Technical Appendix Beetsma-Bovenberg

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Notation:

We will use the following conventions: wage setters’ expectations formed at the start of the first period are denoted with a superscript “e”, while the mathematical expectation taken at the start of the first period is denoted by $E_0[.]$. Whenever the need to explicitly distinguish mathematical and subjective expectations is absent, we denote expectations at the start of the first period by superscript “e”, in order to save space.

Take a generic variable $z_{it}$. An upperbar denotes the cross-country average of the variable: $\bar{z}_t = \frac{1}{n} \sum_{i=1}^{n} z_{it}$. The exceptions are $\bar{x}_t = \frac{1}{n} \sum_{i=1}^{n} \bar{x}_{it}$ and $\bar{g}_t = \frac{1}{n} \sum_{i=1}^{n} \bar{g}_{it}$. A hat above a variable denotes its deviation the cross-country average: $\hat{z}_{it} = z_{it} - \bar{z}_t$. The exceptions are $\hat{x}_{it} = \hat{x}_{it} - \bar{x}_t$ and $\hat{g}_{it} = \hat{g}_{it} - \bar{g}_t$.

A Derivation of the supply curves

Start from the production function of the representative firm in country $i$ in period $t$ in levels:

$$Y_{it} = AL_{it}^\eta, \quad 0 < \eta < 1,$$

where $Y_{it}$ is the level of production, $L_{it}$ is the level of labor input and $A$ is a constant which may depend on the capital stock, which is assumed fixed (in the short run). The profit function of the firm is:

$$(1 - \tau_{it}) P_t A L_{it}^\eta - W_{it} L_{it},$$

where $P_t$ is the price level in the union and $W_{it}$ is the level of nominal wage. The first-order condition yields:

$$\eta (1 - \tau_{it}) P_t AL_{it}^{\eta-1} = W_{it},$$

hence, in logs,

$$\log \eta + p_t - \tau_{it} + \log A + (\eta - 1) l_{it} = w_{it},$$

where $l_{it} = \log L_{it}$, $w_{it} = \log W_{it}$ and $p_t = \log P_t$ and where we have used that $\log (1 - \tau_{it}) \approx -\tau_{it}$ if $\tau_{it}$ is small. Rewriting this yields:

$$l_{it} = \frac{1}{\eta - 1} [\tau_{it} - \log (\eta A) + w_{it} - p_t]. \quad (1)$$
Also take logs of the production function, to yield:

\[ y_{it} = \log A + \eta l_{it}, \]

where \( y_{it} = \log Y_{it} \) and substitute (1) into this expression, so that:

\[ y_{it} = \log A + \nu \log (\eta A) + \nu (p_t - w_{it} - \tau_{it}), \quad (2) \]

where we have defined \( \nu \equiv \eta / (1 - \eta) \).

Next, assume that workers aim at a target real wage \( r_{it} \) (this generalizes the assumption in the main text). Hence, they minimize \( E_0 [w_{it} - (r_{it} + p_t)]^2 \). For the nominal wage of period 1, we then have \( w_{i1} = r_{i1} + p_{i1} \), where we observe that wage setters in each country form the same expectation of the price level, because prices are the same across the monetary union at each moment. Hence, we can rewrite (2) for \( t = 1 \) as:

\[
y_{i1} = \log A + \nu \log (\eta A) - \nu r_{i1} + \nu (p_1 - E_0 (p_1) - \tau_{i1})
= \log A + \nu \log (\eta A) - \nu r_{i1} + \nu ([p_1 - p_0] - (E_0 (p_1) - p_0) - \tau_{i1})
= \log A + \nu \log (\eta A) - \nu r_{i1} + \nu (\pi_1 - E_0 (\pi_1) - \tau_{i1}).
\]

For the nominal wage of period 2, we have \( w_{i2} = r_{i2} + E_0 (p_2) + p_1 - E_0 (p_1) \). Hence, \( w_{i2} - p_2 = r_{i2} + E_0 (p_2) - E_0 (p_1) - p_2 + p_1 = r_{i2} + [E_0 (\pi_2) - \pi_2] \). Hence, we can write (2) for \( t = 2 \) as:

\[
y_{i2} = \log A + \nu \log (\eta A) - \nu r_{i2} + \nu (\pi_2 - E_0 (\pi_2) - \tau_{i2}).
\]

Suppose that society’s, and thus, the government’s, blisspoint for output is \( \tilde{y}_{it} \). To arrive at the expressions in the main text, define \( x_{it} \equiv y_{it} - \log A - \nu \log (\eta A) + \nu r_{it} \) and \( \tilde{x}_{it} \equiv \tilde{y}_{it} - \log A - \nu \log (\eta A) + \nu r_{it} \).

**B Derivation of the second-best equilibrium**

Both monetary and fiscal policymaking are conducted under commitment. The commitment structure for monetary policy is as follows. At the start of period 1, the ECB announces and commits to inflation for both period 1 and period 2. The private sector believes the announcements of the ECB and sets, at the start of period 1, inflation expectations accordingly for both periods. In other words, and this is crucial, the private sector will not update its inflation expectations at the start of the second period.

The commitment problem in fiscal policy manifests itself in debt policy. Hence, we assume that at the start of the first-period governments commit themselves to a set of debt levels they will choose in that period when policies are selected. First, we compute the outcomes of inflation, output and public spending in each of the two periods, given these debt levels. Then, we compute the optimal debt levels for the governments to commit to.
B.1 The ECB’s first-order conditions

The ECB’s Lagrangian, to be minimized over $\pi_1$, $\pi_1^e$, $\pi_2$ and $\pi_2^e$, is:

$$L^{ECB} = L_1^{ECB} + \beta L_2^{ECB},$$

where

$$2L_1^{ECB} = \alpha_1 \pi_1 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu (\pi_1 - \pi_1^e - \tau_{i1}) - \bar{x}_{i1} \right\}^2 + 2\theta_1 \{ \pi_1 - \pi_1^e \},$$

and

$$2L_2^{ECB} = \alpha_2 \pi_2 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu (\pi_2 - \pi_2^e - \tau_{i2}) - \bar{x}_{i2} \right\}^2 + 2\theta_2 \{ \pi_2 - \pi_2^e \},$$

where $\theta_1$ and $\theta_2$ are the Lagrange multipliers associated with the constraints in periods 1 and 2, respectively, that the expectations formed by wage setters, $\pi_i^e$, be equal to realized inflation. Both expectations $\pi_1^e$ and $\pi_2^e$ are formed at the start of period 1.

The ECB’s first-order conditions with respect to $\pi_1$ and $\pi_1^e$ can be written as, respectively:

$$\alpha_1 \pi_1 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu (\pi_1 - \pi_1^e - \tau_{i1}) - \bar{x}_{i1} \right\} + \theta_1 = 0,$$

$$\frac{1}{n} \sum_{i=1}^{n} \nu \left\{ \nu (\pi_1 - \pi_1^e - \tau_{i1}) - \bar{x}_{i1} \right\} + \theta_1 = 0,$$

which can be combined to give:

$$\alpha_1 \pi_1 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu \left( \pi_1 - \pi_1^e - \tau_{i1} \right) - \bar{x}_{i1} \right\} - \frac{1}{n} \sum_{i=1}^{n} \nu \left( \pi_1 - \pi_1^e - \tau_{i1} \right) - \bar{x}_{i1} = 0 \quad (3)$$

The ECB’s first-order conditions with respect to $\pi_2$ and $\pi_2^e$ can be written as, respectively:

$$\alpha_2 \pi_2 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu (\pi_2 - \pi_2^e - \tau_{i2}) - \bar{x}_{i2} \right\} + \theta_2 = 0,$$

$$\frac{1}{n} \sum_{i=1}^{n} \nu \left( \pi_2 - \pi_2^e - \tau_{i2} \right) - \bar{x}_{i2} + \theta_2 = 0.$$
We combine these two equations to:

\[
\alpha \pi_2 + \frac{1}{n} \sum_{i=1}^{n} \left\{ \nu \left[ \nu \left( \pi_2 - \pi^e_2 - \tau_{i2} \right) - \tilde{x}_{i2} \right] \right\} = 0.
\]

\[
\frac{1}{n} \sum_{i=1}^{n} \nu \left[ \nu \left( \pi_2 - \pi^e_2 - \tau_{i2} \right) - \tilde{x}_{i2} \right] = 0.
\]

### B.2 Solution of second-period outcomes

We solve now for the second-period outcomes, given the debt level committed to. The loss function to be minimized by the government of country \(i\) in the second period is:

\[
\alpha \pi_2^2 + \nu \left[ \nu \left( \pi_2 - \pi^e_2 - \tau_{i2} \right) - \tilde{x}_{i2} \right]^2 + \alpha_g \left[ - (1 + \rho) d_{i1} + \tau_{i2} - \tilde{g}_{i2} \right]^2.
\]

The first-order condition with respect to \(\tau_{i2}\) can be written as:

\[
\nu \left[ \nu \left( \pi_2 - \pi^e_2 - \tau_{i2} \right) - \tilde{x}_{i2} \right] = \alpha_g \left[ - (1 + \rho) d_{i1} + \tau_{i2} - \tilde{g}_{i2} \right].
\]

Hence, (4) and (5) together with the second-period government budget constraint form the relevant system to be solved in the second period. The solution is obtained in two steps. In step 1 we solve for the cross-country average component of the solution, while in step 2 we solve for the country-specific component of the solution.

Because there are no shocks, expectations and realizations coincide, so that the system formed by (4), (5) and the second-period budget constraint reduces to:

\[
\alpha \pi_2 = 0,
\]

\[
-\nu^2 \left( \tau_{i2} + \frac{\tilde{x}_{i2}}{\nu} \right) = \alpha_g \left[ - (1 + \rho) d_{i1} + \tau_{i2} - \tilde{g}_{i2} \right],
\]

\[
g_{i2} = - (1 + \rho) d_{i1} + \tau_{i2}.
\]

The solution of this system of equations is — we present the solution of \(\tilde{x}_{i2} - x_{i2}\) instead of \(\tau_{i2} + \frac{\tilde{x}_{i2}}{\nu}\), because \(\tilde{x}_{i2} - x_{i2}\) enters the welfare loss function directly:

\[
\pi_2 = 0,
\]

\[
\tilde{x}_{i2} - x_{i2} = \left[ \frac{1}{\nu} \right] \left[ K_{i2} + (1 + \rho) d_{i1} \right],
\]

\[
g_{i2} = - (1 + \rho) d_{i1} + \tau_{i2}.
\]
\( \bar{g}_{i2} - g_{i2} = \left[ \frac{1/\alpha_g}{S} \right] [K_{i2} + (1 + \rho) d_{i1}] \),

where

\[ S \equiv \frac{1}{\nu^2} + \frac{1}{\alpha_g}, \]

as in the main text. Hence, government \( i \)'s second-period loss, conditional upon the debt level, is:

\[ L_{i2}^F \equiv \frac{1}{S} [K_{i2} + (1 + \rho) d_{i1}]^2. \]

**B.3 Solution of first-period outcomes**

We solve now for the first-period outcomes, \textit{given} the debt level. The loss function to be minimized by government \( i \) in the first period is:

\[
\alpha_x \pi_1^2 + [\nu (\pi_1 - \pi_1^e - \bar{x}_{i1}) - \bar{x}_{i1}]^2 + \alpha_g \left[ - (1 + \rho) d_{i0} + \tau_{i1} + d_{i1} - \bar{g}_{i1} \right]^2.
\]

The first-order condition with respect to \( \tau_{i1} \) can be written as:

\[
-\nu \left[ \nu (\pi_1 - \pi_1^e - \bar{x}_{i1}) - \bar{x}_{i1} \right] + \alpha_g (g_{i1} - \bar{g}_{i1}) = 0,
\]

Because there are no shocks, expectations and realizations coincide, and the relevant system formed by (3), (7) and the first-period budget constraint reduces to:

\[
\alpha_x \pi_1 = 0,
\]

\[
-\nu^2 \left( \tau_{i1} + \frac{\bar{x}_{i1}}{\nu} \right) = \alpha_g (g_{i1} - \bar{g}_{i1}),
\]

\[
g_{i1} = -(1 + \rho) d_{i0} + \tau_{i1} + d_{i1}.
\]

The solution of this system of equations is:

\[
\pi_1 = 0,
\]

\[
\bar{x}_{i1} - x_{i1} = \left[ \frac{1/\nu}{S} \right] [K_{i1} + (1 + \rho) d_{i0} - d_{i1}],
\]

\[
\bar{g}_{i1} - g_{i1} = \left[ \frac{1/\alpha_g}{S} \right] [K_{i1} + (1 + \rho) d_{i0} - d_{i1}].
\]

Hence, government \( i \)'s first-period loss, conditional upon the debt level, is:

\[ L_{i1}^F \equiv \frac{1}{S} [K_{i1} + (1 + \rho) d_{i0} - d_{i1}]^2. \]
B.4 The optimal debt levels

The debt level should be chosen so as to minimize \( L_{11}^F + \beta L_{12}^F \). Hence, we need to minimize:

\[
\frac{1}{2s} \left\{ [K_{11} + (1+\rho) d_{i0} - d_{i1}]^2 + \beta [K_{i2} + (1+\rho) d_{i1}]^2 \right\}.
\]

over \( d_{i1} \). This yields:

\[
d_{i1} = \frac{[K_{11} + (1+\rho) d_{i0} - K_{i2}] + (1 - \beta^*) K_{i2}}{1 + \beta^*(1+\rho)}.
\] (8)

where, as in the main text,

\( \beta^* = \beta (1+\rho) \).

B.5 Solutions for inflation, output and public spending

The solutions for inflation, output and public spending in periods 1 and 2 are:

\[ \pi_1 = \pi_2 = 0, \]

\[ \tilde{x}_{i1} - x_{i1} = \left[ \frac{1}{s} \right] \left[ \frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)} \right] F_1, \tilde{x}_{i2} - x_{i2} = \frac{1}{\pi^*}(\tilde{x}_{i1} - x_{i1}), \]

\[ \tilde{g}_{i1} - g_{i1} = \left[ \frac{1}{\alpha_2} \right] \left[ \frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)} \right] F_1, \tilde{g}_{i2} - g_{i2} = \frac{1}{\pi^*}(\tilde{g}_{i1} - g_{i1}) \]

C Derivation of the discretionary equilibrium

We now solve the model for the case of a centralized, discretionary monetary policy and discretionary, decentralized fiscal policies. We allow for the possibility of a (possibly time-varying) inflation target. The case of pure discretion is obtained when the inflation target is restricted to zero in both periods. The model is solved through backwards induction.

C.1 Period 2

We compute the second-period policy outcomes, conditional on the first-period debt choices.

In period 2 the ECB minimizes over \( \pi_2 \):

\[
\frac{1}{2s} \left\{ \alpha_2 (\pi_2 - \pi_2^*)^2 + \frac{1}{\tau} \sum_{i=1}^{n} \left[ \nu (\pi_2 - \pi_2^* - \tau_{i2}) - \tilde{x}_{i2} \right]^2 \right\}.
\]

The ECB’s first-order condition is:
\[ \alpha \pi (\pi_2 - \pi_2^e) + \frac{1}{n} \sum_{i=1}^{n} [\nu (\pi_2 - \pi_2^* - \tau_{i2}) - \bar{x}_{i2}] = 0. \] (9)

The fiscal authority of country \( i \) minimizes over \( \tau_{i2} \):

\[ \frac{1}{2} \left\{ \alpha \pi_2^2 + [\nu (\pi_2 - \pi_2^* - \tau_{i2}) - \bar{x}_{i2}]^2 + \alpha_g [- (1 + \rho) d_{i1} + \tau_{i2} - \bar{y}_{i2}]^2 \right\}. \]

The first-order condition is:

\[ -\nu [\nu (\pi_2 - \pi_2^* - \tau_{i2}) - \bar{x}_{i2}] + \alpha_g (\bar{y}_{i2} - \bar{y}_{i2}) = 0. \] (10)

Furthermore, we can write the second-period government budget constraint as:

\[ K_{i2} + (1 + \rho) d_{i1} = (\tau_{i2} + \bar{x}_{i2}/\nu) + (\bar{g}_{i2} - \bar{g}_{i2}). \] (11)

The relevant system to be solved for the second-period outcomes is (9), (10) and (11). Because the wage setter’s expectation over second-period inflation is formed at the start of the first period, and realized second period inflation depends on debt accumulation in the first period, the solution for the expectation of second-period inflation depends on the expectation of debt formed at the start of the first period. Thus, we first (Step 1) compute the solution of the start-of-the-first period expectation of this system and then (Step 2) compute the solution for the system in deviations from this start-of-the-first period expectation.

C.1.1 Step 1: solution in terms of start-of-first-period expectations

Take start of first-period expectations of (9), (10) and (11). The system to be solved is then given by:

\[ \alpha \pi (\pi_2 - \pi_2^* - \tau_{i2}) = \nu^2 (\bar{\tau}_{2}^e + \bar{x}_{i2}/\nu), \] (12)

\[ \nu^2 (\pi_2^e + \bar{x}_{i2}/\nu) = \alpha_g (\bar{y}_{i2} - \bar{y}_{i2}), \] (13)

\[ K_{i2} + (1 + \rho) d_{i1} = (\tau_{i2} + \bar{x}_{i2}/\nu) + (\bar{g}_{i2} - \bar{g}_{i2}). \] (14)

Rewrite equation (12) to give:

\[ \pi_2^e = \pi_2^* + \left[ \frac{\nu^2}{\alpha \pi} \right] (\bar{\tau}_{2}^e + \bar{x}_{i2}/\nu). \] (15)

Take cross-country averages of (13) and (14) and combine the result to eliminate \( \bar{y}_2 - \bar{y}_{i2}^e \). We end up with:

\[ \bar{\tau}_{2}^e + \bar{x}_{i2}/\nu = \left[ \frac{1/\nu^2}{S} \right] [\bar{K}_2 + (1 + \rho) \bar{d}_1], \] (16)

where \( S \) was defined in (6), conform the main text. Combine (16) with (15) to give:

\[ 7 \]
\[ \pi^e_2 = \pi^s_2 + \left[ \frac{1/\alpha_\pi}{S} \right] [K_2 + (1 + \rho) \bar{d}^1_i]. \quad (17) \]

Furthermore, we can combine (13) and (14) to yield
\[ K_{i2} + (1 + \rho) \bar{d}^e_i = \left( 1 + \frac{\nu^2}{\alpha g} \right) (\tau^e_{i2} + \bar{x}_{i2}/\nu). \]

Rewrite the result to yield:
\[ \tau^e_{i2} + \bar{x}_{i2}/\nu = \left[ \frac{1/\alpha_\pi}{S} \right] [K_{i2} + (1 + \rho) \bar{d}^e_i]. \quad (18) \]

Combine this with (13) to find that:
\[ \bar{g}_{i2} - g^e_{i2} = \left[ \frac{1/\alpha_\pi}{S} \right] [K_{i2} + (1 + \rho) \bar{d}^e_i]. \quad (19) \]

Finally,
\[ \bar{x}_{i2} - \bar{x}^e_{i2} = \left[ \frac{1/\nu}{S} \right] [K_{i2} + (1 + \rho) \bar{d}^e_i]. \quad (20) \]

C.1.2 Step 2: solution in terms of deviations from start-of-first-period expectations

Subtract (12), (13) and (14) from (9), (10) and (11), respectively. This yields the system:
\[ \left( \alpha_\pi + \nu^2 \right) \pi^d_2 = \nu^2 \bar{\bar{y}}^d_{i2}, \quad (21) \]
\[ -\nu^2 \left( \pi^d_2 - \tau^d_{i2} \right) + \alpha_g g^d_{i2} = 0, \quad (22) \]
\[ (1 + \rho) d^d_{i1} = \tau^d_{i2} - g^d_{i2}, \quad (23) \]

where we use superscript $^d$ to denote the deviation of the actual value from the start-of-the-first-period expectation, e.g. $\bar{\bar{y}}^d_{i2} = \bar{y}_{i2} - \bar{y}^e_{i2}$.

Step 2a: Take cross-country averages of (21), (22) and (23), to give:
\[ \bar{\pi}^d_2 = \left[ \frac{1/\alpha_\pi}{1/\alpha_\pi + 1/\nu^2} \right] \bar{\bar{y}}^d_{i2}, \]
\[ \bar{g}^d_2 = \left( \nu^2/\alpha_g \right) \left( \pi^d_2 - \bar{\bar{y}}^d_2 \right), \]
\[ (1 + \rho) \bar{d}^d_{i1} = \bar{\bar{y}}^d_2 - \bar{g}^d_2. \]

The solution is:
\[ \pi^d_2 = \left[ \frac{1/\alpha_\pi}{\nu} \right] (1 + \rho) \bar{d}^d_{i1}, \quad (24) \]
where we have used that \( \bar{x}_2^d = \nu \left( \pi_2^d - \bar{x}_2^d \right) \).

**Step 2b:** Subtract cross-country averages from (22) and (23), to give:

\[
\nu^2 \bar{x}_2^d + \alpha_g \bar{y}_2^d = 0,
\]

\[
(1 + \rho) \bar{d}_1^1 = \bar{r}_1^d - \bar{g}_2,
\]

where using the definitions, for example \( \bar{r}_2^d = \bar{r}_2^d - \bar{x}_2^d = (\tau_{i2} - \tau_{i2}^e) - (\bar{r}_2 - \bar{r}_2^e) \).

This is solved to yield:

\[
\bar{x}_2^d = - \left[ \frac{1/\nu}{1/\nu + \alpha_g} \right] (1 + \rho) \bar{d}_1^1,
\]

\[
\bar{g}_2 = - \left[ \frac{1/\alpha_g}{1/\alpha_g + \nu} \right] (1 + \rho) \bar{d}_1^1.
\]

**C.1.3 The complete second-period solution:**

We can now compose the complete solutions of the relevant second-period variables. First, using (17) and (24), we compute \( \pi_2 = \pi_2^e + \pi_2^d \). Second, using (20), (25) and (27), one obtains \( \bar{x}_2 - x_{i2} = \bar{x}_{i2} - x_{i2}^e - \bar{x}_2^d - \bar{x}_{i2}^d \). Finally, using (19), (26) and (28), one has \( \bar{g}_{i2} - g_{i2} = \bar{g}_{i2} - g_{i2}^e - \bar{g}_2^d - \bar{g}_{i2}^d \). Hence,

\[
\bar{x}_{i2} = x_{i2}^e + \left[ \frac{1/\alpha_s}{1/\alpha_s + \nu} \right] \left\{ \bar{K}_2 + (1 + \rho) \left[ \bar{d}_1^1 - \left( 1 - \frac{S}{P} \right) \bar{d}_1^d \right] \right\},
\]

\[
\bar{K}_2 = K_{i2}^e + (1 + \rho) \left[ d_{i1} - \left( 1 - \frac{S}{P} \right) d_{i1}^d \right],
\]

\[
\bar{d}_1^1 = d_{i1}^e + (1 + \rho) \left[ \bar{d}_1^1 - \left( 1 - \frac{S}{P} \right) \bar{d}_1^d \right],
\]

\[
\bar{d}_1^d = d_{i1}^d - \frac{1}{\nu} \left[ \frac{1}{\alpha_g} \sum_{i=1}^{n} \left\{ \bar{K}_{i2} + (1 + \rho) \left[ d_{i1} - \left( 1 - \frac{S}{P} \right) d_{i1}^d \right] \right\} \right],
\]

\[
\bar{g}_{i2} - g_{i2} = \left[ \frac{1/\alpha_g}{1/\alpha_g + \nu} \right] \left\{ \bar{K}_{i2} + (1 + \rho) \left[ d_{i1} - \left( 1 - \frac{S}{P} \right) d_{i1}^d \right] \right\}. \tag{31}
\]

Using (29), (30) and (31), the ECB’s and government i’s second-period losses are, respectively:

\[
L_{2}^{ECB} = \frac{1}{2} \left[ \frac{1/\alpha_s}{1/\alpha_s + \nu} \right] \left\{ \bar{K}_2 + (1 + \rho) \left[ \bar{d}_1^1 - \left( 1 - \frac{S}{P} \right) \bar{d}_1^d \right] \right\}^2,
\]

\[
= \frac{1}{2} \left[ \frac{1/\alpha_s}{1/\alpha_s + \nu} \right] \sum_{i=1}^{n} \left\{ \bar{K}_{i2} + (1 + \rho) \left[ d_{i1} - \left( 1 - \frac{S}{P} \right) d_{i1}^d \right] \right\}^2, \tag{32}
\]

\[
L_{i2}^{E} = \frac{1}{2} \alpha_s \left\{ \bar{K}_2 + (1 + \rho) \left[ \bar{d}_1^1 - \left( 1 - \frac{S}{P} \right) \bar{d}_1^d \right] \right\}^2
\]

\[
+ \frac{1}{2} \left[ \frac{1/\alpha_s}{1/\alpha_s + \nu} \right] \sum_{i=1}^{n} \left\{ \bar{K}_{i2} + (1 + \rho) \left[ d_{i1} - \left( 1 - \frac{S}{P} \right) d_{i1}^d \right] \right\}^2. \tag{33}
\]
Note that, if expectations were formed at the start of each period, then the terms with $d^i_1$ would drop out. The terms with $d^i_1$ are the source of the time-inconsistency problem with fiscal policy.

**C.2 Period 1**

The ECB minimizes over $\pi_1$:

$$\frac{1}{2} \left\{ \alpha_\pi (\pi_1 - \pi^*_1)^2 + \frac{1}{n} \sum_{i=1}^n \left[ \nu (\pi_1 - \pi^c_1 - \tau_{i1}) - \bar{x}_{i1} \right]^2 \right\} + \beta L^{ECB}_2.$$  

Hence, the first-order condition is:

$$\alpha_\pi (\pi_1 - \pi^*_1) + \frac{1}{n} \sum_{i=1}^n \left[ \nu (\nu (\pi_1 - \pi^c_1 - \tau_{i1}) - \tilde{x}_{i1}) \right] = 0. \tag{34}$$

The government of country $i$ minimizes over $\tau_{i1}$ and $d_{i1}$:

$$\frac{1}{2} \left\{ \alpha_\pi_\pi^2 + \left[ \nu (\pi_1 - \pi^c_1 - \tau_{i1}) - \bar{x}_{i1} \right]^2 + \alpha_g \left[ - (1 + \rho) d_{i0} + \tau_{i1} + d_{i1} - \tilde{g}_{i1} \right]^2 + \beta L^G_{i2} \right\}. \tag{35}$$

The first-order conditions are:

$$-\nu \left[ \nu (\pi_1 - \pi^c_1 - \tau_{i1}) - \bar{x}_{i1} \right] + \alpha_g (g_{i1} - \tilde{g}_{i1}) = 0, \tag{36}$$

where

$$\partial L^F_{i2} / \partial d_{i1} \tag{37}$$

$$= \left\{ \alpha_\pi^2 + \left[ \frac{1}{\nu} \right] \left\{ K_{i1} + (1 + \rho) \left[ \frac{\nu}{\bar{x}} d_{i1} + \frac{\nu}{\bar{x}} \right] \left[ 1 - \frac{\nu}{\bar{x}} \right] \right\} \right\} * \frac{1}{n} (1 + \rho) \frac{1}{\nu}$$

$$+ \left\{ K_{i2} + (1 + \rho) \left[ d_{i1} - \frac{\nu}{\bar{x}} \right] \right\} * (1 + \rho) \frac{1}{n} \left[ \left( \frac{\nu}{\bar{x}} \right) + \frac{1}{n} \left( \frac{\nu}{\bar{x}} \right) \right].$$

Finally, we can rewrite the first-period government budget constraint as:

$$K_{i1} + (1 + \rho) d_{i0} = (\tau_{i1} + \bar{x}_{i1}/\nu) + (\tilde{g}_{i1} - g_{i1}) + d_{i1}. \tag{38}$$

The system of first-order conditions to be solved is thus given by (34), (35), (36) and (38). The solution is obtained in two steps. In step 1 we solve for the solution in terms of cross-country averages. In step 2 we solve for the country-specific component of the solution.

Realizing that, in equilibrium, expectations and realizations coincide, we can write the system (34), (35), (36) and (38) as:

$$\alpha_\pi (\pi_1 - \pi^*_1) = \nu^2 \left( \bar{x}_1 + \bar{x}_1 / \nu \right), \tag{39}$$

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\[ \nu^2 (\tau_{11} + \bar{x}_{11}/\nu) = \alpha_g (\bar{g}_{11} - g_{11}), \] \hspace{1cm} (40)

\[ \alpha_g (\bar{g}_{11} - g_{11}) = \beta \left\{ \pi_2^* + \left[ \frac{1}{2} \alpha_s \right] \left[ K_2 + (1 + \rho) \bar{d}_1 \right] \right\} \frac{1}{n} (1 + \rho) \frac{1}{p} + \beta \left[ K_2 + (1 + \rho) \bar{d}_1 \right] (1 + \rho) \frac{1}{p} \left[ 1 \left( \frac{n-1}{n} \right) + \frac{1}{n} \left( \frac{\nu}{\bar{\nu}} \right) \right], \]

\[ K_{11} + (1 + \rho) \tilde{d}_0 = (\tau_{11} + \bar{x}_{11}/\nu) + (\bar{g}_{11} - g_{11}) + \bar{d}_1. \] \hspace{1cm} (42)

**Step 1:** Take cross-country averages across (39)-(42) to yield the following system:

\[ \alpha_{\pi} (\pi_{11} - \pi_{11}^*) = \nu^2 (\bar{\tau}_1 + \bar{x}_1/\nu), \] \hspace{1cm} (43)

\[ \nu^2 (\bar{\tau}_1 + \bar{x}_1/\nu) = \alpha_g (\bar{g}_1 - \bar{g}_1), \] \hspace{1cm} (44)

\[ \alpha_g (\bar{g}_1 - \bar{g}_1) = \beta \left\{ \pi_2^* + \left[ \frac{1}{2} \alpha_s \right] \left[ K_2 + (1 + \rho) \bar{d}_1 \right] \right\} \frac{1}{n} (1 + \rho) \frac{1}{p} + \beta \left[ K_2 + (1 + \rho) \bar{d}_1 \right] (1 + \rho) \frac{1}{p} \left[ 1 \left( \frac{n-1}{n} \right) + \frac{1}{n} \left( \frac{\nu}{\bar{\nu}} \right) \right], \]

\[ \bar{K}_1 + (1 + \rho) \bar{d}_0 = (\bar{\tau}_1 + \bar{x}_1/\nu) + (\bar{g}_1 - \bar{g}_1) + \bar{d}_1. \] \hspace{1cm} (46)

Combine (44) with (46) to eliminate \( \bar{g}_1 - \bar{g}_1 \) and obtain after rewriting:

\[ (\bar{\tau}_1 + \bar{x}_1/\nu) = \left[ \frac{1}{2} \frac{\nu^2}{\bar{\nu}} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right]. \] \hspace{1cm} (47)

Hence,

\[ \pi_{11} = \pi_{11}^* + \left[ \frac{1}{2} \frac{\nu^2}{\bar{\nu}} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right], \] \hspace{1cm} (48)

\[ \bar{g}_1 - \bar{g}_1 = \left[ \frac{1}{2} \frac{\nu^2}{\bar{\nu}} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right]. \] \hspace{1cm} (49)

Hence, combining this last equation with (45) to eliminate \( \bar{g}_1 - \bar{g}_1 \) and rewriting yields:

\[ \bar{K}_1 \left[ \frac{1}{n} \left( \frac{\nu}{\bar{\nu}} \right) \right] + \bar{d}_0 \] \hspace{1cm} (50)
or (using the definition of $P$

$$K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 = \beta^* \frac{1}{\pi} \bar{S}^* \bar{\pi}_2^* + \beta^* [K_2 + (1 + \rho) \bar{d}_1]. \quad (51)$$

Hence,

$$\bar{d}_1 = \frac{(K_1 + (1 + \rho) \bar{d}_0 - K_2)}{1 + \beta^* (1 + \rho)} - \frac{1}{\pi} \beta^* \frac{S/P}{1 + \beta^* (1 + \rho)} \bar{\pi}_2^*. \quad (52)$$

Step 2: Subtract (44)-(46) from (40)-(42). This gives the following system:

$$\nu^2 (\bar{\tau}_{1i} + \bar{x}_{1i}/\nu) = \alpha_g (\bar{\hat{g}}_{i1} - \bar{g}_{i1}), \quad (53)$$

$$\alpha_g (\bar{\hat{g}}_{i1} - \bar{g}_{i1}) = \beta^* \left[ \frac{P^*/P}{S} \right] \left[ \bar{K}_{i2} + (1 + \rho) \bar{d}_{i1} \right], \quad (54)$$

$$\bar{K}_{i1} + (1 + \rho) \bar{d}_{i0} = (\bar{\tau}_{1i} + \bar{x}_{1i}/\nu) + (\bar{\hat{g}}_{i1} - \bar{g}_{i1}) + \bar{d}_{i1}, \quad (55)$$

where, as in the main text,

$$P^* = \left[ (n - 1)/n \right] (1/\alpha_g) + 1/\nu^2 + 1/\alpha_g.$$

Equations (53) and (55) can be combined to yield the outcomes conditional on $\bar{d}_{i1}$:

$$\bar{\tau}_{1i} + \bar{x}_{1i}/\nu = \left[ \frac{1/\nu^2}{\alpha_g} \right] \left[ \bar{K}_{i1} + (1 + \rho) \bar{d}_{i0} - \bar{d}_{i1} \right]. \quad (56)$$

Hence,

$$\bar{\hat{d}}_{i1} = \left[ \bar{K}_{i1} + (1 + \rho) \bar{d}_{i0} - \bar{K}_{i2} \right] + [1 - \beta^* (P^*/P)] \bar{K}_{i2} \quad \frac{1 + \beta^* (1 + \rho) (P^*/P)}{1}. \quad (59)$$

Because $P^*/P < 1$, debt dispersion across countries is blown up: it is easy to show that $\bar{d}_{i1} > \bar{d}_{i1}^S$ if $\bar{F}_i > 0$ and $\bar{d}_{i1} < \bar{d}_{i1}^S$ if $\bar{F}_i < 0$.

C.3 Complete outcomes for inflation, output and spending under pure discretion

We now present the complete outcomes for inflation, output and spending in both periods under pure discretion. Under pure discretion, inflation targets (and debt targets) are absent. Hence, the outcomes are:
\[ \pi_1 = \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{\beta^*(1+p)}{1+\beta^*(1+p)} \right] \tilde{F}, \]

\[ \tilde{x}_{i1} - x_{i1} = \left[ \frac{1/\nu}{S} \right] \left[ \frac{\beta^*(1+p)}{1+\beta^*(1+p)} \right] \tilde{F} + \left[ \frac{1/\nu}{S} \right] \left[ \frac{\beta^*(1+p)/(P^*/P)}{1+\beta^*(1+p)/(P^*/P)} \right] \tilde{F}_i, \]

\[ \tilde{g}_{i1} - g_{i1} = \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{\beta^*(1+p)}{1+\beta^*(1+p)} \right] \tilde{F} + \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{\beta^*(1+p)/(P^*/P)}{1+\beta^*(1+p)/(P^*/P)} \right] \tilde{F}_i, \]

\[ \pi_2 = \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{\beta^*(1+p)}{1+\beta^*(1+p)} \right] \tilde{F}, \]

\[ \tilde{x}_{i2} - x_{i2} = \left[ \frac{1/\nu}{S} \right] \left[ \frac{1+\rho}{1+\beta^*(1+p)} \right] \tilde{F} + \left[ \frac{1/\nu}{S} \right] \left[ \frac{1+\rho}{1+\beta^*(1+p)/(P^*/P)} \right] \tilde{F}_i, \]

\[ \tilde{g}_{i2} - g_{i2} = \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{1+\rho}{1+\beta^*(1+p)} \right] \tilde{F} + \left[ \frac{1/\alpha_s}{S} \right] \left[ \frac{1+\rho}{1+\beta^*(1+p)/(P^*/P)} \right] \tilde{F}_i. \]

D Ex-post fiscal coordination

Fiscal policies are chosen so as to minimize

\[ V_U = \frac{1}{n} \sum_{i=1}^{n} V_{S,i}, \quad (60) \]

Because spillovers can only take place via debt policy, the computation largely parallels that of the uncoordinated, discretionary equilibrium. Conditional on the choices of debt, the second-period policy outcomes are still given by (29) - (31).

The choices of \( \tau_{i1} \) and \( d_{i1}, \forall i \), minimize:

\[ \frac{1}{2n} \sum_{i=1}^{n} \left\{ \alpha_\pi \pi_1^2 + \left[ \nu (\pi_1 - \pi_1^*) - \tilde{x}_{i1} \right]^2 + \alpha_\rho \left[ - (1 + \rho) d_{i0} + \tau_{i1} + d_{i1} - \tilde{g}_{i1} \right]^2 + 2 \beta L_{i2}^F \right\}. \]

The first-order conditions are (35), \( \forall i \), (38), \( \forall i \), and

\[ \frac{1}{n} \alpha_\rho (\tilde{g}_{i1} - g_{i1}) = \beta \left( \frac{1}{n} \sum_{i=1}^{n} L_{i2}^F \right) / \partial d_{i1}, \forall i, \]

where
To sum up, the full set of first-order conditions for the first period is:

\[
\alpha_\pi (\pi_1 - \pi_1^*) + \frac{1}{\alpha} \sum_{i=1}^{n} \nu (\nu (\pi_1 - \pi_1^* - \tau_{1i}) - \bar{x}_{1i}) = 0, \tag{61}
\]

\[
-\nu [\nu (\pi_1 - \pi_1^* - \tau_{1i}) - \bar{x}_{1i}] + \alpha_\rho (g_{1i} - \bar{g}_{1i}) = 0, \tag{62}
\]

\[
\alpha_\rho (\bar{g}_{1i} - g_{1i}) = \beta^* \left\{ \frac{1}{\alpha} \pi_2^* + \frac{1}{\alpha} \{ K_{12} + (1 + \rho) [d_{1i} - (1 - \kappa) \hat{d}_1^i] \} \right\}, \tag{63}
\]

\[
K_{1i} + (1 + \rho) d_{10} = (\tau_{1i} + \bar{x}_{1i}/\nu) + (\bar{g}_{1i} - g_{1i}) + d_{1i}. \tag{64}
\]

Realizing that expectations and realizations coincide, (61), (62), (63) and (64) reduce to:

\[
\alpha_\pi (\pi_1 - \pi_1^*) = \nu^2 (\bar{x}_{1i} + \bar{x}_{1i}/\nu), \tag{65}
\]

\[
\nu^2 (\tau_{1i} + \bar{x}_{1i}/\nu) = \alpha_\rho (\bar{g}_{1i} - g_{1i}), \tag{66}
\]

\[
\alpha_\rho (\bar{g}_{1i} - g_{1i}) = \beta^* \left\{ \frac{1}{\alpha} \pi_2^* + \frac{1}{\alpha} \{ K_{12} + (1 + \rho) d_{1i} \} \right\}, \tag{67}
\]
\[ K_{i1} + (1 + \rho) d_{i0} = (\tau_{i1} + \bar{x}_{i1}/\nu) + (\bar{g}_{i1} - g_{i1}) + d_{i1}. \]  

We solve first for the cross-country average solution (Step 1), then for the country-specific components of the outcomes (Step 2).

**Step 1:** Take cross-country averages across (65)-(68) to yield the following system:

\[ \alpha_\pi (\pi_1 - \pi_1^\ast) = \nu^2 (\bar{\tau}_1 + \bar{x}_1/\nu), \]  

\[ \nu^2 (\bar{\tau}_1 + \bar{x}_1/\nu) = \alpha_g (\bar{g}_1 - \bar{g}_1), \]  

\[ \alpha_g (\bar{g}_1 - \bar{g}_1) = \beta^* \{ \frac{1}{\beta} \pi_2^* + \frac{1}{\beta} [K_2 + (1 + \rho) \bar{d}_1] \}, \]  

\[ \bar{K}_1 + (1 + \rho) \bar{d}_0 = (\bar{\tau}_1 + \bar{x}_1/\nu) + (\bar{g}_1 - \bar{g}_1) + \bar{d}_1. \]  

Combine (70) with (72) to eliminate \( \bar{g}_1 - \bar{g}_1 \) and obtain after rewriting:

\[ (\bar{\tau}_1 + \bar{x}_1/\nu) = \left[ \frac{1}{\nu^2} \right] [K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1]. \]  

Hence,

\[ \pi_1 = \pi_1^\ast + \left[ \frac{1}{\beta^\ast} \right] [K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1], \]  

\[ \bar{g}_1 - \bar{g}_1 = \left[ \frac{1}{\beta^\ast} \right] [K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1]. \]  

Hence, combining this last equation with (71) to eliminate \( \bar{g}_1 - \bar{g}_1 \) and rewriting yields:

\[ \left[ \frac{1}{\beta^\ast} \right] [K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1] = \beta^* \{ \frac{1}{\beta} \pi_2^* + \frac{1}{\beta} [K_2 + (1 + \rho) \bar{d}_1] \}, \]  

or

\[ K_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 = \beta^* \frac{\bar{s}}{\beta^*} \pi_2^* + \beta^* [K_2 + (1 + \rho) \bar{d}_1]. \]  

Hence,

\[ \bar{d}_1 = \frac{[K_1 + (1 + \rho) \bar{d}_0 - K_2] + (1 - \beta^*) K_2}{1 + \beta^*(1 + \rho)} - \beta^* \left[ \frac{s}{1 + \beta^*(1 + \rho)} \right] \pi_2^*. \]  

**Step 2:** Subtract (70)-(72) from (66)-(68). This gives the following system:

\[ \nu^2 (\bar{\tau}_{i1} + \bar{x}_{i1}/\nu) = \alpha_g (\bar{g}_{i1} - \bar{g}_{i1}), \]  

\[ \alpha_g (\bar{g}_{i1} - \bar{g}_{i1}) = \beta^* \left[ \frac{1}{\beta} \right] [K_{i2} + (1 + \rho) \bar{d}_{i1}], \]  

\[ \bar{K}_{i1} + (1 + \rho) \bar{d}_{i0} = (\bar{\tau}_{i1} + \bar{x}_{i1}/\nu) + (\bar{g}_{i1} - \bar{g}_{i1}) + \bar{d}_{i1}. \]
Equations (79) and (81) can be combined to yield outcomes conditional on \( \hat{d}_{i1} \):

\[
\hat{\tau}_{i1} + \tilde{x}_{i1}/\nu = \left[ \frac{1/\nu^2}{\alpha} \right] \left[ \hat{K}_{i1} + (1 + \rho) \hat{d}_{i0} - \hat{d}_{i1} \right]. \tag{82}
\]

\[
\tilde{g}_{i1} - \hat{g}_{i1} = \left[ \frac{1/\alpha}{\nu} \right] \left[ \hat{K}_{i1} + (1 + \rho) \hat{d}_{i0} - \hat{d}_{i1} \right]. \tag{83}
\]

Combining the latter equation with (80) gives:

\[
\hat{K}_{i1} + (1 + \rho) \hat{d}_{i0} - \hat{d}_{i1} = \beta^* \left[ \hat{K}_{i2} + (1 + \rho) \hat{d}_{i1} \right]. \tag{84}
\]

Hence,

\[
\hat{d}_{i1} = \frac{\hat{K}_{i1} + (1 + \rho) \hat{d}_{i0} - \hat{d}_{i1}}{1 + \beta^* (1 + \rho)}. \tag{85}
\]

E Proof of Propositions 1 and 3

The case of Proposition 1 is obtained when \( n = 1 \), in which case fiscal decentralization coincides with ex-post fiscal coordination.

The second-best outcomes for inflation are \( \pi_1 = \pi_2 = 0 \). Substitute the proposed debt target, \( \bar{d}_S \), for \( \bar{d}_1 \) into (17), so that:

\[
\pi_2 = \pi^*_2 + \left[ \frac{1/\alpha}{\nu} \right] \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_S^2 \right].
\]

Hence, setting \( \pi^*_2 = -\left[ \frac{1/\alpha}{\nu} \right] \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_S^2 \right] = -\left[ \frac{1/\alpha}{\nu} \right] \left[ \frac{1 + \rho}{1 + \beta^* (1 + \rho)} \right] \tilde{F} \) produces \( \pi_2 = 0 \). Substitute \( \tilde{d}_S^2 \) for \( \tilde{d}_1 \) into (48), so that:

\[
\pi^*_1 = \pi^*_1 + \left[ \frac{1/\alpha}{\nu} \right] \left[ \tilde{K}_1 + (1 + \rho) \tilde{d}_0 - \tilde{d}_S^2 \right].
\]

Hence, setting \( \pi^*_1 = -\left[ \frac{1/\alpha}{\nu} \right] \left[ \tilde{K}_1 + (1 + \rho) \tilde{d}_0 - \tilde{d}_S^2 \right] = -\left[ \frac{1/\alpha}{\nu} \right] \left[ \frac{1 + \rho}{1 + \beta^* (1 + \rho)} \right] \tilde{F} \) produces \( \pi_1 = 0 \).

Substitute \( \bar{d}_S^2 = \bar{d}_1^2 \) for \( \bar{d}_1 \) and 0 for \( \bar{d}_S^2 \) into (31). Then, replace \( \tilde{d}_S^2 \) by the cross-country average of (8). The resulting outcome for \( \tilde{g}_2 - \tilde{g}_1 \) coincides with the corresponding second-best outcome reported in Table 1 (remember that under the assumptions of Proposition 1, \( \tilde{F}_i \) is zero). Next, substitute \( \bar{d}_S^2 \) for \( \bar{d}_1 \) into (49) and use the cross-country average of (8) to find that \( \tilde{g}_1 \) coincides with its second-best outcome. In a similar way, one can check that the outcomes for \( \tilde{x}_{i1} - x_{i1} \) and \( \tilde{x}_{i2} - x_{i2} \) coincide with their second-best values. This completes the proof that under the proposed targets all outcomes coincide with their second-best values and, hence, the resulting equilibrium coincides with the second-best equilibrium.
Proof of Proposition 2

Here, we provide the generalization of the proof of Proposition 2 with the extensions also to the cases of decentralized fiscal decision making with homogeneous and heterogeneous countries. The case of Proposition 2 is obtained when \( n = 1 \), in which case fiscal decentralization coincides with ex-post fiscal coordination.

We derive the optimal inflation targets \( \pi_1^* \) and \( \pi_2^* \) in the absence of debt targets.

To this end, we need the expressions for the first-period policy outcomes, given the inflation targets and debt accumulation:

\[
\pi_1 = \pi_1^* + \left[ \frac{1}{s} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right],
\]

\[
\bar{g}_i - g_i = \left[ \frac{1}{s} \right] \left[ K_i + (1 + \rho) d_{i0} - d_{i1} \right],
\]

\[
\bar{x}_i - x_i = \left[ \frac{1}{s} \right] \left[ K_i + (1 + \rho) d_{i0} - d_{i1} \right].
\]

These expressions are obtained with the help of equations (47), (48), (49), (56) and (57).

We need to minimize over \( \pi_1^* \) and \( \pi_2^* \) the loss function

\[
\frac{1}{n} \sum_{i=1}^{n} (L_{i1}^F + \beta L_{i2}^F),
\]

where

\[
L_{i1}^F = \frac{1}{2} \alpha \pi \left\{ \pi_1^* + \left[ \frac{1}{s} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right] \right\}^2
\]

\[
+ \frac{1}{2\pi} \left[ K_i + (1 + \rho) d_{i0} - d_{i1} \right]^2,
\]

and \( L_{i2}^F \) is given by (33) with \( \bar{d}_1 = 0 \), because, as perceived from the start of the game when \( \pi_1^* \) and \( \pi_2^* \) are chosen, expectations and realizations coincide.

The first-order condition for minimization over \( \pi_1^* \) is:

\[
\frac{1}{n} \sum_{i=1}^{n} \alpha \pi \left[ \pi_1^* + \left( \frac{1}{s} \right) \left( \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right) \right] = 0.
\]

Hence,

\[
\pi_1^* = - \left[ \frac{1}{s} \right] \left[ \bar{K}_1 + (1 + \rho) \bar{d}_0 - \bar{d}_1 \right].
\]

Whatever the equilibrium value \( \bar{d}_1 \) is, setting \( \pi_1^* \) according to (87) with this equilibrium value substituted gets rid of the inflation bias in period 1.

The first-order condition for minimizing \( \pi_2^* \) is more complicated. We can write

\[
\frac{1}{n} \sum_{i=1}^{n} \frac{\partial L_i^F}{\partial \pi_2} + \beta \frac{1}{n} \sum_{i=1}^{n} \frac{\partial L_i^F}{\partial \pi_2}.
\]

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where

\[
\frac{\partial L^{\text{Fi}}_1}{\partial \pi^*_2} = \left\{ \alpha \pi \left[ \pi^*_2 + \frac{1}{\alpha S} \right] \left( K_2 + (1 + \rho) \tilde{d}_1 \right) * \left( \frac{1}{\alpha S} \right) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right\} 
\]

\[\quad + \frac{1}{S} [K_{i1} + (1 + \rho) d_{i0} - d_i] * \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \]

\[= - \frac{1}{S} [K_{i1} + (1 + \rho) d_{i0} - d_i] \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right), \tag{88}\]

where the last equality has used (87) and where (using (52)):

\[
\frac{\partial \tilde{d}_1}{\partial \pi^*_2} = \frac{\partial \tilde{d}_1}{\partial \pi^*_2} = - \frac{1}{S} \beta^* \left[ \frac{S/P}{1 + \rho (1 + \rho)} \right] < 0.
\]

Hence,

\[
\frac{1}{S} \sum_{i=1}^{n} \frac{\partial L^{\text{Fi}}_1}{\partial \pi^*_2} = - \frac{1}{S} [K_{i1} + (1 + \rho) d_{i0} - d_i] \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right), \tag{89}\]

which (if \(\tilde{d}_1\) is not too large) is positive, indicating that a relaxation (increase) of the second-period inflation target raises the average first-period welfare loss because it leads to lower debt. Next,

\[
\frac{1}{n} \sum_{i=1}^{n} \frac{\partial L^{\text{Fi}}_1}{\partial \pi^*_2} = \frac{1}{n} \sum_{i=1}^{n} \left\{ \alpha \pi \left[ \pi^*_2 + \frac{1}{\alpha S} \right] \left( K_2 + (1 + \rho) \tilde{d}_1 \right) * \left( \frac{1}{\alpha S} \right) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right\} 
\]

\[= \frac{1}{n} \sum_{i=1}^{n} \left\{ \alpha \pi \left[ \pi^*_2 + \frac{1}{\alpha S} \right] \left( K_2 + (1 + \rho) \tilde{d}_1 \right) * \left[ 1 + \frac{1}{\alpha S} (1 + \rho) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right] \right\} \tag{90}\]

\[= \alpha \pi \left[ \pi^*_2 + \frac{1}{\alpha S} \right] \left( K_2 + (1 + \rho) \tilde{d}_1 \right) * \left[ 1 + \frac{1}{\alpha S} (1 + \rho) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right] \]

\[+ \frac{1}{S} [K_{i1} + (1 + \rho) d_{i0} - d_i] * (1 + \rho) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right), \]

where we have used that \(\partial d_{i1}/\partial \pi^*_2 = \partial \tilde{d}_1/\partial \pi^*_2\). Hence,

\[
\frac{1}{n} \sum_{i=1}^{n} \frac{\partial L^{\text{Fi}}_1}{\partial \pi^*_2} + \beta \frac{1}{n} \sum_{i=1}^{n} \frac{\partial L^{\text{Fi}}_2}{\partial \pi^*_2} \]

\[= \frac{1}{S} \left\{ - [K_{i1} + (1 + \rho) d_{i0} - d_i] + \beta^* \left[ K_2 + (1 + \rho) \tilde{d}_1 \right] \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right\} \]

\[+ \beta \alpha \pi \left[ \pi^*_2 + \frac{1}{\alpha S} \right] \left( K_2 + (1 + \rho) \tilde{d}_1 \right) * \left[ 1 + \frac{1}{\alpha S} (1 + \rho) \left( \frac{\partial \tilde{d}_1}{\partial \pi^*_2} \right) \right] \tag{91}\]

Note that, because \(\partial d_1/\partial \pi^*_2\) is a constant (independent of \(\pi^*_2\)), the right-hand side of (91) is a linear function of \(\pi^*_2\) (remember that \(d_1\) is linear in \(\pi^*_2\), by (52)).
The solution for \( d \) is zero for the value of \( T \) that makes the second line of (91) positive if \( d_1 < d_1^S \), zero if \( d_1 = d_1^S \), and negative if \( d_1 > d_1^S \).

Hence, the third line of (91) is zero for the value of \( \pi_*^S \) that solves (using (52)):

\[
\pi_*^S = - \left( \frac{1}{S} \right) \left\{ \tilde{K}_2 + (1 + \rho) \left[ \frac{\beta^*}{\beta^* + \rho} \right] \right\}.
\]

The solution for \( \pi_2^{**} \) of this equation is given by

\[
\pi_2^{**} = \left\{ 1 - \frac{1}{n} \left[ \frac{1}{\rho} \right] \frac{\beta^*}{\beta^* + \rho} \right\}^{-1} \bar{d}_1^S + \frac{\beta^*}{\beta^* + \rho} \left[ \frac{1}{\rho} \right] \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S < 0.
\]

If \( \pi_*^S = \pi_*^{**} \), then \( d_1 > d_1^S \) (by (52)) and, hence, the right-hand side of (91) is negative.

Now, consider the target \( \pi_*^S = \pi_*^{opt} = - \left( \frac{1}{S} \right) \left\{ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right\} > \pi_*^{**} \).

We can then write (using (52)):

\[
\begin{aligned}
\bar{d}_1 &= \tilde{d}_1^S + \frac{1}{n} \beta^* \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \\
&= \tilde{d}_1^S + \frac{1}{n} \beta^* \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S.
\end{aligned}
\]

Hence,

\[
\begin{aligned}
\tilde{K}_2 + (1 + \rho) \tilde{d}_1 &= \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S + \frac{1}{n} \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right] \\
&= \left\{ 1 + \frac{1}{n} \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \right\} \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right] \\
&= \left\{ 1 + \frac{1}{n} \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \right\} \left[ \frac{1}{\rho} \right] \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S.
\end{aligned}
\]

and

\[
\begin{aligned}
\beta \alpha \pi \left\{ \pi_*^S + \left( \frac{1}{S} \right) \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right] \right\} &= -\beta \left[ \frac{1}{n} \right] \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right] + \\
&= \left[ \frac{1}{S} \right] \left\{ 1 + \frac{1}{n} \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \right\} \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right] \\
&= \left[ \frac{1}{S} \right] \left[ \frac{1}{\rho} \right] \left[ \frac{1}{\rho} \right] \left[ \tilde{K}_2 + (1 + \rho) \tilde{d}_1^S \right],
\end{aligned}
\]

and

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Hence, the right-hand side of (91) can be written as:

\[
\begin{align*}
&\frac{1}{\alpha_S} \left[ \frac{1}{P} \right] \left[ \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right] F \left( \frac{\partial L}{\partial \pi_i} \right) \\
&\frac{\beta^*}{S} \left[ \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right] F \left( \frac{\partial L}{\partial \pi_i} \right) + \\
&\frac{1}{\alpha_S} \left[ \frac{1}{P} \right] \left[ \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right] F \left( \frac{\partial L}{\partial \pi_i} \right) \left[ \beta^* \left( \frac{S/P}{1+3^*(1+\rho)} \right) + \frac{\beta^*}{1+3^*(1+\rho)} \right].
\end{align*}
\]

Hence, the sign of this expression is:

\[
\operatorname{sign} \left\{ \frac{1}{\alpha_S} \left[ \frac{1}{P} \right] \left[ \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right] F \left( \frac{\partial L}{\partial \pi_i} \right) + \frac{\beta^*}{S} \left[ \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right] F \left( \frac{\partial L}{\partial \pi_i} \right) \left[ \beta^* \left( \frac{S/P}{1+3^*(1+\rho)} \right) + \frac{\beta^*}{1+3^*(1+\rho)} \right] \right\}
\]

\[
\begin{align*}
&\operatorname{sign} \left\{ \left[ 1 + \frac{1}{\alpha_S} \right] \left( \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right) + \frac{1}{\alpha_S} \beta^* \left[ \frac{S/P}{1+3^*(1+\rho)} \right] + \frac{1}{\alpha_S} \beta^* + 1 \right\} \\
&\operatorname{sign} \left\{ -\frac{1}{\alpha_S} \left( \frac{S/P}{1+3^*(1+\rho)} \right) + \frac{1}{\alpha_S} \beta^* \right\} + 1 \right\}.
\end{align*}
\]

Remember that \( P = S + 1/\alpha_S \). Hence,

\[
\frac{1}{\alpha_S} \left[ \frac{S}{P} + \left( \frac{1}{\alpha_S} \right) \left( \frac{3^*(1+\rho)}{1+3^*(1+\rho)} \right) \right] + 1 > 0.
\]

Hence, (86) attains a strict, global minimum for a value of \( \pi^*_2 \) between \( \pi^{**}_2 \) and \( \pi^{*\text{opt}}_2 \). The optimal value of \( \pi^*_2 \) is denoted by \( \pi^{**}_2 \).

Now, concentrate on the case in which countries are heterogeneous. To find the optimal \( \pi^*_2 \) from the perspective of an individual country \( i \) (it is easy to check that the preferred \( \pi^*_1 \) is the same for all countries), we note that (by (88), (89) and (90)):

\[
\frac{\partial L^F}{\partial \pi^*_2} + \beta \frac{\partial L^F}{\partial \pi^*_2} = \frac{1}{n} \left\{ K_{1i} + (1 + \rho) \hat{d}_{i0} - \hat{d}_{i1} \right\} + \beta^* \left[ K_{i2} + (1 + \rho) \hat{d}_{i1} \right] \left( \frac{\partial L}{\partial \pi^*_2} \right). \quad (92)
\]

The third line of (92) is zero when \( \hat{d}_{i1} = \hat{d}^S_{i1} \). However, if \( \tilde{F}_i > 0 \), then the third line of (92) is negative because \( \hat{d}_{i1} > \hat{d}^S_{i1} \) (by (59)) and \( \frac{\partial L}{\partial \pi^*_2} < 0 \). Vice versa, if
$\hat{F}_i < 0$. Because the second line of (91) is linear and increasing in $\pi_2^*$, while the final line does not depend on $\pi_2^*$, the optimal $\pi_2^*$ from $i$’s perspective exceeds $\pi_2^{**}$ if $\hat{F}_i > 0$, while the opposite is the case if $\hat{F}_i < 0$. 