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DOI
10.1103/PhysRevD.106.062002

Publication date
2022

Document Version
Final published version

Published in
Physical Review D. Particles, Fields, Gravitation, and Cosmology

Citation for published version (APA):

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Download date: 23 Dec 2023
Search for gravitational waves from Scorpius X-1 with a hidden Markov model in O3 LIGO data

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(Received 26 January 2022; accepted 26 August 2022; published 21 September 2022)

Results are presented for a semicoherent search for continuous gravitational waves from the low-mass x-ray binary Scorpius X-1, using a hidden Markov model (HMM) to allow for spin wandering. This search improves on previous HMM-based searches of Laser Interferometer Gravitational-Wave Observatory data by including the orbital period in the search template grid, and by analyzing data from the latest (third) observing run. In the frequency range searched, from 60 to 500 Hz, we find no evidence of gravitational radiation. This is the most sensitive search for Scorpius X-1 using a HMM to date. For the most sensitive subband, starting at 256.06 Hz, we report an upper limit on gravitational wave strain (at 95% confidence) of $h_0^{95\%} = 6.16 \times 10^{-26}$, assuming the orbital inclination angle takes its electromagnetically restricted value $\iota = 44^\circ$. The upper limits on gravitational wave strain reported here are on average a factor of $\sim 3$ lower than in the second observing run HMM search. This is the first Scorpius X-1 HMM search with upper limits that reach below the indirect torque-balance limit for certain subbands, assuming $\iota = 44^\circ$.

DOI: 10.1103/PhysRevD.106.062002

I. INTRODUCTION

Rotating neutron stars are promising candidates for continuous-wave searches with terrestrial gravitational wave (GW) detectors such as the second-generation Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) [1–5], Advanced Virgo [4], and the Kamioka Gravitational-Wave Detector [6]. Continuous GWs from neutron stars are emitted by an oscillating quadrupole moment, which can be produced in various ways, including elastic strain [7,8], magnetic gradients [9–11], r-modes [12–14], or nonaxisymmetric circulation of the superfluid interior [15–18]. These mechanisms emit GWs at specific multiples of the spin frequency $f_\star$ [1]. Low-mass x-ray binaries (LMXBs) have been targeted by previous LIGO searches, [19–24] because they may emit GWs relatively strongly while existing in a state of rotational equilibrium, in which the accretion torque balances the GW torque [25–27]. Under torque-balance conditions, the characteristic GW strain $h_0$ is proportional to the square root of the x-ray flux, implying that the brightest LMXB, Scorpius X-1 (Sco X-1), is also a strong GW emitter [1,27].

Continuous-wave searches directed at Sco X-1 have been performed with data from the first (O1) and second (O2) observing runs of LIGO [19–21,23,28–31]. No signal has been detected to date, but astrophysically interesting upper limits have been obtained. For O1, a hidden Markov model (HMM) pipeline [29] obtained an upper limit at 95% confidence level of $h_0^{95\%} = 8.3 \times 10^{-25}$ in the 100–200 Hz frequency range, while a cross-correlation (CrossCorr) pipeline [28,32] achieved $h_0^{95\%} = 2.3 \times 10^{-25}$ in the same frequency range. For O2, the HMM pipeline [21] obtained $h_0^{95\%} = 3.47 \times 10^{-25}$, in the 100–200 Hz frequency range, while CrossCorr [23] improved on its O1 results by a factor of $\sim 1.8$. All of these upper limits on $h_0^{95\%}$ are marginalized over the neutron star spin inclination $\iota$, assuming an isotropic prior. If instead one assumes an electromagnetically informed prior, $\iota = 44^\circ \pm 6^\circ$ [33], the O2 upper limits obtained by CrossCorr reduce to $\sim 10^{-26}$ [23]. As with any observation the upper limits are conditional on the signal model. CrossCorr and the HMM assume different phase evolution, so the foregoing $h_0^{95\%}$ values cannot be compared directly (see Sec. VB), although they are broadly indicative of course. Now, the third observation run (O3), which is longer and more sensitive than O1 and O2, offers an opportunity to repeat the O1 and O2 searches with improved sensitivity. The improved HMM search is the subject of this paper.

Searching for LMXBs presents two challenges. First, $f_\star$ (and hence the GW frequency) wanders stochastically in objects where it is measured electromagnetically, due to fluctuations in the hydromagnetic accretion torque [34–37]. Second, in some LMXBs including Sco X-1, which do not exhibit x-ray pulsations, $f_\star$ is not measured electromagnetically [1,37]. Hence a GW search must cover a wide band (width $\sim 1$ kHz) looking for an unknown, wandering, quasimonochromatic tone [38,39]. HMM tracking is a tried and tested method for searches of the above sort, with a long history of practical use in telecommunications [40].
and remote sensing [41]. HMM tracking has been used in numerous searches for GWs, e.g. for Sco X-1 in O1 [29] and O2 [21], young supernova remnants in O3 [42], accreting millisecond x-ray pulsars in O2 [22] and O3 [24], all sky searches [43], and long-duration transients [44].

The \( J \)-statistic [45], a frequency domain matched filter, is used in tandem with the HMM described in Refs. [21,29,45,46]. The outline of the rest of the paper is as follows. Section II explains briefly the HMM formulation used and the \( J \)-statistic. In Sec. III, the search pipeline and parameter space are described. In Secs. IV and V, we present the search results and upper limits, respectively. We conclude in Sec. VI.

II. HMM ALGORITHM

In Sec. II A we review the HMM formalism used to track the wandering GW emission frequency from one time step to the next, according to a user-selected set of transition probabilities. For each time step we calculate the likelihood of a signal being present given the data, via a maximum likelihood matched filter called the \( J \)-statistic, which is reviewed in Sec. II B.

A. Hidden state structure and automaton

A HMM is a probabilistic finite state automaton characterized by a hidden state variable \( q(t) \), which takes the discrete values \( \{q_1, \ldots, q_{N_q}\} \), and an observable state variable \( o(t) \), which takes the discrete values \( \{o_1, \ldots, o_{N_o}\} \). The automaton jumps between states at discrete time epochs \( \{t_1, \ldots, t_{N_T}\} \). The probability of being in hidden state \( q(t_{n+1}) \) at time \( t_{n+1} \) depends only upon the state at the previous time step \( t_n \). This is known as a Markov process.

To complete the model, two matrices are defined. First, the transition probability matrix \( A_{q(t_{n+1})q(t_n)} \), which relates the probability of a state \( q(t_n) \) to jump to \( q(t_{n+1}) \), takes the form

\[
A_{q(t_{n+1})q(t_n)} = \frac{1}{3} \left[ \delta \left[ q(t_{n+1}) \middle| q(t_n) \right] + \delta \left[ q(t_{n+1}) \middle| q(t_n) + \Delta q \right] \right],
\]

where \( \delta \) is the Kronecker delta, with \( i \) and \( j \) given by the terms in square brackets, respectively. Equation (1) defines the signal model as a piecewise constant function that jumps by \(-1, 0, \) or \(1\) frequency bin \( \Delta q \) at each discrete transition time. See Sec. III A for the details on the search frequency bin size. The other matrix is the emission probability matrix \( L_{o(t_n)q(t_n)} \) that relates the likelihood of observing \( o(t_n) \) if the hidden state variable is \( q(t_n) \). \( L_{o(t_n)q(t_n)} \) is constructed from the matched-filter \( J \)-statistic, which we review in Sec. II B. In our GW application, \( f_s(t) \) maps onto \( q(t) \), noting that \( f_s(t) \) is hidden because it cannot be measured electromagnetically for Sco X-1.

The data, specifically the Fourier transform of the time series output by the detector, map onto \( o(t) \). The total observation time \( t_{\text{obs}} \) is divided into \( N_T \) segments of duration \( T_{\text{drift}} \) rounded down to the nearest integer, i.e. \( N_T = \left\lfloor \frac{t_{\text{obs}}}{T_{\text{drift}}} \right\rfloor \). \( T_{\text{drift}} \) is chosen to prevent \( f_s(t) \) from wandering by more than one frequency bin per time step. The rate of spin wandering is unknown \textit{a priori} in Sco X-1, as \( f_s(t) \) cannot be measured electromagnetically. The observed x-ray flux variability can be used to estimate the stochastic variation in \( f_s(t) \) [38,47], and from this we make an informed choice of \( T_{\text{drift}} \), as in previous searches.

The probability that the hidden state follows some path \( Q = \{q(t_1), \ldots, q(t_{n_T})\} \) given some observed data \( O = \{o(t_0), \ldots, o(t_{N_T})\} \) is then given by the product of the transition and emission probabilities for each step, viz.

\[
P(Q|O) = L_{o(t_{N_T})q(t_{N_T})} A_{q(t_{N_T})q(t_{N_T-1})} \cdots L_{o(t_1)q(t_1)} A_{q(t_1)q(t_0)} \Pi_{q(t_0)},
\]

where we define the prior probability of being in some initial state \( q(t_0) \) at time \( t_0 \) as \( \Pi(q(t_0)) \). In this paper we assign equal probability to all initial states, with \( \Pi(q(t_0)) = 1/N_Q \).

We seek the path \( Q^* \) that maximizes \( P(Q^*|O) \) in Eq. (2). But it is computationally inefficient to consider all the \( N_Q^{N_T+1} \) possible paths. A robust and computationally efficient way to find \( Q^* \) is the Viterbi algorithm [48]. This recursive algorithm exploits the principle of optimality to find \( Q^* \) given \( O \). A comprehensive description of the algorithm can be found in Appendix A of Ref. [29].

In this paper we use as detection statistic the log-likelihood of the most likely path \( \mathcal{L} = \ln P(Q^*|O) \).

B. \( J \)-statistic

The emission probability \( L_{o(t_n)q(t_n)} \) relates the observed data \( o(t_n) \), collected in the interval \( t_n \leq t \leq t_n + T_{\text{drift}} \), to the hidden states \( q(t_n) \). In this paper, we express \( L_{o(t_n)q(t_n)} \) in terms of the \( J \)-statistic [21,45]. The \( J \)-statistic is a maximum likelihood, frequency domain, matched-filter which tracks the orbital phase of the neutron star in its binary system. It is an extension of the traditional \( \mathcal{F} \)-statistic [49], which is a matched filter for a biaxial rotor [50]. The Doppler shift due to the binary motion disperses the \( \mathcal{F} \)-statistic power into orbital sidebands of the GW carrier frequency. Although the \( \mathcal{F} \)-statistic can be used to produce matched filters to account for the Doppler shift due to the binary motion, the \( J \)-statistic is used for computational efficiency. Section III in Ref. [45] presents the detailed derivation of the \( J \)-statistic.

In general, to account for the dispersed power, the \( J \)-statistic is constructed from matched filters of the suggestive form

\[
G(f) = \mathcal{F}(f) \otimes B(f),
\]

where \( \otimes \) denotes convolution. The matched filters are the periodograms of the individual sidebands and are found by the windowed Fourier transform (WFT) of the data.
with

\[ B(f) = \sum_{s=-m}^{m} J_s(2\pi f_o a_0) \exp(-i s \phi_a) \delta\left(f - \frac{s}{P}\right), \]

where \( \otimes \) denotes convolution, \( J_s(x) \) is the Bessel function of the first kind of order \( s \), \( m = \text{ceil}(2\pi f_o a_0) \), and \( f_0 \) is the central frequency of the subband (see Sec. III A for details). Equation (4) assumes the GW signal is produced by a biaxial rotor in a circular Keplerian orbit and requires three binary orbital parameters: the projected semimajor axis \( a_0 \), the orbital phase at a reference time \( \phi_a \), and \( P \) is the binary orbital period.

### III. SEARCH IMPLEMENTATION

In this section we discuss the practical details of the search. Sections III A and III B define the parameter domain and grid, respectively. Section III C sets out the workflow. Section III D justifies the selection of the false alarm probability in terms of the number of search templates. Section III E specifies the primary and secondary data products ingested by the search.

#### A. Sco X-1 parameters

The \( J \)-statistic depends on the location of the source, described by the right ascension \( \alpha \) and declination \( \delta \). It also depends on the three binary orbital elements: \( P, a_0, \text{and } \phi_a \). The time of passage through the ascending node \( T_{\text{asc}} \) is linked to \( \phi_a \) via \( \phi_a = 2\pi T_{\text{asc}} / P (\text{mod } 2\pi) \). Henceforth we use \( T_{\text{asc}} \) instead of \( \phi_a \). Some of these parameters have been measured electromagnetically for Sco X-1. Their values and uncertainties are summarized in Table I.

The last electromagnetic measurements [51] for the time of ascension \( T_{\text{asc,ref}} = 974.416.624 \) Global Positioning System (GPS) time are dated November 22 04:00:15 Greenwich Mean Time 2010, here denoted \( T_0 \). We forward-propagate \( T_{\text{asc,ref}} \) to the start of O3, \( T_{O3,0} = 1238166483 \) GPS time, viz.

\[ T_{\text{asc,0}} = T_{\text{asc,ref}} + N_{\text{orb}} P_0. \]

where \( P_0 = 68023.86048 \) s is the central value of the orbital period in Table I, and \( N_{\text{orb}} = \lceil \frac{T_{O3,0} - T_0}{P_0} \rceil \) is the number of full orbits between the reference time \( T_0 \) and \( T_{O3,0} \). The original uncertainties for \( T_{\text{asc,ref}} \) and \( P \) are also propagated using Eq. (5). This is illustrated in Fig. 1. The propagation maps the original uncertainty ellipses of \( T_{\text{asc,ref}} - P \) (upper panel in Fig. 1) to the present uncertainty ellipses \( T_{\text{asc}} - P \) (lower panel in Fig. 1). Propagating the original uncertainties via Eq. (5) creates correlations between the uncertainties of \( T_{\text{asc}} \) and \( P \).

The propagated priors on \( T_{\text{asc}} \) and \( P \) change throughout the search duration as their correlation grows with time. The lower panel of Fig. 1 shows the change from the start of O3 marked as solid color lines, to its end, \( T_{O3,\text{end}} = 1269361423 \) GPS time, shown as dotted lines. The search has been designed to cover the whole \( 3\sigma \) region of the propagated \( T_{\text{asc}} - P \) space, from start to end of O3.

For \( a_0 \), we cover the range \( 1.45 \leq a_0 / (1 \text{ s}) \leq 3.25 \), following the electromagnetic measurements presented in [51]. We can write this range equivalently as \( a_0 = 2.35 \pm 3\sigma_{a_0} \), with \( a_0 = 2.35 \) and \( \sigma_{a_0} = 0.3 \) s.

The coherence time is set to \( T_{\text{drift}} = 10 \) d. The latter choice is justified astrophysically: it is the characteristic timescale of the random walk in \( f_\ast \) inferred from accretion-driven fluctuations in the x-ray flux of Sco X-1 [19,38,47]. It also matches the value used in previous published searches for Sco X-1, enabling direct comparison with historical results [19,21,28,29]. The resolution in frequency space \( \Delta f_{\text{drift}} \), i.e. the size of the frequency bins, is set by the coherence time as \( \Delta f_{\text{drift}} = 1 / (2 T_{\text{drift}}) = 5.787037 \times 10^{-5} \) Hz.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Search range</th>
<th>EM data</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right ascension</td>
<td>( \alpha )</td>
<td>16 h 19 m 55.0850 s</td>
<td>Y</td>
<td>[52]</td>
</tr>
<tr>
<td>Declination</td>
<td>( \delta )</td>
<td>(-15\text{°38}’24.99)</td>
<td>Y</td>
<td>[52]</td>
</tr>
<tr>
<td>Orbital inclination angle</td>
<td>( \iota )</td>
<td>44 \pm 6°</td>
<td>Y</td>
<td>[33]</td>
</tr>
<tr>
<td>Projected semimajor axis</td>
<td>( a_0 )</td>
<td>2.35 \pm 0.3 s</td>
<td>Y</td>
<td>[51]</td>
</tr>
<tr>
<td>Orbital period</td>
<td>( P )</td>
<td>68 023.86048 \pm 0.0432 s</td>
<td>Y</td>
<td>[51]</td>
</tr>
<tr>
<td>GPS time of ascension</td>
<td>( T_{\text{asc}} )</td>
<td>1 238 149 477.03488 \pm 200 s</td>
<td>Y</td>
<td>[51]</td>
</tr>
<tr>
<td>Frequency</td>
<td>( f )</td>
<td>60–500 Hz</td>
<td>N</td>
<td>...</td>
</tr>
</tbody>
</table>
of $N_{\text{orb}}$. To be conservative, we choose to propagate the uncertainties on $T_{\text{asc,ref}}$ to the end of O3 with

$$
\sigma_{T_{\text{asc}}} = \sqrt{\sigma_{T_{\text{asc,ref}}}^2 + (N_{\text{orb,end}}\sigma_P)^2},
$$

where one has $N_{\text{orb,end}} = \left[\frac{T_{\text{asc,ref}} - T_0}{P_0}\right]$, and $\sigma_{T_{\text{asc}}} = 50$ and $\sigma_P = 0.0432$ s are the uncertainties for $T_{\text{asc,ref}}$ and $P$, respectively, the result $\sigma_{T_{\text{asc}}} = 200$ s is included in Table I. Although the uncertainties are propagated to the end of O3, the central value for the time of ascension is only propagated to the start of O3, $T_{\text{asc,0}} = 1238149477.03488$ s, as shown in Sec. III A.

We cover the parameter space by using a rectangular grid defined by the limits $(a_0 \pm 3\sigma_{a_0}, T_{\text{asc}} \pm 3\sigma_{T_{\text{asc}}}, P_0 \pm 3\sigma_P)$. We set the spacing of the grid points by selecting an acceptable maximum mismatch $\mu_{\text{max}}$, following [53]. This mismatch represents the fractional loss in signal-to-noise ratio between the search with the true parameters and the nearest grid point. For the search we use $\mu_{\text{max}} = 0.1$. The number of grid points needed to cover the $(a_0, T_{\text{asc}}, P)$ space, for a given $\mu_{\text{max}}$, are calculated using Eq. (71) of Ref. [53], namely

$$
N_{a_0} = \left[3\sqrt{2}\mu_{\text{max}}^{-1/2} f \sigma_{a_0}\right],
$$

$$
N_{T_{\text{asc}}} = \left[6\pi^2\sqrt{2}\mu_{\text{max}}^{-1/2} f a_0 P^{-1} \sigma_{T_{\text{asc}}}\right],
$$

$$
N_P = \left[\pi^2\sqrt{6}\mu_{\text{max}}^{-1/2} f a_0 T_{\text{drift}} N_T P^{-2} \sigma_P\right].
$$

For O3 the number of contiguous semicoherent segments is $N_T = 36$. To be conservative when applying Eqs. (7)–(9), we use the highest frequency in each subband for $f$, the highest $a_0 = \bar{a}_0 + 3\sigma_{a_0}$, and the lowest $P = P_0 - 3\sigma_P$. Table II lists the number of grid points in selected subbands. For $T_{\text{asc}}$ and $P$ we search only the points defined by Eqs. (7)–(9) that lie within the start (color lines) and end (dotted lines) ellipses in the bottom panel of Fig. 1.

### C. Workflow

The workflow for the search is illustrated in Fig. 2, as a flowchart.

<table>
<thead>
<tr>
<th>Subband (Hz)</th>
<th>$N_{a_0}$</th>
<th>$N_{T_{\text{asc}}}$</th>
<th>$N_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>767</td>
<td>149</td>
<td>5</td>
</tr>
<tr>
<td>160</td>
<td>2031</td>
<td>394</td>
<td>12</td>
</tr>
<tr>
<td>260</td>
<td>3296</td>
<td>640</td>
<td>19</td>
</tr>
<tr>
<td>360</td>
<td>4560</td>
<td>885</td>
<td>27</td>
</tr>
<tr>
<td>460</td>
<td>6331</td>
<td>1228</td>
<td>37</td>
</tr>
</tbody>
</table>
At the outset, the time series from the detector is converted into short Fourier transforms (SFTs) lasting $T_{\text{SFT}} = 1800$ s. The corresponding data, for each frequency subband, are divided into $N_T$ blocks of duration $T_{\text{block}} = 10d$. All of the SFTs in a single block are used to calculate an $F$-statistic atom [54] with the fixed parameters $\alpha$ and $\delta$ in Table I. The process is repeated for all the blocks, generating $N_T$ atoms. The $F$-statistic atoms do not depend on the orbital parameters so they are stored in a look-up table. The steps in this paragraph conclude in the blue parallelogram denoted “$N_T F$-statistic atoms” in Fig. 2.

In every subband, each $F$-statistic atom is fed into the $J$-statistic in Eq. (4) for a triad of orbital parameters $(a_0, T_{\text{asc}}, P)$. The Viterbi algorithm (see [48] or Sec. II B of Ref. [45]) finds the optimal frequency path connecting the $J$-statistic blocks and its associated log-likelihood. In Fig. 2, the latter steps extend from the orange rectangle “Calculate the $J$-statistic” up to the blue parallelogram marked “log-likelihood and optimal path for $(a_0, T_{\text{asc}}, P)_i$.”

For a subband centered on the frequency $\bar{f}$, the search scans over all binary templates $(a_0, T_{\text{asc}}, P)_i$ with $1 \leq i \leq N_{\text{tot}}(\bar{f})N_{T_{\text{asc}}}(\bar{f})N_P(\bar{f})$, calculated using Eqs. (7)–(9). This step is illustrated as the green oval denoted “Used all templates?” in Fig. 2. For each $(a_0, T_{\text{asc}}, P)_i$, an optimal path and its associated log-likelihood are recorded. Following the loop over all binary templates, the optimal path with highest log-likelihood, denoted $\max (\mathcal{L})$, is selected in the blue parallelogram marked “$\max (\mathcal{L})$ and assoc. optimal path” in Fig. 2. If $\max (\mathcal{L})$ is higher than the detection threshold (see Sec. III D), then the subband is recorded as a candidate and passed through a hierarchy of vetoes (see Sec. IV B), via the “yes” output of the upper green oval. Subbands with $\max (\mathcal{L})$ below the detection threshold are used to calculate GW strain upper limits via the “no” output of the upper green oval. Vetoed subbands are not used to calculate GW upper limits.

**D. False alarm probability and detection threshold**

A subband is registered as a candidate, when $\max (\mathcal{L})$ exceeds a threshold $\mathcal{L}_{\text{th}}$, corresponding to a user-selected false alarm probability. As the distribution of $\max (\mathcal{L})$ in pure noise is unknown, we rely on Monte Carlo simulations to determine $\mathcal{L}_{\text{th}}$ in each subband of the search. To estimate the distribution of $\max (\mathcal{L})$ in pure noise, we generate synthetic Gaussian data using the lalappMakefakedata_v5 program in the LIGO Scientific Collaboration Algorithm Library [55]. The synthetic data are generated for a subband centered on the frequency $\bar{f}$, with $\alpha$ and $\delta$ copied from Table I. Then the search workflow described in Sec. III C is applied. To avoid needless computation, we limit the grid to $N_{a_0} = 322, N_{T_{\text{asc}}} = 35$, and $N_P = 5$ in every subband, independent of $\bar{f}$.

In general, $\mathcal{L}_{\text{th}}$ depends on the number of generated log-likelihoods per subband, i.e. $N_{\text{tot}} = N_{\bar{f}}N_{a_0}N_{T_{\text{asc}}}N_P$. We describe the false alarm probability $\alpha_{N_{\text{tot}}}$ of a subband with $N_{\text{tot}}$ log-likelihoods in terms of the probability of a false alarm in a single terminating frequency bin per orbital template $\alpha$ as...
\[ \alpha_{\text{tot}} = 1 - (1 - \alpha)^{N_{\text{tot}}} \] (10)

The probability density function of the log-likelihood per subband \( p(\mathcal{L}) \) for the most likely path given an orbital template is observed to follow an exponential tail, \( p(\mathcal{L}) = A \exp[-\lambda(\mathcal{L} - \mathcal{L}_{\text{tail}})] \) in noise. The cutoff \( \mathcal{L}_{\text{tail}} \) and the \( \lambda \)-parameter are obtained empirically. From our synthetic Gaussian trials we obtain \( \lambda = 0.23 \) across all subbands. The normalization \( A = \alpha_{\text{tot}} / N_{\text{tot}} \), where \( N_{\text{tot}} \) is the number of log-likelihoods above \( \mathcal{L}_{\text{tail}} \), is calculated from the \( N_{\text{tot}} \) samples used to generate \( p(\mathcal{L}) \). The probability \( \alpha \) in Eq. (10) is given by

\[ \alpha = \int_{\mathcal{L}_{\text{th}}}^{\infty} p(\mathcal{L}) d\mathcal{L}. \] (11)

Combining \( p(\mathcal{L}) \), Eq. (10), and Eq. (11) yields for \( \mathcal{L}_{\text{th}} \)

\[ \mathcal{L}_{\text{th}} = \mathcal{L}_{\text{tail}} - \frac{1}{\lambda} \log \left( \frac{\alpha_{\text{tot}}}{N_{\text{tail}}} \right), \] (12)

provided Eq. (10) is Taylor expanded to first order, given \( N_{\text{tot}} \gg 1 \). Note that \( \mathcal{L}_{\text{th}} \) is an implicit function of the subband frequency through \( N_{\text{tail}} \).

Historically HMM Sco X-1 searches [21,29] use \( \alpha_{\text{tot}} = 0.01 \), a choice we adopt here to avoid excessive follow-up of candidates and ease the comparison with previous HMM Sco X-1 searches. A false alarm probability of 1% per subband applied to a total of 725 subbands implies we should expect \( \sim 7 \) false alarms from the search. Searches with electromagnetically constrained \( f_* \) such as Refs. [22,24] allow for \( \alpha_{\text{tot}} = 0.3 \), given the reduced search space. Appendix A in Ref. [24] presents the detailed procedure to set thresholds using Monte Carlo simulations.

E. O3 data

The search uses all the O3 dataset, starting April 1, 2019, 15:00 UTC and finishing March 27, 2020, 17:00 UTC. The dataset is divided in two. The first part (O3a) spans April 1, 2019 to October 1, 2019 followed by a month-long commissioning break. The second part (O3b) was intended to span November 1, 2019 to April 30, 2020 but was suspended in March 2020 due to the COVID-19 coronavirus pandemic. SFTs are generated from the “C01 calibrated self-gated” dataset, specifically designed to remove loud glitches from the strain data, following the procedure in Ref. [56].

Due to the month-long commissioning break between O3a and O3b, two out of \( N_T \) segments have no SFTs. The two segments are dated October 8, 2019, 15:00 UTC and October 15, 2019, 15:00 UTC, respectively. We replace them by segments with uniform log-likelihood across all frequency bins, to allow the HMM to connect data from O3a with data from O3b while accommodating spin wandering. Every time atoms are created, such as in Secs. III D, IV B 2, and IV B, the relevant missing atoms are replaced with uniform log-likelihood atoms.

FIG. 3. Candidates plotted as a function of their terminating frequency bin \( q^*(T_{\text{asc}}) \) (horizontal axis, units in hertz) and the orbital parameters \( a_0 \) (vertical axis in left panel, units in seconds), offset from the central time of ascension \( T_{\text{asc}} - T_{\text{asc}0} \) (vertical axis in central panel; units in seconds) and offset from the central period \( P - P_0 \) (vertical axis in right panel; units in seconds). The color scale indicates the \( \max(\mathcal{L}) \) obtained for the candidate. Candidates marked with purple squares are eliminated by the single IFO veto, while red circles mark the ones eliminated by the known lines veto. The candidate with no marking survives both the single IFO and known lines vetoes, but is eliminated by its absence when using noise-subtracted data (see Sec. IV A).
IV. O3 ANALYSIS

A. Candidates

The results of the search are plotted in Fig. 3. On the horizontal axis we show the terminating frequency bin of the optimal path, i.e. $q^{+}(t_{N_{f}})$ per subband, that satisfies $\max(L) > L_{th}$, defining preliminary signal candidates. On the vertical axes we graph the orbital parameters $a_{0}$ (left panel), $T_{\text{asc}}$ (middle panel), and $P$ (right panel).

Figure 3 contains 35 candidates with $\max(L) > L_{th}$. To eliminate false alarms we use the hierarchy of vetoes employed in Ref. [21]. The vetoes discard candidates that (i) lie near an instrumental noise line (known line veto) or (ii) appear in one interferometer (IFO) but not the other (single IFO veto). In previous searches, other vetoes, e.g. candidates that appear in half of the observation time ($T_{\text{obs}}/2$ veto), have been used in addition to (i) and (ii) [21,22,24,29,42]. Additional vetoes are unnecessary in this paper; all but one of the candidates are vetoed by (i) and (ii), and the survivor is eliminated by other means, as discussed below. The outcomes of the vetoes in the 35 subbands containing outliers are recorded in Table III.

The sole candidate that passes vetoes (i) and (ii), contained in the subband starting at 60.05 Hz, is suspiciously close to the known 60 Hz noise line due to the United States of America power grid [57]. Additionally, several other candidates appear near harmonics of 60.05 Hz, for instance 119.52 and 179.60 Hz. When we search the subband using the C01 calibrated self-gated 60 Hz subtracted dataset, which uses the algorithm described in Ref. [58] to subtract the 60 Hz noise line, the candidate disappears.

B. Vetoes

1. Known lines

Narrow band noise features in the IFO are caused by a plethora of reasons, such as the suspension system or the electricity grid [59,60]. As noise lines artificially increase the output of the $F$-statistic, subbands that contain them tend to be flagged as candidates. In response, we veto any candidate whose optimal frequency path $f(t_{i})$ satisfies

$$|f(t_{i}) - f_{\text{line}}| < \frac{2\pi a_{0} f_{\text{line}}}{P},$$

for any epoch $t_{i}$ in the search. Here $f_{\text{line}}$ is the frequency of the noise line. We refer to the vetted known lines list in Ref. [61]. This test vetoes 30 out of the 35 candidates. We note that the number of remaining candidates, after eliminating those caused by noise lines, is consistent with our original number of expected candidates.

![Table III. Candidates for the O3 search. The first column corresponds to the minimum frequency in the subband that contains the candidate. The second column is the log-likelihood of the candidate. The other columns record the outcome of the two vetoes used in Secs. IV B 1 and IV B 2. A candidate that passes or fails a veto is marked with a ✓ or X, respectively. H and L are the Hanford-only and Livingston-only max($L$) values. The remaining candidate, contained in the 60.05 Hz subband, is eliminated when using the C01 calibrated self-gated 60 Hz subtracted dataset.](https://example.com/table_iii.png)

2. Single IFO

A plausible astrophysical signal that has escaped detection in prior searches would likely be weak enough to need data from both IFOs to be detectable, or strong enough to be seen in both, given their comparable sensitivities in most frequency bands. In contrast, instrumental artifacts are unlikely to appear simultaneously in both IFOs. Let $\max(L)_{a}$ and $\max(L)_{b}$ denote the
log-likelihoods in each IFO, and let \( \max(\mathcal{L})_b \) denote the log-likelihood of the original candidate. There are four possible outcomes:

1. If one finds \( \max(\mathcal{L})_a < \mathcal{L}_b \) and \( \max(\mathcal{L})_b > \mathcal{L}_a \), and the optimal path \( f_b(t_i) \), associated with \( \max(\mathcal{L})_b \), satisfies
   \[
   |f_b(t_i) - f_b(t_i)| < \frac{2\pi a_0 f_L}{P}
   \]  
   for any epoch \( t_i \) in the search, where \( f_b(t_i) \) is the optimal path associated with \( \max(\mathcal{L})_b \) then the candidate is consistent with an instrumental artifact in detector \( b \). It is vetoed.
2. If one finds \( \max(\mathcal{L})_a < \mathcal{L}_{th} \) and \( \max(\mathcal{L})_b > \mathcal{L}_{th} \), but the candidate does not satisfy Eq. (14), then it is saved for further postprocessing. Such a candidate could be a faint astrophysical signal which needs both IFOs to be detected.
3. If one finds \( \max(\mathcal{L})_a > \mathcal{L}_{th} \) and \( \max(\mathcal{L})_b > \mathcal{L}_{th} \), then the candidate could be a strong astrophysical signal. It could also imply a common noise source in both detectors. The candidate is flagged for postprocessing.
4. If one finds \( \max(\mathcal{L})_a < \mathcal{L}_{th} \) and \( \max(\mathcal{L})_b < \mathcal{L}_{th} \), the candidate could be a weak astrophysical signal that needs both detectors to appear above threshold. This candidate is also saved for postprocessing.

This test vetoes four out of the remaining five candidates, with the last candidate eliminated by its nonappearance in noise-subtracted data, as discussed in Sec. IV.A.

V. FREQUENTIST UPPER LIMITS

A. Procedure

Subbands without candidates are used to place upper limits on the gravitational wave strain detectable at a 95% confidence level \( h_0^{95\%} \). We use the approach in the historical HMM Sco X-1 searches [21,29] to set frequentist upper limits. This is done to facilitate comparison with previous upper limits.

To capture the variation of the wave strain as a function of the inclination angle \( \iota \), we define the effective strain [47]

\[
h_0^{eff} = h_0 2^{-1/2} [(1 + \cos^2 \iota)/2]^2 + \cos^2 \iota]^{1/2}.
\]  
We note that Eq. (15) allows us to rescale \( h_0^{95\%} \) for the circularly polarized case, \( |\cos \iota| = 1 \), to any other inclination angle \( \iota \). For example, if we assume the electromagnetically measured inclination of Sco X-1 orbit as \( \iota = 44^\circ \) then Eq. (15) yields \( h_0^{eff,95\%} = 1.35 h_0^{95\%} \).

To set frequentist upper limits in a subband with central frequency \( f \), we start by generating 100 copies of the O3 data for this subband. A Sco X-1-like signal, i.e. using the astrophysical parameters in Table I, is injected into each copy of the subband. The parameters \( \{\psi, a_0, T_{asc}, P\}_{inj} \) used to create the injected signal are drawn from uniform distributions within the range given by their respective 3\( \sigma \) error bars. We make sure the injected \( T_{asc,inj} \) and \( P_{inj} \) values lie inside the propagated priors shown in Fig. 1, second panel. The injected frequency \( f_{inj} \) is uniformly selected from the interval \( f \pm \delta f \) with \( \delta f = 0.05 \) Hz. The interval \( f \pm \delta f \) is chosen for simplicity. The initial value of \( h_0 \) is chosen such that there is at least one frequency path with \( \max(\mathcal{L}) > \mathcal{L}_{th} \). We progressively reduce \( h_0 \), holding \( \{\psi, a_0, T_{asc}, P\}_{inj} \) constant, until the signal is no longer detectable. We record the last detectable amplitude as \( h_0^{min} \). The procedure is repeated for all copies of the subband, choosing a new set of injection parameters \( \{\psi, a_0, T_{asc}, P\}_{inj} \) per copy. Finally \( \{h_0^{min1}, \ldots, h_0^{min100}\} \) are sorted in ascending order and the 95th becomes \( h_0^{95\%} \). The injections have \( |\cos \iota| = 1 \), so we use Eq. (15) to convert \( h_0^{95\%} \) to other polarizations.

B. Upper limits

The limits on \( h_0^{95\%} \) are plotted in Fig. 4. We present three cases, as in Ref. [21]: circular polarization \( \iota = 0 \) (blue dots in Fig. 4), \( \iota = 44^\circ \) (following the electromagnetic measurements in Ref. [51]; denoted by green dots in Fig. 4), and unknown polarization (orange dots in Fig. 4). In the latter context, unknown polarization means we marginalize over all possible polarizations assuming a uniform distribution in \( \cos \iota \) from \(-1 \) to \( 1 \).

The upper limits from the O3 search are on average \( \sim 3 \) times lower than those from the O2 HMM search [21]. For the subband starting at 256.06 Hz we obtain the lowest \( h_0^{95\%} \), given by \( 4.56 \times 10^{-26} \), \( 6.16 \times 10^{-26} \), and \( 9.41 \times 10^{-26} \) for circular, electromagnetically restricted (\( \iota = 44^\circ \)), and unknown polarizations, respectively. Compared to the most sensitive subbands in previous HMM Sco X-1 searches, the lowest \( h_0^{95\%} \) is a factor of \( \sim 13 \) lower than in O1 data [29] and \( \sim 3 \) lower than in O2 data [21].

It is tempting to compare the results presented in this section with the results of searches with other pipelines, such as CrossCorr O1 [28] and O2 [23]. Such comparisons can be broadly indicative, if done informally. However, the \( h_0^{95\%} \) values output by different pipelines cannot be compared directly, when the pipelines assume different phase models, as foreshadowed in Sec. I. The phase evolution assumed by CrossCorr is given by Eq. (4.15) in Ref. [20], viz. \( \Phi_k = \Phi_0 + 2\pi f_0 \{t_k - d_k - \alpha \sin[2\pi (t_k - t_{asc})]/P\} \), where \( K \) indexes jointly the detector and the selected time interval, with \( t_k \) the midpoint of the latter, and \( d_k \) is the projected distance from the solar system barycenter to the detector. In previously published CrossCorr implementations [20,23,28,32], it is assumed that \( f_0 \) stays constant throughout the total observation time \( T_{obs} \). Section VA of Ref. [20] explores how the foregoing phase evolution can be generalized in future CrossCorr implementations. On the other hand, the HMM executes an unbiased random walk in \( f(t) \), which is piecewise constant in the coherent blocks.
\[ t_n \leq t \leq t_n + T_{\text{drift}}, \] as presented in Sec. II A. The different phase evolution for CrossCorr and the HMM translates into upper limits that are specific to the two different sets of phase paths.

### C. Torque-balance upper limit

Torque balance assumes the spin-down torque due to gravitational wave emission balances the accretion spin-up torque. From this assumption theoretical upper limits on gravitational wave strain can be estimated from x-ray observations. Following Eq. (4) in Ref. [23], the amplitude of the GW signal produced in torque equilibrium \( h_{0\text{eq}} \) is

\[
h_{0\text{eq}} = 3.4 \times 10^{-26} \left( \frac{R}{10 \text{ km}} \right)^{1/2} \left( \frac{1.4 M_\odot}{M} \right)^{1/4} \left( \frac{r_m}{10 \text{ km}} \right)^{1/4} \times \left( \frac{F_X}{3.9 \times 10^{-7} \text{ erg cm}^{-2} \text{s}^{-1}} \right)^{1/2} \left( \frac{600 \text{ Hz}}{f_{\text{GW}}} \right)^{1/2}. \tag{16}
\]

In Eq. (16), \( F_X \) is the x-ray flux, \( M \) is the fiducial neutron star mass, \( r_m \) is the lever arm, \( R \) is the neutron star radius, and \( f_{\text{GW}} \) is the GW frequency. Equation (16) assumes the maximum accretion luminosity is completely radiated as x rays, i.e. \( X = 1 \) in Eq. (4) of Ref. [23]. To calculate Eq. (16), we use \( r_m = R = 10 \text{ km} \), plotted as a solid red line in Fig. 4, or \( r_m = R_A \), where \( R_A \) is the Alfvén radius, which corresponds approximately to the inner edge of the accretion disk [29,36]. This is given by

\[
R_A = 35 \left( \frac{B_*}{10^9 G} \right)^{1/4} \left( \frac{R}{10 \text{ km}} \right)^{12/7} \times \left( \frac{1.4 M_\odot}{M} \right)^{1/7} \left( \frac{10^{-8} M_\odot \text{ yr}^{-1}}{\dot{M}} \right)^{2/7} \text{ km}. \tag{17}
\]

where \( B_* \) is the polar magnetic field strength at the stellar surface, \( G \) is Newton’s gravitational constant, and \( \dot{M} \) is the accretion rate set to the Eddington limit \( 2 \times 10^{-8} M_\odot \text{ yr}^{-1} \), for a fiducial neutron star with mass \( M = 1.4 M_\odot \) and radius \( R = 10 \text{ km} \) [62,63]. This limit is plotted in Fig. 4 as the dashed red line.

The electromagnetic inclination \( \iota = 44^\circ \) produces upper limits that dip under the theoretical torque-balance limits (red lines in Fig. 4) for the first time in the HMM Sco X-1 search history. The CrossCorr search pipeline achieved this milestone in the O2 search; see Fig. 1 in Ref. [23]. Again, the reader is reminded that different pipelines assume different signal models, and upper limits conditional on different signal models cannot be compared directly.

FIG. 4. Frequentist effective wave strain upper limits at 95% confidence as a function of subband frequency, for three scenarios: circular polarization with \( \iota = 0 \) (blue stars), \( \iota \approx 44^\circ \) based on the electromagnetic measurements (see Ref. [51]; orange dots), and a flat prior on \( \cos \iota \) (green dots). Indirect torque-balance upper limits (see Sec. V C) for two torque lever arms are also shown: the stellar radius (red solid line) and the Alfvén radius (dashed red line).
VI. CONCLUSIONS

In this paper we search the LIGO O3 data for continuous GWs from the LMXB Sco X-1, using a hidden Markov model, combined with the maximum-likelihood $J$-statistic and a binary template grid that includes the projected semimajor axis $a_{\nu}$, time of ascension $T_{\text{asc}}$, and orbital period $P$. The binary orbital elements are constrained via electromagnetic observations, but the spin frequency is unknown. Monte Carlo simulations are used to establish a detection threshold $\theta_{\text{th}}$, with a false alarm probability $\alpha_{\text{N}} = 0.01$, per subband. The search is conducted in the range 60–500 Hz, partitioned into subbands of width $\Delta f_{\text{sub}} = 0.606$ Hz. The subbands with an optimal path satisfying $\max(L) > L_{\text{th}}$ are passed through a hierarchy of vetoes. One candidate survives the vetoes, but this candidate is eliminated when using the C01 calibrated self-gated 60 Hz subtracted dataset.

The most sensitive subband, starting at 256.06 Hz, yields $h_{0}^{95\%} = 4.56 \times 10^{-26}, 6.16 \times 10^{-26}$, and $9.41 \times 10^{-26}$ for circular, electromagnetically restricted ($i = \delta 44^{\circ}$), and unknown polarizations, respectively.

The above results improve on the two previous HMM Sco X-1 searches [21,29] by using data from O3 and including the orbital period $P$ in the searched template grid. For comparison, the most sensitive subband in the O2 HMM search, 194.6 Hz, obtained $h_{0}^{95\%} = 1.42 \times 10^{-25}$ for $i = 0$ [21], while for the same subband and polarization the present search obtains $h_{0}^{95\%} = 5.40 \times 10^{-26}$. On average our upper limits are a factor of $\sim 3$ below the O2 HMM results. The present search sets the lowest upper limits for the HMM searches, beating for first time the torque-balance limit for the electromagnetically restricted $i = 44^{\circ}$ case.

Other LMXBs are not as bright in x rays as Sco X-1, but they are important targets too. Some LMXBs emit x-ray pulsations, so that $f_*$ is measured to high precision electromagnetically, an important advantage. However, the gravitational wave frequency emitted by such objects may be displaced from $f_*$ and wander randomly with respect to it. A HMM-based search is well placed to track such wandering. Searches for LMXBs with electromagnetically constrained rotation frequencies have been performed in O2 [22] and O3 [24] data. Reference [24] reported strain upper limits in the range $5.1 \times 10^{-26} \leq h_{0}^{95\%} \leq 1.1 \times 10^{-25}$ for its 20 candidates. Such searches offer considerable promise in future observing runs.

ACKNOWLEDGMENTS

This material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation. The authors also gratefully acknowledge the support of the Science and Technology Facilities Council (STFC) of the United Kingdom, the Max-Planck-Society (MPS), and the State of Niedersachsen/Germany for support of the construction of Advanced LIGO and construction and operation of the GEO 600 detector. Additional support for Advanced LIGO was provided by the Australian Research Council. The authors gratefully acknowledge the Italian Istituto Nazionale di Fisica Nucleare (INFN), the French Centre National de la Recherche Scientifique (CNRS), and the Netherlands Organization for Scientific Research (NWO), for the construction and operation of the Virgo detector and the creation and support of the EGO consortium. The authors also gratefully acknowledge research support from these agencies as well as by the Council of Scientific and Industrial Research of India, the Department of Science and Technology, India, the Science and Engineering Research Board (SERB), India, the Ministry of Human Resource Development, India, the Spanish Agencia Estatal de Investigación (AEI), the Spanish Ministerio de Ciencia e Innovación and Ministerio de Universidades, the Conselleria de Fons Europeus, Universitat i Cultura and the Direcció General de Política Universitaria i Recerca del Govern de les Illes Balears, the Conselleria d’Innovació, Universitats, Ciència i Societat Digital de la Generalitat Valenciana, and the CERCA Programme Generalitat de Catalunya, Spain, the National Science Centre of Poland, and the European Union—European Regional Development Fund; Foundation for Polish Science (FNPI), the Swiss National Science Foundation (SNSF), the Russian Foundation for Basic Research, the Russian Science Foundation, the European Commission, the European Social Funds (ESF), the European Regional Development Funds (ERDF), the Royal Society, the Scottish Funding Council, the Scottish Universities Physics Alliance, the Hungarian Scientific Research Fund (OTKA), the French Lyon Institute of Origins (LIO), the Belgian Fonds de la Recherche Scientifique (FRS-FNRS), Actions de Recherche Concertées (ARC) and Fonds Wetenschappelijk Onderzoek—Vlaanderen (FWO), Belgium, the Paris Île-de-France Region, the National Research, Development and Innovation Office Hungary (NKFIH), the National Research Foundation of Korea, the Natural Science and Engineering Research Council Canada, Canadian Foundation for Innovation (CFI), the Brazilian Ministry of Science, Technology, and Innovations, the International Center for Theoretical Physics South American Institute for Fundamental Research (ICTP-SAIFR), the Research Grants Council of Hong Kong, the National Natural Science Foundation of China (NSFC), the Leverhulme Trust, the Research Corporation, the Ministry of Science and Technology (MOST), Taiwan, the U.S. Department of Energy, and the Kavli Foundation. The authors gratefully acknowledge the support of the NSF, STFC, INFN, and CNRS for provision of computational resources. This work was supported by MEXT, JSPS Leading-edge Research Infrastructure Program, JSPS Grant-in-Aid for Specially Promoted Research 26000005, JSPS Grant-in-Aid for
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