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Chapter 13

Idealized Modeling of Psychological Dynamics

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13.1 Introduction

Formal theories are among the greatest accomplishments of science. Such theories have ample advantages—they make unambiguous predictions, provide insights on how to influence the system under study, and their strength and weaknesses are easier to analyze than is the case for verbal models. In addition to being useful as data-analytical models, network models can also provide good starting points for the development of formal theories as they can also be used as data-generating models. Using such models, we can investigate dynamics by varying relevant parameters in the network. In this chapter, we introduce the Ising model (Ising, 1925) as such a data-generating model and discuss dynamics that are captured by the Ising model.

The Ising model is a simple model of interactions between nodes in a network and has been originally developed for the study of magnets. Due to its simplicity and hence broad applicability, the Ising model has become one of the most well-known models in statistical physics, which studies the behavior of higher-level (or macroscopic) properties emerging from local interactions of lower-level (or microscopic) properties. Since its conception it has been applied to many different areas where we can make a distinction between micro- and macroscopic properties, such as predator-prey dynamics (Kim et al., 2005),

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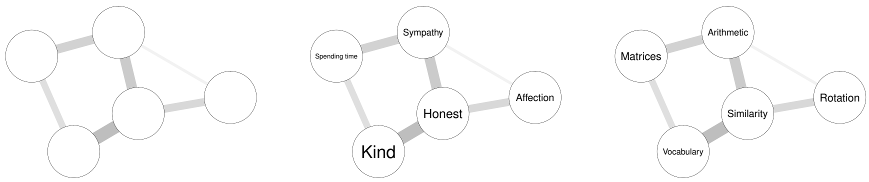

Figure 13.1. Illustration of the difference between the general Ising model and specific theories using the Ising model. The left network represents the general Ising model and the middle and right networks represent specific theories on attitudes and intelligence, respectively.

population dynamics (Galam et al., 1982), opinion dynamics (Vazquez et al., 2003), and neuroscience (Fraiman et al., 2009). For example, in Ising models of opinion dynamics, one studies how societal trends emerge at the macroscopic level from local interactions between individuals at the microscopic level. Due to its simplicity on the one hand and its richness for modeling dynamics, the Ising model has proven to be extremely useful to generate insights across many different areas. As illustrated by these applications, the Ising model is a general 'content-free' model. The dynamics of a network are specified by the Ising model but the content of the nodes are not. To turn the Ising model into a formalized theory in a given area, one needs to specify what the different nodes represent and the connections between them. This difference between the general Ising model and specific theories is illustrated in Figure 13.1. The left plot shows a network with unspecified nodes, representing the general Ising model. The middle and right plots show an attitude network and an intelligence network, respectively, which can be modeled using the dynamics specified by the Ising model.

The outline for this chapter is the following: First, we discuss the basics of the Ising model. Second, we illustrate emerging dynamics of the Ising model using the example of attitudes. Third, we illustrate how the Ising model can be utilized to model cross-sectional phenomena like positive manifolds using the example of general intelligence.

13.2 Basics of the Ising model

The Ising model set the stage for the general development of Markov random field models, and it is likely still the most used Markov random field model for binary variables (Kindermann, Snell, et al., 1980). Nodes (or spins in statistical physics terms) in the Ising model can be either in the state '-1' or '1.' The collection of all nodes' states is called the configuration of an Ising model. For example, the configuration for a five-node Ising model could take the form of {-1, -1, 1, 1, 1}. The probability of the different nodes assuming a certain configuration depends on two factors. First, the probability of a configuration is high when this configuration has low *energy*. Second, the influence of energy on probability is moderated by the *temperature* of the Ising model. Low temperature leads to a stronger correspondence between energy and probability: zero temperature implies that the system can only be in the lowest energy state, while infinitely high temperature implies that all configurations are equally likely no matter the energy. Temperature thus determines how randomly or deterministic a system behaves.

Temperatures between 0 and ∞ describe systems that are neither fully deterministic nor completely random (e.g., the system generally gravitates toward low energy, but can also show instances where it moves to higher energy states). Lower temperature results in a relatively more ordered system, while higher temperature results in a relatively more random system. For a technical explanation of calculating probabilities from the Ising model, see Technical Box 13.1.

The energy of an Ising model is affected by two classes of parameters. The first class of parameters contains couplings that describe the interactions between the nodes. These couplings can range from $-\infty$ to ∞, with zero indicating the absence of a coupling

The probability of a configuration in the Ising model can be calculated using Equation 13.1:

$$P(x) = \frac{e^{-\beta H(x)}}{Z}, \tag{13.1}$$

where $H(x)$ represents the energy of a given configuration, β represents the inverse temperature of the system, and Z is the sum of all configurations' energies. As can be seen in Equation 13.1, temperature determines the extent to which differences in energy translate to differences in probability.

The energy $H(x)$ of a configuration can be calculated using Equation 13.2:

$$H(x) = -\sum_{<i,j>} \omega_{ij} x_i x_j - \sum_i \tau_i x_i, \tag{13.2}$$

where $H(x)$ represents the energy of a given configuration of the system of n distinct nodes $1, \ldots, i, j, \ldots, n$ that are in state x. ω represents the coupling between nodes and τ represents the external field of the nodes.

To run dynamical simulations on the Ising model, we can adapt Equation 13.2 in the following way:

$$H(x_i) = -\sum_j \omega_{ij} x_i x_j - \tau_i x_i,$$

where $H(x_i)$ represents the energy of a given node x_i, ω_{ij} represents the coupling of this node with a different node x_j, and τ_i represents the external field of the node. The advantage of adapting Equation 13.2 in this way is that we now can calculate the energy of each node separately. To run a dynamical simulation on the Ising model, in each iteration one node is randomly picked. Then the energy of this node is calculated and compared to the energy of the node if it were in its opposite state. The node then flips with this probability:

$$P(x_i \rightarrow x_i') = 1/(1 + e^{\beta \Delta H(x_i x_i')}),$$

where $P(x_i \rightarrow x_i')$ represents the probability that the node flips its state, x_i' represents the opposite state of the node, and $\Delta H(x_i x_i')$ represents the difference in energy between the current state of the node and its opposite state. This setup of dynamically simulating the Ising model is called Glauber dynamics (Glauber, 1963).

Technical Box 13.1. Calculating probabilities of the Ising model.

between two nodes. The energy of two nodes connected by a positive coupling is lower and probability therefore higher when the nodes are in the same state than when they are in different states, while the energy of two nodes connected by a negative coupling is lower when the nodes are in different states than when they are in the same state. The magnitude of the couplings determines how high the difference in energy and probability is between nodes that are in consistent states with the couplings and nodes that are in inconsistent states with the couplings. The second class of parameters is an external field that describes the disposition of the nodes to be in a given state. This external field can also range from $-\infty$ to ∞, with zero indicating that a node considered on its own has no disposition to be in a given state. The energy of a node with a positive external field is lower when the node is in the '1' state than when it is in the '-1' state. A node with a positive external field therefore has higher probability to be in the '1' state. In contrast, the energy of a node with a negative external field is lower when the node is in the '-1' state than when it is in the '1' state. A node with a negative external field therefore has higher probability to be in the '-1' state. The magnitude of the external field determines how high the difference in energy and probability is between nodes that are in a consistent state with their external field and nodes that are in an inconsistent state with their external field.

13.3 Idealized simulations of attitude dynamics

In this section, we illustrate some important dynamics of the Ising model using attitude networks as an example. In the network theory of attitudes (Dalege et al., 2016; Dalege et al., 2018), attitudes (the liking or disliking of an attitude object) are treated as higher level properties emerging from lower level beliefs, feelings, and behaviors. For example, a positive attitude towards a person might emerge from beliefs like judging the person to be kind and honest, feelings like affection and sympathy, and behaviors like spending a lot of time with this person. These lower level attitude elements can be represented as nodes in an Ising model, with the state of the nodes indicating whether these beliefs, feelings, and behaviors are present or absent. For example, judging a person as honest can be represented as the '1' state of the corresponding node, while not judging a person as honest can be represented as the '-1' state. The overall attitude is then the sum score of all nodes (e.g., judging the person as kind and honest, feelings of affection and sympathy, and spending a lot of time with this person). Couplings between attitude elements represent direct interactions between attitude elements that are of inferential nature and that increase the consistency of the attitude network. Different attitude elements that are similar to each other provide information on each other, which makes it more likely that if you endorse one belief you will also endorse a similar belief of the same valence. For example, one might infer that a person is honest, because one has observed this person to act in caring manner. Conversely, if attitude elements are similar but of different valence, they inhibit each other. For example, one might infer that a person is not honest, because one has observed this person to act in a egoistic way. The external field represents information that is external to the network. This external information can stem from outside the person (e.g., observing a person to be honest increases the external field of this attitude element) or from internal dispositions of the person (e.g., a person who is very trusting in general has a positive external field for judging a new acquaintance as honest). The (inverse) temperature of the attitude network subsumes several processes that increase consistency of attitudes, such as directing attention to the attitude object or importance of the attitude object. Low temperature indicates that the attitude object is of high personal importance, while high temperature indicates that one pays little attention to the attitude object. This simple setup is surprisingly successful in explaining several effects in the attitude literature. In the remainder of this section, we first illustrate how the Ising model can be used to model polarization. We then introduce the concept of hysteresis and provide a simple illustration of this phenomenon.

Polarization in the attitude literature refers to the phenomenon that attitudes become more extreme, with both the number of very negative and very positive attitudes increasing. Typical determinants of attitude polarization are the amount of thought directed at the attitude object (Tesser, 1978) and heightened subjective importance of the attitude object (Howe & Krosnick, 2017). These determinants can be straightforwardly modeled by decreasing the temperature of an Ising model. The reason to use temperature as an analogue of what happens when an individual thinks about an attitude object is that decreasing temperature increases the inferences between attitude elements. We assume that making such inferences requires attention. As can be seen in Figure 13.2 (top panels), decreasing the temperature in an Ising model indeed makes it more likely to find the system in polarized configurations, an individual is more likely to have a polarized attitude if they direct much attention to the attitude. However, the simulation we used here is static—there are no time dynamics and the distributions shown in the top panels of Figure 13.2 represent equilibrium distributions. For the next simulations, we will use a dynamic version of the Ising model, see Technical Box 13.1.

Using the dynamic simulator introduced in Technical Box 13.1, we can also model the process by which an individual transitions from a neutral attitude to a polarized attitude. To do so, we first run several iterations on the dynamic simulator with high temperature (representing that an individual pays little attention to an attitude object) and then decrease the temperature (representing that the individual thinks about the attitude object). As can be seen in Figure 13.2 (bottom panels), the dynamical simulation mirrors the results of the equilibrium distributions: under high temperature, we see a normal distribution around the sum score of 0 for the equilibrium distribution and the dynamical simulation shows a network fluctuating around the sum score of 0. Under low temperature, we see a bimodal distribution for the equilibrium distribution and the dynamical simulation shows a network transitioning between the extreme sum scores of -10 and 10. This also illustrates an important aspect of the Ising model: The processes modeled by the Ising model are egordic. Ergodicity implies that the results of taking an infinite number of single samples of identical systems are indistinguishable from the results of letting one of these systems run indefinitely.

Using the dynamic simulator for the Ising model, we now turn to a simulation of hysteresis. Hysteresis is one of the central characteristics of a dynamical system and in essence means that the current state of the system is not only dependent on the forces acting on the system momentarily but also on the history of the system. In the case of attitudes, hysteresis implies that the attitude someone holds does not only depend on the information

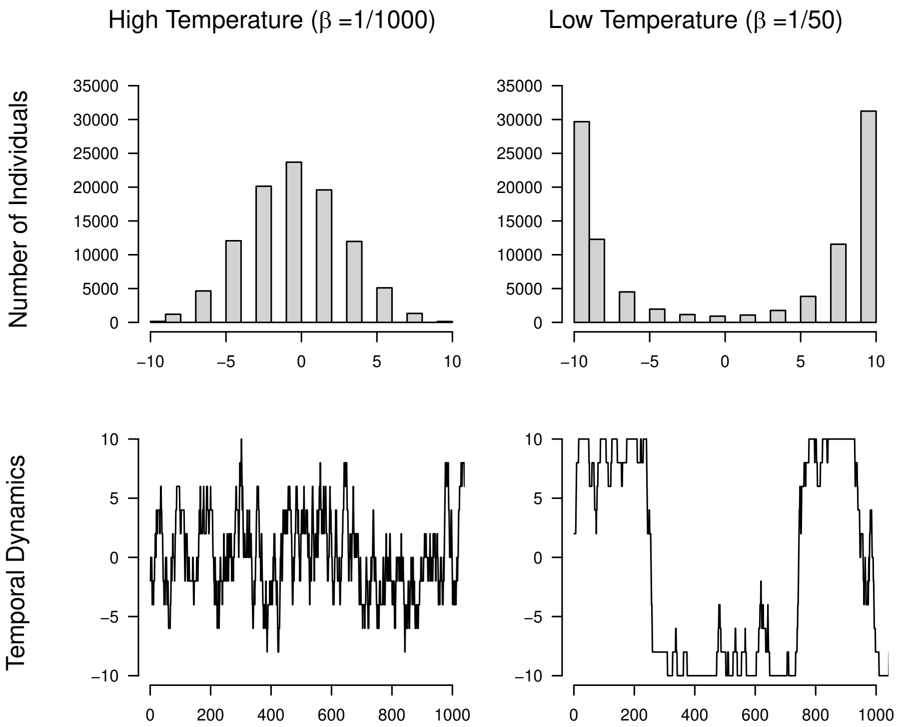

Figure 13.2. Illustration of modeling polarization using a ten-node Ising model. The upper plots show distributions of sum scores for equilibrium distributions for 10,000 networks under high (left) and low (right) temperature. The lower plots show sum scores for 1,000 time points of a network under high (left) and low (right) temperature.

someone receives, but also on the order that one receives the information (van der Maas et al., 2003). For example, if one meets a person and the first few interactions are positive, it requires a disproportionate amount of negative interactions to change one's mind. In contrast, if one had first experienced the negative interactions, one's attitude would be considerably more negative.

To model hysteresis, we again make use of our dynamic simulator. We start the simulation either with a negative or positive field acting on all nodes (implying that a person first had mostly negative or positive interactions with another person) and then increasingly shift the external field to the opposite value. We do this for different values of temperature. As can be seen in Figure 13.3, we observe hysteresis only if the temperature is sufficiently low. This implies that only attitudes of sufficiently high importance show hysteresis. If we don't care about an attitude object, we shift our attitudes in accordance with the information that we receive. In contrast, if the attitude object is important to us, it takes a disproportionate amount of information to change our attitudes. If a subjectively important attitude changes, however, the change is extreme. For example, an ardent supporter of a politician will not simply become indifferent if they receive negative information about that politician, but will change to a fierce opponent if the negative information cannot be discarded anymore.

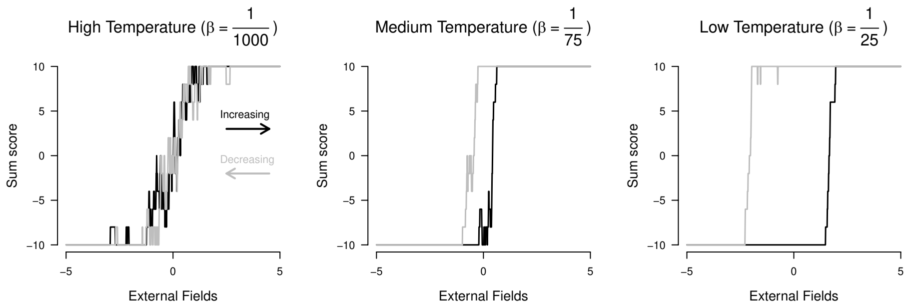

Figure 13.3. Illustration of modeling hysteresis using a ten-node Ising model. The left plot shows the dynamics of a high-temperature Ising model, the middle plot shows the dynamics of a moderate-temperature Ising model, the right plot shows the dynamics of a low-temperature Ising model. Dark grey lines show dynamics where the external fields changed from negative to positive and light grey lines show dynamics where the external fields changed from positive to negative. Larger gaps between the lines indicate stronger hysteresis effects.

13.4 Modeling cross-sectional phenomena in intelligence research

So far, we focused on how the Ising model can be used as a model of time-dependent dynamics. Many phenomena in psychology, however, are concerned with patterns in cross-sectional (single measurement) data rather than with patterns that develop over time. In this section, we discuss how the Ising model can be used to model such cross-sectional phenomena. Many prominent psychological phenomena were discovered by analysing cross-sectional data. For example, general intelligence is studied by analyzing the cognitive test scores (e.g., IQ tests) across individuals. The empirical phenomena gleaned from these cross-sectional analyses inspired the classic g-factor theory of general intelligence (Spearman, 1904, 1927). A prominent example is the pattern of exclusively positive correlations between cognitive test scores known as the positive manifold (Spearman, 1904). Another important example is the block structure in observed correlation matrices (Carroll, 1993). A well-known example of a block structure in correlation matrices is the finding that common personality tests result in five blocks with high correlations within and lower correlations between blocks. This finding underlies the Big-Five theory of personality, which holds that these blocks of high correlations represent fundamental traits of personality (e.g., extraversion). Latent trait theories such as the g-factor theory of general intelligence and the Big-Five theory of personality have a long history in psychometrics. However, recent work revealed that the phenomena that inspired latent trait theories could also be studied using networks such as the Ising model.

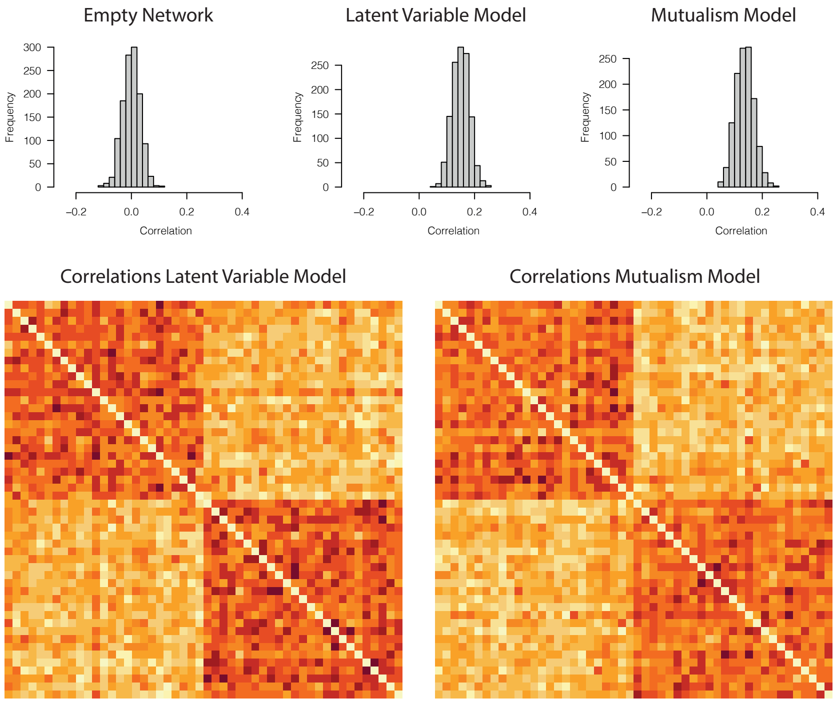

Figure 13.4. Illustration of the positive manifold and block structures emerging from latent variable models and the Ising model. The upper plots show histograms for the correlations between nodes for an empty network (left), a latent variable model where the external fields are correlated (middle), and the mutualism model where nodes are connected by couplings (right). The lower plots show heatmaps of the correlations for a latent variable model with two latent factors (left) and for a mutualism model with two distinct clusters (right).

In contrast to latent-trait theories, the mutualistic theory of general intelligence gives a dynamic account of phenomena such as the positive manifold (van der Maas et al., 2006) and revolves around the Lotka-Volterra mutualism model (Murray, 2002). We provide a rough translation of mutualism theory using the Ising model to explore with our dynamic simulator introduced in Technical Box 13.1. Mutualism theory starts with a conceptual network of low-level processes, abilities, and skills as nodes (spins in the Ising model). The network's couplings reflect mutual relations between the processes, abilities, and skills, and the external fields govern their activation. For example, a child's vocabulary acquisition feeds of improvements in short-term memory, which facilitates breaking down complex problems into manageable mental chunks. In the mutualism model, an increasing external field models a skill's growth. These external fields are random variables (vary across persons), but the couplings are fixed parameters. Van der Maas et al. (2006) start with a situation in which there are no couplings between the network's skills and uncorrelated external fields. This setup would imply that there are no interactions between skills, and that dispositions of skills are also independent from each other (e.g., an individual who has a high disposition for reading skills is not more likely to have a high disposition for mathematical skills than an individual who has a low disposition for reading skills). Fixing the external fields to zero (or another reasonable value) and running our dynamic simulation on this network set-up will generate data that lack any structural relations between processes, abilities, and skills (c.f., the correlation in the upper left plot of Figure 13.4). Thus, there is no positive manifold. Van der Maas et al. (2006) then discuss two different scenarios that generate the positive manifold. Note that in all simulations in this section, temperature is set to 1.

The second scenario correlates the thresholds. Correlating the thresholds models a scenario in which individuals who have high disposition for a given skill are also more likely to have a high disposition for a different skill. Basically, some individuals are just more intelligent than other individuals across the board. See Technical Box 13.2 for the technical details of the simulations in this section. If the couplings are absent (i.e., are all equal to zero) in the presence of correlated thresholds, this set-up is consistent with a latent variable model known as the Rasch model (Rasch, 1960). In other words, running our dynamic simulation on this network set-up will generate data that are consistent with the positive manifold. The upper middle plot of Figure 13.4 confirms that this is the case.

Things become more interesting with the third scenario of van der Maas et al. (2006). Here the thresholds are again uncorrelated (i.e., fixed), but the couplings ω_{ij} are set to a positive value. This scenario reflects the core idea of mutualism theory that "…cognitive processes have mutual beneficial or facilitating relations" (van der Maas et al., 2006, p. 845). Remarkably, running our dynamic simulation on this network set-up generates

Here we provide the technical details of the simulations on the positive manifold. Thresholds were correlated using the following equation:

$$\tau_{ip} = g_p + \mu_i.$$

This equation decomposes the threshold in a person effect g_p that offers a constant contribution to all skills of person p and an effect μ_i that is specific to the particular skill i.

We use the following model for two separate external fields:

$$\tau_{ip} = \begin{cases} g_p + l_p + \mu_i & \text{if skill } i \text{ is in the language domain.} \\ g_p + m_p + \mu_i & \text{if skill } i \text{ is in the mathematics domain.} \end{cases}$$

To model two different clusters, we use two types of couplings, the within domain coupling ω^W and the between domain coupling ω^B with $\omega^W > \omega^B$, such that

$$\omega_{ij} = \begin{cases} \omega^W & \text{if skills } i \text{ and } j \text{ belong to the same domain.} \\ \omega^B & \text{if skills } i \text{ and } j \text{ belong to different domains.} \end{cases}$$

Technical Box 13.2. Modeling the positive manifold with the Ising model.

data that also show the positive manifold (c.f., upper right plot in Figure 13.4). Thus, we have shown that a phenomenon like the positive manifold can emerge from the positively reinforcing relations between processes in the network. Our simple dynamic simulator offers an alternative explanation of the positive manifold that does not require the latent *g*-factor. Recent work established a formal mathematical relation between the two scenarios (Epskamp et al., 2018; Marsman et al., 2018).

We now turn to the situation that some cognitive domains form clusters. The typical block structure in correlation matrices of intelligence tests reflects this phenomenon. In the latent trait scenario—Scenario two—we explain the phenomenon by introducing an additional latent trait for each cognitive domain. Suppose that there are two cognitive domains: language and mathematics. We model these two latent traits by having separate external fields for the traits. In the absence of couplings, this set-up would be consistent with a latent variable model known as the multidimensional item response theory model. Running our dynamic simulator on this network set-up generates data that reflect the block pattern in the correlation matrix (c.f., lower left plot in Figure 13.4).

In the network scenario—Scenario three—the block pattern in the observed correlation matrix reflects a community structure in the network. A community is a collection of nodes that share high couplings (i.e., have stronger mutual relations). Here, the cognitive domains form the communities, and we differentiate the couplings between skills of the same domain from the couplings between skills of different domains. Running the dynamic simulation on the network in this third set-up will generate data with a block pattern in observed correlations (c.f., lower right plot in Figure 13.4). Again, we needed no latent traits to generate data consistent with the phenomenon.

With our rough translation of the mutualism model to the Ising model and our simple dynamic simulator, we can generate established cross-sectional phenomena of general intelligence. Even though this offers an alternative to the classic theory for these phenomena, the model is overly simplistic. For example, why is the network's structure the same for each individual? Given our idiosyncratic developmental trajectories, that does not automatically make sense. And what about developmental phenomena? The ideas above culminated in a novel network theory of general intelligence that builds on mutualism theory and addresses these concerns (Savi et al., 2019). This theory consists of two parts. The first part, wired intelligence, uses a specific formulation of the Ising model to account for heterogeneity (Marsman & Huth, 2021). Specifically, it uses a set-up in which the network's topology is random (varies across persons) while ensuring that we have a single Ising model cross-sectionally. That is, the particular Ising model is itself an emergent property. In this way, we can model individual networks while retaining consistency with cross-sectional phenomena. In the second part, wiring intelligence, the individual's network is grown so that developmental phenomena emerge. Savi et al. (2019) discuss two developmental phenomena. The age de-differentiation hypothesis or increasing positive manifold reflects the gradual increase of a common structure in intelligence across individuals; correlations between skills increase with age. In Savi et al. (2019), this property emerges from simply growing the individual networks. Furthermore, the authors show that we could use particular growth strategies to generate data consistent with a second developmental phenomenon. The Matthew effect reflects accumulating individual differences due to minor differences in early success in, for instance, reading. However, more research is needed to establish exactly if and how such phenomena emerge in our cognitive development.

13.5 Conclusion

In this chapter, we discussed the Ising model as an idealized data-generating model for psychological networks. We first showed how the Ising model can be used to model emergent dynamics of networks. Using attitude networks, we illustrated that simple Ising models can reproduce phenomena like polarization and hysteresis. A central theme in these illustrations is that the temperature of an Ising model has fundamental implications for the emergent behavior of Ising models. Low temperature results in networks becoming more deterministic, which in turn results in the emergence of polarization and hysteresis. High temperature, in contrast, results in networks behaving more randomly and their emergent dynamics are more simple than the dynamics of low temperature networks. We then showed that the Ising model can also be utilized to model cross-sectional phenomena. Using the example of general intelligence, we showed that Ising models can reproduce positive manifolds. Additionally, introducing communities in an Ising model results in block structures in correlation matrices. The simulations in this chapter are deliberately simple, but more complex models can be built using simple models like the Ising model as a starting point. For example, nested Ising models can be developed for the study of attitude networks in which individual attitude networks are embedded in a social network (Galesic et al., 2021; van der Maas et al., 2020). We hope that our introduction of the Ising model as an idealized data-generating model for psychological networks will inspire further investigations into the formalization of psychological theories. Such efforts can improve the testability and applicability of psychological theories—ultimately contributing to transform psychological science to a more mature science.

13.6 Exercises

Conceptual

13.1. Think of a research area within psychology not covered in this chapter and investigate whether the Ising model could be used as an idealized model for this area. What would the spins, couplings, external field, and temperature represent? Do you think the Ising model might be a promising model for this area? What are reasons that speak for the Ising model and what are reasons that speak against the Ising model in this area?

13.2. Choosing either the example of attitudes or general intelligence, provide three reasons why the Ising model is *not* a good model for this example.

13.3. Think of a finding within psychology that could be related to hysteresis. Discuss why the Ising model might or might not be suited to model this finding.

13.4. Think of a research area within psychology other than intelligence where a positive manifold has been observed as well. Discuss whether a latent trait theory or a network theory might be best suited to explain this positive manifold.

13.5. Search the literature for the phenomenon of 'critical slowing down.' Do you think this phenomenon could be modeled with an Ising model?

True or false

13.6. When temperature of an Ising model is low, the correlations between nodes are high.

13.7. Ising models always show hysteresis.

13.8. Highly important attitudes (modeled as low temperature Ising models) are more stable than unimportant attitudes (modeled as high temperature Ising models).

13.9. The Ising model can only be used to model within-person processes.

13.10. A network theory of intelligence is better suited to explain the positive manifold found for intelligence tests than a latent trait theory.

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