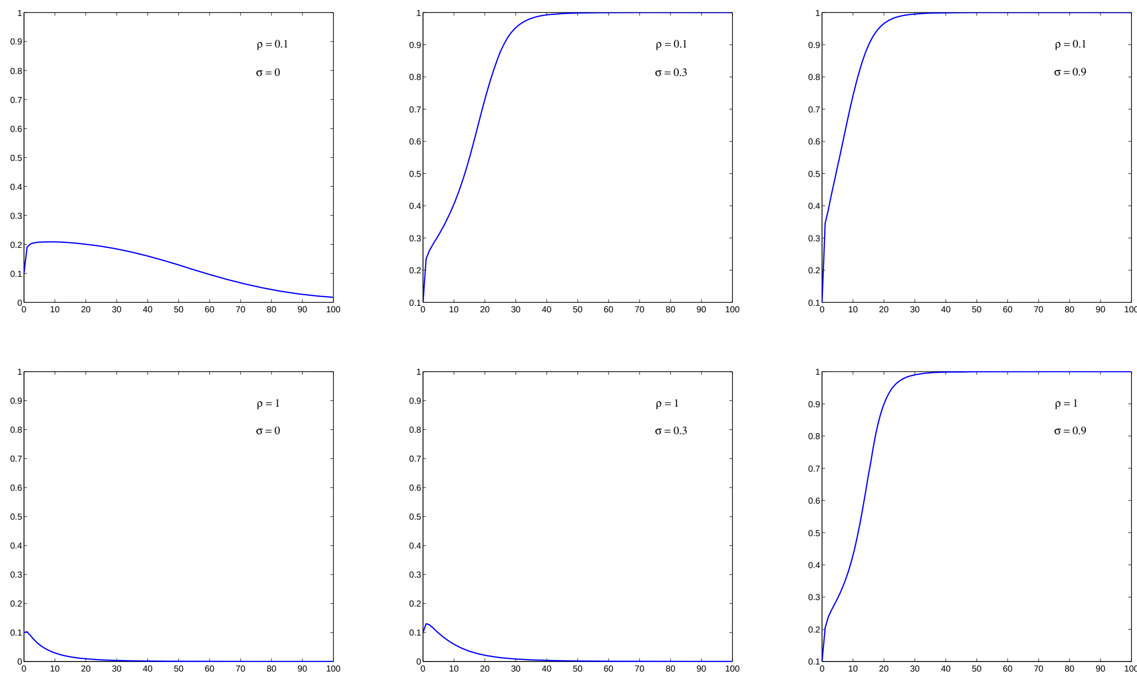


Figure 5.14: Time series of $x_t$ (share of clean technology) with policy $I$. Scenario B: $h_c = 1.5$, $h_d = 1$, $\psi_c = 1.5$, $\psi_d = 1$. Top: $\rho = 0.1$ (weak externalities). Bottom: $\rho = 1$ (strong externalities). Left: $\sigma = 0$. Centre: $\sigma = 0.3$. Right: $\sigma = 0.9$. Other parameters are $\beta = 1$, $\lambda_{c0} = 1$, $\lambda_{d0} = 2$, $\zeta = 0.5$.

six conditions the clean technology overcomes the dirty one, and the system converges to the socially desirable equilibrium $x = 1$. Without environmental policy the market always converges to the sub-optimal equilibrium $x = 0$. When externalities are weak, even a medium level of policy stringency ($\sigma = 0.3$) is sufficient to unlock the market from the dirty technology (top-centre panel). With strong externalities a stronger effort is needed (bottom-left panel). From the analysis of scenarios $A$ and $B$ we draw the following conclusions: an environmental policy alone is not able to foster permanently the clean technology. Neither a faster rate of progress of the clean technology is able to do that. Only the combination of faster progress and environmental policy achieves this goal, and tips the market from the sub-optimal to the desirable equilibrium. Stronger network externalities and/or social interactions make this process more difficult and call for a tougher environmental policy.

The second environmental policy that we consider, policy $II$, is based on learning curves (5.10). Let assume that in each period a government subsidizes the clean technology proportionally to the profitability gap in the previous period. The gap at time $t$ becomes:

$$\lambda_t^{\sigma II} = \lambda_t - \sigma\lambda_{t-1}^{\sigma II}, \tag{5.24}$$

where $\lambda_t$ is again given by Eq. (5.23). The profitability gap (5.24) should be compared to Eq. (5.20) of policy $I$. It is convenient to re-write $\lambda_t$ as $\lambda_t = \lambda_0 + \Delta\Psi_t$, with $\lambda_0$ the initial condition, and $\Delta\Psi_t$ the differential endogenous technological progress of the two technologies (second and third term of Eq. 5.23):

$$\Delta\psi_t = \psi_d\left(h_d\sum_{j=1}^{t}(1-x_j)\right)^{\zeta} - \psi_c\left(h_c\sum_{j=1}^{t}x_j\right)^{\zeta}, \tag{5.25}$$

with the assumption $\Delta\psi_0 = 0$. The profitability gap $\lambda_t^{\sigma II}$ can then be expressed as follows:

$$\lambda_t^{\sigma II} = \lambda_0 + \Delta\Psi_t - \sigma\lambda_{t-1}^{\sigma II}. \tag{5.26}$$

By iterative substitution of lagged terms, we get to the following expression for $\lambda_t^{\sigma II}$:

$$\lambda_t^{\sigma II} = \lambda_0\sum_{i=0}^{t}(-\sigma)^i + \sum_{j=0}^{t}(-\sigma)^j\Delta\Psi_{t-j}. \tag{5.27}$$

The first term in the right hand side is a geometric series. If $\sigma < 1$ (but positive, by definition) it is equal to $\lambda_0\frac{1-(1-\sigma)^{t+1}}{1+\sigma}$, and in an infinite time it converges to $\frac{\lambda_0}{1+\sigma}$. The intuition is that policy $II$ has a "contrarian" attitude, as Eq. (5.24) shows, and by reducing the technological gap it tends to stabilize it.[1] In the meantime $\Delta\psi_t$ continues to evolve due to (endogenous) technological progress, as described by Eq. (5.25), growing

[1]If $\sigma = 1$, this term is equal to $\lambda_0$ when $t$ is even, and zero otherwise. Values of $\sigma$ larger than one make the geometric series non convergent, but they are not realistic in that $\sigma > 1$ would mean a complete reversal of the technological gap in only one period of policy action.

positive or negative, or converging to a finite value (see Proposition 5.4.1). In all cases where the gap $\Delta\psi$ diverges, the policy intervention gets amplified by such differential technological progress, as indicated by the second term of the right hand side in Eq. (5.27): environmental policy and technological progress do not simply add together, but interact dynamically. Such nonlinear interaction is the main feature of this model of technology competition with environmental policy.

The difference of utilities (5.3) for the model with policy $II$ is:

$$u_{d,t} - u_{c,t} = \lambda_t^{\sigma II} + \rho(1 - 2x_t), \tag{5.28}$$

and according to Eq. (5.2) the map of the share $x_t$ becomes

$$x_t = \frac{1}{1 + e^{\beta[\lambda_t^{\sigma II} + \rho(1-2x_{t-1})]}} \equiv f_t^{\sigma II}(x_{t-1}). \tag{5.29}$$

These can be compared to Eq. (5.21) and Eq. (5.22) of policy $I$, to Eq. (5.3) and Eq. (5.4) of Section 5.2 (basic model) to Eq. (5.6) and Eq. (5.7) of Section 5.3 (environmental policy) and to Eq. (5.12) and Eq. (5.12) of Section 5.4 (technological progress).

We do for policy $II$ the same set of simulations of policy $I$. Fig. 5.15 reports the results for scenario $A$ (equal rate of progress for the clean and the dirty technologies), and Fig. 5.16 refers to scenario $B$ (faster progress for the clean technology). The simulation results for policy $II$ are in line with the results for policy $I$, which indicates that the two policies are not substantially different. Policy $II$ is characterized by transitory oscillations of the market share $x_t$, and that unlocking of the market from the dirty technology occurs somewhat more slowly than with policy $I$, when $\sigma = 0.9$. This two considerations would suggest to prefer policy $I$, although our simulation experiments are by no means exhaustive, having considered only a few particular - realistic though - settings and scenarios. The choice between one type of policy or the other is probably going to be dictated by pragmatic reasons, as for instance considerations on whether it is

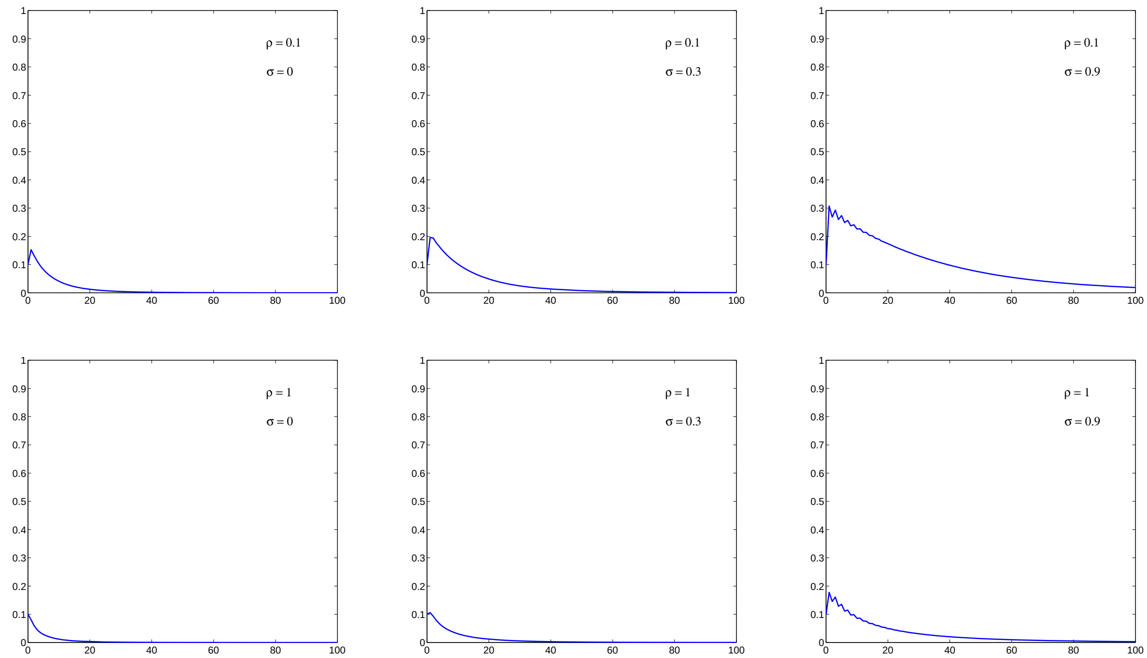


Figure 5.15: Time series of $x_t$ (share of clean technology) with policy $II$. Scenario A: $h_c = h_d = 1$, $\psi_c = \psi_d = 1$. Top: $\rho = 0.1$ (weak externalities). Bottom: $\rho = 1$ (strong externalities). Left: $\sigma = 0$. Centre: $\sigma = 0.3$. Right: $\sigma = 0.9$. Other parameters are $\beta = 1$, $\lambda_{c0} = 1$, $\lambda_{d0} = 2$, $\zeta = 0.5$.

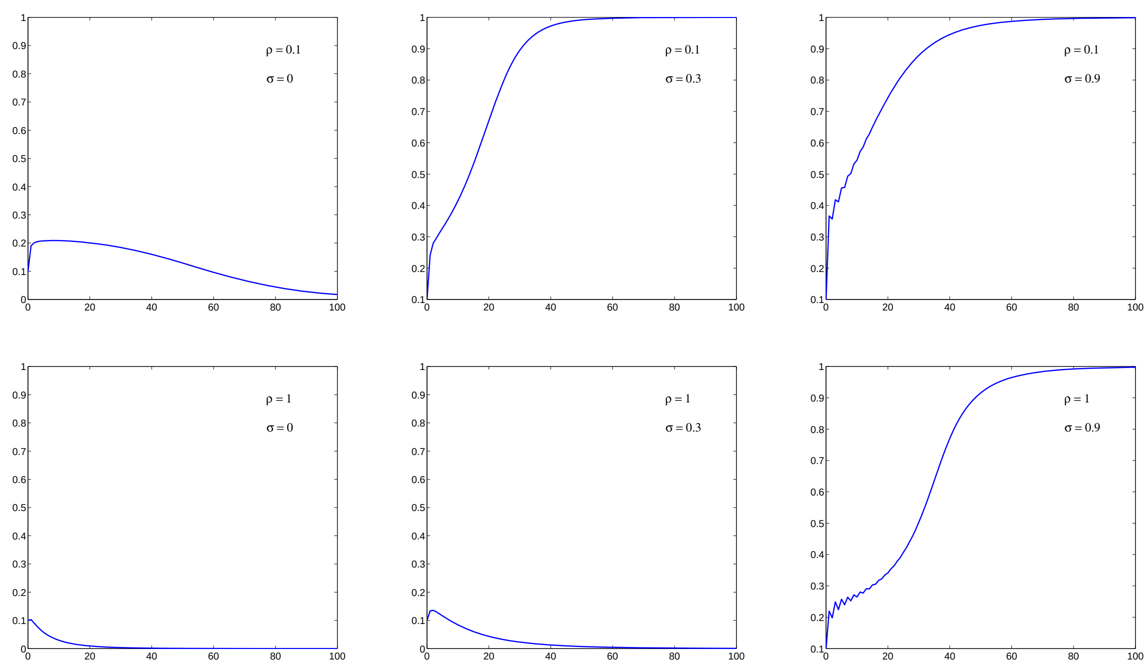


Figure 5.16: Time series of $x_t$ (share of clean technology) with policy $II$. Scenario B: $h_c = 1.5$, $h_d = 1$, $\psi_c = 1.5$, $\psi_d = 1$. Top: $\rho = 0.1$ (weak externalities). Bottom: $\rho = 1$ (strong externalities). Left: $\sigma = 0$. Centre: $\sigma = 0.3$. Right: $\sigma = 0.9$. Other parameters are $\beta = 1$, $\lambda_{c0} = 1$, $\lambda_{d0} = 2$, $\zeta = 0.5$.

easier to measure the market share $x$ or the profitability gap $\lambda$.

## 5.6 Conclusion

The present chapter proposes the following contributions: a discrete choice model of technological competition, with social interactions beside network externalities as explanation of positive feedback in technology choices; the analysis of the interplay between technology competition and technological progress, and consequently the effect of social interactions and/or network externalities on technological progress; finally, an environmental economics application of the technology competition model, with the introduction of an environmental policy, with and without technological progress.

There are some limitations in this model and in its various extensions. First of all, entry of new technologies is excluded, and competition is limited to the initial pool of technologies. On the theoretical side, there is the limitation inherent to adopting a "mean-field" approach, where the population of agents is indefinitely large and their interactions are random and homogeneous. Any network structure is missing here, such as possible reference groups, institutions and large corporations that can influence agents decisions. Finally, technologies are described in a very stylised way, without any sector or market specific feature. On the other hand, because of this abstraction, the model proposed here is not limited to competing technologies, but can also describe the competition of different freeware products that are based on the same technology, such as web browsers for instance. More generally it addresses all situations of product and firm competition where a price is not defined or not relevant, and other causes lie behind shares dynamics beside product performance, such as network externalities and social interactions.

The first and basic version of the model focuses on the equilibria of the model, with a series of analytical results and numerical observations about the qualitative changes (bifurcations) in dynamics and the transitions from one to multiple equilibria that follow from changes in the parameters. In particular, one parameter measures the effect of social interactions and network externalities. Social interactions can be important in technology competition, especially when network externalities are weak, as it is the case in the competition of hi-tech products as web browsers, for instance.

A first extension (Section 5.3) introduces an environmental dimension in the model, with a policy that subsidizes the clean technology. This version of the model is relevant in all cases where the competing technologies present some degree of pollution. In general an environmental policy shifts the market equilibrium reducing the share of dirty technology. In cases of multiple equilibria, the environmental policy can flip the market from the "sub-optimal" to the socially desirable equilibrium where the clean technology is dominant. There are cases where a tougher policy produces cycles of period two, with the clean and the dirty technology alternating as the dominant technology. This is the result of a policy that just "follows" pollution, without creating the conditions for systematic catch-up of clean technologies through technological progress.

In order to account for possible stickiness in agents decision we introduce asynchronous updating of strategies. This is relevant in all cases where switching technology is difficult and costly, as in power generation, for instance. The main result here is that although asynchronous updating stabilises the system by reducing the amplitude of oscillations, it may trigger chaotic behaviour by making the map of the system non-monotonic.

The model is extended in a different direction in Section 5.4, with technological progress. With this extension we propose a discrete choice model that combines technological competition and technological growth. The main focus of this section are the effects of social interactions and network externalities on technological progress. There are cases where these externalities lower technological progress overall.

Technological progress and environmental policy are brought together in a further extension which combines Section 5.3 and Section 5.4. This version of the model gives the following main results: an environmental policy alone is not capable of unlocking the market from the dirty technology. In order to tip the system to the socially desirable equilibrium, the clean technology must have a higher rate of progress. This calls for an innovation policy besides the environmental policy. Moreover, a tougher environmental policy is needed whenever the positive feedback of social interactions and network externalities is stronger. This fact suggests to work also on the side of network externalities, in order to lower policy costs. This can be done for instance by removing technological standards and by making technological infrastructures more flexible.

# Appendix

## 5.A Analysis of equilibria in the basic model

Consider the map (5.4) for the basic model of Section 5.2:

$$f(x) = \frac{1}{1 + e^{\beta[\lambda+\rho(1-2x)]}}. \tag{5.30}$$

The first derivative of $f$ is:

$$f'(x) = \frac{2\beta\rho e^{\beta[\lambda+\rho(1-2x)]}}{\{1 + e^{\beta[\lambda+\rho(1-2x)]}\}^2}. \tag{5.31}$$

Since $f$ is continuous in $[0,1]$ and $f(x) \in [0,1]$ $\forall x \in [0,1]$, then $f$ has at least one fixed point $x = f(x) \in [0,1]$, which is proved by applying the Bolzano's theorem to the function $g(x) = f(x) - x$. This means that at least one equilibrium exists. Moreover, since $f'(x) > 0$ for all $x \in [0,1]$, $f(x = 0) > 0$ and $f(x = 1) < 1$, there is at least one stable equilibrium, by the Mean-value theorem.

The second derivative of the map (5.4) is:

$$f''(x) = \frac{4\rho\beta^2 e^{\beta[\lambda+\rho(1-2x)]}\big[e^{\beta[\lambda+\rho(1-2x)]} - 1\big]}{\{1 + e^{\beta[\lambda+\rho(1-2x)]}\}^3}. \tag{5.32}$$

The condition $f''(x) = 0$ gives the inflection point $\hat{x} \equiv \frac{\rho+\lambda}{2\rho}$, with $f''(x) > 0$ in $[0, \hat{x})$ and $f''(x) < 0$ in $(\hat{x}, 0]$. The inflection point $\hat{x}$ does not depend on $\beta$. If $\lambda > \rho$, then $\hat{x}$ is outside the interval $[0,1]$, and there can not be more than one fixed point for $f$. Similarly, if $\lambda < -\rho$. This is why $-\rho < \lambda < \rho$ is a necessary condition for multiple equilibria of $f$.

The steepness of function $f$ in the inflection point is $f'(\hat{x}) = \frac{\rho\beta}{2}$. Since this is the point where $f'$ is maximum, $\rho\beta > 2$ is a necessary condition for multiple equilibria.

## 5.B Environmental policy with pollution tax

In Section 5.3 we study the competition of "dirty" and "clean" technologies in the presence of subsidies for the clean technology. Consider an environmental policy that is enforced through a pollution tax, instead. Assume the tax is proportional to the pollution level. If $\tau$ is the pollution tax rate, the cost of clean and dirty technologies are $c_c = c_{c0} + \tau e_c x$ and $c_d = c_{d0} + \tau e_d(1-x)$, where $e_c$ and $e_d$ are the pollution intensities of the clean and the dirty technologies, respectively. By assumption, $e_d > e_c$. The difference in profitability is $\lambda^\tau(x) = \lambda_0 + \tau[(e_c + e_d)x - e_d]$ and the difference in utility becomes $u_d - u_c = \lambda_0 + \rho(1-2x) + \tau[(e_c + e_d)x - e_d]$. Using $\lambda^\tau$ instead of $\lambda$ in Eq. (5.4), the map of the system becomes

$$f_\tau(x) = \frac{1}{1 + e^{\beta\{\lambda_0 + \rho(1-2x) + \tau[(e_c+e_d)x - e_d]\}}}. \tag{5.33}$$

This is the same type of function that we obtain with subsidies, (Eq. 5.7). Without policy (or with zero emission) the map $f_\tau$ coincides with the map (5.4) of the basic model.

## 5.C Analysis of equilibria for the extended models

The map of the basic model (5.4) and the maps of the extensions (5.7) and (5.33) can be written in the following general form:

$$f_{a,b}(x) = \frac{1}{1 + e^{a-bx}}. \tag{5.34}$$

The first derivative of this map is

$$f'_{a,b}(x) = \frac{be^{a-bx}}{(1 + e^{a-bx})^2}. \tag{5.35}$$

The value of $b$ determines whether the map is upward or downward sloping. In the case of the basic model we have $b = 2\beta\rho$, which means the map $f$ is always upward sloping. In the case of an environmental policy with subsidies (Eq. 5.7) we have $b = \beta(2\rho - \sigma)$. Consequently the map $f^{\sigma}$ is downward sloping whenever $\sigma > 2\rho$. The model with a pollution tax (Eq. 5.33) has $b = \beta[2\rho - \tau(e_c + e_d)]$. In this case the map $f^{\tau}$ is downward sloping as soon as $\tau > \frac{2\rho}{e_c+e_d}$. In both cases, a higher policy effort (larger subsidies $\sigma$ or higher tax $\tau$) has the following effect:

- weak policy effort ($b > 0$): increasing the effort ($\sigma$ or $\tau$) a transition occurs from three steady states, two of which are stable, to one stable steady state.

- strong policy effort ($b < 0$): increasing the effort ($\sigma$ or $\tau$) is destabilizing, with a transition from a stable equilibrium to a stable period 2 cycle.

Network externalities as well as social interactions have an opposite effect, because a larger $\rho$ increases the value of the first derivative. Put differently, the environmental policy counters the positive feedback of these externalities. This is because such policy has been modelled with an effort inversely proportional to the share of the technology that it aims at fostering (Eq. 5.5).

The second derivative of (5.34) is

$$f''_{a,b}(x) = b^2 e^{a-bx} \frac{(e^{a-bx} - 1)}{(e^{a-bx} + 1)^3}. \tag{5.36}$$

The second derivative is zero in the flex point $\hat{x} = \frac{a}{b}$, where the first derivative $f'_{a,b}(\hat{x}) = \frac{b}{4}$ is maximum in absolute terms. For the basic model we have:

$$\hat{x} = \frac{\lambda_0 + \rho}{2\rho}, \qquad f'(\hat{x}) = \frac{\beta\rho}{2}. \tag{5.37}$$

For the model with subsidies we have:

$$\hat{x}_\sigma = \frac{\lambda_0 + \rho - \sigma}{2\rho - \sigma}, \qquad f'_\sigma(\hat{x}) = \frac{\beta(2\rho - \sigma)}{4}. \tag{5.38}$$

The model with a pollution tax presents:

$$\hat{x}_\tau = \frac{\lambda_0 + \rho - \tau e_d}{2\rho - \tau(e_c + e_d)}, \qquad f'_\tau(\hat{x}) = \frac{\beta(2\rho - \tau(e_c + e_d))}{4}. \tag{5.39}$$

The effect of the intensity of choice is the following:

- weak policy effort ($b > 0$, map upward sloping): increasing $\beta$ makes the map more S-shaped, possibly leading to two stable steady states.

- strong policy effort ($b < 0$, map downward sloping): increasing $\beta$ makes the map more similar to an inverse S, eventually leading to period 2 cycles.

Not only the value of the derivative, but also the position of the flex point is important to dictate the dynamics of the system. The effect of policy effort on the flex point is given by the following derivative:

$$\frac{d\hat{x}}{d\sigma} = \frac{\lambda_0 - \rho}{(2\rho - \sigma)^2}. \tag{5.40}$$

No matter whether the map is upward or downward sloping, the effect of raising subsidies is to shift $\hat{x}_\sigma$ to the right whenever $\lambda_0 > \rho$, and to the left otherwise. The effect of this shift on the stability of equilibria is ambiguous, though, because it depends on whether the map $f_\sigma$ is upward or downward sloping.