Muon performance studies in ATLAS towards a search for the Standard Model Higgs boson
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In this chapter we will cover the production and decay of the well known $Z$ boson and the predicted Standard Model Higgs boson at hadron collider experiments. First the elementary particles and forces in the Standard Model are introduced in Section 2.1. The weak force and one of its carriers, the $Z$ boson, will be addressed in more detail in Section 2.1.1. After that, Section 2.2 introduces the mechanism of electroweak symmetry breaking, a necessary ingredient in incorporating particle masses into the Standard Model. This mechanism also predicts the existence of the Higgs boson.

The production of the $Z$ boson and its decay channels are covered in Section 2.3, where we also derive an expression for the cross section for $pp \rightarrow q\bar{q} \rightarrow \gamma^* / Z \rightarrow \mu^- \mu^+$. This is the principal result that is used in the muon momentum resolution studies of Section 5.3. The simulation of production and decay of $Z$ bosons in ATLAS is discussed in Section 2.4. In Section 2.5 we turn our attention to the production and decay of the Standard Model Higgs boson at hadron colliders. We also briefly discuss the simulation of the dominant production mechanism of the Higgs at the LHC in that section.

2.1 Elementary particles and forces in the Standard Model

The current understanding of elementary particles and forces is described within a theoretical framework called the Standard Model [5–7]. The Standard Model was developed by numerous physicists from the early 1960s onwards and has been extremely successful in describing and predicting the elementary particles and their interactions.

The Standard Model describes particle interactions as a result of local gauge symmetries [8]. It incorporates three of the four known fundamental forces: the electromagnetic force, the strong force and the weak force. Gravity is too weak at the scales with which particle physics is concerned and is not incorporated in the Standard Model. Figure 2.1 shows a schematic overview of the Standard Model particles and interactions. We will discuss the particles and their interactions in more detail below.
Production and decay of the $Z$ and Higgs boson at hadron colliders

Figure 2.1: Elementary particle couplings are represented by lines.

Force particles

Each of the fundamental forces is mediated by a fundamental particle: the hypothetical graviton for gravity, the photon for electromagnetism, the gluon for the strong force and the $W^\pm$ and $Z$ bosons for the weak force. A particle experiences a force if it couples to the respective force mediator, or force carrier. All force carrier particles are bosons, i.e. particles with integer spin.

Matter particles

The elementary building blocks of ordinary matter are leptons and quarks. These are classified as fermions, particles with half integer spin. The quantum properties of a fermion determine with which force(s) it interacts: fermions with electric charge interact electromagnetically, with color charge strongly and with weak isospin weakly.

Leptons are fermions that interact through the weak force. The electron is a charged lepton that therefore interacts both through the electromagnetic and the weak force.

Protons and neutrons are examples of composite particles, each containing three quarks. Particles consisting of quarks are called hadrons, which explains the naming of the Large Hadron Collider (LHC), a proton-proton collider.

All fermions can be divided in three so-called generations of families, as illustrated in Figure 2.2. A particle from a higher generation has the same quantum numbers as the related particle from a lower generation and differs only in mass. For example, the up-, charm- and top-quark have identical quantum numbers but the up- and top-quark differ about five orders of magnitude in mass.
All particles and interactions can be described by imposing local gauge invariance on the Standard Model Lagrangian. In the Standard Model the electromagnetic and the weak force are unified in one *electroweak* description. However, this description leaves the particles massless, something that conflicts with experimental data. In 1964 several papers were written to address this issue \[\text{[9–11]}\]. A mechanism of *electroweak symmetry breaking* was proposed, now commonly known as the Higgs mechanism.\[1\] This mechanism introduces a new field, the Higgs field, and predicts the existence of a new particle, the Higgs boson. Elementary particles acquire mass by interactions with the Higgs field. The search for the Higgs boson is a major motivation for building the LHC. The mechanism of electroweak symmetry breaking is discussed in Section 2.2. For a more thorough review of the Higgs mechanism the reader is referred to refs. \[\text{[5–7]}\].

![Figure 2.2: The particles of the Standard Model.](image)

### 2.1.1 A closer look at the weak force

**Force carriers**

Weak interactions are mediated by $W^\pm$ and $Z$ gauge bosons. Weak interactions involving a $W^\pm$ boson are referred to as charged weak interactions. Neutral weak interactions involve a $Z$ boson. The $Z$ boson couples to particles with weak isospin.

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\[1\] It is also known as the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism, to mention all major contributors.
Both the $W^\pm$ and $Z$ bosons are massive particles, limiting the weak interactions to short ranges. They have first been observed by the UA1 and UA2 experiments at the CERN SPS \[12\]―\[14\]. The mass of the $Z$ has been measured at $e^+e^-$ experiments at LEP and at the Stanford Linear Collider (SLC) to be $91.1876\pm0.0021$ GeV with a full width of $2.4952\pm0.0023$ GeV \[15\]. It decays shortly after being produced.

### QED as an example for the theory of weak interactions

The theory of weak interactions was first modeled on a very successful other theory in particle physics: quantum electrodynamics (QED). QED is the quantum field theory of relativistic electrodynamics. It is the first theory in which full agreement between special relativity and quantum mechanics was obtained. Because of the success of QED it served as an example for the construction of the theory of weak interactions. Calculating probabilities for a process to occur in QED was simplified by the introduction of the Feynman rules. An introduction to these rules is beyond the scope of this thesis, but we will briefly mention the steps involved.

First the physics process is represented in a Feynman diagram, such as Figure 2.3. Two incoming particles, or Dirac fields $\psi_A$ and $\psi_B$, annihilate to a new particle, the propagator, which subsequently decays to two particles. Every vertex is assigned a factor $ig e\gamma^\mu$, where $\gamma^\mu$ represents the Dirac matrices and $g_e$ represents the coupling strength. A massless propagator is assigned a factor $ig e\gamma_{\mu\nu}/q^2$, where in this case $q^2$ is the sum of the two incoming four-momenta.

![An example of a Feynman diagram.](image)

The probability for the process to occur is proportional to the square of the matrix element, $\mathcal{M}$. This matrix element is calculated with Equation 2.1 using the incoming and outgoing particles and the assigned factors in the Feynman diagram.\footnote{An example of such a process is $e^+e^-$ annihilation into a photon, which subsequently decays to an $e^+e^-$ pair. This process is also known as Bhabha scattering.} This is a simplified example. Actually, matrix elements for all possible Feynman diagrams (i.e. with the same incoming and outgoing particles) need to be summed before taking the square.\footnote{This is a simplified example. Actually, matrix elements for all possible Feynman diagrams (i.e. with the same incoming and outgoing particles) need to be summed before taking the square.}
2.1 Elementary particles and forces in the Standard Model

\[ iM = \frac{i\gamma^\mu}{q^2} \bar{\psi}_B g_\mu \gamma^\nu \psi_A \]

\[ = -\frac{i\gamma^\mu}{q^2} g_{\mu\nu} \bar{\psi}_B \gamma^\mu \psi_A \bar{\psi}_C \gamma^\nu \psi_D. \] (2.1)

In QED, the expression for the matrix element contains terms of the form $\bar{\psi} \gamma^\mu \psi$. Similar calculation rules have been applied on weak interaction processes, but experiments revealed a remarkable difference which required a different underlying theory.

**V-A structure of weak interactions**

It turns out from experiment that weak interactions are parity violating, i.e. the weak force couples differently to particles with left-handed chirality than to particles with right-handed chirality. This feature is not understood fundamentally but follows from various measurements dating back to 1957 [16][17]. Mathematically, a Dirac field $\psi$ can be projected into its left-handed and right-handed components using the $\gamma^5$ matrix as

\[ \psi_L = \frac{1}{2} (1 - \gamma^5) \psi, \] (2.2)

\[ \psi_R = \frac{1}{2} (1 + \gamma^5) \psi, \] (2.3)

where $\psi_L$ and $\psi_R$ denote the left-handed and right-handed component, respectively. In the ultra-relativistic limit chirality is equal to helicity, which is more easily interpreted: a particle with left-handed helicity has its spin in opposite direction of its momentum.

The first description of charged weak interactions, i.e. processes involving $W^\pm$, used terms with $\bar{\psi} \gamma^\mu \psi$ in its matrix element calculations. This pure vector description turned out to be wrong. New terms of the form $\bar{\psi} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \psi$ were required, showing equally strong vector ($\gamma^\mu$) and axial-vector ($\gamma^\mu \gamma^5$) components. This is referred to as the V-A structure of weak interactions. Comparing this V-A structure to Equation 2.2 shows that the $W$ boson only couples to the left-handed component of $\psi$.

For charged weak interactions the vector and axial-vector components are equally strong. For neutral weak interactions, i.e. processes involving $Z$, this is not the case and we have $\gamma^\mu \frac{1}{2} (c_V^f - c_A^f \gamma^5)$ where $c_V$ and $c_A$ are the vector and axial-vector coupling respectively. These couplings are defined as

\[ c_V^f = T_f^3 - 2Q_f \sin^2 \theta_W, \] (2.4)

\[ c_A^f = T_f^3, \] (2.5)

where $T_f^3$ is the third component of the weak isospin and $Q_f$ is the electric charge of the fermion. The angle $\theta_W$ is the weak mixing angle. This angle is related to the masses of the weak gauge bosons via $\cos \theta_W = M_W / M_Z$. Table 2.1 shows values for $Q_f$, $T_f^3(= c_A^f)$ and $c_V^f$ for different fermions $f$.

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4Using Dirac matrices $\gamma^\mu$ and with $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

5The first experiment indicating parity violation used the decay of polarized cobalt nuclei and showed a clear preference for electrons to be emitted in a direction opposite to that of the nuclear spin.
Production and decay of the Z and Higgs boson at hadron colliders

\[
\begin{array}{|c|ccc|}
\hline
f & Q_f & T_f^3 (= c_A^f) & c_V^f \\
\hline
\nu_e, \nu_\mu, \nu_\tau & 0 & \frac{1}{2} & \frac{1}{2} \\
e^-, \mu^-, \tau^- & -1 & -\frac{1}{2} & \approx -0.04 \\
u, c, t & \frac{2}{3} & \frac{1}{2} & \approx 0.19 \\
d, s, b & -\frac{1}{3} & -\frac{1}{2} & \approx -0.35 \\
\hline
\end{array}
\]

\textbf{Table 2.1:} Values for } Q_f, T_f^3 = c_A^f \text{ and } c_V^f \text{ for different fermions.}

The quantum properties } T_f^3 \text{ and } Q_f \text{ are of importance for Z decays. The Z boson is an unstable particle and will decay shortly after being produced. The decay rate for a Z decaying into a fermion anti-fermion pair can be calculated from the quantum properties of the involved fermions. The third component of the weak isospin and the charge of the particular fermion determine the Z decay rate via

\[
\Gamma(Z \to f \bar{f}) = N_C \frac{\sqrt{2} G_F M_Z^3}{6\pi} \cdot \left[ (T_f^3 - Q_f \sin^2 \theta_W)^2 + (Q_f \sin \theta_W)^2 \right], \quad (2.6)
\]

where } G_F \text{ is the Fermi coupling constant and } N_C \text{ is the number of colors } \text{[15]. For } q\bar{q} \text{ final states } N_C = 3, \text{ otherwise } N_C = 1.

\section{2.2 Electroweak symmetry breaking}

The Standard Model describes the strong and electroweak interactions by an } SU(3)_C \times SU(2)_L \times U(1)_Y \text{ gauge symmetry. The } SU(3)_C \text{ gauge symmetry describes the strong interactions and will not be discussed in this thesis. The gauge symmetry } SU(2)_L \times U(1)_Y \text{ is the unified electroweak description of the weak force and electromagnetism. The Standard Model Lagrangian for the electroweak interactions is invariant under local gauge transformations.}^6 \text{ Requiring a local symmetry results in four gauge fields (} W_{1,2,3} \text{ and } B \text{) and four gauge bosons (} W^\pm, Z \text{ and } \gamma \text{) plus interaction terms. Another consequence of this invariance is that the gauge bosons are required to be massless. For the electromagnetic part of the theory this is not a problem as the photon is indeed massless. However, the limited range of the weak interaction clearly indicates that the } W \text{ and } Z \text{ bosons are massive and that the theoretical description is incomplete.}

One solution is to add mass terms for the gauge bosons in the Lagrangian by hand. For the } W \text{ bosons, the } Z \text{ boson and the photon one expects mass terms of the form } -M_W^2 (W_\mu)^2, -\frac{1}{2} M_Z^2 B_\mu^2 \text{ and } -\frac{1}{2} M_\gamma^2 A_\mu^2 \text{ respectively. These terms are not gauge invariant and make the theory non-renormalizable. This is clearly not a successful approach.}

\footnote{This is highly non-trivial. In fact, it is because it happens to describe the observed interactions that this local gauge invariance requirement keeps on being used.}

\footnote{SU(2) gives } 2^2 - 1 = 3 \text{ gauge fields, } U(1) \text{ the other } 1^2 = 1.
2.2 Electroweak symmetry breaking

Higgs mechanism

A solution to this problem was provided by Peter Higgs in 1964 [10] and is known as the Higgs mechanism. It introduces two complex fields in an isospin doublet $\phi$ and adds a term, $L_{\text{scalar}}$, to the Standard Model Lagrangian for these fields.

$$L_{\text{scalar}} = (D^\mu \phi)^\dagger (D_\mu \phi) - (\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2) / V(\phi)$$

(2.7)

where the isospin doublet has the form

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i \phi_2 \\ \phi_3 + i \phi_4 \end{pmatrix}.$$  

(2.8)

The covariant derivative $D_\mu$ contains the weak hypercharge $Y$ and the $SU(2)_L$ and $U(1)_Y$ gauge couplings $g$ and $g'$.

$$D_\mu = \partial_\mu + ig \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu + ig' Y B_\mu,$$

(2.9)

where $\vec{\tau}$, $\vec{W}_\mu$ and $B_\mu$ represent the Pauli matrices, the $SU(2)_L$ gauge fields and the $U(1)_Y$ gauge fields respectively. A detailed discussion about the Higgs mechanism is beyond the scope of this thesis. However, it is instructive to know why this mechanism is referred to as electroweak symmetry breaking.

For simplicity, imagine a classical scalar field $\phi$ with a potential given by

$$V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4,$$

(2.10)

where $\lambda > 0$. The corresponding terms for this field $\phi$ in the Lagrangian are

$$L_\phi = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4.$$  

(2.11)

Equation 2.11 is symmetric in $\phi$ and describes a particle with mass $\mu$. By choosing $\mu^2 < 0$ we see that the particle mass is imaginary. However, we need to evaluate the perturbations around the minimum of the potential, which is at $v = \pm \sqrt{\mu / \lambda}$. If we now introduce a quantum field $h$ which is centered at the vacuum, i.e. $h = \phi - v$, we can rewrite the Lagrangian in terms of $h$ as

$$L_h = \frac{1}{2} (\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4 - \frac{1}{4} \lambda v^4.$$  

(2.12)

A positive mass term, $m_h = \sqrt{2 \lambda v^2}$, appears for the corresponding particle, i.e. the Higgs boson. The Lagrangian is still symmetric in the field $\phi$ but the vacuum is no longer symmetric in the field $h$. This is referred to as spontaneous symmetry breaking.
Mass of the gauge bosons

We return to the problem of massless gauge bosons, resulting from the requirement of local gauge invariance. Equation 2.7 gives the term that is added to the Standard Model Lagrangian by the Higgs mechanism. The choice of potential results in the symmetry breaking, as was shown before. We will now focus our discussion on the kinetic term, \((D^\mu \phi)^\dagger (D_\mu \phi)\), which will give rise to the mass of the gauge bosons.

The isospin doublet that breaks \(SU(2)_L \times U(1)_Y\) but leaves \(U(1)_Q\) invariant, and thus the photon massless, is

\[
\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},
\]  

which corresponds to Equation 2.8 for \(\phi_1 = \phi_2 = \phi_4 = 0\) and \(\phi_3 = v + h\).

Using Equations 2.9 and 2.13 to calculate \((D^\mu \phi)^\dagger (D_\mu \phi)\) results in terms proportional to \(v^2\), \(vh\) and \(h^2\). Here we will only discuss the terms \(\propto v^2\) because these give rise to the masses of the gauge bosons. Neglecting other terms and recognizing \(Y\phi_0 = 1\) we have

\[
(D^\mu \phi_0)^\dagger (D_\mu \phi_0) = \frac{1}{8} v^2 \left[ g^2 (W_1^2 + W_2^2) + (-gW_3 + g'B_\mu)^2 \right] + \mathcal{O}(vh) + \mathcal{O}(h^2). \tag{2.14}
\]

No mass terms can be identified in this equation. In fact, the gauge fields \(W_1\) and \(W_2\) mix to form well defined masses, i.e. the \(W^+\) and \(W^-\) bosons observed in experiments: \(W^\pm = \frac{1}{\sqrt{2}} (W_1 \mp iW_2)\). Similarly the gauge fields \(W_3\) and \(B_\mu\) mix to form the \(Z\) boson, \(Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gW_3 - g'B_\mu)\), and the photon \(A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'W_3 + gB_\mu)\). Inserting this in Equation 2.14 gives

\[
(D^\mu \phi_0)^\dagger (D_\mu \phi_0) = \frac{1}{8} v^2 \left[ 2g^2 W^+ W^- + (g^2 + g'^2) Z_\mu^2 + 0 \cdot A_\mu^2 \right]. \tag{2.15}
\]

From this equation, the mass terms can immediately be identified as \(M_{W^\pm} = \frac{1}{2} vg\), \(M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}\) and \(M_\gamma = 0\). Exactly the result that Higgs and others were trying to achieve: three massive gauge bosons and one massless gauge boson for the electroweak symmetry group.

While the terms proportional to \(vh\) and \(h^2\) can be neglected for the discussion on gauge boson masses, they are important terms for other reasons. These terms describe the interactions between the gauge bosons and the Higgs field. The gauge fields enter quadratically, indicating a three-point interaction and four-point interaction of the Higgs to two gauge bosons for terms with \(vh\) and \(h^2\), respectively. One example of a three-point interaction is \(H \rightarrow ZZ\). This is exactly the interaction that motivates our search for a Standard Model Higgs in a four muon final state with two intermediate \(Z\) bosons (cf. Chapter 7). The strength of the coupling, and thereby the decay rate, is determined by the mass of the Higgs boson and the mass of the gauge boson. The mass of the Higgs boson is unknown, but once it is known the predicted coupling strengths can be tested.

\footnote{Note that \(W^+ W^- = \frac{1}{2} (W_1^2 + W_2^2)\).}
2.2 Electroweak symmetry breaking

Mass of the fermions

The fermion mass terms cannot be added by hand in the Standard Model Lagrangian. Like was the case for the gauge bosons, these mass terms of the form $-m_f \bar{\psi} \psi$ would not be local gauge invariant. This can be seen by expanding $\psi$ into its left-handed and right-handed components as

$$-m_f \bar{\psi} \psi = -m_f (\bar{\psi}_R + \bar{\psi}_L)(\psi_R + \psi_L) = -m_f (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R),$$  \hspace{1cm} (2.16)$$

where we use $\bar{\psi}_R \psi_R = \bar{\psi}_L \psi_L = 0$. The left-handed doublet and the right-handed singlet transform differently under $SU(2)_L \times U(1)_Y$ and therefore Equation 2.16 is not gauge invariant under local $SU(2)_L \times U(1)_Y$ transformations.

However, by using the Higgs field ($\phi$) that we introduced before, we can construct a Yukawa interaction term that does remain gauge invariant under local $SU(2)_L \times U(1)_Y$ transformations.

$$\lambda_f (\bar{\psi}_R \phi \psi_L + \bar{\psi}_L \phi \psi_R),$$  \hspace{1cm} (2.17)$$

where $\lambda_f$ is the Yukawa coupling. Taking the electron as an example we can work out the resulting electron term for the Lagrangian from Equation 2.17

$$\mathcal{L}_e = -\lambda_e \left[ \bar{e}_R \phi \psi_L + \bar{e}_L \phi \psi_R \right] = -\lambda_e \frac{(v + h)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R) = -\lambda_e \frac{(v + h)}{\sqrt{2}} \bar{e} e = -\frac{\lambda_e v}{\sqrt{2}} \bar{e} e - \frac{\lambda_e}{\sqrt{2}} h \bar{e} e.$$  \hspace{1cm} (2.18)$$

The first term in Equation 2.18 gives the electron mass, $m_e = \lambda_e v / \sqrt{2}$. The second term describes the electron-Higgs interaction. As was the case for the gauge bosons, this interaction is proportional to the electron mass, since $\lambda_e / \sqrt{2} = m_e / v$.

Mass of the Higgs boson

An expression for the mass of the Higgs boson followed from our discussion above and is given by $m_h = \sqrt{2 \lambda v^2}$. From muon decay experiments it is derived that $v \approx 246$ GeV \cite{15}. Since the Higgs self-coupling $\lambda$ is a free parameter, the mass of the Higgs boson is not predicted in the Standard Model. Direct searches at LEP and Tevatron exclude, at 95\% confidence level, a Standard Model Higgs boson with a mass of $M_H < 114.4$ GeV and $156 < M_H < 177$ GeV, respectively \cite{18,19}.\hspace{1cm}
In addition to experimental limits, there are also theoretical bounds on the Higgs mass. A detailed discussion about the argumentation behind these bounds is beyond the scope of this thesis, but we will briefly mention some arguments.

The Standard Model Higgs boson plays an important role in the cancellation of high-energy divergences in the amplitude for elastic scattering processes of longitudinally polarized massive gauge bosons. Due to this cancellation of divergences the theory remains unitary and renormalizable. Requiring perturbation theory to remain valid puts an upper bound on the Higgs mass of about 700 GeV.[9]

The self-coupling, $\lambda$, is not a constant but depends on the energy scale, $Q^2$. The evolution function, $\beta_\lambda \equiv d\lambda/d(\ln Q^2)$, can be evaluated at very large and very small values of $\lambda$, i.e. $\lambda \gg g, g'$ and $h_t$.[10] or $\lambda \ll g, g'$ and $h_t$. For $\lambda \gg g, g'$ and $h_t$ there is some cut-off scale $\Lambda_1$ at which $\lambda(\Lambda_1)$ becomes infinite. This so-called Landau pole defines an upper bound on the Higgs mass.

For $\lambda \ll g, g'$ and $h_t$ there is some cut-off scale $\Lambda_2$ at which $\lambda(\Lambda_2)$ becomes negative. The requirement of a positive value for $\lambda$ is referred to as vacuum stability and provides a lower bound on the Higgs mass. Figure 2.4 shows the upper and lower bounds as a function of the energy scale $\Lambda$. The allowed region derived from the running of the self-coupling is indicated in the figure.

![Figure 2.4](image.png)

**Figure 2.4:** Theoretical limits on the Standard Model Higgs boson mass, derived from the running of the self-coupling. A top quark mass of 175 GeV is assumed.[20]

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[9]: A consequence of this is that if we don’t find the Higgs, or something else that cancels these divergences and fits the Standard Model, then the Standard Model must be wrong!

[10]: $h_t$ is the top quark Yukawa coupling.
2.3 Production and decay of the $Z$ boson at hadron colliders

The muon performance studies that are presented in Chapter 5 make use of $Z \rightarrow \mu^– \mu^+$ events in ATLAS. Before addressing this $Z$ decay channel we look at the production of $Z$ bosons in hadron colliders. Afterwards we discuss the $Z$ decay channels in general, and the $\mu^– \mu^+$ decay channel in particular. Since we will be looking at these di-muon final states we cover the di-muon invariant mass distribution. The shape of this distribution is referred to as the di-muon invariant mass lineshape and is discussed in some detail.

2.3.1 Production of the $Z$ boson

Many different particles can be produced in hadronic collisions such as at the proton-proton ($pp$) collisions of the LHC and at the proton-antiproton ($p\bar{p}$) collisions of the Tevatron. The main figures of merit for an accelerator are its center of mass energy, $\sqrt{s}$, and instantaneous luminosity, $\mathcal{L}$. The instantaneous luminosity in a particle collider is a measure for the number of collisions per unit area per second. The number of produced particles per second, $N$, in a collision is proportional to $\mathcal{L}$ and the particle production cross section, $\sigma(\sqrt{s})$, via

$$N = \mathcal{L} \sigma(\sqrt{s}).$$

(2.19)

For fixed $\mathcal{L}$ the particle production cross sections can be compared for various values of $\sqrt{s}$. Figure 2.5 shows an example of such a comparison with the expected number of events per second on the right vertical axis [21]. It shows an increase of one order of magnitude in the production of $Z$ bosons going from the Tevatron center of mass energy to the LHC design center of mass energy of 14 TeV.

The $Z$ boson couples weakly to all fermions because all fermions carry weak isospin. In hadron colliders and at leading order the only possible s-channel Feynman diagram for $Z$ production is from quark-antiquark ($q\bar{q}$) annihilation. This annihilation process can also produce a virtual photon $\gamma^*$, since quarks couple to the electromagnetic force carrier. Therefore we get two possible Feynman diagrams, which are often shown in one diagram as in Figure 2.6. Drell and Yan were the first to explain this $q\bar{q}$ annihilation process, using the parton model [22].

Whether the $q\bar{q}$ annihilation in Figure 2.6 results in a virtual photon or a $Z$ boson depends on the energy available in the hard scattering process. This in turn depends on the proton momentum fractions, $x$, carried by the quark and anti-quark. The momentum fractions for quarks and gluons are modeled by parton distribution functions (PDFs). Figure 2.7 shows the distribution function $f(x,Q^2)$ multiplied by $x$ for a proton at two energy scales $Q^2$. For anti-protons the quarks and anti-quarks in Figure 2.7 are inverted.

At the Tevatron ($p\bar{p}$ collider) the $q\bar{q}$ annihilation mostly occurs between two valence quarks. At the LHC ($pp$ collider) Drell-Yan $Z$ production is only possible with an anti-quark from the sea quarks, i.e. quarks from virtual $q\bar{q}$ pairs that arise from gluon splitting. Sea quarks carry on average a lower momentum fraction than the valence quarks (cf. Figure 2.7).
The calculation of the $Z$ production cross section involves a summation over all quarks with sufficient energy to produce a $Z$. Since at the LHC a sea quark is required, the production cross section would be reduced compared to the Tevatron, for equal $\sqrt{s}$. However, at the center of mass energy of the LHC more sea quarks with sufficient energy to produce a $Z$ are available. This effect dominates by far and causes an increased $Z$ production cross section at the LHC compared to the Tevatron.
2.3 Production and decay of the $Z$ boson at hadron colliders

**Figure 2.7:** NNLO parton distribution functions for a proton [21].

### 2.3.2 $Z$ decay channels

When studying $Z$ decay channels it is important to remember that the $Z$ boson couples weakly to all fermions, $f$. Conservation of charge and weak isospin only allows for $Z \to f\bar{f}$ decays to occur, e.g. $Z \to \mu^+\mu^-$. Table 2.2 shows the decay modes of the $Z$ and their respective measured decay widths [15]. The observed differences can be explained by the quantum properties of the fermions, as was shown in Equation 2.6.

<table>
<thead>
<tr>
<th>$Z$ decay modes</th>
<th>$\Gamma_i/\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-e^+$</td>
<td>$3.363 \pm 0.004%$</td>
</tr>
<tr>
<td>$\mu^-\mu^+$</td>
<td>$3.366 \pm 0.007%$</td>
</tr>
<tr>
<td>$\tau^-\tau^+$</td>
<td>$3.367 \pm 0.008%$</td>
</tr>
<tr>
<td>$\nu\bar{\nu}$</td>
<td>$20.00 \pm 0.06%$</td>
</tr>
<tr>
<td>$q\bar{q}$</td>
<td>$69.91 \pm 0.06%$</td>
</tr>
</tbody>
</table>

*Table 2.2: $Z$ boson decay modes and their relative decay widths.*

Although most $Z$ bosons decay into hadrons ($q\bar{q}$) this is not a very clean experimental signature. QCD backgrounds overwhelm the signal from hadronic $Z$ decays. In contrast, the excellent momentum resolution of electrons and muons in ATLAS results in a clear signal from the leptonic $Z$ decays. The implementation of lepton triggers in ATLAS helps in selecting interesting events for this analysis. Requiring isolated electrons and muons reduces the QCD background in these decay channels to a negligible level.
Production and decay of the \(Z\) and Higgs boson at hadron colliders

Using the properties of the \(Z\) boson, one of the standard candles in particle physics, extensive performance studies are possible. For example the di-muon invariant mass spectrum around the mass of the \(Z\) boson can be used to study the muon momentum resolution as described in Section 5.3. For this purpose a theoretical description of the expected di-muon invariant mass distribution, or lineshape, is required.

### 2.3.3 The cross section for \(pp \rightarrow q\bar{q} \rightarrow \gamma^*/Z \rightarrow \mu^-\mu^+\)

The di-muon invariant mass lineshape is a direct reflection of the production cross section \(\sigma(pp \rightarrow q\bar{q} \rightarrow \gamma^*/Z \rightarrow \mu^-\mu^+)\). To obtain the theoretical prediction for this lineshape we follow the Feynman rules. At leading order there are two Feynman diagrams that contribute to this process, shown in Figure 2.8. Therefore we have two matrix elements which have to be summed and then squared, giving us three terms in the production cross section formula: a pure photon term, a pure \(Z\) term and an interference term.

![Feynman diagram for Drell-Yan production in a proton-proton collision, followed by a decay into two muons.](image)

**Figure 2.8:** Feynman diagram for Drell-Yan production in a proton-proton collision, followed by a decay into two muons.

In reference [7] a detailed overview of the calculation is given. The total cross section \(\sigma\) for producing a lepton pair is obtained by weighting the subprocess cross section \(\sigma(q\bar{q} \rightarrow l^+l^-)\) with the parton distribution functions \(f(x)\) as

\[
\sigma = \sum_q \int dx_1 dx_2 f_q(x_1) f_{\bar{q}}(x_2) \sigma(q\bar{q} \rightarrow l^+l^-). \tag{2.20}
\]

The subprocess cross section depends on the energy available in the hard scatter, \(\hat{s}\), and is given by

\[
\hat{\sigma}(q\bar{q} \rightarrow l^+l^-)(\hat{s}) = \frac{4\pi\alpha^2}{3\hat{s}} \frac{1}{N} \left( Q_q^2 - 2Q_q c_V^q c_V^q \chi_1(\hat{s}) + (c_A^q)^2 + (c_A^q)^2 \right),
\]

\[
\tag{2.21}
\]
where $\alpha$ and $N$ are the electromagnetic coupling and an overall color averaging factor, respectively.

$\chi_1(\hat{s})$ and $\chi_2(\hat{s})$ are defined as

\begin{align*}
\chi_1(\hat{s}) &= \kappa \frac{\hat{s}(\hat{s} - M_Z^2)}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}, \\
\chi_2(\hat{s}) &= \kappa^2 \frac{\hat{s}^2}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2},
\end{align*}

where $M_Z$ and $\Gamma_Z$ are the mass and full width of the $Z$ boson and $\kappa = \frac{\sqrt{2} G_F M_Z}{\sqrt{\pi} \alpha}$ with $G_F$ the Fermi coupling constant. Section 5.3 shows how Equation 2.20 is applied in ATLAS: how the structure functions are modeled by fits to simulation and how detector resolution effects are included.

We see that Equation 2.21 indeed shows three terms:

- a pure photon term ($\propto \frac{\hat{s}}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$);
- a pure $Z$ term ($\propto \frac{\hat{s}}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$);
- an interference term ($\propto \frac{\hat{s} - M_Z^2}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$).

To a good approximation $\sqrt{\hat{s}}$ can be taken to be the measured di-muon invariant mass. For $\hat{s} \ll M_Z^2$ the photon term dominates. For $\hat{s} \approx M_Z^2$ the $Z$ term dominates.

### 2.4 Simulation of $Z \rightarrow \mu^- \mu^+$ in ATLAS

To obtain a good understanding of the production and decay of $Z$ bosons in ATLAS data the observations are compared to simulated samples. The simulation needs to incorporate up-to-date knowledge on production processes and decay channels, but also a detailed description of the detector used in the analysis.

The simulation software is integrated into the ATLAS software framework, ATHENA, and uses the Geant4 simulation for its detector description. The simulation is generally divided into three steps:

1) event generation - simulating the collision and its collision products;
2) detector simulation - describing the detector geometry and how the collision products interact with the (in)active detector material;
3) digitization - translating particle-detector interactions into times, amplitudes and electronics signals such as voltages, currents, bits and bytes as they would be measured by the detector in real collisions.

The file formats after step three are identical to the ones after data taking. This allows for a direct comparison between data and simulation by running particle reconstruction algorithms (cf. Section 3.5) on these identical file formats. A more detailed description of the simulation steps can be found in ref. 25. The different steps involved in event generation are discussed in more detail below.
Production and decay of the $Z$ and Higgs boson at hadron colliders

**Event generation**

Each collision event, including production and decay of intermediate particles such as the $Z$ boson, can be split up in several stages according to the factorization theorem \[26\]. The description of a proton-proton collision includes:

- the structure of the incoming protons;
- the hard scattering process;
- parton showering;
- hadronization.

In Section \[2.3\] we discussed the importance of the structure of the incoming protons in the context of parton distribution functions (PDFs). In the case of Drell-Yan production the available energy in the hard scattering determines whether a virtual photon or a $Z$ boson can be created. The hard scattering is the core process of the hadronic collision and describes the interaction of the incoming partons.

In all stages of the collision process stochastic processes are involved. Monte Carlo (MC) generators are widely used to model this stochastic behavior. Simulation samples in ATLAS are therefore commonly referred to as ‘Monte Carlo’ samples\[11\].

Several types of MC generators are used in ATLAS studies. A distinction is made between general purpose event generators that perform the full simulation chain and specialized matrix element generators. For a detailed description of the (differences between) MC generators the reader is referred to refs. \[25,27\]. The following generators are most common for $Z \rightarrow \mu^-\mu^+$ studies in ATLAS:

- **PYTHIA**: a widely used general purpose event generator capable of performing the full simulation chain. A wide range of hard scattering processes are available at leading order (LO) calculation. For higher order corrections the parton shower approach is used;

- **HERWIG**: a similar, and also widely used, general purpose event generator. HERWIG differs from PYTHIA in the modeling of the parton showers and the hadronization;

- **MC@NLO**: a matrix element generator including full next-to-leading order (NLO) calculations for QCD processes during the hard scattering process. It includes higher order approximations of the parton shower and can be interfaced with HERWIG;

- **ALPGEN**: a matrix element generator that can generate LO matrix elements for up to six additional final state partons. Spin correlations are also taken into account and interfaces to PYTHIA and HERWIG exist.

\[11\] One will also hear people mentioning “Monte Carlo data”, which I think is wrong and confusing.
Depending on the order of calculation parton showering techniques can be applied in the event generation. Parton showering models are applied for LO calculations. Perturbative corrections from the emission of gluons by the (anti-)quarks\[^{12}\] are approximated by parton showering models: a gluon splits into a $q\bar{q}$ pair which in turn radiates gluons after which the splitting of the gluons can occur again until a cut-off energy is reached.

Particle decays are modeled using measured and predicted particle masses, lifetimes and branching ratios. The decay of the $Z$ boson is simulated using the decay widths discussed in Section 2.3.2. However, in the configuration of the event generator a choice for a specific decay channel can be made. This effectively selects only the required decay channel to be in the generator output and reduces the computing time needed for the further steps of detector simulation and digitization.

If the decay products contain (anti-)quarks the parton showering can also be applied at that stage. Eventually only color free quark composite particles, i.e. hadrons, are allowed in nature. These states should be formed from the produced (anti-)quarks and gluons. This process of hadronization involves non-perturbative processes and cannot be calculated analytically.

### 2.5 Production and decay of the Higgs boson at the LHC

The mass of the Higgs boson is an unknown parameter in the Standard Model. Depending on its mass, the production cross-section for the Higgs boson varies. The main production mechanism for the Standard Model Higgs at the LHC is via gluon-gluon ($gg$) fusion. Figure 2.9 shows a leading order Feynman diagram for this process.

![Feynman diagram for Higgs production via gluon-gluon fusion.](image)

**Figure 2.9:** Feynman diagram for Higgs production via gluon-gluon fusion.

The second largest production mechanism is via vector boson fusion (VBF). In addition to these two largest mechanisms several associated production mechanisms exist in which the Higgs boson is produced in association with other particles. Table 2.3 shows production cross sections at several assumptions for the Higgs mass \[^{28}\]. Figure 2.10(a) shows the total production cross section and the sub-leading contributions.

The Higgs boson is an unstable particle and will decay shortly after being produced. Many possible decay channels exist for the Standard Model Higgs boson. In this thesis we will focus on the decay channel where the Higgs boson decays into two $Z$ bosons,

\[^{12}\]The incoming (anti-)quarks can emit photons and/or gluons because of their electromagnetic and color charge, respectively. Gluon emission dominates in hadronic collisions.
Production and decay of the $Z$ and Higgs boson at hadron colliders

which subsequently decay into two muons each. A four muon final state could then be detected in the Muon Spectrometer. Figure 2.10(b) shows the branching ratios for various decay channels of the Higgs boson, including the decay to two $Z$ bosons.

<table>
<thead>
<tr>
<th>Production</th>
<th>$M_H = 114$ GeV</th>
<th>130 GeV</th>
<th>150 GeV</th>
<th>200 GeV</th>
<th>300 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg$ fusion</td>
<td>18</td>
<td>14</td>
<td>10</td>
<td>5</td>
<td>2.4</td>
</tr>
<tr>
<td>VBF</td>
<td>1.35</td>
<td>1.15</td>
<td>0.96</td>
<td>0.64</td>
<td>0.30</td>
</tr>
<tr>
<td>WH</td>
<td>0.78</td>
<td>0.49</td>
<td>0.30</td>
<td>0.10</td>
<td>0.020</td>
</tr>
<tr>
<td>ZH</td>
<td>0.42</td>
<td>0.28</td>
<td>0.17</td>
<td>0.061</td>
<td>0.012</td>
</tr>
<tr>
<td>$t\bar{t}H$</td>
<td>0.11</td>
<td>0.077</td>
<td>0.049</td>
<td>0.018</td>
<td>0.0047</td>
</tr>
</tbody>
</table>

Table 2.3: Higgs production cross-sections in pb for various Higgs masses.

Figure 2.10: Higgs production cross section and branching ratio at various masses [28].

Simulation of the Higgs boson at the LHC

The simulation of the Higgs production is performed by several event generators. POWHEG generates events with next to leading order (NLO) calculations and can be easily interfaced to a shower generator [29]. Within ATLAS POWHEG is currently the event generator of preference for studies of signals where the Higgs boson decays to lepton final states. Therefore, our studies of Chapter 7 are performed with samples where POWHEG is interfaced to PYTHIA. We obtain the shapes of Higgs mass distributions from these simulated samples but scale these according to a recent update of Higgs production cross sections and branching ratios [28]. This will give us the most up to date prediction of the expected number of Higgs bosons in our studies.