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2 **Supplementary Information for**

3 **Understanding the co-evolution of mask-wearing and epidemics : a network perspective**

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8 **This PDF file includes:**

9 Figs. S1 to S28

10 Tables S1 to S2

11 SI References

12 In the Supplementary document, we provide (i) formal definition of the network model and the ODE model; (ii)
 13 an overview of the existing work on complex contagions, the threshold model and multi-contagion frameworks; (iii)
 14 Methodology in selecting baseline parameters; (iv) Results for the robustness of the critical transitions for the network
 15 model; (v) Additional results for the ODE model.

16 **Reproducibility:** The analysis code are publicly available at:
 17 <https://github.com/BridgelessAlexQiu/Mask-Disease-Multilayer>.

18 The Network Model of Dueling Contagions

19 Let $\mathcal{A} = \{a_1, a_2\}$ be the set of two contagions in which a_1 is the social contagion *mask-wearing*, and a_2 is a *disease*.
 20 We model the dissemination of the dueling contagions using a *graphical dynamical system* (1). Specifically, a graphical
 21 dynamical system \mathcal{S} is defined as a pair $(\mathcal{M}, \mathcal{F})$ where

- 22 (i) $\mathcal{M} = \{G_1, G_2\}$ is a two-layer network for which $G_1 = (V, E_1)$ is the the social layer, and $G_2 = (V, E_2)$ is the the
 23 biological layer. Layers G_1 and G_2 are defined over the **same** synthetic population V of size n , and they differ
 24 by the set of edges. The contagion a_i spreads only on G_i , $i = 1, 2$.
- 25 (ii) $\mathcal{F} = \{f_{i,j} : a_i \in \mathcal{A}, v_j \in V\}$ is a collection of functions for which $f_{i,j}$ is the **local transition function** of
 26 vertex $v_j \in V$, $1 \leq j \leq n$ for the contagion a_i , $i = 1, 2$. In general, $f_{i,j}$ specifies how v_j updates its state for the
 27 contagion a_i .

28 An individual is associated with two states: (i) a binary *social* state which indicates his/her decision in mask-wearing
 29 (i.e., state M represents wearing a mask, and state NM represents not wearing a mask), and (ii) a *disease* state from
 30 $\{S, I, R\}$. At each time step, all individuals update their social and biological states *synchronously*. In the following
 31 sections, we define the local functions (i.e., update rules) for the social contagion and the biological contagion.

32 **Update rules for the social contagion.** The dynamics of the social contagion a_1 is captured by the threshold model (2).
 33 Specifically, we consider the following three factors in triggering an individual v to wear a mask at time* $t > 1$:

- 34 (i) *Peer pressure:* Let $\psi_1^{(t-1)}(v) \in [0, 1]$ be the fraction of v 's neighbors in G_1 who wear masks at time $t - 1$.
- 35 (ii) *Fear:* Let $\psi_2^{(t-1)} \in [0, 1]$ be the fraction of infected individuals by the disease at time $t - 1$.
- 36 (iii) *Prosociality:* Let $x(v) \in \{0, 1\}$ be an indicator random variable, for which $x(v) = 0$ if v is not prosocial, and
 37 $x(v) = 1$ if v is prosocial. Note that $x(v)$ is an initial condition of v that does not change with time.

38 In addition, an individual v has two thresholds defined as follows:

- 39 (i) *Sensitivity to peer pressure:* Let $\tau_1(v) \in [0, 1]$ be the peer pressure threshold such that v chooses to a mask at
 40 time t if $\psi_1^{(t-1)}(v) \geq \tau_1(v)$, that is, the fraction of the v 's mask-wearing neighbors at time $t - 1$ is at least $\tau_1(v)$.
- 41 (ii) *Fear for the disease:* Let $\tau_2(v) \in [0, 1]$ be the fear threshold such that v chooses to a mask at time t if
 42 $\psi_2^{(t-1)} \geq \tau_2(v)$, that is, the fraction of the overall infections is at least $\tau_2(v)$.

43 At each time step t , $t > 1$, an individual v chooses to wear a mask if and only if at least one of the following three
 44 conditions is satisfied: (i) $\psi_1^{(t-1)}(v) \geq \tau_1(v)$, (ii) $\psi_2^{(t-1)} \geq \tau_2(v)$, or (iii) $x(v) = 1$. In other words, an individual's
 45 mask-wearing behavior can be triggered by either peer pressure, fear, or prosociality. If none of the above conditions
 46 are satisfied, however, v chooses not to wear a mask at time t . Please see the related work section for additional
 47 motivation of the model.

*We assume that the dynamics starts on time step 1.

48 **The local function.** To formally state the social dynamics, let $\mathbb{B}_1 = \{M, NM\}$ be the domain of the two states (i.e.,
 49 **Mask** or **No-Mask**) for the social contagion a_1 . The local function $f_{1,v}(v, t)$ computes the mask-wearing state of $v \in V$
 50 at time step t . In particular,

$$f_{1,v}(v, t) = \begin{cases} M & \text{if } [\psi_1^{(t-1)}(v) \geq \tau_1(v)] \vee [\psi_2^{(t-1)} \geq \tau_2(v)] \vee x(v) \\ NM & \text{Otherwise} \end{cases}$$

51 where \vee represent the logical OR function.

52 **Update rules for the disease.** The dynamics of disease a_2 adapts the SIR model under a simple extension for which
 53 it accounts for the effectiveness of mask-wearing in decreasing the disease infection rate. Let the disease *baseline*
 54 *transmission probability* $p \in [0, 1]$ be the probability of infection for a susceptible individual s in contact with an
 55 infected neighbor i , under the condition that **both i and s do not wear masks**. At a time step $t \geq 1$, if i or s
 56 wears a mask, however, the infection probability for s decreases as follows

- 57 (i) If only the susceptible vertex s wears a mask at time t , p is decreased to αp , where $\alpha \in [0, 1]$ is a discounting
 58 factor.
- 59 (ii) If only the infected vertex i wears a mask at time t , p is decreased to βp , where $\beta \in [0, 1]$ is a discounting factor.
- 60 (iii) If both i and s wear masks at time t , the transmission probability becomes $\alpha\beta p$, where both discounting factors
 61 are applied.

Table S1. Transmission probability under effectiveness of mask-wearing

State of <i>susceptible</i>	State of <i>infected</i>	Transmission Probability
NO-MASK	NO-MASK	p
MASK	NO-MASK	αp
NO-MASK	MASK	βp
MASK	MASK	$\alpha\beta p$

63 The parameters α and β capture the effectiveness of masks. Note that α , β , and p are constants over the course of
 64 the pandemic.

65 **The local function.** To formally define the disease dynamics, let $\mathbb{B}_2 = \{S, I, R\}$ the set of disease states. The local
 66 function $f_{2,v}(v, t)$ is a probabilistic function that determines the state of $v \in V$ for a_2 at time step t . In particular,

- 67 (i) If v is susceptible at time $t - 1$, $f_{2,v}(\cdot)$ captures the probabilistic process for infection. Given a susceptible vertex
 68 $v \in V$, let $d_1^{(t-1)}$ be the number of v 's mask-wearing infected neighbors in G_2 , at time $t - 1$. Let $d_2^{(t-1)}$ denote
 69 the number of v 's mask-free infected neighbors in G_2 , at time $t - 1$. Then the probability of v gets infected at
 70 time t is defined as

$$Pr(v \text{ gets infected}) = \begin{cases} 1 - [(1 - \alpha\beta p)^{d_1^{(t-1)}} \cdot (1 - \alpha p)^{d_2^{(t-1)}}] & \text{If } v \text{ wears a mask at time } t \\ 1 - [(1 - \beta p)^{d_1^{(t-1)}} \cdot (1 - p)^{d_2^{(t-1)}}] & \text{Otherwise} \end{cases}$$

- (ii) If v is infected at time $t - 1$, then $f_{2,v}(\cdot)$ captures the probabilistic process for recovery. Under each time step,
 each infected vertex recovers with a fixed probability $r \in [0, 1]$, that is,

$$Pr(v \text{ recovers}) = r$$

- 71 (iii) If v is recovered, it remains recovered for all succeeding time steps.

72 Related Work

73 **Complex contagions and threshold models.** Social contagions such as *mask-wearing* behavior, public opinions, and
74 social norms can be broadly described as *complex contagions* whose adoptions require multiple sources of influence (3).
75 Centola and Macy (3) first introduced the concept of complex contagions in 2007. Specifically, based on Watts and
76 Strogatz’s small-world network model (4), Centola and Macy showed that having long ties in the network could
77 hinder the spread of complex contagions (3). Complex contagions have since been widely investigated in areas such as
78 social science (5, 6), public health (7, 8), and political science (9, 10). We refer reader to the work by Guilbeault et
79 al. (11) for a detailed review.

80 Complex contagion processes are commonly studied using *threshold models* (2, 11) where an individual adopts a
81 behavior if the fraction of immediate connections engaging in the behavior exceeds a personal threshold. Granovetter (2)
82 first proposes a threshold model to analyze the dynamics of collective human behaviors. The work by Granovetter (2)
83 considers the problem of determining the number of people taking each action (e.g., the number of people adopting/not
84 adopting the contagion) when the dynamics converges. He also investigates the stability of the convergence after
85 perturbations of the initial threshold distribution. Since its introduction by Granovetter, variations of the threshold
86 model on networks have been proposed and studied across numerous domains. Watts in his classic work (6) proposes
87 a simply yet powerful model to examine the cascades of contagions on random networks. In Watts’ model (6),
88 individuals update their states based on the threshold rule, where the heterogeneous thresholds are selected from
89 a distribution. Further, state updates happen in a random asynchronous order, and individuals’ adoptions of the
90 contagion are permanent throughout the dynamics (i.e., once an individual adopts the contagion, he/she continues to
91 adopt it over the succeeding time steps). Based on the proposed model (6), Watts explores the correlations between
92 the connectivity of a network and *rare* but *large* global cascades that are triggered by only small sets of initially active
93 vertices. Watts shows that when the network is sparse, the cascade is modulated mainly by the overall connectivity of
94 the network, where high-degree vertices are key triggers. In contrast, if the network is sufficiently dense, then the
95 cascade is modulated by the local connectivity of individuals, where vertices with degrees around the average are
96 key triggers. Our social contagion model can be seen as an extension of Watts’ model (6), with the following key
97 distinctions: (i) Watts’ dynamics is irreversible, that is, once an individual contracts the contagion (e.g., wears a
98 mask), the adoption is permanent throughout the entire dynamics. In contrast, the mask-wearing states of individuals
99 are *reversible* under our model, such that a person chooses **not** to wear a mask at a time step if none of the three
100 conditions are satisfied. (ii) individuals update states in random asynchronous order under Watts’ model, whereas
101 our model considers a synchronous update scheme. (iii) our model further intertwines the social dynamics with the
102 disease dynamics. We remark that the first distinction already implies a significant difference between the two models.
103 For example, one primary cause of the observed critical transition in our model is the distinct time-series dynamics
104 between the two regimes, shown in Fig 4 of the main manuscript. Note that, however, the oscillating dynamics of
105 infection and mask adoption (i.e., Fig 4 (a) and (c)) will not be observed if the mask-adoption is not reversible.

106 In another work by Dodds and Watts (12, 13), they propose a diffusion scheme that incorporates the mechanisms
107 of independent cascade models (i.e., independent interaction models as stated in (13)) and threshold models. In
108 particular, an individual v has a state in $\{S, I, R\}$ (or $\{S, I\}$ which depends on the model set up), and for each contact
109 with another infected individual, v receives a “dose” with some probability p . Each individual v then maintains
110 a *memory* of the accumulated doses over the past T contacts, and v contracts the contagion if *the accumulated*
111 *doses exceed a personal threshold*. They identify three classes of the dynamics (i.e., epidemic threshold, vanishing
112 critical mass, and critical mass) and investigate transitions between the classes (13). Watts and Dodds (14) later
113 studied the opinion formation processes by modeling the cascade of influence using a threshold scheme. They find
114 that global cascades are often triggered by the group of people who can be easily influenced (i.e., their threshold
115 values are relatively low), rather than the influential ones (14). Further, they consider various roles (e.g., initiators,
116 early adopters) of people with different influential levels (e.g., influencers, hyperinfluentials) and investigate the

117 corresponding effects on global cascades.

118 Ramazi et al. (15) use the threshold model to study the agent decision-making processes in coordination/anti-
119 coordination games. Note that when agents are anti-coordinating, a vertex v becomes active if the fraction of *inactive*
120 neighbors is at least a personal threshold of v . Ramazi et al. (15) show that under the asynchronous dynamics, the
121 system almost always converges to an equilibrium in finite time, irrespective to the underlying network topology.
122 Kuhlman et al. (16) introduce a “bi-threshold” model to study contagions (e.g., smoking behavior) where the adoption
123 and the abandonment of the contagion requires different threshold. In particular, each individual v has two thresholds
124 τ_{up} and τ_{down} , where v becomes active when at least τ_{up} neighbors are active, and v becomes inactive when at least
125 τ_{down} neighbors are inactive. Kuhlman et al. (16) explore the complexity of different problems (e.g., convergence) in
126 this model and performed simulation on real-world datasets. Huang et al. (17) propose a variation of the threshold
127 model that account for the effect of “persuasion”. Each individual in this model (17) has two thresholds where the
128 first threshold τ_1 is the adoption threshold (i.e., an individual becomes active if the fraction of active neighbors is at
129 least τ_1), and the second threshold τ_2 is the “persuasion threshold”. In particular, a vertex becomes a persuader if
130 the fraction of active neighbors is at least τ_2 , and a persuader can always successfully transmit the contagion to its
131 neighbors (17). They show that the network becomes less robust under the influence of persuaders. The threshold
132 model have also been studied in numerous contexts including evolution of discrete dynamical systems (18), networked
133 games (19), disaster management (20), diffusion of innovations (21) and marketing (22).

134 **Multi-contagion models.** Many existing methods model the concurrent spread of social and biological contagions. Fu
135 et al. (23) study the coupling of vaccination behaviors and the H1N1 flu epidemic in multiplex networks. In their
136 model, an individual is in one of the following states $\{S, I, R, V\}$ with some probability at each time step. The model
137 is then a Markov chain over state space of 4^n state. They found that the vaccination decisions are affected by the
138 vaccination states of neighbors as well as the prevalence of the disease (23). We stretch differences between Fu’s
139 model and our proposed model. First, the social and biological states are mutually exclusive in Fu et al., whereas in
140 our proposed model, an individual simultaneously has a social state and a biological state. At the same time, the role
141 of peer pressure in making vaccination decisions for a vertex is probabilistic, with a positive correlation to the number
142 of vaccinated neighbors. In contrast, our model considers the threshold behaviors in obeying social norms. Moreover,
143 fear is modeled probabilistically in Fu et al. (23) such that more infected neighbors yield a higher probability of taking
144 vaccines. Note that such a generalization assumes the same fear threshold for all vertex. In our model, however, we
145 give the option to individualize each vertex with a different threshold of fear and peer pressure. Lastly, Fu et al. (23)
146 do not consider factors such as prosociality in triggering people to take vaccines (23).

147 Salathé and Bonhoeffer (24) also study the impact of vaccine opinions on the disease outbreak. Specifically,
148 each vertex is assigned an initial opinion about vaccination, either negative or positive. The opinion dynamic is a
149 probabilistic process (happens after opinion initialization) such that a vertex v changes its opinion under a probability
150 that is positively correlated to v ’s neighbors with opposite opinions. After the opinion formation, people with positive
151 opinions are vaccinated and become immune to the disease. The disease dynamic happens after vaccination which
152 is again a probabilistic process specified by a transmission rate (24). Campbell and Salathé (25) study the spread
153 of health behaviors and disease over the same network. Their model consists of two periods for which social and
154 biological contagions spread in different periods, that is, their dynamics are not concurrent. In adopting the social
155 behavior (refusal to take vaccines), they consider a threshold model based on the number of exposures to the neighbors
156 who have adopted the social behavior. They also account for the exposures from other sources such as the media (25).
157 Note that other factors such as prosociality or fear are not considered. After the social dynamic reaches a fixed point,
158 the second period begins for which the disease spreads under the SIR disease model. Their results suggest that under
159 the interplay of the social and disease dynamics, the structure of the underlying social network has a strong influence
160 over outbreak size. In particular, they find that the outbreak size is maximized under small-world networks (25).

161 Funk et al. (26) investigate how responsive behaviors to an ongoing epidemic can influence the disease dynamic. In
162 particular, they model the spread of awareness of the disease among the population by considering the number of
163 hops the epidemic-related information has traveled before reaching an individual. The information that undergoes
164 more passages then has a lower discounting factor on the transmissibility of the disease (26). Granell et al. (27)
165 consider a microscopic Markov chain approach to model the interrelation between social and biological dynamics over
166 multiplex networks. In particular, an individual becomes aware of the disease (which then takes countermeasures)
167 either by contacting awarded neighbors or getting infected by the disease. Moreover, awareness can be forgotten with
168 some probability. Lastly, the infectivity of disease-aware individuals gets discounted because of the health behaviors
169 which they have adopted. In another work by Funk et al. (28) on modeling the spread of awareness and diseases,
170 they use the SIS model for the social dynamic and the SIRS model for the disease dynamic. To incorporate the
171 interplay between the two contagions, the infection rate gets discounted for those with disease awareness. An aligned
172 work was conducted by Hébert-Dufresne (29) who also consider the concurrent spread of awareness and disease on
173 highly clustered networks. In particular, the social dynamics of awareness dampens the infection rate, and the disease
174 dynamics increases the transmission rate of awareness. They make an interesting case where an increase in the
175 clustering of networks could incur larger epidemics.

176 Epstein et al. (30) model the interacting dynamics of fear and the disease. In their method, an individual contracts
177 fear with a fixed per-contact probability from neighbors who are infected or who are afraid of the disease (or both).
178 In response to fear, people are removed from the circulation, and those who have been removed are put back into
179 circulation at some given rate. Overall, they observe multiple waves of infections under the proposed dynamics. Gross
180 et al (31) tackle the problem from another perspective for which a susceptible can take countermeasures by breaking a
181 link to an infected neighbor and rewiring to some other susceptible under a given probability. Their result shows that
182 under the adaptive rewiring, the network could be updated to two clusters of susceptible and infected, respectively.
183 Cencetti and Battiston (32) study a multiplex network model where contagions can disseminate across different layers.
184 They show that when the two layers are drastically different, the rate of the diffusion process is maximized when the
185 cross-layer interaction is high. They also suggest that the diffusion rate is determined by the layer with the least
186 velocity. Alvarez-Zuzek et al. (33) also investigate the co-evolution of the opinion about vaccination (as a social
187 contagion) and a disease over multiplex networks. In particular, they simulate the diffusion of the opinion about
188 vaccination using the M-model, which consists of two concurrent processes: persuasion and compromise. On the
189 other hand, the disease model is the SIRV model under the effectiveness of vaccination. They find that as the vaccine
190 gets more effective, the number of recovered individuals decreases since fewer infections occur. They also suggest a
191 threshold value of the vaccine effectiveness, such that the epidemic will not persist if the vaccine effectiveness is above
192 the threshold. In contrast, the epidemic will always develop despite the vaccine effectiveness when the transmission
193 probability is high enough (33). Jovanovski et al. (34) study the multi-contagion SIS model over multiplex networks.
194 They analyze the bounds of the basic reproduction number of the proposed dynamic and characterize the dependence
195 of the epidemic threshold to the network structure.

196 Many existing models study the interplay between human behaviors and diseases from economic perspectives.
197 Toxvaerd (35) proposes a framework where individuals can perform social distancing under a non-cooperative decision-
198 making pattern. Toxvaerd shows that the social dynamic reaches an equilibrium when the disease prevalence is at its
199 highest. Another work by Toxvaerd (36) considers a model under a decentralized decision pattern where individuals
200 can overexpose to disease. The results show a negative correlation between induced infection risks increases and
201 prevalence, subsequently, a high disease prevalence leads to a drop in the overall welfare. David et al. (37) extend the
202 SEIR model to explore how testing and quarantine impact the disease dynamic. In particular, they establish that a
203 large range of testing under targeted quarantine can effectively lessen the negative influence on the economy. There is
204 a plethora of other models that investigate the dueling dynamics and economic impacts (38–40).

205 **Behavioral responses.** We give a short overview of the literature supporting the factors that in our model triggers
206 individuals' mask-wearing behaviors: *peer pressure*, *fear for infection*, and *prosociality*.

207 **Peer pressure.** Extensive literature supports that behaviors are shaped by social norms (41, 42). Particularly,
208 research has shown that peer pressure plays an important role in people's mask-wearing decisions. For instance,
209 Bokemper et al. (43) conducted a study via online survey over U.S and Italy to understand the role of peer influence
210 in mask-wearing. Their results show that more people wearing masks enforce the social norm, leading to more
211 compliance among the population. Moreover, people sometimes judge those who do not wear masks which further
212 increases the social pressure (43). Another study by Hansstein and Echegaray (44) shows that college students wear
213 masks because it is "socially appropriate" to do so. In particular, their adaption of this prosocial behavior is greatly
214 influenced by friends and roommates. Similar results were given by Barcelo and Sheen (45) which suggests that
215 people are more likely to wear masks if they live in a region where mask-wearing is common. Lastly, DeJonckheere et
216 al. (46) conducted a survey for 1,087 youth. They considered different factors in mask-wearing. In particular, they
217 show that 15.7% of the participants wear masks because of peer pressure.

218 Social conformity can be captured by **threshold models**. In the classic work by Granovetter, a simple model is
219 considered where each people have a threshold of peer influences, and they decide to take action if the number of
220 peers who take actions exceeds the threshold (2). In another study, Wolf et al. (47) conducted social experiments on
221 college students in Germany to understand their decision-making process. They found that students use quorum
222 thresholds as a strategy, and this threshold is set based on the decisions made by people around them.

223 **Fear for infection.** Fear is an adaptive human response to unprecedented diseases like COVID-19 (48). The fear
224 for COVID-19 is triggered by different sources, including the media coverage of the number of confirmed cases and the
225 death toll. Bendau et al conduct a survey on 6,233 participants in Germany, where they find a positive correlation
226 between COVID-related media consumption and anxiety and depression. Furthermore, those with pre-existing fear
227 for health conditions are more likely to spend time watching the news which leads to a higher level of fear (49).
228 Similar results are suggested by Mertens et al. (50) where they give online surveys to 439 participants, and found that
229 media usage is associated with an increased level of fear. Other factors that cause fear include the health of others,
230 personal health, Food shortage, etc. Liu et al. (51) study the social enforcement process over multiplex networks.
231 They identify the co-existence of extinction and outbreak states both analytically and experimentally. More relevant
232 to our work, Liu et al. (51) also show the existence of a critical transition for the bi-stability. Fitzpatrick et al. (52)
233 conduct online surveys on over 10,368 individuals in U.S. Overall, they find that the general population is fearful
234 for COVID. In particular, they suggest that the level of fear is higher and more concentrated in urban areas and
235 areas with high numbers of cases, that is, the fear is not equally distributed. Also, certain demographics such as
236 female, Asian, Hispanic, etc have a higher level of fear (52). Different scales are designed to evaluate the level of fear
237 experienced by individuals. In particular, Ahorsu et al. (48) introduce a seven-item scale where they show a high
238 correlation between fear of COVID and depression.

239 People put on face masks in response to the fear for COVID-19. Huynh (53) conducted a survey on 345 individuals
240 in Vietnam and the results suggest a positive correlation between the fear for COVID-19 and the likelihood of wearing
241 masks. It is also shown that wearing masks can reduce the fear of infection and encourage people to participate in
242 daily activities (54). Nakayachi et al. (55) surveyed 1,000 participants to examine the factors that incentivized mask
243 adoption. Apart from a strong correlation between peer influence and mask usage, feeling relief from fear and anxiety
244 is the second most prominent factor that triggers mask-wearing (55).

245 **Prosociality.** Prosocial behaviors are defined as actions with motivations for promoting human welfare. Research
246 has shown that prosocial individuals exist in many societies (56–58). In this work, we assume that prosocial people
247 put on masks voluntarily.

248 Methodology in Selecting Baseline Parameters

249 This section presents the methodology in choosing the baseline values for social and disease parameters shown in
250 Table (S2). In general, we assume that a fraction of the population is of the prosocial type. Further, We allow fear
251 and peer pressure thresholds of individuals to be heterogeneous. Specifically, the peer pressure/fear threshold of each
252 individual is assigned uniformly at random from a range.

253 **The fraction of the prosocial population.** Given that the U.S. has no mask-wearing habit, we assume that no
254 individuals wear masks on the first day of the epidemic. To model society with a low prosociality level, we set 1% of
255 the population to be *prosocial individuals* who wear masks voluntarily during the epidemic period. In our numerical
256 experiments, we vary the population's prosocial level to characterize different societies.

257 **The range of fear threshold.** As shown in the related work section, extensive research stretches the critical role
258 of fear in the dueling of social and disease dynamics. However, to the best of our knowledge, they do not provide
259 information regarding the *thresholds of fear* considered in our model. To get insight for a lower bound of fear threshold,
260 we attempted to refer to the number of infections in the U.S. at the time (Mar 13, 2020) when the White House
261 declared the national emergency[†]. However, the corresponding number[‡] is only 1,268, which is statistically negligible
262 compared to the U.S. population. Nevertheless, we set the lower bound of fear to 0.001, a value that accounts for the
263 minuscule fraction of infection referred above, yet, still statistically significant. As for an *upper bound for fear*, at the
264 time of writing, the number of reported cases of COVID in the U.S. is 36,118,278, which is roughly 10% of the entire
265 population, and this number is enough to trigger a very high adoption rate of mask-wearing (under other measures
266 such as government mandates). Thus, we set the upper bound of fear to 0.15 to account for the stubborn people who
267 are inflexible in adopting masks.

268 **The range of peer pressure threshold.** In social science studies, Wolf et al. (47) suggest that the peer influence
269 thresholds could range from 0.1 to 0.8. Moreover, He et al. (59) and Kahane (60) found that there always exists
270 stubborn people who are unlikely to wear masks. Overall, we set the lower and the upper bound of the peer pressure
271 to 0.3 and 1.0, respectively. Specifically, the lower bound 0.3 accounts for those easily influenced by peers, and the
272 upper bound 1.0 accounts for the stubborn individuals.

273 **Discounting factor for masks.** There are plenty of existing studies on the *effectiveness of masks* in preventing
274 disease spread. Brienen et al.(61) show that the infectious risk after wearing masks decreases almost linearly w.r.t.
275 the mask effectiveness. Leung et al. (62) conducted studies on mask-wearing as a mitigation strategy for COVID
276 and flu. They screened 3,363 individuals. For the group without face masks, up to 30% and 36% of the participants
277 were infected by COVID and influenza virus, respectively. On the other hand, the infection rates for the group with
278 masks are reduced to 0% and 4%. Eikenberry et al. (63) estimate the inward mask efficiency to be up to 80% for
279 cloth masks, 70% to 90% for surgical masks, and over 95% for N95's. Patel et al. suggest a similar result where 80%
280 to 90% of particles released from the infectious source can be captured by masks (64). All the results suggest the
281 high effectiveness of masks. As for the differences between sources and receiver efficacy, Diza et al. (65) show that
282 wearing masks at the source is at least 3 times more effective than wearing masks at the receiver. Overall, we set the
283 receiver discounting factor $\alpha = 0.3$ and the source discounting factor $\beta = 0.1$ to capture (i) the high effectiveness of
284 masks, and (ii) the difference between masks on sources and receivers. Lastly, following epidemiological literature, we
285 set the expected *infectious period* to 9 days ($r = 1/9$) (66, 67).

286 Despite the explicitness of the baseline parameter values, we employ various parameter settings in experiments
287 to account for diverse testing scenarios. In particular, the critical transition of dueling dynamics is observed under
288 different ranges of fear and peer pressure thresholds, different percentages of prosocial people in society, and different
289 mask effectiveness.

[†] www.ajmc.com/view/a-timeline-of-covid19-developments-in-2020

[‡] www.statista.com/statistics/1103185/cumulative-coronavirus-covid19-cases-number-us-by-day/

Table S2. Baseline parameters

Parameter	Description	Baseline value	Ref.
p_s	Fraction of prosocial population	0.01	Assumed
p_i	Fraction of population wearing masks on day 1	0.00	Assumed
l_f	A lower bound of the threshold for fear	0.001	Assumed
u_f	An upper bound of the threshold for fear	0.15	Assumed
l_p	A lower bound of the threshold for peer pressure	0.3	(47, 68)
u_p	An upper bound of the threshold for peer pressure	1.0	(59, 60, 69)
α	Discounting factor for a susceptible individual wearing a mask	0.3	(62–64, 70)
β	Discounting factor for an infected wearing a mask	0.1	(62–64, 70)
r	Recovery rate	1/9	(67)

290

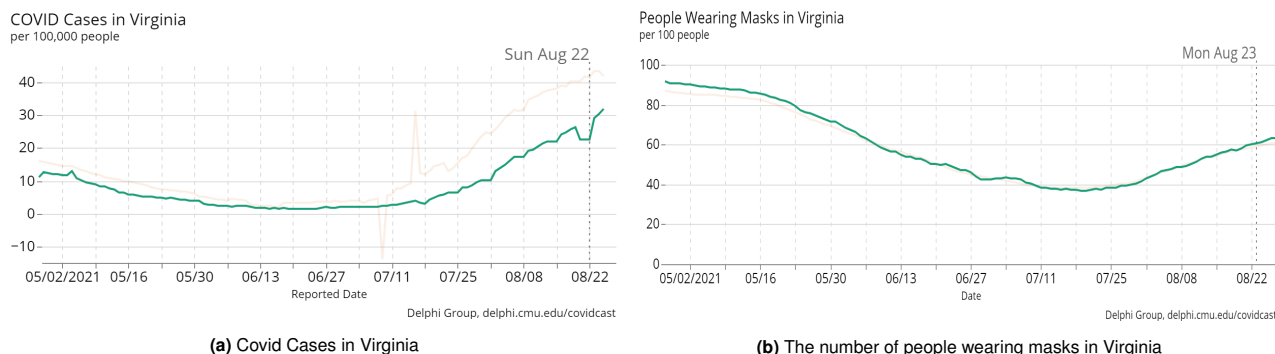


Fig. S1. The number of Covid cases and the number of people wearing masks in Virginia from 05/02/2021 to 08/22/2021. The plot is generated by COVIDCast. Overall, we observe an oscillation of infection and mask adoption.

292 **Robustness of the critical transition**

293 In this section, we demonstrate the robustness of the critical transition. Specifically, we study its persistence under
 294 various parameter settings, network structures, and model extensions.

295 **Robustness with respect to model parameters.** We explore the influence of social and disease parameters on the critical
 296 transitions of the joint dynamics. In particular, we consider variations of (i) percentage of prosocial individuals in a
 297 society, (ii) ranges of peer pressure thresholds, (iii) ranges of fear thresholds, (iv) mask effectiveness, (v) network
 298 density, and (vi) initialization of only high/low degree nodes to be infected.

299 Figure S2 depicts the coupling of social and disease dynamics in four societies with different percentages of
 300 prosocial individuals: 0.01, 0.1, 0.2, 0.3, respectively. Overall, the system exhibits a critical transition under various
 301 prosocial percentages. Furthermore, the critical transition is more prominent in society with a low fraction of prosocial
 302 individuals. Specifically, the critical transition diminishes under a society with 30% of the prosocial population. This
 303 correlation is expected since the critical transition is driven by the level of mask usage among the population.

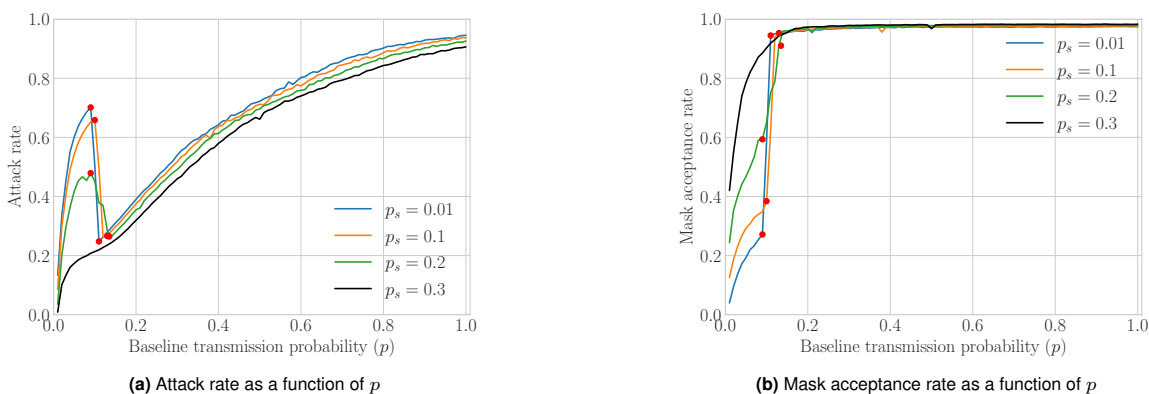


Fig. S2. Dueling dynamics under different prosocial percentages p_s . We consider four societies with various percentages of prosocial individuals: $p_s = 0.01, 0.1, 0.2, 0.3$, respectively. Panel (a) depicts the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All other parameters are set to their baseline values shown in Table S2. Overall, the critical transition persists before p_s reaching 0.3. We also observe a diminishing magnitude of critical transition as the fraction of prosocial individuals increases.

304 Results on the effect of other parameters are shown in Figure S3 to Fig S8. Here, we summarize the main
 305 findings. **The critical transition of the dueling dynamics are observed under a broad spectrum of model
 306 parameters.** In particular, the critical transition is more pronounced when (i) there are fewer prosocial people in the

307 society; (ii) people are more fearful of the disease; (iii) people are less sensitive to peer pressure; (iv) masks are more
 308 effective. Moreover, the critical transition persists irrespective of the connectivity of initially infected individuals (i.e.,
 309 degrees).

310 We also conducted an *ablation study* to determine if the critical transition is a combined effect of both *peer pressure*
 311 and *fear*. Specifically, we remove one component (i.e., *peer pressure* or *fear*) from the network model and explore the
 312 attack rate as a function of baseline transmission probability. The result is shown in Fig S9. Overall, the dueling
 313 dynamics do not undergo critical transitions in the absence of fear or peer pressure, suggesting the necessity of both
 314 components.

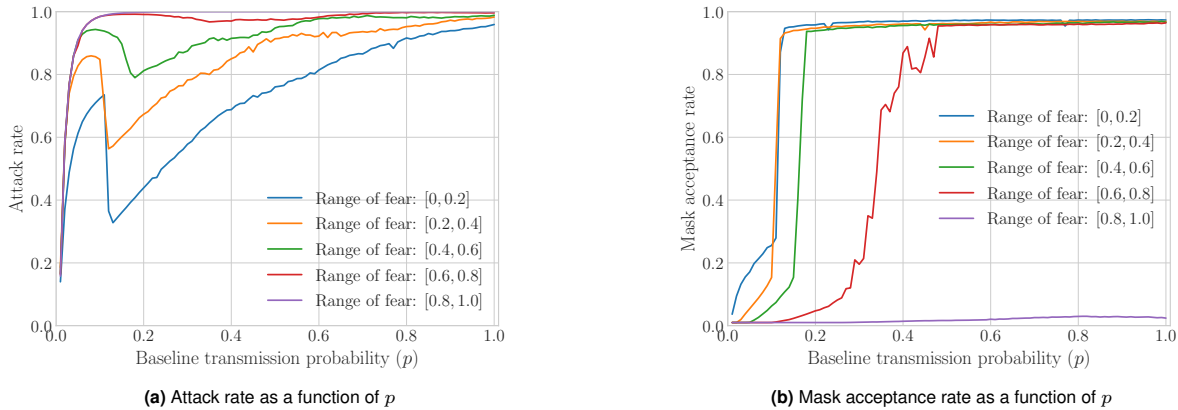


Fig. S3. Dueling dynamics under different levels of fear. We consider varying ranges of *fear* thresholds in the population: $[0, 0.2]$, $[0.2, 0.4]$, $[0.4, 0.6]$, $[0.6, 0.8]$, $[0.8, 1.0]$. In particular, the fear threshold for each individual is chosen uniformly at random within a specified range. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All other parameters are set to their baseline values shown in Table S2. Overall, we observe that the critical transition is more pronounced when people are more fearful of the disease.

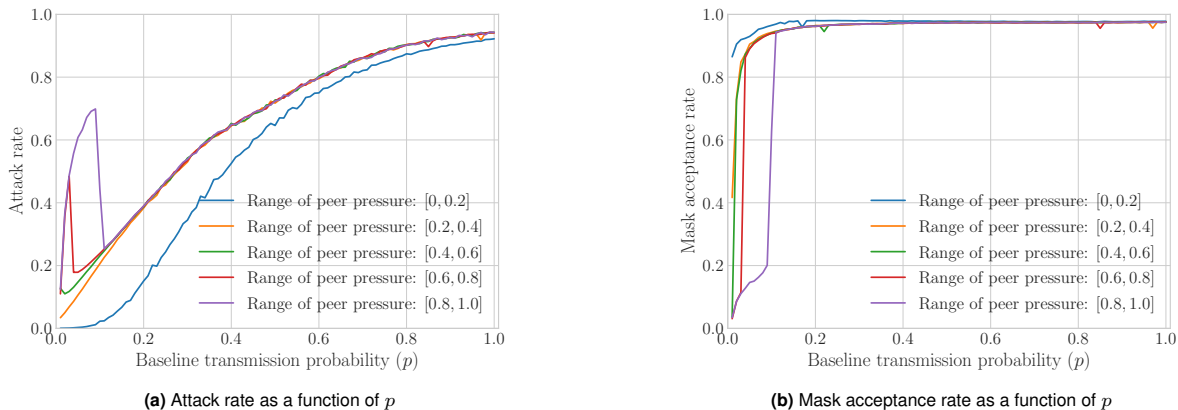


Fig. S4. Dueling dynamics under different levels of peer pressure. We consider varying ranges of *peer pressure* in the population: $[0, 0.2]$, $[0.2, 0.4]$, $[0.4, 0.6]$, $[0.6, 0.8]$, $[0.8, 1.0]$. In particular, the peer pressure threshold for each individual is chosen uniformly at random within a specified range. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All other parameters are set to their baseline values shown in Table S2. Overall, the critical transition is more pronounced when people are less sensitive to the peer pressure.

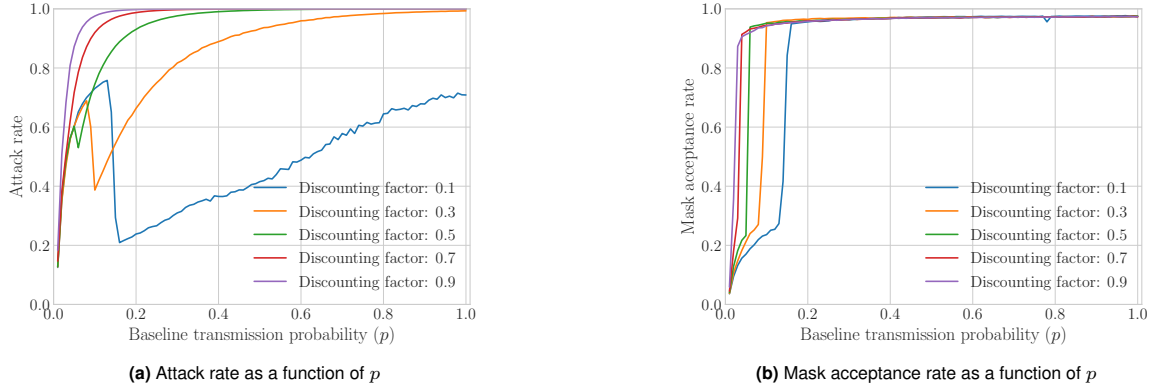


Fig. S5. Dueling dynamics under different mask effectiveness. We consider masks with different discounting factors (α and β are set to be the same) from 0.1 to 0.9. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All other parameters are set to their baseline values shown in Table S2. Overall, the critical transition is more pronounced when masks are more effective.

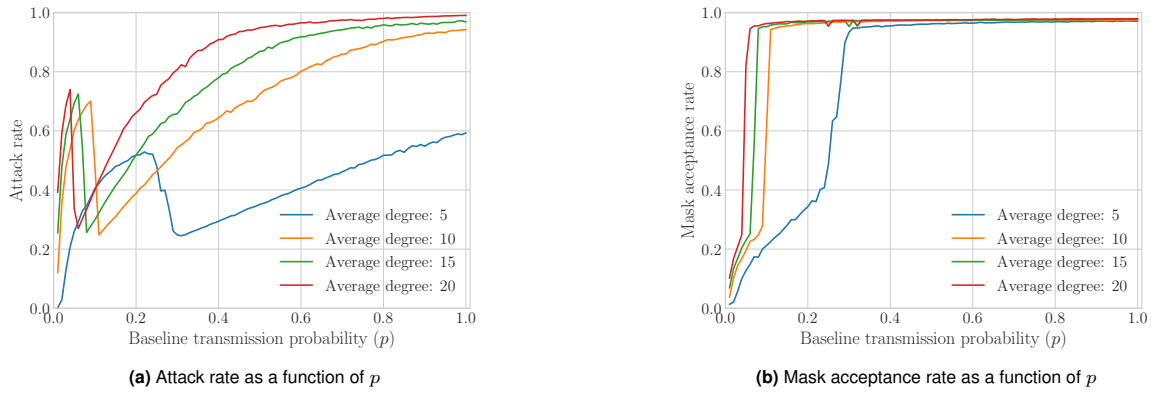


Fig. S6. Dueling dynamics under various averaged degrees. We vary the averaged degrees of the two-layer networks from 5 to 20. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All other parameters are set to their baseline values shown in Table S2. We observe that the critical transition is more pronounced when the network density increases.

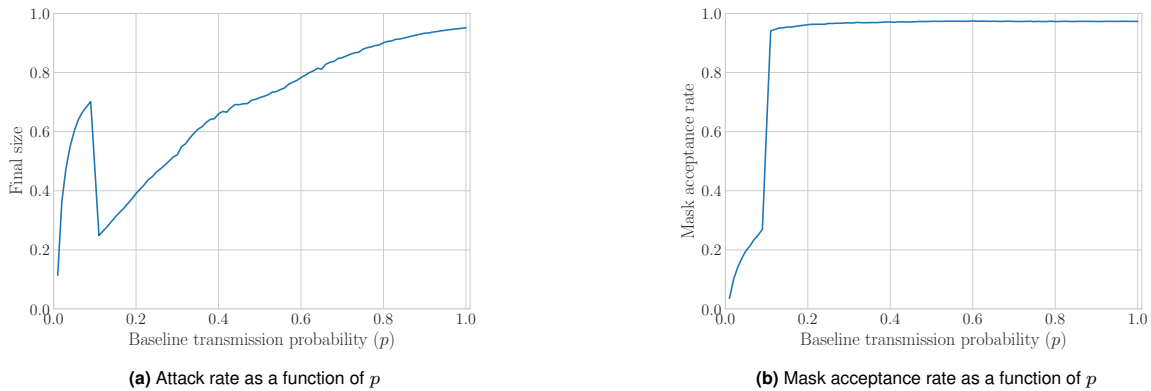


Fig. S7. Dueling dynamics under initializations of high degree nodes. The plot depicts the coupling of social and disease dynamic when the 10 initially infected nodes are randomly chosen from the top 20% of the population with the **highest** degrees. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All parameters are set to their baseline values shown in Table S2. We observe that the critical transition persists under this setting.

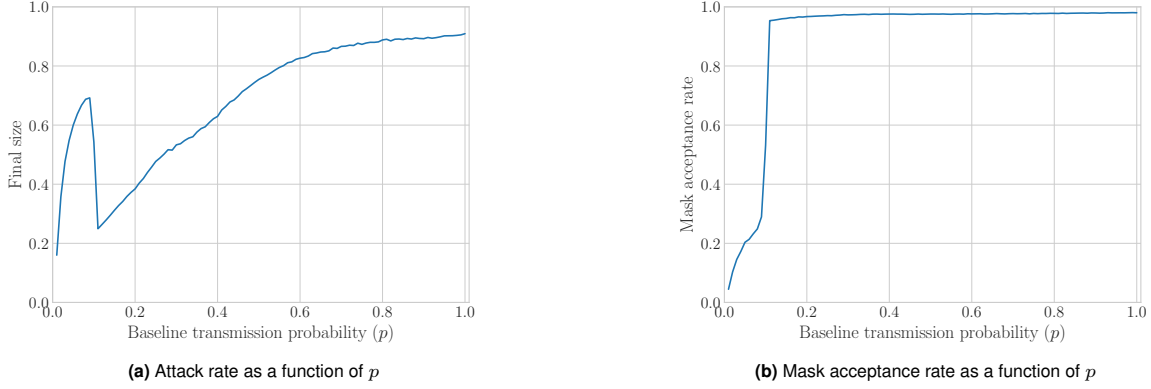


Fig. S8. Dueling dynamics under initializations of low degree nodes. The coupling of social and disease dynamic when the 10 initially infected nodes are randomly chosen from the top 20% of the population with the **lowest** degrees. Panel (a) shows the attack rate as a function of the baseline transmission probability p . Panel (b) plots the mask acceptance rate as a function of p . All parameters are set to their baseline values shown in Table S2. Similar to the setting in Fig S7, we also observe a critical transition when low degree nodes are initially infected.

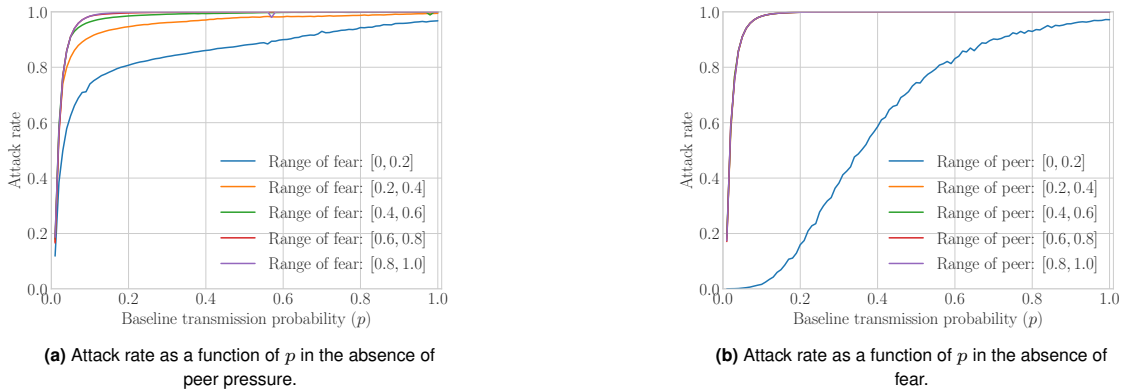


Fig. S9. Ablation study of the dueling dynamics. We investigate if the critical transition is a joint effect of fear and peer pressure. The figure shows the attack rate as functions of the transmission probability with either fear or peer pressure component removed from the model. In particular, panel (a) depicts the disease dynamic in the absence of peer pressure, under different levels of fear in the population. Panel (b) shows the disease dynamic in the absence of fear fraction, under different levels of peer pressure in the population. All other parameters are set to their baseline values shown in Table S2. Overall, the dynamics do not undergo critical transitions in the absence of fear or peer pressure, suggesting the necessity of both components.

315 **Robustness with respected to network structures.** We conducted experiments on two classes of networks: (i) power-law
 316 networks with a small number of high degree nodes, and (ii) Erdős–Rényi networks where most vertices have similar
 317 degrees. Specifically, we simulate the dueling dynamics on power-law networks of sizes up to 100,000 with exponents
 318 γ from 2.35 to 3.39. Overall, the critical transition occurs in all the tested networks. Fig S10 (b) illustrates the critical
 319 transition under power-law networks of size 100,000 with varying γ . Overall, we observe the corresponding attack
 320 rate at the first tipping point increases as γ increases.

321 Fig S10 (a) shows the the disease dynamic for Erdős–Rényi graphs of size 100,000 with varying average degrees
 322 \bar{d} from 1 to 9. In particular, we observe an emergence of the critical transition as the average degree increases.
 323 This occurs partly due to the increased the social dynamic strength as the network gets well connected. We further
 324 explored Erdős–Rényi networks with higher average degrees, and the critical transition persisted. We also remark
 325 that the critical transition exists even when the two network layers are drastically different. That is, each layer is a
 326 newly generated random network of the same class (instead of undergoing edge perturbations over the same baseline
 327 network).

328 Overall, our numerical explorations suggest that the emergence of the critical transition is mainly driven by

329 the coupling between the behavioral response model and the epidemic dynamics. While some effects on the joint
 330 dynamics are observed, the critical transition is shown to be robust to varying characteristics of the underlying
 331 network structures.

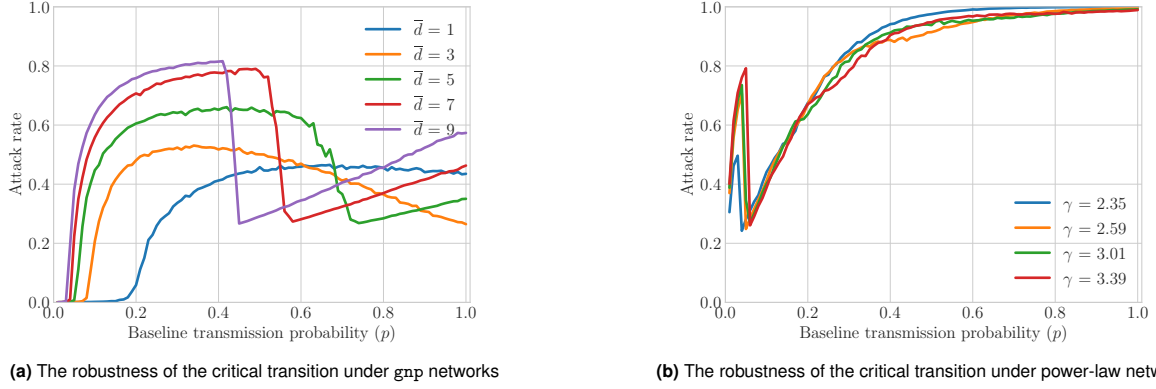


Fig. S10. The robustness of the critical transition under different network structures. The figure depicts the baseline transmission probability as a function of attack rate, under power-law and gnp network, respectively. Panel (a) shows the results on gnp networks networks of size 100, 000, with average degree \bar{d} from 1 to 9. Panel (b) depicts the results on power-law networks of size 100, 000 with exponents γ from 2.35 to 3.39. All parameters are set to their baseline values shown in Table S2.

332 **Results for Model extensions.** We study the persistence of the critical transition under two extensions of the behavior
 333 model. In the first extension, we incorporate the *habit formation process* into the mask-wearing dynamics. Specifically,
 334 each individual v is associated with $\epsilon_v \in [0, 1]$, denoting the strength of the mask-wearing habit. If v chooses to wear
 335 a mask (either due to fear or peer pressure) on any given day, he or she continues to wear a mask the next day with
 336 probability ϵ_v . Conversely, if v takes off the mask, the social dynamic switches back to the threshold-based model.
 337 The result is shown in Fig S11. Overall, we observe critical transitions of the disease dynamic under various habit
 338 strengths of the population. In particular, the critical transition exhibits a negative correlation to the habit strength,
 339 such that the lower the strength, the sharper the critical transition.

340 In the second extension, we introduce the asymptomatic state to the model. In particular, a fraction of the infected
 341 population is *asymptomatic*, and only *symptomatic* infections can trigger fear in the population. The results are
 342 shown under Fig S12 and Fig S13, where we investigate the dueling dynamics under different ratios of symptomatic to
 343 asymptomatic infections. Overall, the critical transition persists under a broad range of symptomatic to asymptomatic
 344 ratios.

345 **Habit formation.** Let $\epsilon_v \in [0, 1] : v \in V$ be the *strength* of the mask-wearing habit for an individual v such that if v
 346 chooses to wear a mask (due to fear or peer pressure) on a given day, then with probability ϵ_v , he continuous to wear
 347 a mask in the next day due to the habit. Let $a_4^{(t)}(v)$ be the indicator variable such that v wears a mask at time t due
 348 to the habit, then an individual wears a mask at time t if $a_1 \vee a_2^{(t)}(v) \vee a_3^{(t)}(v) \vee a_4^{(t)}(v)$ is satisfied. If v takes off
 349 the mask at any time, the social dynamic switches back to the threshold-based model. We remark that the original
 350 dueling model is a special case of this extend model under $\epsilon_v = 0, \forall v \in V$.

351 The result is shown in Fig S11. Overall, we observe critical transitions of the disease dynamic under various habit
 352 strengths. Specifically, the critical transition exhibits a negative correlation with the habit strength such that the
 353 lower the strength, the sharper the critical transition.

354 **Only symptomatic infections trigger fear.** We consider an extension of the model where a fraction of the infected population
 355 is *asymptomatic*, and we only consider the total number of *symptomatic* infections in triggering fear. The results
 356 are shown under Figure S12 and Figure S13 where we investigate the dueling dynamics under different ratios of

357 symptomatic to asymptomatic infections. Overall, the critical transition persists under a broad range of symptomatic
358 to asymptomatic ratios.

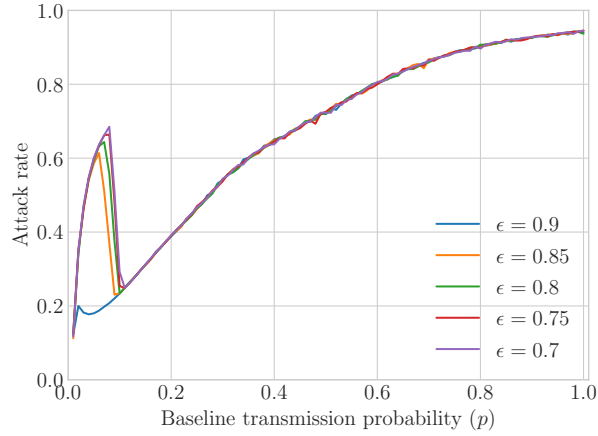


Fig. S11. critical transition under different habit strengths ϵ (the probability of continuing wearing a mask). The plot shows the transmission probability as a function of attack rate under different habit strengths from 0.9 to 0.7. All other parameters are set to their baseline values shown in Table S2.

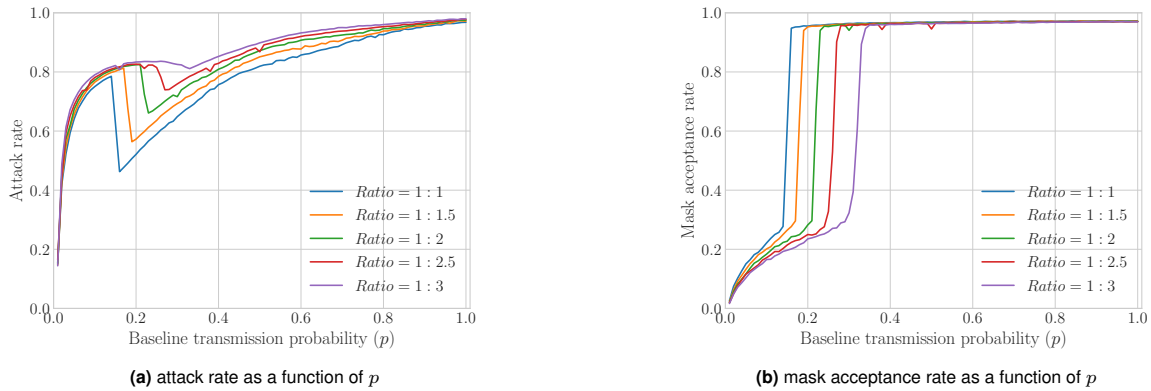


Fig. S12. Dueling dynamics under different symptomatic to asymptomatic infections ratios. We consider different ratios of **symptomatic to asymptomatic** infections where there are more asymptomatic cases. Panel (a) shows the attack rate as a function of transmission probability p . Panel (b) plots the mask acceptance rate as a function of transmission probability. All other parameters are set to their baseline values shown in Table S2.

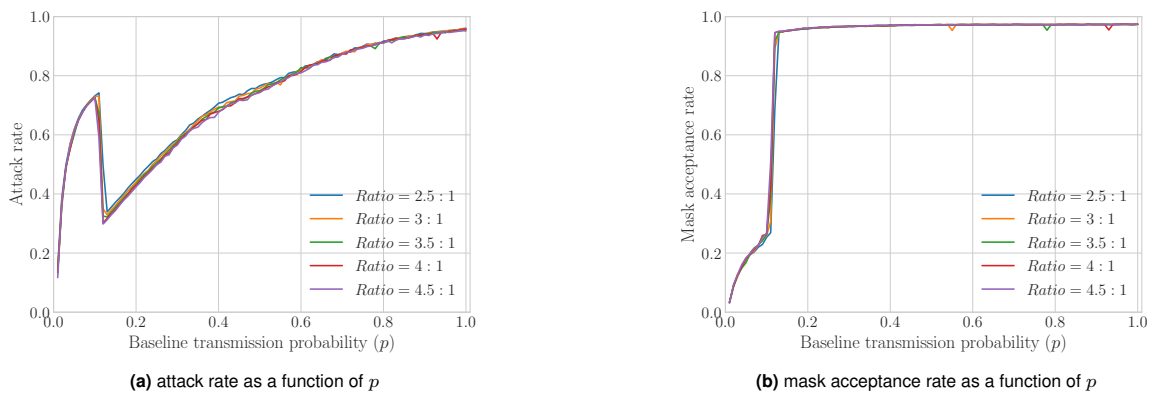


Fig. S13. Dueling dynamics under different symptomatic to asymptomatic infections ratios. We consider different ratios of **symptomatic to asymptomatic** infections where there are more symptomatic cases. Panel (a) shows the attack rate as a function of transmission probability p . Panel (b) plots the mask acceptance rate as a function of transmission probability. All other parameters are set to their baseline values shown in Table S2.

359 **Robustness on real-world networks.** Lastly, we investigate the robustness of the critical transition on a realistic
 360 social contact network. It is a digital twin of Manassas city, Virginia, social contact network. The network has about
 361 35,000 nodes and an average degree of 14. The digital twin is generated by (i) constructing synthetic individuals and
 362 locations; (ii) assigning daily activity sequences to each individual; (iii) mapping each activity of each individual to a
 363 location with the associated visiting time interval; (iv) deriving edges between people from their physical proximity
 364 when they visit the same location at the same time. The detailed steps of the construction of such a digital twin for
 365 any county or state level region in the U.S.A. are described in (71) and more details of the methodology can be found
 366 in (72–74). Here we consider the unweighted version of the Manassas network with the same parameter values in
 367 Table S2. The results are shown in Figure S14. Overall, the critical transition of the dueling dynamics persists in the
 368 Manassas social contact network. Note that the observed critical transition is less pronounced compared with other
 369 synthetic networks’ results. Remark that the magnitude of a critical transition (i.e., absolute difference of values on
 370 the y-axis between the two tipping points) is affected by the parameter setting; thus, we can produce sharper critical
 371 transitions by varying the model parameters.

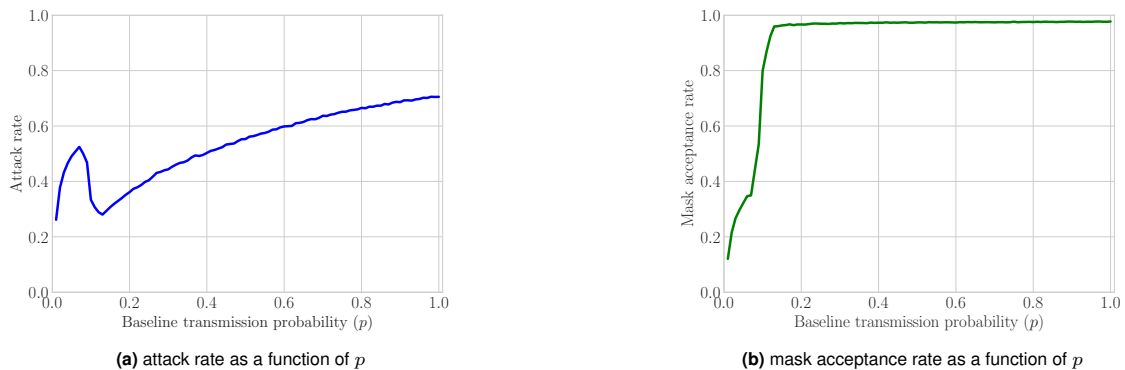


Fig. S14. Dueling dynamics in Manassas networks. The plot depicts the coupling of social and disease dynamic for the Manassas networks of size 35,028 with an average degree of 14. Panel (a) shows the attack rate as a function of transmission probability p . Panel (b) plots the mask acceptance rate as a function of transmission probability. All parameters are set to their baseline values shown in Table S2. Overall, we observe a critical transition of the social and biological dynamics. Remark that the critical transition is sensitive to the parameter setting, as we can make it more pronounced by changing the model parameters.

372 **A different view of the time-series dynamics between the two regimes**

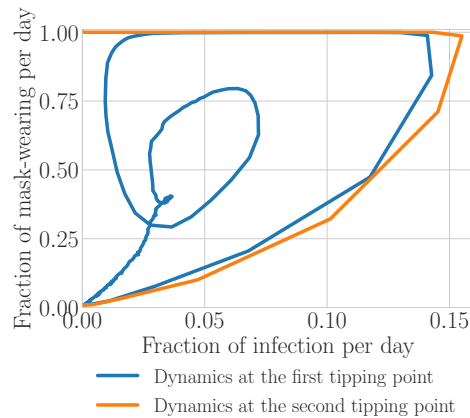


Fig. S15. A different view of the time-series dynamics between the two regimes. We depict the time-series dynamics of infection and mask adoption at the first and the second tipping point, respectively. See for further explanation and discussion using a compartmental model.

373 **Robustness of the causes of the critical transition**

374 As show in the main manuscript, one cause of the critical transition is the distinct time-series dynamics of the two
 375 regimes. Specifically, the infection and mask adoption dynamics exhibits multiple waves when the disease baseline
 376 transmission probability p is in the first regime. In contrast, infection and mask adoption dynamics undergoes only a
 377 single wave when p is in the second regime. This section demonstrates the robustness of this underlying mechanism
 378 driving the critical transition. In particular, the study is performed under different parameter setting with variations
 379 on (i) range of peer pressure thresholds; (ii) range of fear thresholds, and (iii) fraction of the prosocial population.
 380 Overall, when critical transitions appear, we observe the contrasting dynamics under all tested parameters. Example
 381 time-series dynamics are at the first/second tipping points of the critical transition, shown from Fig S16 to S18.

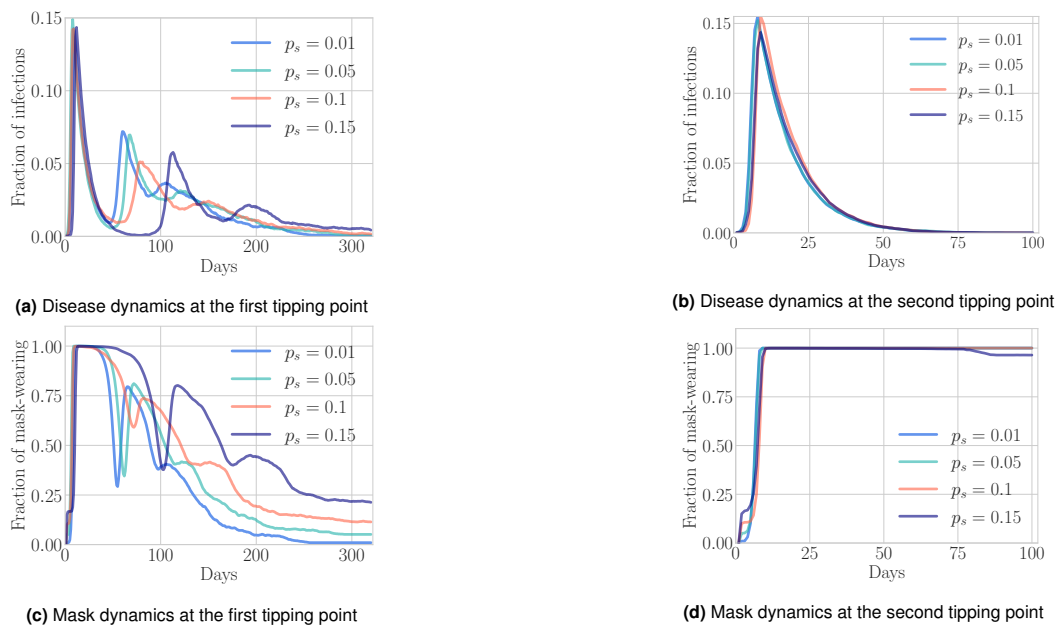


Fig. S16. The time-series dynamics of disease and mask adoption at the tipping points under varying prosocial percentages p_s . Panels (a) and (c) depict the time-series dynamics at the first tipping point. Panels (b) and (d) depict the time-series dynamics at the second tipping point. The percentage of prosocial population ranges from 0.01 to 0.15. The baseline transmission probability is fixed to the values at the tipping points. All other parameters are set to their baseline values shown in Table S2.

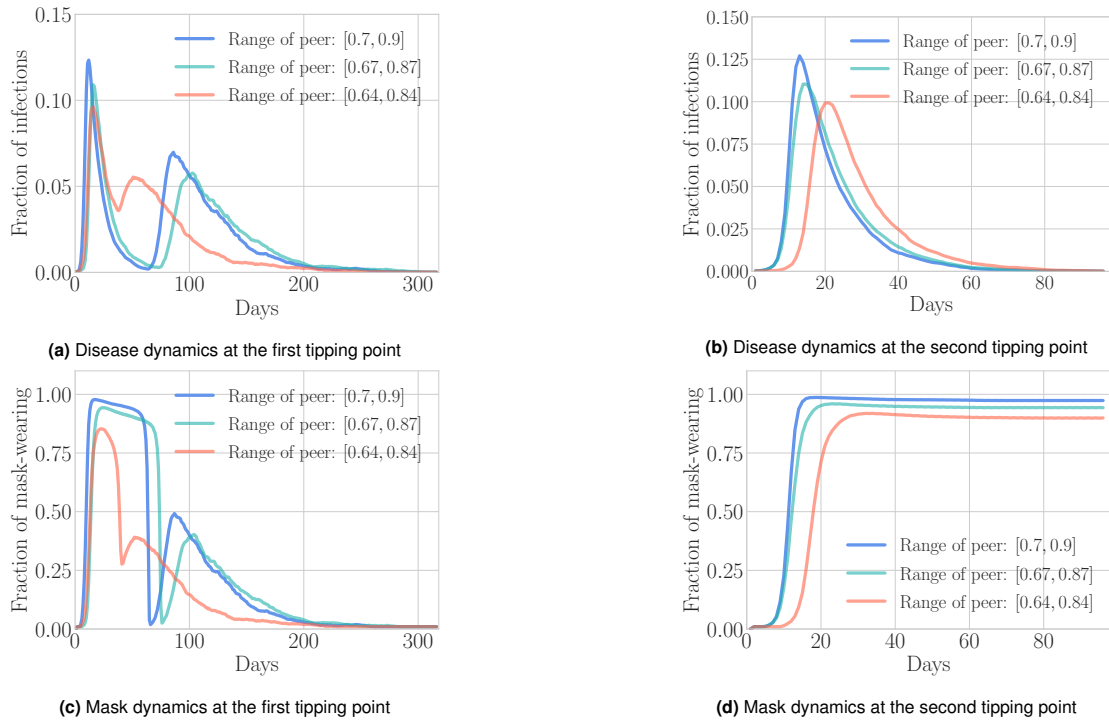


Fig. S17. The time-series dynamics of disease and mask adoption at the tipping points under varying ranges of peer pressure thresholds. Panels (a) and (c) depict the time-series dynamics at the first tipping point. Panels (b) and (d) depict the time-series dynamics at the second tipping point. The range of peer pressure threshold varies from $[0.7, 0.9]$ to $[0.64, 0.84]$. The baseline transmission probability is fixed to the values at the tipping points. All other parameters are set to their baseline values shown in Table S2.

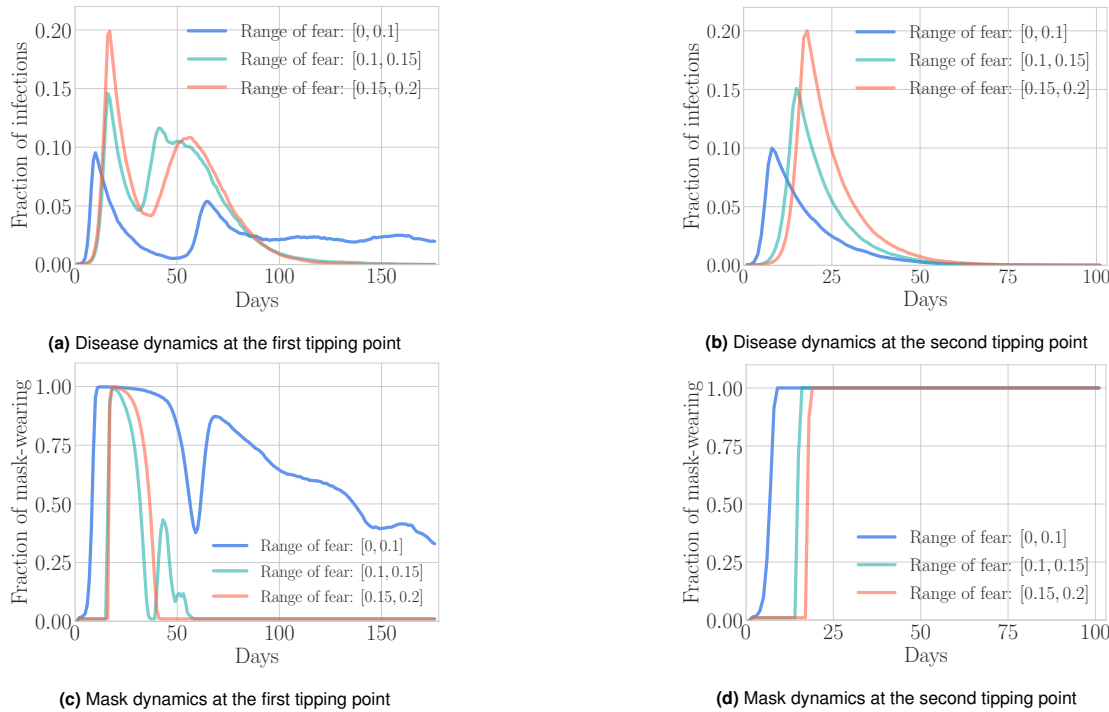


Fig. S18. The time-series dynamics of disease and mask adoption at the tipping points under varying ranges of fear thresholds. Panels (a) and (c) depict the time-series dynamics at the first tipping point. Panels (b) and (d) depict the time-series dynamics at the second tipping point. The range of fear threshold varies from $[0, 0.1]$ to $[0.15, 0.2]$. The baseline transmission probability is fixed to the values at the tipping points. All other parameters are set to their baseline values shown in Table S2.

Enforcement of mask-wearing tames epidemic waves

We further explore the infection dynamic under public mask-wearing mandates. In particular, when the disease prevalence starts to lessen, we enforce mask-wearing individuals to continue wearing masks in spite of the reduction in peer pressure and fear. Subsequently, as shown in Figure S19, we can effectively suppress the disease revival and bring the anticipated oscillation of infection down to only a single wave. This observation highlights the importance of continuing mask mandate despite a low level of disease prevalence and social stigma.

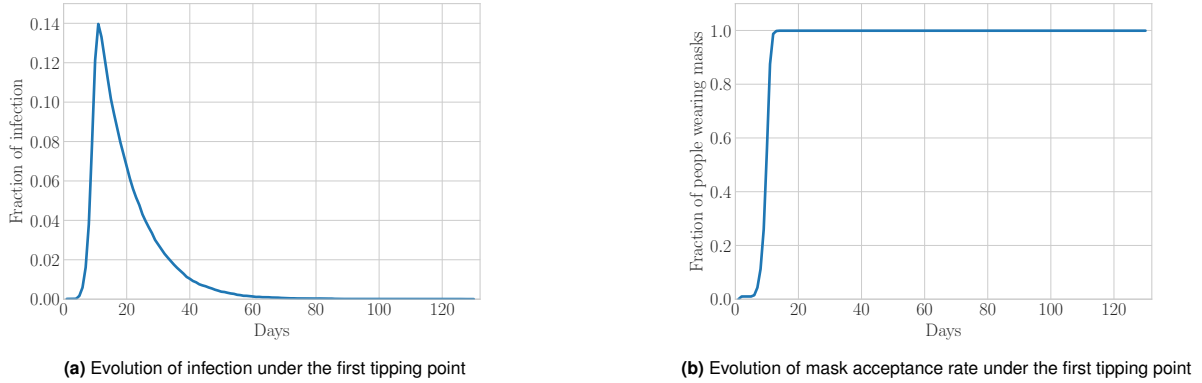


Fig. S19. The co-evolution of the disease and mask adoption at the first tipping points under continued mask mandate. We fix the transmission probability p of the disease to the value at the first tipping point. Furthermore, we enforce mask mandates when the disease prevalence starts to decrease. Panel (a) shows the infection dynamic, and panel (b) depicts the change in mask adoption percentage. All parameters are set to their baseline values shown in Table S2. Overall, the disease dynamic exhibits only a single wave, and we observe no oscillation of infection and mask adoptions. Furthermore, we sampled a large number of transmission probabilities within the third regime and observed no oscillations. The single-wave dynamics contrast the multiwave dynamics without mask mandate.

Critical transitions and multiwave infections in a mean-field model

In this section we use a parsimonious mean-field model of disease transmission, coupled with a phenomenological threshold condition approach to model a population's behavioral response. Our goal is to show that the critical transition phenomena of the attack rate are robust to the modeling framework used. That is, we show that the proposed mean-field approach to a well-mixed population can exhibit qualitatively similar results than the proposed multiplex network model.

Consider a population composed of two behaviorally distinct groups: individuals do not complying with precautionary behaviors (N_1), and individuals complying with precautionary behaviors (N_2), where the total population is given by $N = N_1 + N_2$. Assume the contagion process follows a Susceptible (S_i), Infected (I_i) and Recovered (R_i), structure in each of the behavior groups. Non-compliant susceptible individuals get infected by contacting: (i) non-compliant infected individuals at rate $pS_1 \frac{I_1}{N}$, and (ii) compliant infected individuals at rate $pS_1 \frac{\sigma_i I_2}{N}$, where $0 \leq \sigma_i \leq 1$ stands for the precautionary efforts of infected compliant individuals. Similarly, compliant susceptible individuals get infected by making contacts with: (i) non-compliant infected individuals at rate $\sigma_s p \frac{I_1}{N}$, and (ii) compliant infected individuals at rate $\sigma_s p \frac{\sigma_i I_2}{N}$, where σ_s stands for a reduced susceptibility of compliant susceptible individuals. Infected individuals, regardless of their behavior, recover at rate γ .

We model *fear of infection* by using a threshold condition based on the disease prevalence level. Non-compliant individuals start following precautionary behaviors after the disease attains a pre-defined prevalence threshold τ . After which, people from the non-compliant group (N_1), regardless of their health-status, move to the compliant group (N_2) at a rate φ . Moreover, our fear condition only plays a role whenever the disease prevalence is over the fear threshold ($\varphi > 0$), otherwise there is no recruitment on the compliant populations ($\varphi = 0$). Finally, we assume compliant individuals lose fear of infection and become non-compliant at a rate λ , regardless of their health-status. The proposed model for behavioral adaptations by a prevalence threshold condition is formalized in the following set

$$\begin{aligned}
 \dot{S}_1 &= -(I_1 + \sigma_I I_2) p \frac{S_1}{N} + \lambda S_2 - \varphi S_1, \\
 \dot{I}_1 &= (I_1 + \sigma_I I_2) p \frac{S_1}{N} + \lambda I_2 - \gamma I_1 - \varphi I_1, \\
 \dot{R}_1 &= \lambda R_2 + \gamma I_1 - \varphi R_1 \\
 \dot{S}_2 &= -(I_1 + \sigma_I I_2) \sigma_S p \frac{S_2}{N} - \lambda S_2 + \varphi S_1, \\
 \dot{I}_2 &= (I_1 + \sigma_I I_2) \sigma_S p \frac{S_2}{N} - \lambda I_2 - \gamma I_2 + \varphi I_1, \\
 \dot{R}_2 &= -\lambda R_2 + \gamma I_2 + \varphi R_1
 \end{aligned}
 \tag{A}$$

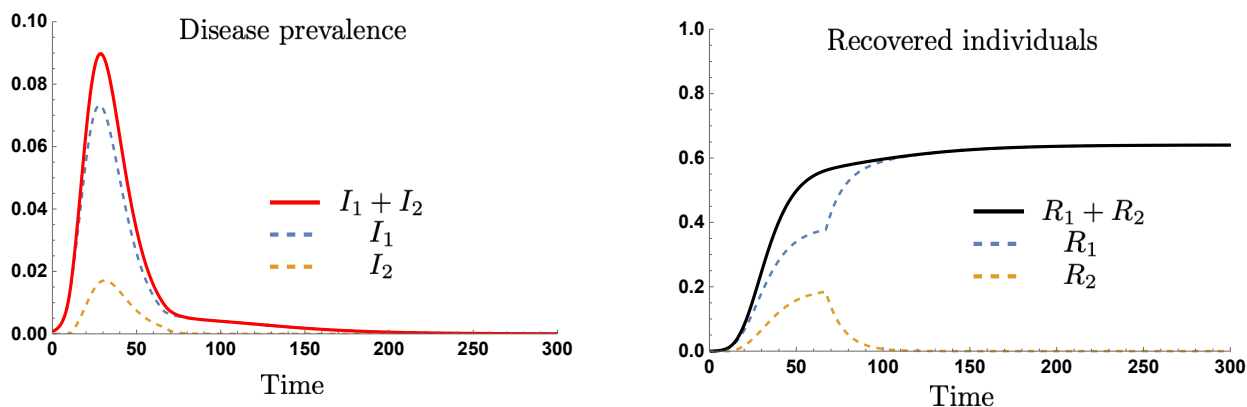
412 where

$$\begin{aligned}
 \dot{N} &= \dot{N}_1 + \dot{N}_2 = 0, \\
 \dot{N}_1 &= \lambda N_2 - \varphi N_1, \\
 \dot{N}_2 &= -\lambda N_2 + \varphi N_1.
 \end{aligned}
 \tag{B}$$

414 Notice that $\Omega = \{S_1, I_1, R_1, S_2, I_2, R_2 \in \mathbb{R}_+^6 : S_1 + I_1 + R_1 + S_2 + I_2 + R_2 \leq N\}$, represents a feasible (positively-invariant
415 and attracting) region for the system Eq. (A).

416 **Results.** Consider an epidemic with $\mathcal{R}_0 = p/\gamma = 2.4$, where $p = 0.48$ and $\gamma = 0.2$. Assume the behavioral response
417 occurs at a threshold prevalence $\tau = 1\%$, after which non-compliant individuals start adopting precautionary behaviors
418 at a rate φ , and return to non-compliance at a rate λ . Susceptible (infected) compliant individuals are assumed to
419 reduce their susceptibility (infectiousness) by a factor of $\sigma_i = 0.1$ ($\sigma_s = 0.2$).

420 Figures S20 and S21 show the evolution of the disease prevalence (panel a) and the epidemic size (panel b) on time.
421 We assume the same disease parameters but different behavioral responses $\varphi = 0.5$ and $\varphi = 0.75$. In S20, we show that
422 even though the disease prevalence is much greater than the fear threshold ($\tau = 0.01$), the population's behavioral
423 response is not fast enough to avoid enough infections during the first epidemic peak ($\varphi = 0.05$). Consequently, the
424 disease prevalence exhibit a big first epidemic peak and a heavy tail, without oscillations.



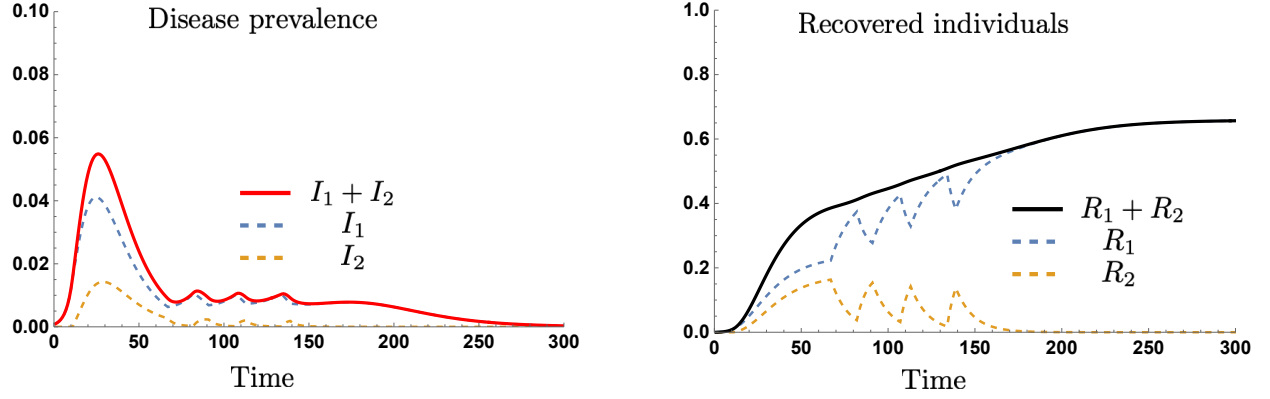
(a) Time course of the infected compliant, infected non-compliant, and total disease prevalence.

(b) Time course of the recovered compliant, non-compliant, and epidemic size.

Fig. S20. The disease dynamics cross the fear threshold once. The adoption of precautionary behaviors is not fast enough to maintain an endemic level of compliant individuals. In consequence, the disease prevalence does not show oscillations on the disease prevalence.

425 In counterpart, S21 show an scenario where, after the disease prevalence hits the fear threshold condition ($\tau = 0.01$),
426 the population comply with precautionary behaviors fast enough ($\varphi = 0.075$), so that the first epidemic peak is highly

427 reduced. In this scenario, after the disease prevalence is reduced below the fear threshold, not anymore people is
 428 recruited on the compliant group ($\varphi = 0$). This produce a resurgence of the epidemic, taking the disease prevalence
 429 over the fear threshold again, followed by the population's behavioral response taking the epidemic prevalence down
 430 again.



(a) Fear threshold activate multiple times. Time course of the compliant non-compliant, and total disease prevalence. (b) Recovered individuals keep changing their behavior. Time course of the group specific and total recovered individuals

Fig. S21. The disease dynamics cross the fear threshold multiple times. The behavioral response rate $\varphi = 0.075$ is enough to produce oscillations on the disease prevalence.

431 Similar to our network model, the mean-field model captures the trade-off between the strength of the behavioral
 432 response (risk reduction), and the disease infectiousness. Whenever there epidemic-behavior dynamics coexist near a
 433 balance, the epidemic would survive for long periods, generating secondary cases in the presence/absence of behavioral
 434 responses, potentially leading to increased attack rate values.

435 Peer-pressure and fear mechanisms in a mean-field model

This section concerns a mean-field model where, as in the main text model, there are two thresholds: a (randomly distributed) fear threshold, τ , and a (randomly distributed) peer-pressure threshold, ω . Our mean-field model is equivalent to assuming a complete network (neighborhood representative of the whole population). Thus, when the disease prevalence is I and the fraction of individuals wearing masks is m , we evaluate the fraction of individuals that want to wear a mask, $F(I, m)$, as a function of I and m . This fraction of the population is made of all of those for which either their fear threshold is lower than I or their peer-pressure threshold is lower than the current m . If we let $\rho_\tau(\tau)$ and $\rho_\omega(\omega)$ be, respectively, the probability density distributions of the thresholds τ and ω , we can write the fraction of individuals that currently want to wear a mask as

$$F(I, m) = \int_{-\infty}^I \rho_\tau(\tau) d\tau + \int_{-\infty}^m \rho_\omega(\omega) d\omega - \int_{-\infty}^m \int_{-\infty}^I \rho_\tau(\tau) \rho_\omega(\omega) d\tau d\omega \quad [C]$$

$$= C_\tau(I) + C_\omega(m) - C_\tau(I)C_\omega(m). \quad [D]$$

436 where we assume the thresholds τ and ω are independent and wrote it in terms of the cumulative distributions of the
 437 thresholds τ and ω , $C_\tau(I)$ and $C_\omega(m)$, respectively.

438 The fraction of mask wearers, m will track this number with some rate r , thus,

$$\dot{m} = r(F(I, m) - m). \quad [E]$$

In our case, we consider uniform distributions of thresholds, i.e., $f \sim U(l_\tau, u_\tau)$ and $p \sim U(l_\omega, u_\omega)$. The cumulative

function of these uniform distributions can be written as

$$C_\tau(I) = \frac{I - l_\tau}{u_\tau - l_\tau} \Theta(I - l_\tau) + \frac{u_\tau - I}{u_\tau - l_\tau} \Theta(I - u_\tau), \text{ and} \quad [\text{F}]$$

$$C_\omega(m) = \frac{m - l_\omega}{u_\omega - l_\omega} \Theta(m - l_\omega) + \frac{u_\omega - m}{u_\omega - l_\omega} \Theta(m - u_\omega), \quad [\text{G}]$$

440 where we use the Heaviside function $\Theta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$. This allows us to study the sign of \dot{m} as a function of m
 441 and I .

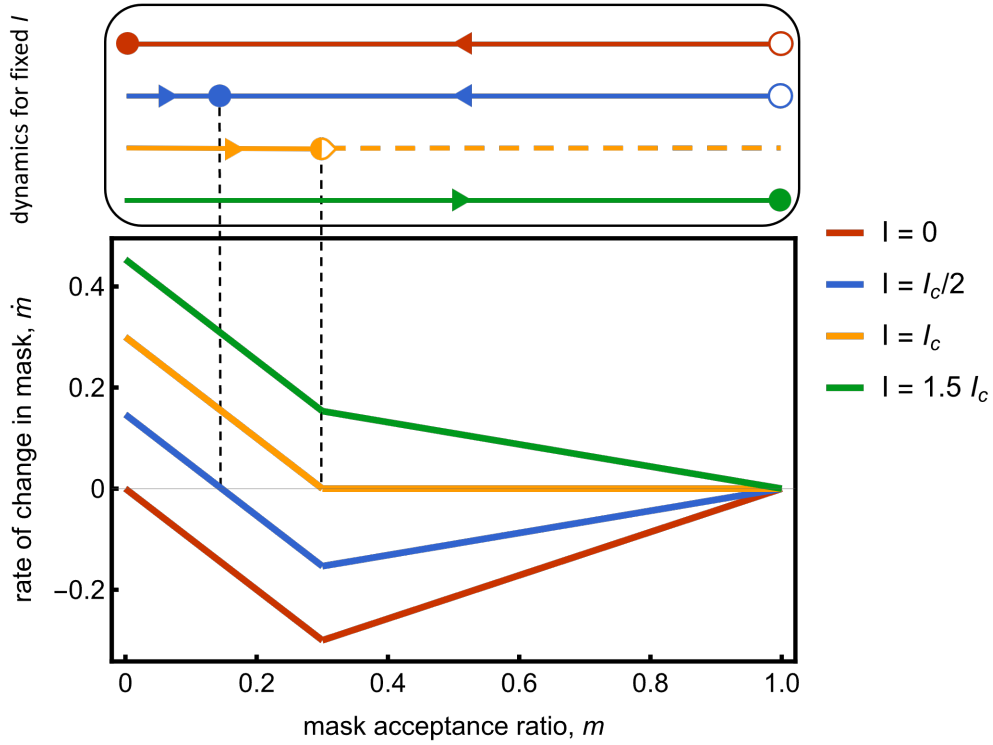


Fig. S22. Dynamics of mask wearing for a fixed level of infection ratio, I . Full circle represent attractors of the dynamics, whereas open circles represent repellers. For $I < I_c = l_\omega + l_\tau(u_\omega - l_\omega)$ mask wearing tends towards a low value. Above I_c , mask wearing tends to 1.

Figure S22 shows the various dynamics for different values of infection ratio, I . For a given value of I , we can find the value for the fraction of the population wearing a mask, m^* , for which mask adoption does not change (full circle at low rates in Figure S22), as a function of the disease prevalence, I . We do so by studying the one-dimensional autonomous system in Eq.(E). For $u_\omega = 1$, we get

$$m^*(I) = \begin{cases} 0 & I \leq l_\tau \\ \frac{l_\tau - I}{l_\tau - u_\tau} & l_\tau < I < l_\tau + l_\omega(u_\tau - l_\tau) \\ 1 & I \geq l_\tau + l_\omega(u_\tau - l_\tau) \end{cases} \quad [\text{H}]$$

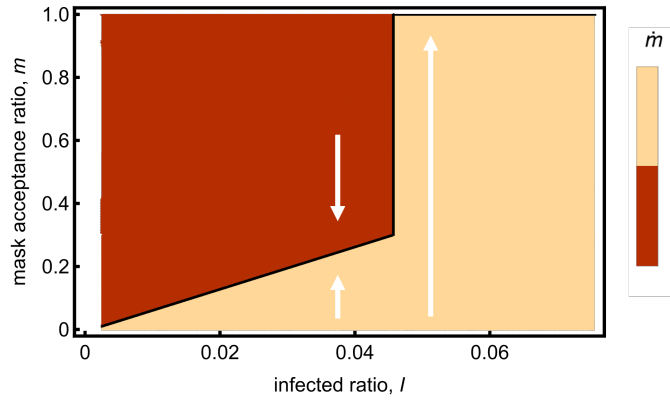


Fig. S23. Bifurcation diagram of mask-wearing behavior for infection ratio, I , as control parameter. A sharp transition occurs at $I = I_c$.

442 In Figure S23, we see that as the infected ratio I is below a critical value, mask adoption tends towards a mixed
 443 equilibrium (between 0 and 1). As the infected ratio goes above I_c , mask adoption grows to 1. For infinite and fully
 444 connected networks, the process is reversible.

445 For the values in the main text, this predicts that mask adoption increases at par with the disease from 0 to about
 446 30% followed by a jump to 100% adoption.

447 To better classify this transition and relate it to traditional critical transitions theory, we can mollify the threshold
 448 distribution to make it infinitely differentiable. We can use a standard mollifier for the uniform distribution (e.g.,
 449 $e^{\frac{-1}{1-x^2}}$) appropriately shifted and scaled, see below in orange). Notice that this smooth scenario is actually more
 450 descriptive of a network setting (as opposed to an infinite well-mixed population) where there is variation in the
 451 neighborhoods experienced by the individuals.

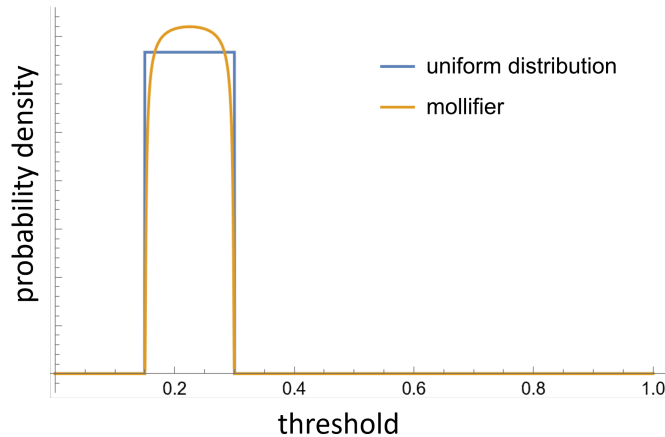


Fig. S24. Mollifier for the uniform distributions

452 Using the mollifier in Figure S24 we obtain the dynamics of Figure S25 and the corresponding bifurcation diagram
 453 of Figure S26.

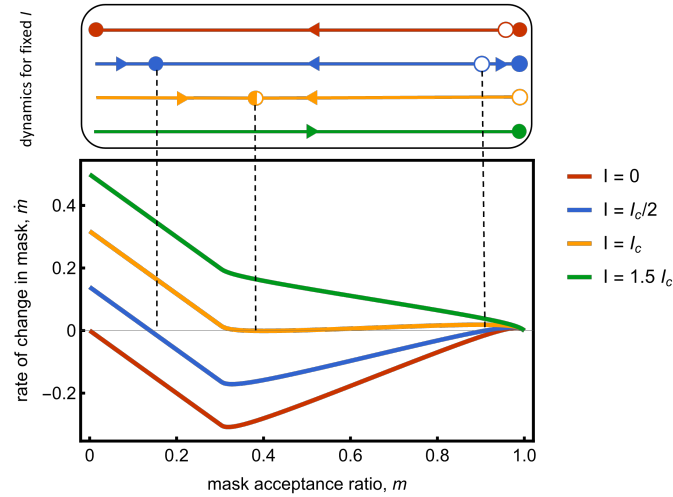


Fig. S25. Dynamics for the mollified system in Figure S22 using the distribution represented in Figure S24.

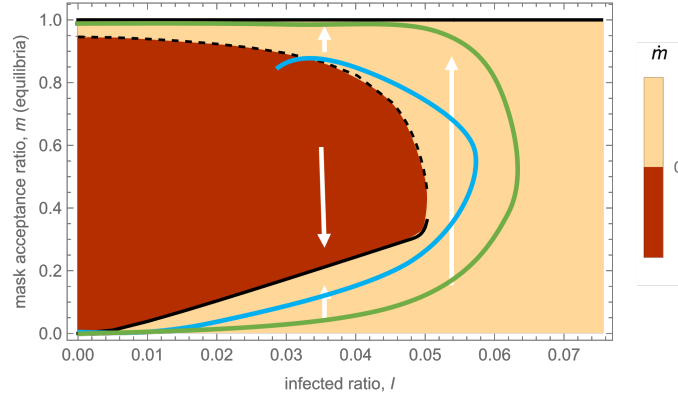


Fig. S26. Bifurcation plot for the mollified system in Figure S22 using the distribution represented in Figure S24. Equivalent to the original in Figure S23. In blue and green we illustrate hypothetical (partial) trajectories in the phase plane: one entering the red region that sees mask adoption going down (blue) and another in which the disease mask drops to zero without having masks rates dropping. Notice that this is not the full phase space since the are also susceptible/recovered states, but the saddle-node region remains constant.

454 Figure S26 makes it clear that the transition is discontinuous (for an infinite system) and corresponds to a
 455 saddle-node bifurcation. It also highlights the hysteresis effect that originates the tipping point.

456 This bifurcation plot converges to the bifurcation plot for the uniform distribution, since the unstable fixed point
 457 moves rapidly towards the top of the plot except for $I = I_c$. For infinite and fully connected populations the process
 458 becomes in practice reversible, with any perturbation to $m = 1$ when $I < I_c$ bringing m to $m^* < 1$. Now, we can see
 459 what happens at the tipping point by looking at the trajectories in this face plane (illustrated in blue and green).
 460 Infection will go up from close to zero to a high value and back, either because mask wearing is high or because the
 461 number of susceptibles is low. As the infection ratio increases, mask adoption increases. When infection ratio goes
 462 above I_c , mask adoption increases abruptly, reducing the increase of the infection ratio. As the infection ration drops,
 463 it can do so either by going through the red region or through the yellow region. When it occurs through the red
 464 region mask acceptance drops and the cycles repeat while there are enough susceptibles. When it occurs through the
 465 yellow region mask wearing remains active until the epidemics is over.

466 Furthermore, when $u_\omega < 1$, mask adoption shows hysteresis, with $I < l_\tau + l_\omega(u_\tau - l_\tau)$ containing another stable
 467 equilibrium at $m^* = 1$. See LuoJun Yang et al. (2022) (75) for discussion on variations of this hysteresis phenomenon
 468 on mask wearing behavior driven by cultural differences.

469 We can further describe the dynamics by coupling it with the disease.
 470 Assuming mask behavior does not correlate with disease state, we can couple it with an SIR dynamics to get

$$\dot{S} = -((1 - m)I + \sigma_I m I) p (1 - m) \frac{S}{N} - ((1 - m)I + \sigma_I m I) \sigma_S p m \frac{S}{N} \quad [\text{I}]$$

$$\dot{R} = \gamma I \quad [\text{J}]$$

$$\dot{I} = -\dot{S} - \dot{R} \quad [\text{K}]$$

$$\dot{m} = r(F(I, m) - m) \quad [\text{L}]$$

471 or simply

$$\dot{S} = -(1 - m(1 - \sigma_I))(1 - m(1 - \sigma_S)) p I \frac{S}{N} \quad [\text{M}]$$

$$\dot{R} = \gamma I$$

$$\dot{I} = -\dot{S} - \dot{R}$$

$$\dot{m} = r(F(I, m) - m)$$

473 Figure S27 shows time series of the disease prevalence and mask wearing for model Eq. (M). The selected simulations
 474 show the oscillation regime that prolong the epidemic period by oscillations of mask wearing.

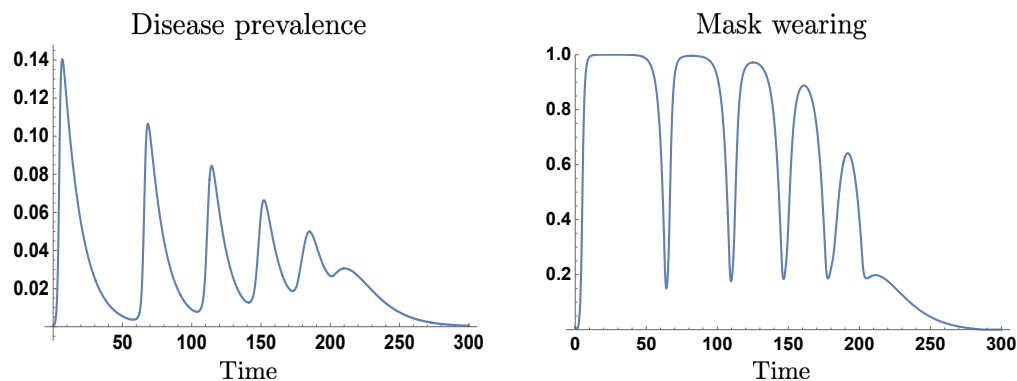


Fig. S27. Time course of the disease prevalence and mask wearing. The mask wearings oscillations promotes epidemic reemergence, increasing the attack rate. Parameters identical to main text, with some probabilities now rates. $p = 0.9$, $r = 10$, $\gamma = 1/9$.

475 Moreover, in Figure S28, our simulations shows that model Eq. (M) also produce a critical transition on the attack
 476 rate, similar to the one observed in the network model.

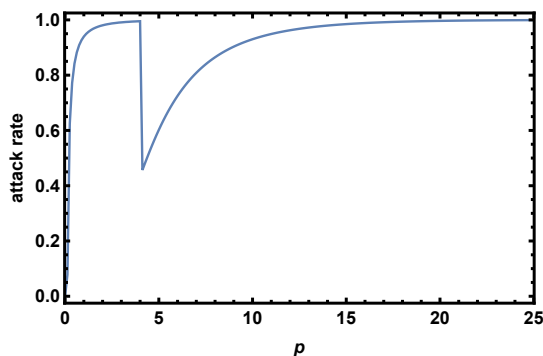


Fig. S28. Attack rate for different transmission probabilities p . The mean-field model including both behavioral mechanisms shows the critical transition observed in the network model.

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