

**Supplemental Material**

This document shows the simulation results for the unequal group sizes given in Table 1; all other settings were identical to the simulation with equal group sizes. The relative group size is constant across conditions: For  $J = 3$  groups relative group sizes are given by  $3/15 = 0.20$ ,  $5/15 = 0.33$ , and  $7/15 = 0.47$ , for group 1, 2, and 3, respectively. In the  $J = 5$  groups case the relative group sizes are  $3/25 = 0.12$ ,  $4/25 = 0.16$ ,  $5/25 = 0.20$ ,  $6/25 = 0.24$ , and  $7/25 = 0.28$ , for group 1, 2, 3, 4, and 5, respectively.

Table 1

*Sample sizes in the simulation with an unbalanced design for  $J = 3$  and 5 groups.*

Number of groups	$n_j$	Condition									
		1	2	3	4	5	6	7	8	9	10
$J = 3$ groups	$n_1$	3	6	12	30	60	120	300	600	1200	3000
	$n_2$	5	10	20	50	100	200	500	1000	2000	5000
	$n_3$	7	14	28	70	140	280	700	1400	2800	7000
$J = 5$ groups	$n_1$	3	6	12	30	60	120	300	600	1200	3000
	$n_2$	4	8	16	40	80	160	400	800	1600	4000
	$n_3$	5	10	20	50	100	200	500	1000	2000	5000
	$n_4$	6	12	24	60	120	240	600	1200	2400	6000
	$n_5$	7	14	28	70	140	280	700	1400	2800	7000
Average sample size	$n$	5	10	20	50	100	200	500	1000	2000	5000

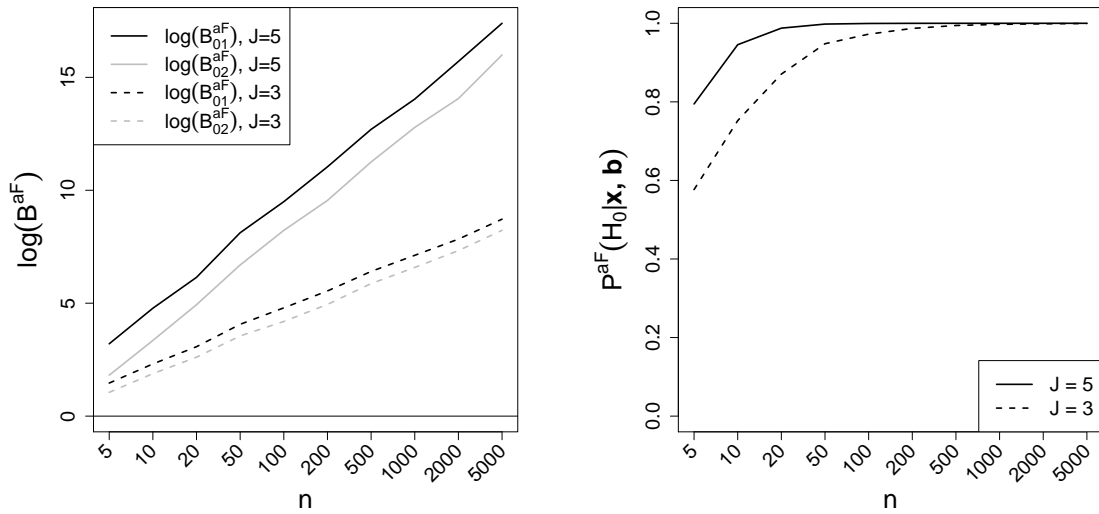


Figure 1. Simulation results for a null population in which all population variances were equal,  $\sigma_1^2 = \dots = \sigma_J^2$ , for  $J = 3$  groups (dashed lines) and  $J = 5$  groups (solid lines). We tested the true hypothesis  $H_0: \sigma_1^2 = \dots = \sigma_J^2$  against the two competing hypotheses  $H_1: \sigma_1^2 < \dots < \sigma_J^2$  and  $H_2: \text{not } \sigma_1^2 < \dots < \sigma_J^2$ . The plots show the median log Bayes factors (left-hand column) testing  $H_0$  against  $H_1$  (black lines) and  $H_0$  against  $H_2$  (gray lines) and the median posterior probability of  $H_0$  (right-hand column) as a function of the average sample size  $n$ .

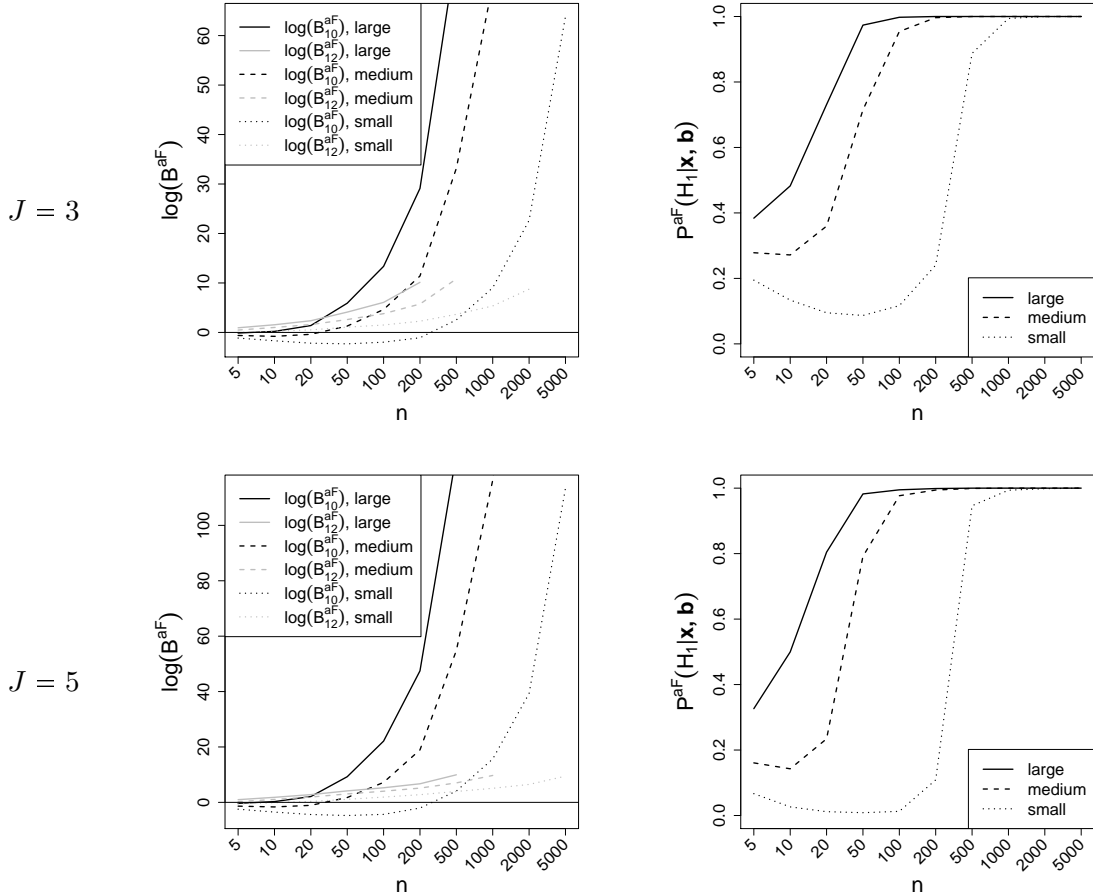


Figure 2. Simulation results for an order population in which the structure of the population variances was  $\sigma_1^2 < \dots < \sigma_J^2$ , with  $J \in \{3, 5\}$ . We considered three effect sizes: small (dotted lines), medium (dashed lines), and large (solid lines). We tested the true hypothesis  $H_1: \sigma_1^2 < \dots < \sigma_J^2$  against the two competing hypotheses  $H_0: \sigma_1^2 = \dots = \sigma_J^2$  and  $H_2: \text{not } \sigma_1^2 < \dots < \sigma_J^2$ . The plots show the median log Bayes factors (left-hand column) testing  $H_1$  against  $H_0$  (black lines) and  $H_1$  against  $H_2$  (gray lines) and the median posterior probability of  $H_1$  (right-hand column) as a function of the average sample size  $n$ . In the log Bayes factors plots the gray lines are discontinued due to numerical reasons (see text).

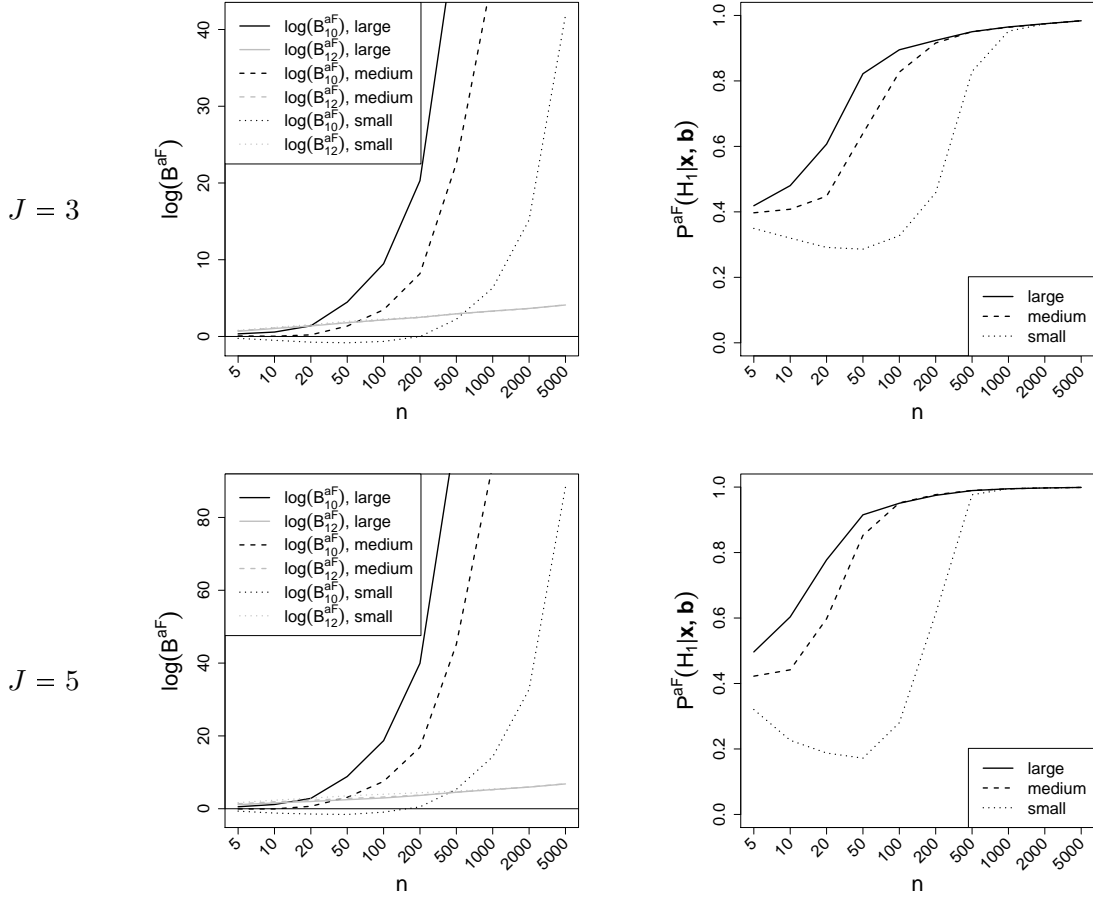


Figure 3. Simulation results for a mixed population. For  $J = 3$  groups the structure of the population variances was  $\sigma_1^2 < \sigma_2^2 = \sigma_3^2$ , whereas for  $J = 5$  groups it was  $\sigma_1^2 < \sigma_2^2 = \sigma_3^2 < \sigma_4^2 = \sigma_5^2$ . We considered three effect sizes: small (dotted lines), medium (dashed lines), and large (solid lines). For  $J = 3$  groups we tested the true hypothesis  $H_1: \sigma_1^2 < \sigma_2^2 = \sigma_3^2$  against the two competing hypotheses  $H_0: \sigma_1^2 = \sigma_2^2 = \sigma_3^2$  and  $H_2: \sigma_1^2 < (\sigma_2^2, \sigma_3^2)$ . For  $J = 5$  groups we tested the true hypothesis  $H_1: \sigma_1^2 < \sigma_2^2 = \sigma_3^2 < \sigma_4^2 = \sigma_5^2$  against  $H_0: \sigma_1^2 = \dots = \sigma_5^2$  and  $H_2: \sigma_1^2 < (\sigma_2^2, \sigma_3^2) < (\sigma_4^2, \sigma_5^2)$ . The plots show the median log Bayes factors (left-hand column) testing  $H_1$  against  $H_0$  (black lines) and  $H_1$  against  $H_2$  (gray lines) and the median posterior probability of  $H_1$  (right-hand column) as a function of the average sample size  $n$ .

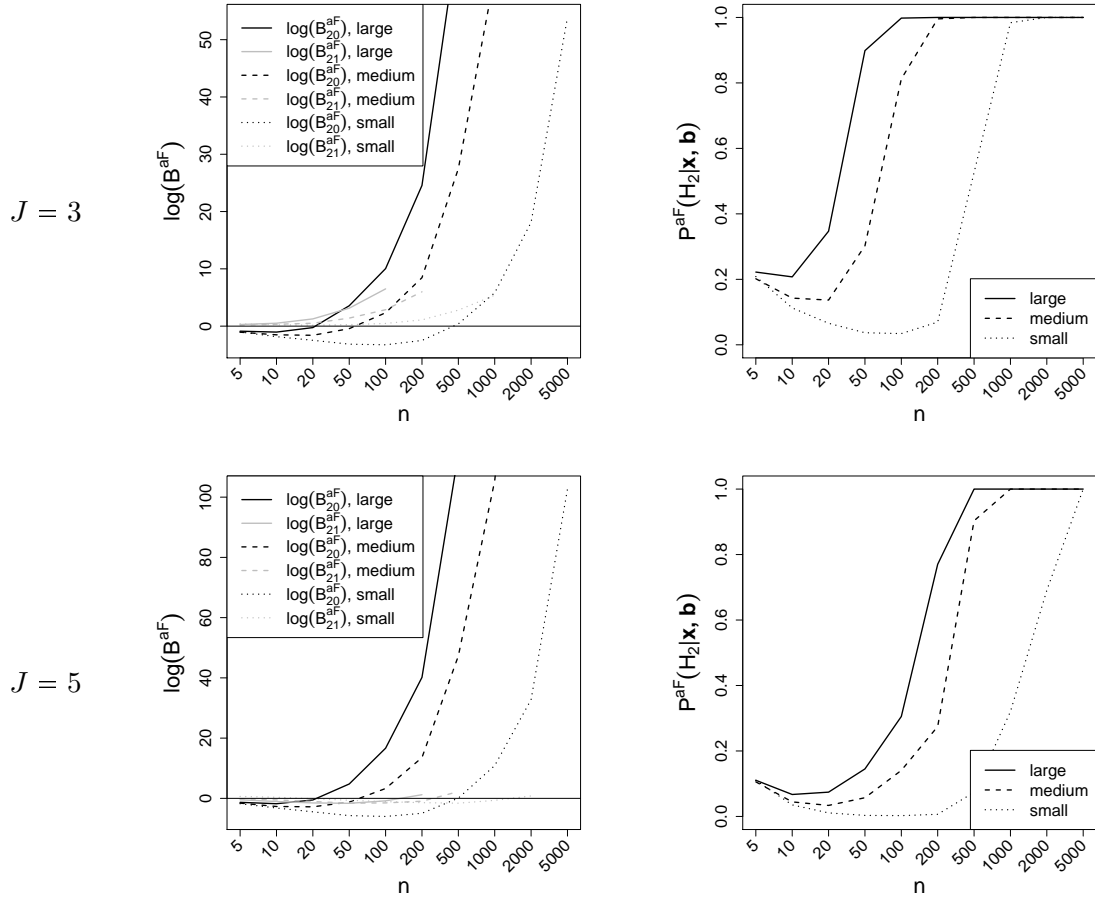


Figure 4. Simulation results for a near order population in which the structure of the population variances was  $\sigma_1^2 < \dots < \sigma_J^2 < \sigma_{J-1}^2$ , with  $J \in \{3, 5\}$ . We considered three effect sizes: small (dotted lines), medium (dashed lines), and large (solid lines). We tested three hypotheses:  $H_0: \sigma_1^2 = \dots = \sigma_J^2$ ,  $H_1: \sigma_1^2 < \dots < \sigma_J^2$ , and  $H_2: \text{not } \sigma_1^2 < \dots < \sigma_J^2$ . Note that the true hypothesis is the complement  $H_2$ . The plots show the median log Bayes factors (left-hand column) testing  $H_2$  against  $H_0$  (black lines) and  $H_2$  against  $H_1$  (gray lines) and the median posterior probability of  $H_2$  (right-hand column) as a function of the average sample size  $n$ . In the log Bayes factors plots the gray lines are discontinued due to numerical reasons (see text).

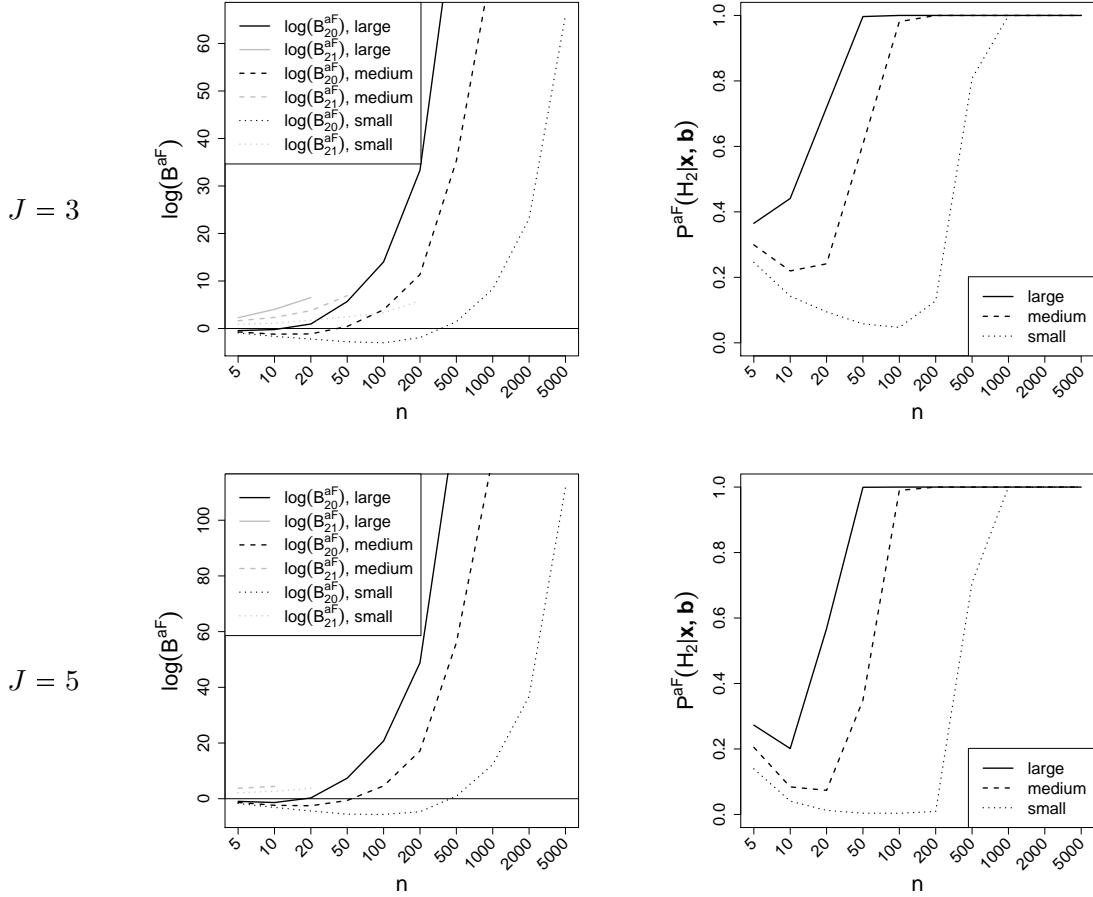


Figure 5. Simulation results for a reverse order population in which the structure of the population variances was  $\sigma_j^2 < \dots < \sigma_1^2$ , with  $J \in \{3, 5\}$ . We considered three effect sizes: small (dotted lines), medium (dashed lines), and large (solid lines). We tested three hypotheses:  $H_0: \sigma_1^2 = \dots = \sigma_j^2$ ,  $H_1: \sigma_1^2 < \dots < \sigma_j^2$ , and  $H_2: \text{not } \sigma_1^2 < \dots < \sigma_j^2$ . Note that the true hypothesis is the complement  $H_2$ . The plots show the median log Bayes factors (left-hand column) testing  $H_2$  against  $H_0$  (black lines) and  $H_2$  against  $H_1$  (gray lines) and the median posterior probability of  $H_2$  (right-hand column) as a function of the average sample size  $n$ . In the log Bayes factors plots the gray lines are discontinued due to numerical reasons (see text).