Appendix for: "Political Shocks, Public Debt and the Design of Monetary and Fiscal Institutions"

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Notation: for a generic variable y, we define $y_t^e \equiv \mathrm{E}_{t-1}[y_t]$ and $y_t^d = y_t - y_t^e$.

A Derivation of infinite-horizon commitment solution.

The central bank selects π_t so as to minimize:

$$\frac{1}{2}\left[\begin{array}{c}\alpha_{\pi M}\left(\pi_t-\pi_t^*\right)+ \\ \left[\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right]^2\end{array}\right]+\theta_t\left[\mathrm{E}_{t-1}\left(\pi_t\right)-\pi_t^e\right]+\beta \mathrm{E}_t\left[L_{t+1}^{CB}\right]. \quad (1)$$

where π_t^* depends only on the shocks μ_t and η_t, so that $\mathrm{E}_{t-1}\left(\pi_t^*\right)=0$. The CB's first-order conditions for π_t and π_t^e can be written as, respectively:

$$\begin{aligned}\alpha_{\pi M}\left(\pi_t-\pi_t^*\right)+\nu\left(\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right)+\theta_t &= 0, \\ \mathrm{E}_{t-1}\left[\nu\left(\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right)\right]+\theta_t &= 0,\end{aligned}$$

which can be combined to give:

$$\begin{aligned} & \alpha_{\pi M}\left(\pi_t-\pi_t^*\right)+\nu\left(\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right)-\mathrm{E}_{t-1}\left[\nu\left(\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right)\right] \\ = & \ 0. \end{aligned} \quad (2)$$

The government, which is of the type η_t in period t selects τ_t and d_t so as to minimise:

$$V_t^{G_t}=\frac{1}{2}\sum_{\xi=t}^{\infty}\beta^{\xi-t}\mathrm{E}_t\left[\begin{array}{c}\alpha_\pi\pi_\xi^2+\left[\nu\left(\pi_\xi-\pi_\xi^e-\tau_\xi\right)-\mu_\xi-\tilde{x}_\xi\right]^2+ \\ \alpha_g\left[-\left(1+\rho\right)d_{\xi-1}+\tau_\xi+d_\xi-\left(\tilde{g}_\xi+\eta_t\right)\right]^2\end{array}\right], \quad (3)$$

where superscript "G_t" indicates that losses are evaluated according to the loss function of the party that is in power in period t. The first-order conditions for τ_t and d_t are:

$$-\nu\left[\nu\left(\pi_t-\pi_t^e-\tau_t\right)-\mu_t-\tilde{x}_t\right]+\alpha_g\left[g_t-\left(\tilde{g}_t+\eta_t\right)\right]=0, \quad (4)$$

$$\alpha_g \left[(\tilde{g}_t + \eta_t) - g_t \right] = \beta \left[\partial \mathrm{E}_t \left(V_{t+1}^{G_t} \right) / \partial d_t \right], \tag{5}$$

$$g_t + (1+\rho)\, d_{t-1} = \tau_t + d_t, \tag{6}$$

and the transversality condition that:

$$\lim_{\xi \to \infty} \left(\tfrac{1}{1+\rho} \right)^{\xi - t} d_{\xi+1} = 0, \tag{7}$$

with probability 1. The complete system of equations to be used to solve for the outcomes is (2), (4), (5), (6) and (7). We solve the system first for *given* debt policies, after which we also solve for the debt policy.

A.1 Derivation of outcomes for given debt policies

We first derive the deterministic components of the outcomes (step 1). Then, we derive the responses to the shocks (step 2).

Step 1: take as-of-the-start-of-period-t expectations of the system (2), (4) and (6) to give:

$$\alpha_{\pi M} \pi_t^e = 0, \tag{8}$$

$$\nu^2 \left(\tau_t^e + \tfrac{\tilde{x}_t}{\nu} \right) + \alpha_g \left(g_t^e - \tilde{g}_t \right) = 0, \tag{9}$$

$$g_t^e + (1+\rho)\, d_{t-1} = \tau_t^e + d_t^e. \tag{10}$$

The solution is:

$$\pi_t^e = 0, \tag{11}$$

$$\tilde{x}_t - x_t^e = \left[\tfrac{1/\nu}{P} \right] \left[K_t + (1+\rho)\, d_{t-1} - d_t^e \right], \tag{12}$$

$$\tilde{g}_t - g_t^e = \left[\tfrac{1/\alpha_g}{P} \right] \left[K_t + (1+\rho)\, d_{t-1} - d_t^e \right], \tag{13}$$

where, as defined in the main text,

$$P = 1/\nu^2 + 1/\alpha_g. \tag{14}$$

Step 2: Subtract (8), (9) and (10) from (2), (4) and (6), respectively, to obtain:

$$\left(\alpha_{\pi M}+\nu^2\right)\pi_t^d-\alpha_{\pi M}\pi_t^*-\nu^2\left(\tau_t^d+\tfrac{\mu_t}{\nu}\right)=0, \tag{15}$$

$$-\nu^2\pi_t^d+\nu^2\left(\tau_t^d+\tfrac{\mu_t}{\nu}\right)+\alpha_g\left(g_t^d-\eta_t\right)=0, \tag{16}$$

$$g_t^d=\tau_t^d+d_t^d. \tag{17}$$

Combine (15) and (16) to eliminate π_t^d and obtain:

$$\tau_t^d+\tfrac{\mu_t}{\nu}=\pi_t^*+\left(1/\alpha_{\pi M}+1/\nu^2\right)\alpha_g\left(\eta_t-g_t^d\right),$$

which can be combined with (17) to give:

$$g_t^d-\eta_t=\left[\tfrac{1/\alpha_g}{P_M^*}\right]\pi_t^*+\left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[d_t^d-\left(\tfrac{\mu_t}{\nu}+\eta_t\right)\right], \tag{18}$$

where, as defined in the main text,

$$P_M^*=1/\alpha_{\pi M}+1/\nu^2+1/\alpha_g. \tag{19}$$

Hence,

$$\tau_t^d+\tfrac{\mu_t}{\nu}=\pi_t^*-\left[\tfrac{1/\alpha_{\pi M}+1/\nu^2}{P_M^*}\right]\left[\pi_t^*+d_t^d-\left(\tfrac{\mu_t}{\nu}+\eta_t\right)\right].$$

Hence,

$$\pi_t^d=\pi_t^*-\left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\pi_t^*+d_t^d-\left(\tfrac{\mu_t}{\nu}+\eta_t\right)\right], \tag{20}$$

and

$$x_t^d=\left[\tfrac{1/\nu}{P_M^*}\right]\left[\pi_t^*+d_t^d-\left(\tfrac{\mu_t}{\nu}+\eta_t\right)\right]. \tag{21}$$

Using (11), (12), (13), (18), (20) and (21) we obtain:

$$\pi_t=\pi_t^e+\pi_t^d=\pi_t^*+\left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu}+\eta_t\right)-d_t^d-\pi_t^*\right], \tag{22}$$

$$\tilde{x}_t-x_t=\left[\tfrac{1/\nu}{P}\right]\left[K_t+(1+\rho)\,d_{t-1}-d_t^e\right]+\left[\tfrac{1/\nu}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu}+\eta_t\right)-d_t^d-\pi_t^*\right], \tag{23}$$

$$(\tilde{g}_t+\eta_t)-g_t=\left[\tfrac{1/\alpha_g}{P}\right]\left[K_t+(1+\rho)\,d_{t-1}-d_t^e\right]+\left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu}+\eta_t\right)-d_t^d-\pi_t^*\right]. \tag{24}$$

A.2 Derivation of the solution for public debt

We are now in a position to characterize debt policy. From now on in Appendix A, we set $\pi_t^* = 0$. To evaluate $\partial \mathrm{E}_t \left(V_{t+1}^{G_t} \right) / \partial d_t$, forward (22), (23) and (24) by one period and substitute the resulting expressions into government G_t's expected loss in period $t+1$,

$$\tfrac{1}{2} \mathrm{E}_t \left[\alpha_\pi \pi_{t+1}^2 + (x_{t+1} - \tilde{x}_{t+1})^2 + \alpha_g \left(g_{t+1} - (\tilde{g}_{t+1} + \eta_t) \right)^2 \right].$$

The derivative with respect to d_t of the expression thus obtained is:

$$\mathrm{E}_t \left[(\tilde{x}_{t+1} - x_{t+1}) (1+\rho) \left[\tfrac{1/\nu}{P} \right] + \alpha_g (\tilde{g}_{t+1} + \eta_t - g_{t+1}) (1+\rho) \left[\tfrac{1/\alpha_g}{P} \right] \right].$$

Hence, combining this with (5), we obtain:

$$\alpha_g (\tilde{g}_t + \eta_t - g_t) = \beta^* \mathrm{E}_t \left[(\tilde{x}_{t+1} - x_{t+1}) \left[\tfrac{1/\nu}{P} \right] + (\tilde{g}_{t+1} + \eta_t - g_{t+1}) \left[\tfrac{1}{P} \right] \right].$$

Combine this with (24) and the one-period forwarded expressions of (23) and (24), to give:

$$\begin{aligned} & \left[\tfrac{1}{P} \right] \left[K_t + (1+\rho) d_{t-1} - d_t^e \right] + \left[\tfrac{1}{P_M^*} \right] \left[\left(\tfrac{\mu_t}{\nu} + \eta_t \right) - d_t^d \right] \\ = \;& \beta^* \mathrm{E}_t \left[(\tilde{x}_{t+1} - x_{t+1}) \left[\tfrac{1/\nu}{P} \right] + (\tilde{g}_{t+1} + \eta_t - g_{t+1}) \left[\tfrac{1}{P} \right] \right]. \end{aligned}$$

Hence,

$$\begin{aligned} & \left[K_t + (1+\rho) d_{t-1} - d_t^e \right] + \left[\tfrac{P}{P_M^*} \right] \left[\left(\tfrac{\mu_t}{\nu} + \eta_t \right) - d_t^d \right] \\ = \;& \beta^* \left\{ \left[K_{t+1} + (1+\rho) d_t - d_{t+1}^e \right] + \eta_t \right\}, \end{aligned} \tag{25}$$

where we have used that $\mathrm{E}_t \left[\eta_{t+1} \right] = 0$. We solve (25) in two steps.

Step 1: take expectations of (25) as of end of period $t-1$:

$$K_t + (1+\rho) d_{t-1} - d_t^e = \beta^* \left[K_{t+1} + (1+\rho) d_t^e - \mathrm{E}_{t-1} d_{t+1} \right]. \tag{26}$$

The solution is:

$$d_t^e = \frac{\left[K_t + (1+\rho) d_{t-1} \right] - \beta^* \left[K_{t+1} - \mathrm{E}_{t-1} (d_{t+1}) \right]}{1 + \beta^* (1+\rho)}. \tag{27}$$

An explicit solution for d_t^e will be derived later on.

Step 2: Subtract (26) from (25):

$$\left[\frac{P}{P_M^*}\right]\left[\left(\frac{\mu_t}{\nu}+\eta_t\right)-d_t^d\right]=\beta^*\left(1+\rho\right)d_t^d+\beta^*\left(\mathrm{E}_{t-1}d_{t+1}-\mathrm{E}_t d_{t+1}\right)+\beta^*\eta_t. \quad (28)$$

Hence,

$$d_t^d=\left[\frac{1}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]\left(\frac{\mu_t}{\nu}+\eta_t\right)+\left[\frac{\beta^*\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]\left[\mathrm{E}_t\left(d_{t+1}\right)-\mathrm{E}_{t-1}\left(d_{t+1}\right)-\eta_t\right]. \quad (29)$$

We can find the final solution for d_t^d as follows. Forward (27) by $\xi \geq 1$ periods. Next, take expectations as of end-of-periods t and $t-1$ and subtract the latter from the former, to obtain:

$$\mathrm{E}_t\left(d_{t+\xi}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi}\right)=\frac{(1+\rho)\left[\mathrm{E}_t\left(d_{t+\xi-1}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi-1}\right)\right]+\beta^*\left[\mathrm{E}_t\left(d_{t+\xi+1}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi+1}\right)\right]}{1+\beta^*(1+\rho)}. \quad (30)$$

Guess that $\mathrm{E}_t\left(d_{t+\xi+1}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi+1}\right)=\varphi\left[\mathrm{E}_t\left(d_{t+\xi}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi}\right)\right], \forall\xi \geq 1$, where φ is a constant to be solved for. Substitute this into (30) and rewrite the result to yield the following equation in φ:

$$\beta^*\varphi^2-\left[1+\beta^*\left(1+\rho\right)\right]\varphi+\left(1+\rho\right)=0.$$

This equation yields two solutions: $\varphi=1+\rho$, which is excluded, because it violates (7),[1] and $\varphi=1/\beta^*$. Using this solution, for $\xi=1$ we can write (30) as:

$$\mathrm{E}_t\left(d_{t+1}\right)-\mathrm{E}_{t-1}\left(d_{t+1}\right)=\frac{1}{\beta^*}d_t^d.$$

Substitute this back into expression (29) for d_t^d and rewrite to give:

$$d_t^d=\left[\frac{1}{1+\left(P_M^*/P\right)\left[\beta^*(1+\rho)-1\right]}\right]\left[\frac{\mu_t}{\nu}+\left(1-\beta^*\left(P_M^*/P\right)\right)\eta_t\right]. \quad (31)$$

[1] We have that $\mathrm{E}_t\left(d_{t+\xi}\right)-\mathrm{E}_{t-1}\left(d_{t+\xi}\right)=\left(1+\rho\right)^{\xi}\left[\mathrm{E}_t\left(d_t\right)-\mathrm{E}_{t-1}\left(d_t\right)\right]$, so that the effect of a shock in period t on public debt "explodes" over time. Taking $\lim_{\xi\to\infty}\left(\frac{1}{1+\rho}\right)^{\xi}\left[E_t\left(d_{t+\xi+1}\right)-E_{t-1}\left(d_{t+\xi+1}\right)\right]=\left(1+\rho\right)\left[\mathrm{E}_t\left(d_t\right)-\mathrm{E}_{t-1}\left(d_t\right)\right]$, which is generally non-zero.

A.2.1 Derivation of an explicit solution for $\mathrm{E}_{t-1}d_t$

Using (12), we have that:

$$\tilde{x}_{t+\xi} - \mathrm{E}_{t-1}x_{t+\xi} = \left[\frac{1/\nu}{P}\right]\left[K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1} - \mathrm{E}_{t-1}d_{t+\xi}\right] \Rightarrow \tag{32}$$

$$\tilde{x}_{t+\xi+1} - \mathrm{E}_{t-1}x_{t+\xi+1} = \left[\frac{1/\nu}{P}\right]\left[K_{t+\xi+1} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi} - \mathrm{E}_{t-1}d_{t+\xi+1}\right], \tag{33}$$

where $\xi \geq 0$. Note that

$$\begin{aligned} & K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1} - \mathrm{E}_{t-1}d_{t+\xi} \\ = \; & \frac{[1+\beta^*(1+\rho)]\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right]-\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right]+\beta^*\left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)} \\ = \; & \beta^*\left[\frac{(1+\rho)\left[K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1}\right] + \left(K_{t+\xi+1} - \mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)}\right]. \end{aligned}$$

where we have used (27) (with forwarding) for $\mathrm{E}_{t-1}d_{t+\xi}$. Again using (27),

$$\begin{aligned} & K_{t+\xi+1} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi} - \mathrm{E}_{t-1}d_{t+\xi+1} \\ = \; & \frac{[1+\beta^*(1+\rho)]\left[K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right]+(1+\rho)\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right]-\beta^*(1+\rho)\left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)} \\ = \; & \frac{(1+\rho)\left[K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1}\right] + \left(K_{t+\xi+1} - \mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)} \\ = \; & \tfrac{1}{\beta^*}\left[K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1} - \mathrm{E}_{t-1}d_{t+\xi}\right]. \end{aligned}$$

Hence, combining this with (32) and (33), one has:

$$\tilde{x}_{t+\xi+1} - \mathrm{E}_{t-1}x_{t+\xi+1} = \tfrac{1}{\beta^*}\left[\tilde{x}_{t+\xi} - \mathrm{E}_{t-1}x_{t+\xi}\right]. \tag{34}$$

Similarly, we find that

$$\tilde{g}_{t+\xi+1} - \mathrm{E}_{t-1}g_{t+\xi+1} = \tfrac{1}{\beta^*}\left[\tilde{g}_{t+\xi} - \mathrm{E}_{t-1}g_{t+\xi}\right]. \tag{35}$$

Taking expectations of the intertemporal government financing requirement as of the end of period $t-1$ and combining the result with (34) and (35), one obtains:

$$
\begin{aligned}
F_t &= \sum_{\xi=0}^{\infty}(1+\rho)^{-\xi}\left[\left(\tilde{x}_{t+\xi}-\mathrm{E}_{t-1}x_{t+\xi}\right)/\nu+\left(\tilde{g}_{t+\xi}-\mathrm{E}_{t-1}g_{t+\xi}\right)\right] \qquad (36)\\
&= \sum_{\xi=0}^{\infty}\left[\tfrac{1}{\beta^*(1+\rho)}\right]^{\xi}\left[\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu+\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right)\right].
\end{aligned}
$$

Hence,

$$
\Longrightarrow \left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu+\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right)=\psi_0^C F_t,
$$

where

$$
\psi_0^C=\tfrac{\beta^*(1+\rho)-1}{\beta^*(1+\rho)}. \qquad (37)
$$

Using (12) and (13), we have that $\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu=\left(\alpha_g/\nu^2\right)\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right)$ and, hence,

$$
\begin{aligned}
\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu &= \left[\tfrac{1/\nu^2}{P}\right]\psi_0^C F_t, \qquad (38)\\
\tilde{g}_t-\mathrm{E}_{t-1}g_t &= \left[\tfrac{1/\alpha_g}{P}\right]\psi_0^C F_t, \qquad (39)
\end{aligned}
$$

We use (38) and (39) to obtain an explicit solution for $\mathrm{E}_{t-1}\left(d_t\right)$. Take expectations of the government financing requirement to give

$$
K_t+(1+\rho)\,d_{t-1}-\mathrm{E}_{t-1}d_t=\left[\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu\right]+\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right). \qquad (40)
$$

Hence, using (38) and (39) and the definition of F_t one has:

$$
K_t+(1+\rho)\,d_{t-1}-\mathrm{E}_{t-1}d_t=\psi_0^C\left[(1+\rho)\,d_{t-1}+G_t\right].
$$

Hence, using the definition of G_t:

$$
\begin{aligned}
\mathrm{E}_{t-1}d_t &= \tfrac{1}{\beta^*}d_{t-1}+K_t-\psi_0^C\sum_{\xi=t}^{\infty}(1+\rho)^{-(\xi-t)}K_\xi\\
&= \tfrac{1}{\beta^*}d_{t-1}+\tfrac{1}{\beta^*(1+\rho)}K_t+\left[\tfrac{1}{\beta^*(1+\rho)}-1\right]\tfrac{1}{1+\rho}\sum_{\xi=t+1}^{\infty}(1+\rho)^{-[\xi-(t+1)]}K_\xi\\
&= \tfrac{1}{\beta^*}d_{t-1}+\tfrac{1}{\beta^*(1+\rho)}G_t-\tfrac{1}{1+\rho}G_{t+1}\\
&= \tfrac{1}{\beta^*}d_{t-1}+\tfrac{(G_t-G_{t+1})+(1-\beta^*)G_{t+1}}{\beta^*(1+\rho)}. \qquad (41)
\end{aligned}
$$

Using (38), (39), (22), (23), (24) and the solution (31) for d_t^d, one obtains the complete solutions for inflation, the output shortfall, the spending shortfall and public debt:

$$\pi_t = \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left[q_2\left(\frac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right], \tag{42}$$

$$\tilde{x}_t - x_t = \left[\frac{1/\nu}{P}\right]\psi_0^C F_t + \left[\frac{1/\nu}{P_M^*}\right]\left[q_2\left(\frac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right], \tag{43}$$

$$(\tilde{g}_t+\eta_t) - g_t = \left[\frac{1/\alpha_g}{P}\right]\psi_0^C F_t + \left[\frac{1/\alpha_g}{P_M^*}\right]\left[q_2\left(\frac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right], \tag{44}$$

$$d_t = \frac{1}{\beta^*}d_{t-1} + \frac{(G_t-G_{t+1})+(1-\beta^*)G_{t+1}}{\beta^*(1+\rho)} + q_1\left[\frac{\mu_t}{\nu}+\left(1-\beta^*\frac{P_M^*}{P}\right)\eta_t\right], \tag{45}$$

where

$$q_1 = \frac{1}{1+\left(P_M^*/P\right)(\beta^*(1+\rho)-1)},\; q_2 \equiv \frac{\left(P_M^*/P\right)[\beta^*(1+\rho)-1]}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)-1]},\; q_3 \equiv \frac{\beta^*\left(P_M^*/P\right)}{1+\left(P_M^*/P\right)(\beta^*(1+\rho)-1)}.$$

B Solutions for the two-period model with commitment

B.1 The case of no targets

B.1.1 Outcomes conditional on debt

Setting $\pi_1^* = \pi_2^* = 0$ and realizing that $d_2^e = d_2^d = 0$, we obtain the first- and second period outcomes for inflation and the output and spending shortfalls directly from (22), (23) and (24):

$$\pi_1 = \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\left(\frac{\mu_1}{\nu}+\eta_1\right)-d_1^d\right], \tag{46}$$

$$\tilde{x}_1 - x_1 = \left[\frac{1/\nu}{P}\right]\left[K_1+(1+\rho)\,d_0-d_1^e\right] + \left[\frac{1/\nu}{P_M^*}\right]\left[\left(\frac{\mu_1}{\nu}+\eta_1\right)-d_1^d\right], \tag{47}$$

$$(\tilde{g}_1+\eta_1)-g_1 = \left[\frac{1/\alpha_g}{P}\right]\left[K_1+(1+\rho)\,d_0-d_1^e\right]+\left[\frac{1/\alpha_g}{P_M^*}\right]\left[\left(\frac{\mu_1}{\nu}+\eta_1\right)-d_1^d\right], \tag{48}$$

and

$$\pi_2 = \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right), \tag{49}$$

$$\tilde{x}_2 - x_2 = \left[\frac{1/\nu}{P}\right]\left[K_2 + (1+\rho)\, d_1\right] + \left[\frac{1/\nu}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right), \tag{50}$$

$$(\tilde{g}_2 + \eta_2) - g_2 = \left[\frac{1/\alpha_g}{P}\right]\left[K_2 + (1+\rho)\, d_1\right] + \left[\frac{1/\alpha_g}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right). \tag{51}$$

B.1.2 Solution for debt

The first-order condition for public debt is given by (25) for $t = 1$ with $d_2^e = 0$ imposed:

$$\begin{aligned} & \left[K_1 + (1+\rho)\, d_0 - d_1^e\right] + \left[\frac{P}{P_M^*}\right]\left[\left(\frac{\mu_1}{\nu} + \eta_1\right) - d_1^d\right] \\ = \;& \beta^* \left\{\left[K_2 + (1+\rho)\, d_1\right] + \eta_1\right\}, \end{aligned} \tag{52}$$

We solve (52) in two steps. In the first step, we take expectations $\mathrm{E}_0[.]$ on both sides of (52) and solve to give:

$$d_1^e = \frac{\left[K_1 + (1+\rho) d_0\right] - \beta^* K_2}{1 + \beta^* (1+\rho)}. \tag{53}$$

In the second step we take difference of (52) and its expectation and solve to give:

$$d_1^d = \left[\frac{1}{1 + \beta^*(1+\rho)\left(P_M^*/P\right)}\right]\frac{\mu_1}{\nu} + \left[\frac{1 - \beta^*\left(P_M^*/P\right)}{1 + \beta^*(1+\rho)\left(P_M^*/P\right)}\right]\eta_1. \tag{54}$$

B.1.3 Final solution and society's expected loss

We substitute the solutions for the public debt components d_1^e and d_1^d back into (46)-(51), to obtain the final solutions for inflation and the output and spending gaps in the two periods:

$$\pi_1 = \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\psi_1\left(\frac{\mu_1}{\nu}\right) + \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left(\frac{2+\rho}{1+\rho}\right)\psi_1 \eta_1, \tag{55}$$

$$\tilde{x}_1 - x_1 = \left[\frac{1/\nu}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right] F_1 + \left[\frac{1/\nu}{P_M^*}\right]\psi_1\left(\frac{\mu_1}{\nu}\right) + \left[\frac{1/\nu}{P_M^*}\right]\left(\frac{2+\rho}{1+\rho}\right)\psi_1 \eta_1, \tag{56}$$

$$(\tilde{g}_1 + \eta_1) - g_1 = \left[\frac{1/\alpha_g}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right] F_1 + \left[\frac{1/\alpha_g}{P_M^*}\right]\psi_1\left(\frac{\mu_1}{\nu}\right) + \left[\frac{1/\alpha_g}{P_M^*}\right]\left(\frac{2+\rho}{1+\rho}\right)\psi_1 \eta_1, \tag{57}$$

$$\pi_2 = \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right), \tag{58}$$

$$\begin{aligned}\tilde{x}_2 - x_2 &= \left[\frac{1/\nu}{P}\right]\left[\frac{1+\rho}{1+\beta^*(1+\rho)}\right]F_1 + \left[\frac{1/\nu}{P}\right]\left[\frac{1}{\beta^*\left(P_M^*/P\right)}\right]\psi_1\left(\frac{\mu_1}{\nu}\right) + \\ &\left[\frac{1/\nu}{P}\right]\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\beta^*\left(P_M^*/P\right)}\right]\psi_1\eta_1 + \left[\frac{1/\nu}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right),\end{aligned} \tag{59}$$

$$\begin{aligned}(\tilde{g}_2 + \eta_2) - g_2 &= \left[\frac{1/\alpha_g}{P}\right]\left[\frac{1+\rho}{1+\beta^*(1+\rho)}\right]F_1 + \left[\frac{1/\alpha_g}{P}\right]\left[\frac{1}{\beta^*\left(P_M^*/P\right)}\right]\psi_1\left(\frac{\mu_1}{\nu}\right) + \\ &\left[\frac{1/\alpha_g}{P}\right]\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\beta^*\left(P_M^*/P\right)}\right]\psi_1\eta_1 + \left[\frac{1/\alpha_g}{P_M^*}\right]\left(\frac{\mu_2}{\nu} + \eta_2\right),\end{aligned} \tag{60}$$

where, as defined in the main text,

$$\psi_1 \equiv \frac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}. \tag{61}$$

We can now use (55)-(60) to compute society's expected loss:

$$\begin{aligned}\mathrm{E}_0\left[V_1^S\right] &= \tfrac{1}{2}\mathrm{E}_0\left[\alpha_\pi\pi_1^2 + (x_1 - \tilde{x}_1)^2 + \alpha_g(g_1 - \tilde{g}_1)^2\right] \\ &\quad + \tfrac{1}{2}\beta\mathrm{E}_0\left[\alpha_\pi\pi_2^2 + (x_2 - \tilde{x}_2)^2 + \alpha_g(g_2 - \tilde{g}_2)^2\right] \\ &= T_1 + T_2 + T_3 + T_4 + T_5,\end{aligned} \tag{62}$$

where T_1,.., T_5 are defined in the main text.

B.2 Debt target combined with inflation targets

Substituting

$$\pi_t^* = \lambda_{0t}\left(\frac{\mu_t}{\nu}\right) + \lambda_{1t}\eta_t,\ t = 1, 2, \tag{63}$$

$$d_1^T = \gamma_0 + \gamma_1\frac{\mu_1}{\nu} + \gamma_2\eta_1, \tag{64}$$

into (22), (23) and (24) yield the following outcomes in periods 1 and 2:

$$\pi_1 = \left[\lambda_{01}\left(\frac{\mu_1}{\nu}\right) + \lambda_{11}\eta_1\right] + \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left[(1 - \gamma_1 - \lambda_{01})\frac{\mu_1}{\nu} + (1 - \gamma_2 - \lambda_{11})\eta_1\right],$$

$$\begin{aligned}\tilde{x}_1 - x_1 &= \left[\frac{1/\nu}{P}\right][K_1 + (1+\rho)\,d_0 - \gamma_0] \\ &+ \left[\frac{1/\nu}{P_M^*}\right]\left[(1-\gamma_1-\lambda_{01})\,\frac{\mu_1}{\nu} + (1-\gamma_2-\lambda_{11})\,\eta_1\right],\end{aligned}$$

$$\begin{aligned}(\tilde{g}_1 + \eta_1) - g_1 &= \left[\frac{1/\alpha_g}{P}\right][K_1 + (1+\rho)\,d_0 - \gamma_0] \\ &+ \left[\frac{1/\alpha_g}{P_M^*}\right]\left[(1-\gamma_1-\lambda_{01})\,\frac{\mu_1}{\nu} + (1-\gamma_2-\lambda_{11})\,\eta_1\right],\end{aligned}$$

and

$$\pi_2 = \left[\lambda_{02}\left(\frac{\mu_2}{\nu}\right) + \lambda_{12}\eta_2\right] + \left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left[(1-\lambda_{02})\,\frac{\mu_2}{\nu} + (1-\lambda_{12})\,\eta_2\right],$$

$$\begin{aligned}\tilde{x}_2 - x_2 &= \left[\frac{1/\nu}{P}\right]\left[K_2 + (1+\rho)\left(\gamma_0 + \gamma_1\frac{\mu_1}{\nu} + \gamma_2\eta_1\right)\right] \\ &+ \left[\frac{1/\nu}{P_M^*}\right]\left[(1-\lambda_{02})\,\frac{\mu_2}{\nu} + (1-\lambda_{12})\,\eta_2\right],\end{aligned}$$

$$\begin{aligned}(\tilde{g}_2 + \eta_2) - g_2 &= \left[\frac{1/\alpha_g}{P}\right]\left[K_2 + (1+\rho)\left(\gamma_0 + \gamma_1\frac{\mu_1}{\nu} + \gamma_2\eta_1\right)\right] \\ &+ \left[\frac{1/\alpha_g}{P_M^*}\right]\left[(1-\lambda_{02})\,\frac{\mu_2}{\nu} + (1-\lambda_{12})\,\eta_2\right],\end{aligned}$$

where we realize that $d_2^e = d_2^d = 0$. Substitute these outcomes into the expected social loss, to yield:

$$\begin{aligned}\mathrm{E}_0\left[V_1^S\right] &= \tfrac{1}{2}\mathrm{E}_0\left[\alpha_\pi\pi_1^2 + (x_1-\tilde{x}_1)^2 + \alpha_g\,(g_1-\tilde{g}_1)^2\right] \\ &+\tfrac{1}{2}\beta\mathrm{E}_0\left[\alpha_\pi\pi_2^2 + (x_2-\tilde{x}_2)^2 + \alpha_g\,(g_2-\tilde{g}_2)^2\right] \\ &= C_1 + C_2 + C_3 + C_4 + (C_{51}+C_{52}) + C_6 + (C_{71}+C_{72})\,, \quad (65)\end{aligned}$$

where

$$C_1 = \tfrac{1}{2}\tfrac{1}{P}\left[\tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right]F_1^2, \tag{66}$$

$$C_2 \equiv \tfrac{1}{2}\alpha_\pi\left[\lambda_{01}+\tfrac{1/\alpha_{\pi M}}{P_M^*}(1-\gamma_1-\lambda_{01})\right]^2\tfrac{\sigma_\mu^2}{\nu^2} + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}(1-\gamma_1-\lambda_{01})^2\tfrac{\sigma_\mu^2}{\nu^2}, \tag{67}$$

$$C_3 = \tfrac{1}{2}\beta^*(1+\rho)\tfrac{1}{P}\gamma_1^2\tfrac{\sigma_\mu^2}{\nu^2}, \tag{68}$$

$$C_4 \equiv \tfrac{1}{2}\alpha_\pi\left[\lambda_{02}+\tfrac{1/\alpha_{\pi M}}{P_M^*}(1-\lambda_{02})\right]^2\tfrac{\sigma_\mu^2}{\nu^2}+\tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}(1-\lambda_{02})^2\tfrac{\sigma_\mu^2}{\nu^2}, \tag{69}$$

$$C_{51} \equiv \tfrac{1}{2}\alpha_\pi\left[\lambda_{11}+\tfrac{1/\alpha_{\pi M}}{P_M^*}(1-\gamma_2-\lambda_{11})\right]^2\sigma_\eta^2 + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}(1-\gamma_2-\lambda_{11})^2\sigma_\eta^2, \tag{70}$$

$$C_{52} \equiv \tfrac{1}{2}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(1-\gamma_2-\lambda_{11})\right]\sigma_\eta^2, \tag{71}$$

$$C_6 = \tfrac{1}{2}\beta^*(1+\rho)\tfrac{1}{P}\gamma_2^2\sigma_\eta^2, \tag{72}$$

$$C_{71} \equiv \tfrac{1}{2}\alpha_\pi\left[\lambda_{12}+\tfrac{1/\alpha_{\pi M}}{P_M^*}(1-\lambda_{12})\right]^2\sigma_\eta^2+\tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}(1-\lambda_{12})^2\sigma_\eta^2, \tag{73}$$

$$C_{72} \equiv \tfrac{1}{2}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(1-\lambda_{12})\right]\sigma_\eta^2. \tag{74}$$

Here, C_1 is the expected loss associated with the deterministic component of the intertemporal government financing requirement,[2] C_2 and C_3 are the expected losses in the first, respectively second, period from imperfect stabilization of μ_1, C_4 is the expected loss from imperfect stabilization of μ_2, $C_{51}+C_{52}$ and C_6 are, respectively, the first- and second-period expected losses from suboptimal stabilization of η_1, and $C_{71}+C_{72}$ is the expected loss from imperfect stabilization of η_2.

B.3 Inflation targets only

Substituting (63) into (22), (23) and (24) yield the following outcomes in periods 1 and 2:

$$\pi_1 = \left[\lambda_{01}\left(\tfrac{\mu_1}{\nu}\right)+\lambda_{11}\eta_1\right]+\left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[(1-\lambda_{01})\tfrac{\mu_1}{\nu}+(1-\lambda_{11})\eta_1-d_1^d\right],$$

$$\tilde{x}_1-x_1 = \left[\tfrac{1/\nu}{P}\right]\left[K_1+(1+\rho)d_0-d_1^e\right]+\left[\tfrac{1/\nu}{P_M^*}\right]\left[(1-\lambda_{01})\tfrac{\mu_1}{\nu}+(1-\lambda_{11})\eta_1-d_1^d\right],$$

[2] Here, we have already made use of the fact that society's optimal γ_0 is (53).

$$(\tilde{g}_1+\eta_1)-g_1=\left[\frac{1/\alpha_g}{P}\right][K_1+(1+\rho)\,d_0-d_1^e]+\left[\frac{1/\alpha_g}{P_M^*}\right]\left[(1-\lambda_{01})\,\tfrac{\mu_1}{\nu}+(1-\lambda_{11})\,\eta_1-d_1^d\right],$$

and

$$\pi_2=\left[\lambda_{02}\left(\tfrac{\mu_2}{\nu}\right)+\lambda_{12}\eta_2\right]+\left[\frac{1/\alpha_{\pi M}}{P_M^*}\right]\left[(1-\lambda_{02})\,\tfrac{\mu_2}{\nu}+(1-\lambda_{12})\,\eta_2\right],$$

$$\tilde{x}_2-x_2=\left[\frac{1/\nu}{P}\right][K_2+(1+\rho)\,d_1]+\left[\frac{1/\nu}{P_M^*}\right]\left[(1-\lambda_{02})\,\tfrac{\mu_2}{\nu}+(1-\lambda_{12})\,\eta_2\right],$$

$$(\tilde{g}_2+\eta_2)-g_2=\left[\frac{1/\alpha_g}{P}\right][K_2+(1+\rho)\,d_1]+\left[\frac{1/\alpha_g}{P_M^*}\right]\left[(1-\lambda_{02})\,\tfrac{\mu_2}{\nu}+(1-\lambda_{12})\,\eta_2\right].$$

As in the case without targets, the first-order condition for public debt can be written as:

$$\alpha_g\,(\tilde{g}_1+\eta_1-g_1)=\beta^*\left[\tfrac{1}{P}\right][K_2+(1+\rho)\,d_1+\eta_1]\,.$$

Hence,

$$\begin{aligned}&[K_1+(1+\rho)\,d_0-d_1^e]+\left[\frac{P}{P_M^*}\right]\left[(1-\lambda_{01})\,\tfrac{\mu_1}{\nu}+(1-\lambda_{11})\,\eta_1-d_1^d\right]\\ =\;&\beta^*\,[K_2+(1+\rho)\,d_1+\eta_1]\,.\end{aligned}$$

As before, taking expectations of this equation, we can solve for d_1^e. Next, subtracting from this equation its expected version, we can solve for d_1^d. The complete solution for public debt is:[3]

$$d_1=\frac{[K_1+(1+\rho)d_0]-\beta^*K_2}{1+\beta^*(1+\rho)}+\left[\frac{1-\lambda_{01}}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]\frac{\mu_1}{\nu}+\left[\frac{1-\lambda_{11}-\beta^*\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]\eta_1. \quad (75)$$

Using this expression for d_1 in the above expressions for inflation, the output shortfall and the spending shortfall, we obtain:

$$\begin{aligned}\pi_1\;=\;&\left[\lambda_{01}\left(\tfrac{\mu_1}{\nu}\right)+\lambda_{11}\eta_1\right]+\left[\frac{1/\alpha_{\pi M}}{P_M^*}\right](1-\lambda_{01})\,\psi_1\frac{\mu_1}{\nu}\\ &+\left[\frac{1/\alpha_{\pi M}}{P_M^*}\right][1+(1-\lambda_{11})\,(1+\rho)]\,\frac{\psi_1}{1+\rho}\eta_1,\end{aligned}$$

[3] Substituting the optimal values for λ_{01} and λ_{11} that we find in Appendix C below, we obtain expression (4.3) in the main text.

$$
\begin{aligned}
\tilde{x}_1 - x_1 &= \left[\tfrac{1/\nu}{P}\right] \tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)} F_1 + \left[\tfrac{1/\nu}{P_M^*}\right] (1-\lambda_{01})\, \psi_1 \tfrac{\mu_1}{\nu} \\
&\quad + \left[\tfrac{1/\nu}{P_M^*}\right] \left[1 + (1-\lambda_{11})(1+\rho)\right] \frac{\psi_1}{1+\rho} \eta_1,
\end{aligned}
$$

$$
\begin{aligned}
(\tilde{g}_1 + \eta_1) - g_1 &= \left[\tfrac{1/\alpha_g}{P}\right] \tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)} F_1 + \left[\tfrac{1/\alpha_g}{P_M^*}\right] (1-\lambda_{01})\, \psi_1 \tfrac{\mu_1}{\nu} \\
&\quad + \left[\tfrac{1/\alpha_g}{P_M^*}\right] \left[1 + (1-\lambda_{11})(1+\rho)\right] \frac{\psi_1}{1+\rho} \eta_1,
\end{aligned}
$$

and

$$
\pi_2 = \left[\lambda_{02} \left(\tfrac{\mu_2}{\nu}\right) + \lambda_{12}\eta_2\right] + \left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right] \left[(1-\lambda_{02}) \tfrac{\mu_2}{\nu} + (1-\lambda_{12})\, \eta_2\right],
$$

$$
\begin{aligned}
\tilde{x}_2 - x_2 &= \left[\tfrac{1/\nu}{P}\right] \left[\tfrac{1+\rho}{1+\beta^*(1+\rho)}\right] F_1 + \left[\tfrac{1/\nu}{P}\right] \tfrac{1}{\beta^*\left(P_M^*/P\right)} (1-\lambda_{01})\, \psi_1 \tfrac{\mu_1}{\nu} \\
&\quad + \left[\tfrac{1/\nu}{P}\right] \tfrac{1}{\beta^*\left(P_M^*/P\right)} \left[(1-\lambda_{11}) - \beta^* \left(P_M^*/P\right)\right] \psi_1 \eta_1 \\
&\quad + \left[\tfrac{1/\nu}{P_M^*}\right] \left[(1-\lambda_{02}) \tfrac{\mu_2}{\nu} + (1-\lambda_{12})\, \eta_2\right],
\end{aligned}
$$

$$
\begin{aligned}
(\tilde{g}_2 + \eta_2) - g_2 &= \left[\tfrac{1/\alpha_g}{P}\right] \left[\tfrac{1+\rho}{1+\beta^*(1+\rho)}\right] F_1 + \left[\tfrac{1/\alpha_g}{P}\right] \tfrac{1}{\beta^*\left(P_M^*/P\right)} (1-\lambda_{01})\, \psi_1 \tfrac{\mu_1}{\nu} \\
&\quad + \left[\tfrac{1/\alpha_g}{P}\right] \tfrac{1}{\beta^*\left(P_M^*/P\right)} \left[(1-\lambda_{11}) - \beta^* \left(P_M^*/P\right)\right] \psi_1 \eta_1 \\
&\quad + \left[\tfrac{1/\alpha_g}{P_M^*}\right] \left[(1-\lambda_{02}) \tfrac{\mu_2}{\nu} + (1-\lambda_{12})\, \eta_2\right].
\end{aligned}
$$

As before, substitute these outcomes into the expected social loss, to yield:

$$
\mathrm{E}_0 \left[V_1^S\right] = C_1 + \tilde{C}_2 + \tilde{C}_3 + C_4 + \left(\tilde{C}_{51} + \tilde{C}_{52}\right) + \tilde{C}_6 + (C_{71} + C_{72}), \qquad (76)
$$

where C_1, C_4, C_{71} and C_{72} are given by (66), (69), (73) and (74), respectively, and

$$\tilde{C}_2 \equiv \tfrac{1}{2}\alpha_\pi \left[\lambda_{01} + \tfrac{1/\alpha_{\pi M}}{P_M^*}\left(1-\lambda_{01}\right)\psi_1\right]^2 \tfrac{\sigma_\mu^2}{\nu^2} + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}\left(1-\lambda_{01}\right)^2 \psi_1^2 \tfrac{\sigma_\mu^2}{\nu^2},$$

$$\tilde{C}_3 = \tfrac{1}{2}\beta\tfrac{1}{P}\left[\tfrac{1}{\beta^*\left(P_M^*/P\right)}\right]^2 \left(1-\lambda_{01}\right)^2 \psi_1^2 \tfrac{\sigma_\mu^2}{\nu^2},$$

$$\tilde{C}_{51} \equiv \tfrac{1}{2}\alpha_\pi \left[\lambda_{11} + \tfrac{1/\alpha_{\pi M}}{P_M^*}\left(\tfrac{1+(1-\lambda_{11})(1+\rho)}{1+\rho}\right)\psi_1\right]^2 \sigma_\eta^2 + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}\left(\tfrac{1+(1-\lambda_{11})(1+\rho)}{1+\rho}\right)^2 \psi_1^2\sigma_\eta^2,$$

$$\tilde{C}_{52} \equiv \tfrac{1}{2}\alpha_g \left[1 - 2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\tfrac{1+(1-\lambda_{11})(1+\rho)}{1+\rho}\right)\psi_1\right]\sigma_\eta^2,$$

$$\tilde{C}_6 \equiv \tfrac{1}{2}\beta\tfrac{1}{P}\left(\tfrac{1-\lambda_{11}-\beta^*\left(P_M^*/P\right)}{\beta^*\left(P_M^*/P\right)}\right)^2 \psi_1^2\sigma_\eta^2.$$

The components of (76) stand for: C_1 is the expected loss associated with the deterministic component of the intertemporal government financing requirement, $\tilde{C}_2$ and $\tilde{C}_3$ are the expected losses in the first, respectively second, period from imperfect stabilization of μ_1, C_4 is the expected loss from imperfect stabilization of μ_2, $\tilde{C}_{51} + \tilde{C}_{52}$ and $\tilde{C}_6$ are, respectively, the first- and second-period expected losses from suboptimal stabilization of η_1, and $C_{71} + C_{72}$ is the expected loss from imperfect stabilization of η_2.

C Proofs of Propositions 1, 2 and 3:

C.1 Proof of Proposition 1

We have to minimize (65) with respect to $\alpha_{\pi M}$, λ_{01}, γ_1, λ_{11}, γ_2, λ_{02} and λ_{12}. Hold $\alpha_{\pi M}$ constant until further notice. All expressions in the other parameters are quadratic and the second-order derivative of (65) in each of these other parameters is always strictly positive. Hence, by solving the first-order conditions, we obtain the optimum.

Taking the first-order condition with respect to λ_{01}, and rewriting, yields:

$$\alpha_\pi \left[P\lambda_{01} + \left(1/\alpha_{\pi M}\right)\left(1-\gamma_1\right)\right] = \left(1-\gamma_1-\lambda_{01}\right) \tag{77}$$

$$\Rightarrow \left(1+\alpha_\pi P\right)\lambda_{01} = \left(1-\alpha_\pi/\alpha_{\pi M}\right)\left(1-\gamma_1\right). \tag{78}$$

Next, take the first-order condition with respect to γ_1 and rewrite to yield:

$$\beta^*\left(1+\rho\right)\tfrac{\left(P_M^*\right)^2}{P}\gamma_1 = \tfrac{\alpha_\pi}{\alpha_{\pi M}}\left[P\lambda_{01} + \tfrac{1}{\alpha_{\pi M}}\left(1-\gamma_1\right)\right] + P\left(1-\gamma_1-\lambda_{01}\right).$$

Combine this expression with (77) and solve to give:

$$\gamma_1 = \frac{(\alpha_\pi/\alpha_{\pi M})+\alpha_\pi P\lambda_{01}}{(\alpha_\pi/\alpha_{\pi M})+\beta^*(1+\rho)\left(P_M^*/P\right)}.$$

Next, combine this expression with (78) and solve to give the solution for λ_{01}:

$$\lambda_{01} = \left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\left[\frac{\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)(P^*/P)}\right]. \tag{79}$$

Substitute this back into the expression of γ_1 and solve to give:

$$\begin{aligned}
\gamma_1 &= \frac{\frac{\alpha_\pi}{\alpha_{\pi M}}+\left(1-\frac{\alpha_\pi}{\alpha_{\pi M}}\right)\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)(P^*/P)}\right]}{(\alpha_\pi/\alpha_{\pi M})+\beta^*(1+\rho)\left(P_M^*/P\right)} \\
&= \left[\frac{1}{1+\beta^*(1+\rho)(P^*/P)}\right]\frac{\frac{\alpha_\pi}{\alpha_{\pi M}}\left[1+\beta^*(1+\rho)\frac{P^*}{P}\right]+\left(1-\frac{\alpha_\pi}{\alpha_{\pi M}}\right)\beta^*(1+\rho)}{\frac{\alpha_\pi}{\alpha_{\pi M}}+\beta^*(1+\rho)\left(1+\frac{1/\alpha_{\pi M}}{P}\right)} \\
&= \left[\frac{1}{1+\beta^*(1+\rho)(P^*/P)}\right]\frac{\frac{\alpha_\pi}{\alpha_{\pi M}}+\beta^*(1+\rho)+\beta^*(1+\rho)\frac{1/\alpha_{\pi M}}{P}}{\frac{\alpha_\pi}{\alpha_{\pi M}}+\beta^*(1+\rho)\left(1+\frac{1/\alpha_{\pi M}}{P}\right)} \\
&= \frac{1}{1+\beta^*(1+\rho)(P^*/P)}.
\end{aligned} \tag{80}$$

Further, note that if $\alpha_{\pi M} = \alpha_\pi$, then $\lambda_{01} = 0$.

Take the first-order condition with respect to λ_{11}, and rewrite to give:

$$P^*\lambda_{11} = \left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)(1-\gamma_2) - \frac{1}{\alpha_\pi}\frac{P_M^*}{P}. \tag{81}$$

Next, take the first-order condition with respect to γ_2, and rewrite to give:

$$\begin{aligned}
&\left[\frac{\alpha_\pi}{\alpha_{\pi M}^2} + P + \left(\frac{\alpha_\pi}{\alpha_{\pi M}} - 1\right)\left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\frac{P}{P^*} + \beta^*(1+\rho)\frac{\left(P_M^*\right)^2}{P}\right]\gamma_2 \\
= \;&\left(\frac{\alpha_\pi}{\alpha_{\pi M}^2} + P\right) + \left(\frac{\alpha_\pi}{\alpha_{\pi M}} - 1\right)\left[\left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\frac{P}{P^*} - \frac{1}{\alpha_\pi}\frac{P_M^*}{P}\right] - P_M^*.
\end{aligned}$$

Working out the coefficient of γ_2 as well as its right-hand side, this expression reduces to:

$$\gamma_2 = \frac{\frac{1}{\alpha_\pi}\left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\frac{1}{PP_M^*}}{1+\beta^*(1+\rho)(P^*/P)}. \tag{82}$$

Substituting this into (81), we can solve for λ_{11}. Finally, we observe that, if $\alpha_{\pi M} = \alpha_\pi$, then $\lambda_{11} = -\frac{1}{\alpha_\pi}\frac{1}{P}$ and $\gamma_2 = 0$.

Taking the first-order condition with respect to λ_{02}, and rewriting, yields:

$$\lambda_{02} = \left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\frac{1}{P^*}. \tag{83}$$

Next, take the first-order condition with respect to λ_{12} and rewrite to give:

$$\lambda_{12} = -\frac{1}{\alpha_{\pi M}} \frac{1}{P}. \tag{84}$$

Finally, we solve for the optimal value of $\alpha_{\pi M}$. To this end, we inspect each of the terms $C_1, .., C_{72}$ that together form (65). The term C_1 does not depend on $\alpha_{\pi M}$.

As regards to the term C_2, we observe from (78) that we can write:

$$1 - \gamma_1 - \lambda_{01} = \left[\frac{\alpha_\pi}{1-\alpha_\pi/\alpha_{\pi M}}\right] P_M^* \lambda_{01}.$$

Hence, we can write:

$$\begin{aligned}
C_2 &= \frac{1}{2}\alpha_\pi \left[\left(\frac{1}{1-\alpha_\pi/\alpha_{\pi M}}\right)\lambda_{01}\right]^2 \frac{\sigma_\mu^2}{\nu^2} + \frac{1}{2}P\left[\left(\frac{\alpha_\pi}{1-\alpha_\pi/\alpha_{\pi M}}\right)\lambda_{01}\right]^2 \frac{\sigma_\mu^2}{\nu^2} \\
&= \frac{1}{2}\alpha_\pi^2 P^* \left[\left(\frac{1}{1-\alpha_\pi/\alpha_{\pi M}}\right)\lambda_{01}\right]^2 \frac{\sigma_\mu^2}{\nu^2} \\
&= \frac{1}{2}\frac{1}{P^*}\left[\frac{\beta^*(1+\rho)(P^*/P)}{1+\beta^*(1+\rho)(P^*/P)}\right]^2 \frac{\sigma_\mu^2}{\nu^2},
\end{aligned} \tag{85}$$

where we have used (79). This expression does not depend on $\alpha_{\pi M}$.

Next, we observe that C_3 does not depend on $\alpha_{\pi M}$, because γ_1 does not depend on $\alpha_{\pi M}$.

We turn now to the term C_4. With the help of (83) we can write this term as:

$$\begin{aligned}
C_4 &= \frac{1}{2}\alpha_\pi \left[\frac{\frac{1}{\alpha_{\pi M}}}{P_M^*} + \frac{P\left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)}{P^* P_M^*}\right]^2 \frac{\sigma_\mu^2}{\nu^2} + \frac{1}{2}\frac{P}{\left(P_M^*\right)^2}\left[\frac{P^* - \left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)}{P^*}\right]^2 \frac{\sigma_\mu^2}{\nu^2} \\
&= \frac{1}{2}\alpha_\pi \left[\frac{\frac{1}{\alpha_{\pi M}}\left(\frac{1}{\alpha_\pi} + P\right) + P\left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)}{P^* P_M^*}\right]^2 \frac{\sigma_\mu^2}{\nu^2} + \frac{1}{2}\frac{P}{(P^*)^2}\frac{\sigma_\mu^2}{\nu^2} \\
&= \frac{1}{2}\left[\alpha_\pi\left(\frac{1/\alpha_\pi}{P^*}\right)^2 + \frac{P}{(P^*)^2}\right]\frac{\sigma_\mu^2}{\nu^2} \\
&= \frac{1}{2}\frac{1}{P^*}\frac{\sigma_\mu^2}{\nu^2},
\end{aligned}$$

which does not depend on $\alpha_{\pi M}$.

Now, turn to the terms C_{51} and C_{52}. First, using (81), work out:

$$\begin{aligned}
1 - \gamma_2 - \lambda_{11} &= (1-\gamma_2) - \left(\frac{1}{\alpha_\pi} - \frac{1}{\alpha_{\pi M}}\right)\frac{1}{P^*}(1-\gamma_2) + \frac{1}{\alpha_\pi}\frac{P_M^*}{PP^*} \\
&= \frac{P_M^*}{P^*}\left(\frac{P^*}{P} - \gamma_2\right).
\end{aligned}$$

Also, using (81),

$$
\begin{aligned}
& \lambda_{11} + \left(\tfrac{1/\alpha_{\pi M}}{P_M^*}\right)(1-\gamma_2-\lambda_{11}) \\
= \; & \tfrac{P}{P_M^*}\lambda_{11} + \left(\tfrac{1/\alpha_{\pi M}}{P_M^*}\right)(1-\gamma_2) \\
= \; & \tfrac{P}{P^* P_M^*}\left(\tfrac{1}{\alpha_\pi} - \tfrac{1}{\alpha_{\pi M}}\right)(1-\gamma_2) - \tfrac{1/\alpha_\pi}{P^*} + \left(\tfrac{1/\alpha_{\pi M}}{P_M^*}\right)(1-\gamma_2) \\
= \; & \tfrac{1/\alpha_\pi}{P^*}(1-\gamma_2) - \tfrac{1/\alpha_\pi}{P^*} \\
= \; & -\tfrac{1/\alpha_\pi}{P^*}\gamma_2.
\end{aligned}
$$

Hence, the first component of C_{51} is minimized by setting $\alpha_{\pi M} = \alpha_\pi$, because in that case $\gamma_2 = 0$, by (82). Next, take the sum of the second term of C_{51} and C_{52}:

$$
\begin{aligned}
& \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}\left(\tfrac{P_M^*}{P^*}\right)^2\left(\tfrac{P^*}{P}-\gamma_2\right)^2\sigma_\eta^2 + \tfrac{1}{2}\alpha_g\sigma_\eta^2 + \tfrac{1}{P_M^*}\tfrac{P_M^*}{P^*}\left(\gamma_2 - \tfrac{P^*}{P}\right)\sigma_\eta^2 \\
= \; & \tfrac{1}{2}\tfrac{P}{(P^*)^2}\left(\tfrac{P^*}{P}-\gamma_2\right)^2\sigma_\eta^2 + \tfrac{1}{2}\alpha_g\sigma_\eta^2 + \tfrac{1}{P^*}\left(\gamma_2 - \tfrac{P^*}{P}\right)\sigma_\eta^2 \\
= \; & \left[\tfrac{1}{2}\tfrac{1}{P} - \tfrac{1}{P^*}\gamma_2 + \tfrac{1}{2}\tfrac{P}{(P^*)^2}\gamma_2^2 + \tfrac{1}{P^*}\gamma_2 - \tfrac{1}{P}\right]\sigma_\eta^2 + \tfrac{1}{2}\alpha_g\sigma_\eta^2 \\
= \; & \left[-\tfrac{1}{2}\tfrac{1}{P} + \tfrac{1}{2}\tfrac{P}{(P^*)^2}\gamma_2^2\right]\sigma_\eta^2 + \tfrac{1}{2}\alpha_g\sigma_\eta^2.
\end{aligned}
$$

This is minimized by setting $\alpha_{\pi M} = \alpha_\pi$, so that $\gamma_2 = 0$, by (82).

The term C_6 is minimized by setting $\alpha_{\pi M} = \alpha_\pi$, so that $\gamma_2 = 0$.

Finally, we turn to minimizing C_{71} and C_{72}. Substituting (84) into the expression for C_{71}, we obtain:

$$
C_{71} \equiv \tfrac{1}{2}\alpha_\pi\left[\left(\tfrac{P}{P_M^*}\right)\lambda_{12} + \left(\tfrac{1/\alpha_{\pi M}}{P_M^*}\right)\right]^2\sigma_\eta^2 + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}\left(\tfrac{P_M^*}{P}\right)^2\sigma_\eta^2 = \tfrac{1}{2}\tfrac{1}{P}\sigma_\eta^2,
$$

which does not depend on $\alpha_{\pi M}$. Finally,

$$
C_{72} \equiv \tfrac{1}{2}\alpha_g\sigma_\eta^2 - \tfrac{1}{P_M^*}(1-\lambda_{12})\sigma_\eta^2 = \tfrac{1}{2}\alpha_g\sigma_\eta^2 - \tfrac{1}{P}\sigma_\eta^2,
$$

which also does not depend on $\alpha_{\pi M}$.

To summarize, we have shown that each component of (65) does not depend on $\alpha_{\pi M}$, after the other parameters have been chosen optimally, or is minimized at $\alpha_{\pi M} = \alpha_\pi$.

C.2 Proof of Proposition 2

We have to minimize (76) with respect to $\alpha_{\pi M}$, λ_{01}, λ_{11}, λ_{02} and λ_{12}. The optimal values for λ_{02} and λ_{12} follow directly from the proof of Proposition 1 and are thus given by (83) and (84). Hold $\alpha_{\pi M}$ constant until further notice. The expressions in λ_{01} and λ_{11} are quadratic and the second-order derivative of (76) in either of these two parameters is always strictly positive. Hence, by solving the first-order conditions, we obtain the optimum.

Differentiating (76) with respect to λ_{01} and using a substantial amount of straightforward algebra yields, as before, the solution (79) for λ_{01}. One can check that this must be the optimal value, because (as we show below) it implies that $\tilde{C}_2$ and $\tilde{C}_3$ equal, respectively, C_2 given by (85) and C_3 given by (68) with (80) substituted. Because the latter two are the corresponding expressions obtained with both optimal debt and inflation targets, they must lead to the lowest stabilization loss associated with μ_1.

The first-order condition for λ_{11} is:

$$
\begin{aligned}
& \alpha_\pi\left[\lambda_{11}+\frac{1/\alpha_{\pi M}}{P_M^*}\left(\frac{1+(1-\lambda_{11})(1+\rho)}{1+\rho}\right)\psi_1\right]\left[1-\frac{1/\alpha_{\pi M}}{P_M^*}\psi_1\right]- \\
& \frac{P}{\left(P_M^*\right)^2}\left[\frac{1+(1-\lambda_{11})(1+\rho)}{1+\rho}\right]\psi_1^2+\frac{1}{P_M^*}\psi_1-\beta\frac{1}{P}\left[\frac{1-\lambda_{11}-\beta^*\left(P_M^*/P\right)}{\left(\beta^*\left(P_M^*/P\right)\right)^2}\right]\psi_1^2 \\
= \; & 0 \\
\Leftrightarrow \; & \alpha_\pi\left[1-\frac{1/\alpha_{\pi M}}{P_M^*}\psi_1\right]\lambda_{11}+\left[\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right]\left[\frac{2+\rho}{1+\rho}\right]\psi_1\left[1-\frac{1/\alpha_{\pi M}}{P_M^*}\psi_1\right] \\
& -\left[\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right]\psi_1\left[1-\frac{1/\alpha_{\pi M}}{P_M^*}\psi_1\right]\lambda_{11}-\frac{P}{\left(P_M^*\right)^2}\frac{2+\rho}{1+\rho}\psi_1^2+\frac{1}{P_M^*}\psi_1 \\
& -\frac{\beta}{P}\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\left(\beta^*\left(P_M^*/P\right)\right)^2}\right]\psi_1^2+\left[\frac{P}{\left(P_M^*\right)^2}+\frac{\beta}{P}\left(\frac{1}{\beta^*\left(P_M^*/P\right)}\right)^2\right]\psi_1^2\lambda_{11} \\
= \; & 0 \\
\Leftrightarrow \; & \lambda_{11}\left[\alpha_\pi-2\left(\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right)\psi_1+\left(\frac{\alpha_\pi/\alpha_{\pi M}^2}{\left(P_M^*\right)^2}\right)\psi_1^2\right]+\left(\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right)\left(\frac{2+\rho}{1+\rho}\right)\psi_1- \\
& \frac{Q_M}{\left(P_M^*\right)^2}\left(\frac{2+\rho}{1+\rho}\right)\psi_1^2+\frac{1}{P_M^*}\psi_1-\frac{\beta}{P}\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\left(\beta^*\left(P_M^*/P\right)\right)^2}\right]\psi_1^2+\frac{P}{\left(P_M^*\right)^2}\frac{1+\beta^*(1+\rho)}{\beta^*(1+\rho)}\psi_1^2\lambda_{11} \\
= \; & 0.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\Leftrightarrow \; & \lambda_{11}\left[\alpha_\pi-2\left(\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right)\psi_1+\left(\frac{Q_M}{\left(P_M^*\right)^2}+\frac{P}{\left(P_M^*\right)^2}\frac{1}{\beta^*(1+\rho)}\right)\psi_1^2\right] \\
= \; & \left[\frac{Q_M}{\left(P_M^*\right)^2}\left(\frac{2+\rho}{1+\rho}\right)+\frac{\beta}{P}\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\left(\beta^*\left(P_M^*/P\right)\right)^2}\right]\right]\psi_1^2-\left[\frac{\frac{\alpha_\pi}{\alpha_{\pi M}}\left(\frac{2+\rho}{1+\rho}\right)+1}{P_M^*}\right]\psi_1.
\end{aligned}
$$

With quite a bit of straightforward algebra, we can write:

$$\alpha_\pi - 2\left(\frac{\alpha_\pi/\alpha_{\pi M}}{P_M^*}\right)\psi_1 + \left(\frac{Q_M}{\left(P_M^*\right)^2} + \frac{P}{\left(P_M^*\right)^2}\frac{1}{\beta^*(1+\rho)}\right)\psi_1^2 = \frac{\alpha_\pi[1+\beta^*(1+\rho)][1+\beta^*(1+\rho)(P^*/P)]}{\left[1+\beta^*\left(P_M^*/P\right)(1+\rho)\right]^2}.$$

Hence,

$$\lambda_{11} = \frac{\left[\frac{Q_M}{\left(P_M^*\right)^2}\left(\frac{2+\rho}{1+\rho}\right) + \frac{\beta}{P}\left[\frac{1-\beta^*\left(P_M^*/P\right)}{\left(\beta^*\left(P_M^*/P\right)\right)^2}\right]\right]\left[\beta^*\frac{P_M^*}{P}(1+\rho)\right]^2 - \left[\frac{\frac{\alpha_\pi}{\alpha_{\pi M}}\left(\frac{2+\rho}{1+\rho}\right)+1}{P_M^*}\right]\left[1+\beta^*\frac{P_M^*}{P}(1+\rho)\right]\left[\beta^*\frac{P_M^*}{P}(1+\rho)\right]}{\alpha_\pi[1+\beta^*(1+\rho)][1+\beta^*(1+\rho)(P^*/P)]}.$$

Some algebra shows that the numerator of this expression can be written as

$$-\left[\beta^*\left(2+\rho\right)/P\right]\frac{\alpha_\pi}{\alpha_{\pi M}}\left[1+\beta^*\left(1+\rho\right)\left(P^*/P\right)\right],$$

so that

$$\lambda_{11} = -\frac{1}{\alpha_{\pi M}}\frac{1}{P}\frac{\beta^*(2+\rho)}{1+\beta^*(1+\rho)}. \tag{86}$$

Finally, we solve for the optimal value of $\alpha_{\pi M}$. To this end, we inspect each of the terms that together form (76). The term C_1 does not depend on $\alpha_{\pi M}$. As regards to the terms $\tilde{C}_2$ and $\tilde{C}_3$, we observe that:

$$\left(1-\lambda_{01}\right)\psi_1 = \frac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)(P^*/P)},$$

and

$$\lambda_{01} + \frac{1/\alpha_{\pi M}}{P_M^*}\left(1-\lambda_{01}\right)\psi_1 = \frac{\beta^*(1+\rho)\frac{1/\alpha_\pi}{P}}{1+\beta^*(1+\rho)(P^*/P)},$$

so that

$$\begin{aligned}\tilde{C}_2 &= \frac{1}{2}\frac{1}{\alpha_\pi}\left[\frac{\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)(P^*/P)}\right]^2\frac{\sigma_\mu^2}{\nu^2} + \frac{1}{2}\frac{P}{\left(P_M^*\right)^2}\left[\frac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)(P^*/P)}\right]^2\frac{\sigma_\mu^2}{\nu^2} \\ &= \frac{1}{2}P^*\left[\frac{\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)(P^*/P)}\right]^2\frac{\sigma_\mu^2}{\nu^2},\end{aligned}$$

and

$$\tilde{C}_3 = \frac{1}{2}\left[\beta^*\left(1+\rho\right)/P\right]\left[\frac{1}{1+\beta^*(1+\rho)(P^*/P)}\right]^2\frac{\sigma_\mu^2}{\nu^2},$$

both of which do not depend on $\alpha_{\pi M}$.

The optimal value of λ_{02} is the same as in the proof of Proposition 2. Hence, C_4 does not depend on $\alpha_{\pi M}$.

Next, we turn to $\tilde{C}_{51}$ and substitute (86) and (61) for λ_{11} and ψ_1, respectively:

$$\begin{aligned}
\tilde{C}_{51} &\equiv \tfrac{1}{2}\alpha_\pi \left[\lambda_{11} + \tfrac{1/\alpha_{\pi M}}{P_M^*}\left(\tfrac{2+\rho}{1+\rho} - \lambda_{11}\right)\tfrac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&\quad + \tfrac{1}{2}\tfrac{P}{\left(P_M^*\right)^2}\left(\tfrac{2+\rho}{1+\rho} - \lambda_{11}\right)^2\left[\tfrac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&= \tfrac{1}{2}\alpha_\pi \left[\lambda_{11} + \left(\tfrac{2+\rho}{1+\rho} - \lambda_{11}\right)\tfrac{(1/\alpha_{\pi M})\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&\quad + \tfrac{1}{2}\tfrac{1}{P}\left(\tfrac{2+\rho}{1+\rho} - \lambda_{11}\right)^2\left[\tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&= \tfrac{1}{2}\alpha_\pi \left[-\tfrac{1}{\alpha_{\pi M}}\tfrac{1}{P}\tfrac{\beta^*(2+\rho)}{1+\beta^*(1+\rho)} + \left(\tfrac{2+\rho}{1+\rho}\right)\tfrac{1+\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)}\tfrac{(1/\alpha_{\pi M})\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&\quad + \tfrac{1}{2}\tfrac{1}{P}\left(\tfrac{2+\rho}{1+\rho}\right)^2\left[\tfrac{1+\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)}\right]^2\left[\tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&= \tfrac{1}{2}\alpha_\pi \left[-\tfrac{1}{\alpha_{\pi M}}\left(\tfrac{2+\rho}{1+\rho}\right)\tfrac{\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)} + \left(\tfrac{2+\rho}{1+\rho}\right)\tfrac{(1/\alpha_{\pi M})\beta^*(1+\rho)/P}{1+\beta^*(1+\rho)}\right]^2 \sigma_\eta^2 + \tfrac{1}{2}\tfrac{1}{P}\left[\tfrac{\beta^*(2+\rho)}{1+\beta^*(1+\rho)}\right]^2 \sigma_\eta^2 \\
&= \tfrac{1}{2}\tfrac{1}{P}\left[\tfrac{\beta^*(2+\rho)}{1+\beta^*(1+\rho)}\right]^2 \sigma_\eta^2,
\end{aligned}$$

which is independent of $\alpha_{\pi M}$.

Now, we consider $\tilde{C}_{52}$, which we can write, after substituting (86) and (61) for λ_{11} and ψ_1, respectively:

$$\begin{aligned}
\tilde{C}_{52} &= \tfrac{1}{2}\alpha_g\sigma_\eta^2 - \left(\tfrac{1}{P_M^*}\right)\left(\tfrac{2+\rho}{1+\rho}\right)\tfrac{1+\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)}\tfrac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\sigma_\eta^2 \\
&= \tfrac{1}{2}\alpha_g\sigma_\eta^2 - \tfrac{\beta^*(2+\rho)/P}{1+\beta^*(1+\rho)}\sigma_\eta^2,
\end{aligned}$$

which is independent of $\alpha_{\pi M}$.

Next, turn to $\tilde{C}_6$. We observe that

$$\begin{aligned}
1 - \lambda_{11} - \beta^*\left(P_M^*/P\right) &= 1 + \tfrac{1}{\alpha_{\pi M}}\tfrac{1}{P}\tfrac{\beta^*(2+\rho)}{1+\beta^*(1+\rho)} - \beta^*\tfrac{P+1/\alpha_{\pi M}}{P} \\
&= 1 + \tfrac{1}{\alpha_{\pi M}}\tfrac{1}{P}\beta^*\left[\tfrac{2+\rho}{1+\beta^*(1+\rho)} - 1\right] - \beta^* \\
&= (1-\beta^*)\left[1 + \tfrac{1}{\alpha_{\pi M}}\tfrac{1}{P}\tfrac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right] \\
&= (1-\beta^*)\left[\tfrac{1+\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)}\right].
\end{aligned}$$

Substitute this into the expression for $\tilde{C}_6$ and rewrite, using (61) for ψ_1, to yield:

$$\begin{aligned}
\tilde{C}_6 &= \tfrac{1}{2}\beta\tfrac{1}{P}\left(1-\beta^*\right)^2 \left[\tfrac{1}{\beta^*\left(P_M^*/P\right)} \tfrac{1+\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)}\right]^2 \left[\tfrac{\beta^*(1+\rho)\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)}\right]^2 \sigma_\eta^2 \\
&= \tfrac{1}{2}\beta\tfrac{1}{P}\left(1-\beta^*\right)^2 \left[\tfrac{1+\rho}{1+\beta^*(1+\rho)}\right]^2 \sigma_\eta^2,
\end{aligned}$$

which does not depend on $\alpha_{\pi M}$ either.

The optimal value of λ_{12} is the same as in the proof of Proposition 2. Hence, C_{71} and C_{72} do not depend on $\alpha_{\pi M}$.

To summarize, we have shown that none of the components of (76) depends on $\alpha_{\pi M}$, after the other parameters have been chosen optimally.

C.3 Proof of Proposition 3

The term T_1 does not depend on $\alpha_{\pi M}$. We can write the term T_2 as:

$$\begin{aligned}
& \tfrac{1}{2}\left[\tfrac{Q_M}{\left(P_M^*\right)^2} + \tfrac{1}{\beta^*(1+\rho)}\tfrac{P}{\left(P_M^*\right)^2}\right]\psi_1^2\left(\tfrac{\sigma_\mu^2}{\nu^2}+\sigma_\eta^2\right) \\
=\ & \tfrac{1}{2}\left[\tfrac{Q_M}{P^2} + \tfrac{1}{\beta^*(1+\rho)}\tfrac{1}{P}\right]\left[\tfrac{\beta^*(1+\rho)}{1+\beta^*\left(P_M^*/P\right)(1+\rho)}\right]^2\left(\tfrac{\sigma_\mu^2}{\nu^2}+\sigma_\eta^2\right) \\
=\ & \tfrac{1}{2}\beta^*\left(1+\rho\right)\left(\tfrac{\sigma_\mu^2}{\nu^2}+\sigma_\eta^2\right)\tfrac{P+\beta^*(1+\rho)Q_M}{\left[P+\beta^*(1+\rho)P_M^*\right]^2}.
\end{aligned}$$

We only need to consider the final factor. The sign of its derivative with respect to $\alpha_{\pi M}$ is given by the expression:

$$\begin{aligned}
& \left[P+\beta^*\left(1+\rho\right)P_M^*\right] * -2\left(\tfrac{\alpha_\pi}{\alpha_{\pi M}^3}\right) * \beta^*\left(1+\rho\right) - \\
& 2\left[P+\beta^*\left(1+\rho\right)Q_M\right] * \beta^*\left(1+\rho\right) * -\left(\tfrac{1}{\alpha_{\pi M}^2}\right) \\
=\ & \left(\tfrac{2}{\alpha_{\pi M}^2}\right)\beta^*\left(1+\rho\right)\left[\begin{array}{c}\left(P+\beta^*\left(1+\rho\right)Q_M\right) \\ -\tfrac{\alpha_\pi}{\alpha_{\pi M}}\left(P+\beta^*\left(1+\rho\right)P_M^*\right)\end{array}\right] \\
=\ & \left(\tfrac{2}{\alpha_{\pi M}^2}\right)\beta^*\left(1+\rho\right)\left[1+\beta^*\left(1+\rho\right)\right]\left[1-\tfrac{\alpha_\pi}{\alpha_{\pi M}}\right]P,
\end{aligned}$$

which is negative if $\alpha_{\pi M} < \alpha_\pi$, zero if $\alpha_{\pi M} = \alpha_\pi$ and positive if $\alpha_{\pi M} > \alpha_\pi$.

The sign of the derivative of the term T_3 is given by:

$$
\begin{aligned}
& P_M^* \left(\frac{\partial Q_M}{\partial \alpha_{\pi M}} \right) - 2Q_M \left(\frac{\partial P_M^*}{\partial \alpha_{\pi M}} \right) \\
= \; & 2 \frac{1}{\alpha_{\pi M}^2} \left[Q_M - \left(\frac{\alpha_\pi}{\alpha_{\pi M}} \right) P_M^* \right] \\
= \; & 2 \frac{1}{\alpha_{\pi M}^2} P \left[1 - \frac{\alpha_\pi}{\alpha_{\pi M}} \right],
\end{aligned}
$$

which is negative if $\alpha_{\pi M} < \alpha_\pi$, zero if $\alpha_{\pi M} = \alpha_\pi$ and positive if $\alpha_{\pi M} > \alpha_\pi$.

We write the term T_4 as follows:

$$T_4 = \frac{1}{2} \left(T_{41} + T_{42} + T_{43} \right) \psi_1^2 \sigma_\eta^2,$$

where

$$T_{41} = \frac{\frac{1}{\alpha_{\pi M}} \left(\frac{\alpha_\pi}{\alpha_{\pi M}} - 1 \right)}{(1+\rho) \left(P_M^* \right)^2}, \; T_{42} = \frac{Q_M}{(1+\rho)^2 \left(P_M^* \right)^2}, \; T_{43} = \beta \frac{1}{P},$$

Hence,

$$\frac{\partial T_4}{\partial \alpha_{\pi M}} = \frac{1}{2} \left(\frac{\partial T_{41}}{\partial \alpha_{\pi M}} + \frac{\partial T_{42}}{\partial \alpha_{\pi M}} + \frac{\partial T_{43}}{\partial \alpha_{\pi M}} \right) \psi_1^2 \sigma_\eta^2 + \left(T_{41} + T_{42} + T_{43} \right) \psi_1 \left(\frac{\partial \psi_1}{\partial \alpha_{\pi M}} \right) \sigma_\eta^2. \tag{87}$$

We observe that $\partial \psi_1 / \partial \alpha_{\pi M} < 0$ and that $\partial T_{43} / \partial \alpha_{\pi M} = 0$. In addition,

$$\frac{\partial T_{41}}{\partial \alpha_{\pi M}} = \frac{1}{\alpha_{\pi M}^2} \frac{\left(1 - 2\frac{\alpha_\pi}{\alpha_{\pi M}} \right) P_M^* + \frac{2}{\alpha_{\pi M}} \left(\frac{\alpha_\pi}{\alpha_{\pi M}} - 1 \right)}{(1+\rho) \left(P_M^* \right)^3} = \frac{1}{\alpha_{\pi M}^2} \frac{1}{(1+\rho) \left(P_M^* \right)^3} \left[-\frac{1}{\alpha_{\pi M}} + \left(1 - 2\frac{\alpha_\pi}{\alpha_{\pi M}} \right) P \right],$$

which is negative for $\alpha_{\pi M} \leq \alpha_\pi$. Further,

$$\frac{\partial T_{42}}{\partial \alpha_{\pi M}} = \frac{\frac{2}{\alpha_{\pi M}^2} \left(1 - \frac{\alpha_\pi}{\alpha_{\pi M}} \right) P}{(1+\rho)^2 \left(P_M^* \right)^2},$$

which is negative if $\alpha_{\pi M} < \alpha_\pi$, zero if $\alpha_{\pi M} = \alpha_\pi$ and positive if $\alpha_{\pi M} > \alpha_\pi$. Hence, T_4 is minimized by making $\alpha_{\pi M} > \alpha_\pi$.

Let us now turn to the term T_5. We have that

$$\frac{\partial T_{51}}{\partial \alpha_{\pi M}} = -\frac{1}{\alpha_{\pi M}^2} \frac{\left[\beta^* (1/P) \right]^2 (1+\rho)(2+\rho)}{\left[1 + \beta^* \left(P_M^* / P \right) (1+\rho) \right]^2} \sigma_\eta^2 < 0.$$

Further, we have that:

$$\frac{\partial T_{52}}{\partial \alpha_{\pi M}} = -\frac{1}{\alpha_{\pi M}^2} \beta \frac{1}{\left(P_M^* \right)^2} \sigma_\eta^2 < 0.$$

This confirms Lemma 3.

D Derivation of infinite-horizon discretionary equilibrium.

D.1 General part of the derivation

In period t the CB minimizes over π_t:

$$V_t^{CB} = \tfrac{1}{2}\left\{\alpha_{\pi M}\pi_t^2 + \left[\nu\left(\pi_t - \pi_t^e - \tau_t\right) - \mu_t - \tilde{x}_t\right]^2\right\} + \beta \mathrm{E}_t\left[V_{t+1}^{CB}\right].$$

Because $\mathrm{E}_t\left[V_{t+1}^{CB}\right]$ does not depend on π_t (the state variables are the countries' debt levels), the CB's first-order condition is:

$$\alpha_{\pi M}\pi_t + \nu\left[\nu\left(\pi_t - \pi_t^e - \tau_t\right) - \mu_t - \tilde{x}_t\right] = 0. \tag{88}$$

The government selects τ_t and d_t so as to minimize (3). Again, the first-order conditions are (4), (5), (6) and (7).

We solve first for the intratemporal allocation. That is, we solve for inflation, taxes and spending as functions of public debt. The system to be solved at this stage is thus (88), (4) and (6).

D.1.1 Derivation of outcomes for given debt policies

We first derive the deterministic components of the outcomes (step 1). Then, we derive the responses to the shocks (step 2).

Step 1: Take as-of-the-start-of-period-t expectations of the system (88), (4) and (6) to give:

$$\alpha_{\pi M}\pi_t^e - \nu^2\left(\tau_t^e + \tfrac{\tilde{x}_t}{\nu}\right) = 0, \tag{89}$$

(9) and (10). The solution of the resulting system is:

$$\pi_t^e = \left[\tfrac{1/\alpha_{\pi M}}{P}\right]\left[K_t + (1+\rho)\, d_{t-1} - d_t^e\right], \tag{90}$$

(12) and (13).

Step 2: Subtract (89), (9) and (10) from (88), (4) and (6), respectively, to give (15), (16) and (17). The solution of this system has been computed before and is given by (18), (20) and (21). Combining these last three equations with (90), (12) and (13), we obtain:

$$\pi_t = \left[\tfrac{1/\alpha_{\pi M}}{P}\right]\left[K_t + (1+\rho)\, d_{t-1} - d_t^e\right] + \left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right], \tag{91}$$

$$\tilde{x}_t - x_t = \left[\tfrac{1/\nu}{P}\right]\left[K_t + (1+\rho)\,d_{t-1} - d_t^e\right] + \left[\tfrac{1/\nu}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right], \tag{92}$$

$$(\tilde{g}_t + \eta_t) - g_t = \left[\tfrac{1/\alpha_g}{P}\right]\left[K_t + (1+\rho)\,d_{t-1} - d_t^e\right] + \left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right]. \tag{93}$$

D.1.2 Derivation of solution for public debt

We are now in a position to characterize debt policy. To evaluate $\partial \mathrm{E}_t\left(V_{t+1}^{G_t}\right)/\partial d_t$, we forward (91), (92) and (93) by one period and substitute the resulting expressions into:

$$\tfrac{1}{2}\mathrm{E}_t\left[\alpha_\pi \pi_{t+1}^2 + (x_{t+1} - \tilde{x}_{t+1})^2 + \alpha_g\left(g_{t+1} - (\tilde{g}_{t+1} + \eta_t)\right)^2\right].$$

The derivative with respect to d_t of the expression thus obtained is:

$$\mathrm{E}_t\left[\begin{array}{c} \alpha_\pi \pi_{t+1}(1+\rho)\left[\frac{1/\alpha_{\pi M}}{P}\right] + (\tilde{x}_{t+1} - x_{t+1})(1+\rho)\left[\frac{1/\nu}{P}\right] + \\ \alpha_g\left(\tilde{g}_{t+1} + \eta_t - g_{t+1}\right)(1+\rho)\left[\frac{1/\alpha_g}{P}\right] \end{array}\right].$$

Hence, combining this with (5), the first-order condition for d_t is:

$$\alpha_g\left(\tilde{g}_t + \eta_t - g_t\right) = \beta^* \mathrm{E}_t\left[\begin{array}{c} \pi_{t+1}\left[\frac{\alpha_\pi/\alpha_{\pi M}}{P}\right] + (\tilde{x}_{t+1} - x_{t+1})\left[\frac{1/\nu}{P}\right] + \\ \left(\tilde{g}_{t+1} + \eta_t - g_{t+1}\right)\left[\frac{1}{P}\right] \end{array}\right].$$

Combine this with the expressions for π_{t+1}, $\tilde{x}_{t+1} - x_{t+1}$ and $\tilde{g}_{t+1} - g_{t+1}$ obtained from (91), (92) and (93) to give,

$$\begin{aligned} & \left[\tfrac{1}{P}\right]\left[K_t + (1+\rho)\,d_{t-1} - d_t^e\right] + \left[\tfrac{1}{P_M^*}\right]\left[\left(\tfrac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right] \\ = \;& \beta^*\left\{\left[\tfrac{Q_M}{P^2}\right]\left(K_{t+1} + (1+\rho)\,d_t - d_{t+1}^e\right) + \left[\tfrac{1}{P}\right]\eta_t\right\}, \end{aligned}$$

because $\mathrm{E}_t\left[\eta_{t+1}\right] = 0$. Here (as in the main text):

$$Q_M = \alpha_\pi/\alpha_{\pi M}^2 + 1/\nu^2 + 1/\alpha_g. \tag{94}$$

Hence,

$$[K_t + (1+\rho)\, d_{t-1} - d_t^e] + \left[\frac{P}{P_M^*}\right] \left[\left(\frac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right] =$$
$$\beta^* \left[\left(\frac{Q_M}{P}\right) \left(K_{t+1} + (1+\rho)\, d_t - d_{t+1}^e\right) + \eta_t\right]. \tag{95}$$

We can solve for public debt in two steps.

Step 1: take expectations of (95) as of (end of) $t-1$:

$$K_t + (1+\rho)\, d_{t-1} - d_t^e =$$
$$\beta^* \left[\left(\frac{Q_M}{P}\right) \left(K_{t+1} + (1+\rho)\, d_t - d_{t+1}^e\right) + \eta_t\right]. \tag{96}$$

The solution is:

$$\mathrm{E}_{t-1} d_t = \frac{[K_t + (1+\rho) d_{t-1}] - \beta^* (Q_M/P)(K_{t+1} - \mathrm{E}_{t-1} d_{t+1})}{1 + \beta^* (1+\rho)(Q_M/P)}. \tag{97}$$

Step 2: Subtract (96) from (95):

$$\left[\frac{P}{P_M^*}\right] \left[\left(\frac{\mu_t}{\nu} + \eta_t\right) - d_t^d\right] = \beta^* (1+\rho) \left(\frac{Q_M}{P}\right) d_t^d + \tag{98}$$
$$\beta^* \left(\frac{Q_M}{P}\right) [\mathrm{E}_{t-1}(d_{t+1}) - \mathrm{E}_t(d_{t+1})] + \beta^* \eta_t,$$

and solve this to yield:

$$d_t^d = \left[\frac{1}{1+\beta^*(1+\rho)\left(P_M^*/P\right)(Q_M/P)}\right] \left(\frac{\mu_t}{\nu} + \eta_t\right) - \left[\frac{\beta^*\left(P_M^*/P\right)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)(Q_M/P)}\right] \eta_t +$$
$$\left[\frac{\beta^*\left(P_M^*/P\right)(Q_M/P)}{1+\beta^*(1+\rho)\left(P_M^*/P\right)(Q_M/P)}\right] [\mathrm{E}_t(d_{t+1}) - \mathrm{E}_{t-1}(d_{t+1})]. \tag{99}$$

Through the final term the shock will be spread out over the entire future horizon.

We can find the final solution for d_t^d as follows. Use (97) forwarded by $\xi - t$ ($\xi \geq t+1$) periods to obtain:

$$\mathrm{E}_t(d_{t+\xi}) - \mathrm{E}_{t-1}(d_{t+\xi}) = \frac{(1+\rho)\left[\mathrm{E}_t\left(d_{t+\xi-1}\right) - \mathrm{E}_{t-1}\left(d_{t+\xi-1}\right)\right] + \beta^*(Q_M/P)\left[\mathrm{E}_t\left(d_{t+\xi+1}\right) - \mathrm{E}_{t-1}\left(d_{t+\xi+1}\right)\right]}{1+\beta^*(1+\rho)(Q_M/P)}. \tag{100}$$

The non-explosive solution is found in the same way as before and is given by:

$$\mathrm{E}_t(d_{t+1}) - \mathrm{E}_{t-1}(d_{t+1}) = \frac{1}{\beta^*(Q_M/P)} d_t^d.$$

Substitute this back into (99) and rewrite to give:

$$d_t^d = \left[\frac{1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]\left[\frac{\mu_t}{\nu} + \left(1-\beta^*\left(P_M^*/P\right)\right)\eta_t\right]. \qquad (101)$$

D.1.3 Derivation deterministic components of $\tilde{x}_{it} - x_{it}$, $\tilde{g}_{it} - g_{it}$ and π_t.

The derivation is similar to that in the case of the second best. Using (92), one has

$$\begin{aligned}
\tilde{x}_{t+\xi} - \mathrm{E}_{t-1}x_{t+\xi} &= \left[\frac{1/\nu}{P}\right]\left[K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1} - \mathrm{E}_{t-1}d_{t+\xi}\right] \Rightarrow \\
\tilde{x}_{t+\xi+1} - \mathrm{E}_{t-1}x_{t+\xi+1} &= \left[\frac{1/\nu}{P}\right]\left[K_{t+\xi+1} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi} - \mathrm{E}_{t-1}d_{t+\xi+1}\right],
\end{aligned}$$

where $\xi \geq t$. Note that

$$\begin{aligned}
& K_{t+\xi} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi-1} - \mathrm{E}_{t-1}d_{t+\xi} \\
= \;& \frac{[1+\beta^*(1+\rho)(Q_M/P)]\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right] - \left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right] + \beta^*(Q_M/P)\left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)(Q_M/P)} \\
= \;& \beta^*\left(Q_M/P\right)\left[\frac{(1+\rho)\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right] + \left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)(Q_M/P)}\right],
\end{aligned}$$

where we have used (97) (with forwarding) for $\mathrm{E}_{t-1}d_{t+\xi}$. Further,

$$\begin{aligned}
& K_{t+\xi+1} + (1+\rho)\,\mathrm{E}_{t-1}d_{t+\xi} - \mathrm{E}_{t-1}d_{t+\xi+1} \\
= \;& \frac{[1+\beta^*(1+\rho)(Q_M/P)]\left[K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right] + (1+\rho)\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right] - \beta^*(1+\rho)(Q_M/P)\left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)(Q_M/P)} \\
= \;& \frac{(1+\rho)\left[K_{t+\xi}+(1+\rho)\mathrm{E}_{t-1}d_{t+\xi-1}\right] + \left(K_{t+\xi+1}-\mathrm{E}_{t-1}d_{t+\xi+1}\right)}{1+\beta^*(1+\rho)(Q_M/P)}.
\end{aligned}$$

Hence,

$$\tilde{x}_{t+\xi+1} - \mathrm{E}_{t-1}x_{t+\xi+1} = \left[\frac{1}{\beta^*(Q_M/P)}\right]\left[\tilde{x}_{t+\xi} - \mathrm{E}_{t-1}x_{t+\xi}\right].$$

For public spending we derive similarly:

$$\tilde{g}_{t+\xi+1} - \mathrm{E}_{t-1}g_{t+\xi+1} = \left[\frac{1}{\beta^*(Q_M/P)}\right]\left[\tilde{g}_{t+\xi} - \mathrm{E}_{t-1}g_{t+\xi}\right].$$

Having derived these recursions, we use (36) to obtain the deterministic components of the final outcomes:

$$\begin{aligned}
& \sum_{\xi=0}^{\infty}(1+\rho)^{-\xi}\left[\left(\tilde{x}_{t+\xi}-\mathrm{E}_{t-1}x_{t+\xi}\right)/\nu+\left(\tilde{g}_{t+\xi}-\mathrm{E}_{t-1}g_{t+\xi}\right)\right]=F_t \\
\Longrightarrow\ & \sum_{\xi=0}^{\infty}\left[\tfrac{1}{\beta^*(1+\rho)(Q_M/P)}\right]^{\xi}\left[\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu+\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right)\right]=F_t \\
\Longrightarrow\ & \left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu+\left(\tilde{g}_t-\mathrm{E}_{t-1}g_t\right)=\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]F_t.
\end{aligned}$$

Hence,

$$\left(\tilde{x}_t-\mathrm{E}_{t-1}x_t\right)/\nu = \left[\tfrac{1/\nu^2}{P}\right]\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]F_t, \tag{102}$$

$$\tilde{g}_t-\mathrm{E}_{t-1}g_t = \left[\tfrac{1/\alpha_g}{P}\right]\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]F_t. \tag{103}$$

D.1.4 Computation of $\mathrm{E}_{t-1}(d_t)$

We compute now $\mathrm{E}_{t-1}(d_t)$. Combining the expectation of the government financing requirement, (40), with (102) and (103) and replacing F_t, one has:

$$K_t+(1+\rho)\,d_{t-1}-\mathrm{E}_{t-1}d_t=\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]\left[(1+\rho)\,d_{t-1}+G_t\right].$$

Hence,

$$\begin{aligned}
\mathrm{E}_{t-1}d_t &= \tfrac{1}{\beta^*(Q_M/P)}d_{t-1}+K_t-\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]\sum_{\xi=t}^{\infty}(1+\rho)^{-(\xi-t)}K_\xi \\
&= \tfrac{1}{\beta^*(Q_M/P)}d_{t-1}+\tfrac{1}{\beta^*(1+\rho)(Q_M/P)}K_t+ \\
&\quad \left[\tfrac{1}{\beta^*(1+\rho)(Q_M/P)}-1\right]\tfrac{1}{1+\rho}\sum_{\xi=t+1}^{\infty}(1+\rho)^{-[\xi-(t+1)]}K_\xi \\
&= \tfrac{1}{\beta^*(Q_M/P)}d_{t-1}+\tfrac{1}{\beta^*(1+\rho)(Q_M/P)}G_t-\tfrac{1}{1+\rho}G_{t+1} \\
&= \tfrac{1}{\beta^*(Q_M/P)}d_{t-1}+\tfrac{(G_t-G_{t+1})+[1-\beta^*(Q_M/P)]G_{t+1}}{\beta^*(1+\rho)(Q_M/P)}.
\end{aligned}$$

D.1.5 The complete outcomes

Using (91), (92), (93), (102), (103) and (101), we have:

$$
\begin{aligned}
\pi_t &= \left[\tfrac{1/\alpha_{\pi M}}{P}\right]\psi_0^P F_t + \left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+p_2\eta_t\right], &(104)\\
\tilde{x}_t - x_t &= \left[\tfrac{1/\nu}{P}\right]\psi_0^P F_t + \left[\tfrac{1/\nu}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+p_2\eta_t\right], &(105)\\
(\tilde{g}_t+\eta_t) - g_t &= \left[\tfrac{1/\alpha_g}{P}\right]\psi_0^P F_t + \left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+p_2\eta_t\right], &(106)\\
d_t &= \tfrac{1}{\beta^*(Q_M/P)}d_{t-1} + \tfrac{(G_t-G_{t+1})+[1-\beta^*(Q_M/P)]G_{t+1}}{\beta^*(1+\rho)(Q_M/P)} + \\
&\quad p_1\left[\tfrac{\mu_t}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_t\right]. &(107)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi_0^P &\equiv \tfrac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)},\ \psi_1^P \equiv \tfrac{\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]},\\
p_1 &\equiv \tfrac{1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]},\ p_2 \equiv \tfrac{\beta^*\left(P_M^*/P\right)}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}.
\end{aligned}
$$

E Derivation of expected social loss in infinite-horizon model

E.1 Commitment

Using (45) and its lags, we can write:

$$
\begin{aligned}
F_t &= (1+\rho)\,d_{t-1} + G_t\\
&= (1+\rho)\left\{\tfrac{1}{\beta^*}d_{t-2} + \tfrac{G_{t-1}-\beta^* G_t}{\beta^*(1+\rho)} + q_1\left[\tfrac{\mu_{t-1}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-1}\right]\right\} + G_t\\
&= (1+\rho)\,\tfrac{1}{\beta^*}d_{t-2} + (1+\rho)\,q_1\left[\tfrac{\mu_{t-1}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-1}\right] + \tfrac{1}{\beta^*}G_{t-1}\\
&= (1+\rho)\,\tfrac{1}{\beta^*}\left\{\tfrac{1}{\beta^*}d_{t-3} + \tfrac{G_{t-2}-\beta^* G_{t-1}}{\beta^*(1+\rho)} + q_1\left[\tfrac{\mu_{t-2}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-2}\right]\right\}\\
&\quad + (1+\rho)\,q_1\left[\tfrac{\mu_{t-1}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-1}\right] + \tfrac{1}{\beta^*}G_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
&= (1+\rho)\left(\tfrac{1}{\beta^*}\right)^2 d_{t-3} + \tfrac{1}{\beta^*}(1+\rho)\,q_1\left[\tfrac{\mu_{t-2}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-2}\right]\\
&\quad + (1+\rho)\,q_1\left[\tfrac{\mu_{t-1}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-1}\right] + \left(\tfrac{1}{\beta^*}\right)^2 G_{t-2}\\
&= \left(\tfrac{1}{\beta^*}\right)^{t-1}\left[(1+\rho)\,d_0 + G_1\right] + (1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*}\right)^{\xi-1} q_1\left[\tfrac{\mu_{t-\xi}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right].
\end{aligned}
$$

We can now combine this with (42), (43) and (44) to give:

$$\begin{aligned}
\pi_t &= \left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[q_2\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right], \\
\tilde{x}_t - x_t &= \left[\tfrac{1/\nu}{P}\right]\psi_0^C(1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*}\right)^{\xi-1}q_1\left[\tfrac{\mu_{t-\xi}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right] \\
&\quad +\left[\tfrac{1/\nu}{P_M^*}\right]\left[q_2\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right]+\left[\tfrac{1/\nu}{P}\right]\psi_0^C\left(\tfrac{1}{\beta^*}\right)^{t-1}\left[(1+\rho)\,d_0+G_1\right], \\
\tilde{g}_t - g_t &= \left[\tfrac{1/\alpha_g}{P}\right]\psi_0^C(1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*}\right)^{\xi-1}q_1\left[\tfrac{\mu_{t-\xi}}{\nu}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right] \\
&\quad +\left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[q_2\left(\tfrac{\mu_t}{\nu}+\eta_t\right)+q_3\eta_t\right]+\left[\tfrac{1/\alpha_g}{P}\right]\psi_0^C\left(\tfrac{1}{\beta^*}\right)^{t-1}\left[(1+\rho)\,d_0+G_1\right] \\
&\quad -\eta_t.
\end{aligned}$$

Let us now compute society's expected loss:

$$\begin{aligned}
&\mathrm{E}_0\left[\alpha_\pi\pi_t^2+(x_t-\tilde{x}_t)^2+\alpha_g(g_t-\tilde{g}_t)^2\right] \\
=\ &\left[\tfrac{1}{P}\right]\left\{\psi_0^C\left(\tfrac{1}{\beta^*}\right)^{t-1}\left[(1+\rho)\,d_0+G_1\right]\right\}^2+ \\
&\left[\frac{\left(\psi_0^C\right)^2}{P}\right](1+\rho)^2\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*}\right)^{2(\xi-1)}q_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)^2\sigma_\eta^2\right]+ \\
&\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\tfrac{\sigma_\mu^2}{\nu^2}+(q_2+q_3)^2\sigma_\eta^2\right]+\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2 \\
\\
=\ &\left[\tfrac{1}{P}\right]\left\{\psi_0^C\left(\tfrac{1}{\beta^*}\right)^{t-1}\left[(1+\rho)\,d_0+G_1\right]\right\}^2+ \\
&\left[\frac{\left(\psi_0^C\right)^2}{P}\right](1+\rho)^2\left[\tfrac{1-(1/\beta^*)^{2(t-1)}}{1-(1/\beta^*)^2}\right]q_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)^2\sigma_\eta^2\right]+ \\
&\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\tfrac{\sigma_\mu^2}{\nu^2}+(q_2+q_3)^2\sigma_\eta^2\right]+\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2.
\end{aligned}$$

Hence,

$$\frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_0\left[\alpha_\pi\pi_t^2+\left(x_t-\tilde{x}_t\right)^2+\alpha_g\left(g_t-\tilde{g}_t\right)^2\right]$$

$$\frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\frac{\left(\psi_0^C\right)^2}{P}\right](1+\rho)^2\left[\frac{1-(1/\beta^*)^{2(t-1)}}{1-(1/\beta^*)^2}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$+\frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\frac{\sigma_\mu^2}{\nu^2}+\left(q_2+q_3\right)^2\sigma_\eta^2\right]+\frac{1}{2}\frac{1}{1-\beta}\alpha_g\left[1-2\left(\frac{1/\alpha_g}{P_M^*}\right)\left(q_2+q_3\right)\right]\sigma_\eta^2$$

$$=\ \frac{1}{2}\frac{1}{1-\beta}\left[\frac{(\beta^*(1+\rho))^2}{(\beta^*)^2-1}\right]\left[\frac{\left(\psi_0^C\right)^2}{P}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$-\frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\frac{(1/\beta^*)^{2(t-1)}}{1-(1/\beta^*)^2}\right]\left[\frac{\left(\psi_0^C\right)^2}{P}\right](1+\rho)^2q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$+\frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\frac{\sigma_\mu^2}{\nu^2}+\frac{1}{n}\left(q_2+q_3\right)^2\sigma_\eta^2\right]+\frac{1}{2}\frac{1}{1-\beta}\alpha_g\left[1-2\left(\frac{1/\alpha_g}{P_M^*}\right)\left(q_2+q_3\right)\right]\sigma_\eta^2,$$

where" " is used to denote that terms not containing $\alpha_{\pi M}$ have been dropped. Note that

$$\frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\frac{(1/\beta^*)^{2(t-1)}}{1-(1/\beta^*)^2}\right]=\frac{1}{2}\frac{1}{1-(1/\beta^*)^2}\sum_{t=1}^{\infty}\left[\frac{1}{\beta^*(1+\rho)}\right]^{t-1}=\frac{1}{2}\frac{(\beta^*)^2}{(\beta^*)^2-1}\frac{1}{\psi_0^C}. \qquad (108)$$

Hence,

$$\frac{1}{2}\frac{1}{1-\beta}\left[\frac{(\beta^*(1+\rho))^2}{(\beta^*)^2-1}\right]\left[\frac{\left(\psi_0^C\right)^2}{P}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$-\frac{1}{2}\frac{[\beta^*(1+\rho)]^2}{(\beta^*)^2-1}\left[\frac{\psi_0^C}{P}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$+\frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\frac{\sigma_\mu^2}{\nu^2}+\left(q_2+q_3\right)^2\sigma_\eta^2\right]+\frac{1}{2}\frac{1}{1-\beta}\alpha_g\left[1-2\left(\frac{1/\alpha_g}{P_M^*}\right)\left(q_2+q_3\right)\right]\sigma_\eta^2$$

$$=\ \frac{1}{2}\frac{1}{1-\beta}\frac{[\beta^*(1+\rho)-1]^2}{(\beta^*)^2-1}\left[\frac{1}{P}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$-\frac{1}{2}\frac{[\beta^*(1+\rho)][\beta^*(1+\rho)-1]}{(\beta^*)^2-1}\left[\frac{1}{P}\right]q_1^2\left[\frac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\frac{P_M^*}{P}\right)^2\sigma_\eta^2\right]$$
$$+\frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left[q_2^2\frac{\sigma_\mu^2}{\nu^2}+\left(q_2+q_3\right)^2\sigma_\eta^2\right]+\frac{1}{2}\frac{1}{1-\beta}\alpha_g\left[1-2\left(\frac{1/\alpha_g}{P_M^*}\right)\left(q_2+q_3\right)\right]\sigma_\eta^2,$$

which equals

$$
\begin{aligned}
& \tfrac{1}{2}\left[\tfrac{1}{P}\right] q_1^2 \left[\tfrac{\sigma_\mu^2}{\nu^2} + \left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right] \tfrac{[\beta^*(1+\rho)-1]^2-(1-\beta)\beta^*(1+\rho)[\beta^*(1+\rho)-1]}{(1-\beta)\left[(\beta^*)^2-1\right]} \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{Q_M}{\left(P_M^*\right)^2}\left[q_2^2\tfrac{\sigma_\mu^2}{\nu^2} + (q_2+q_3)^2\sigma_\eta^2\right] + \tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2 \\
= & \tfrac{1}{2}\left[\tfrac{1}{P}\right] q_1^2 \left[\tfrac{\sigma_\mu^2}{\nu^2} + \left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right] \tfrac{\beta^*(1+\rho)-1}{1-\beta} \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{Q_M}{\left(P_M^*\right)^2}\left[q_2^2\tfrac{\sigma_\mu^2}{\nu^2} + (q_2+q_3)^2\sigma_\eta^2\right] + \tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2
\end{aligned}
$$

$$
\begin{aligned}
= & \tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{\sigma_\mu^2}{\nu^2}\left[\tfrac{\beta^*(1+\rho)-1}{P}q_1^2 + \tfrac{Q_M}{\left(P_M^*\right)^2}q_2^2\right] \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\sigma_\eta^2\left[\tfrac{\beta^*(1+\rho)-1}{P}\left(1-\beta^*\tfrac{P_M^*}{P}\right)^2 q_1^2 + \tfrac{Q_M}{\left(P_M^*\right)^2}(q_2+q_3)^2\right] \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2 \\
= & \tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{\sigma_\mu^2}{\nu^2}\left[\tfrac{\beta^*(1+\rho)-1}{P}q_1^2 + \tfrac{Q_M}{P^2}\left[\beta^*(1+\rho)-1\right]^2 q_1^2\right] \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\sigma_\eta^2\left[\tfrac{\beta^*(1+\rho)-1}{P}\left(1-\beta^*\tfrac{P_M^*}{P}\right)^2 q_1^2 + \tfrac{Q_M}{\left(P_M^*\right)^2}\left(\tfrac{P_M^*}{P}\right)^2\left[\beta^*(2+\rho)-1\right]^2 q_1^2\right] \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3)\right]\sigma_\eta^2 \\
= & \tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{\sigma_\mu^2}{\nu^2}\tfrac{\beta^*(1+\rho)-1}{P}U_1 + \tfrac{1}{2}\tfrac{1}{1-\beta}\sigma_\eta^2 U_2 + \tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\sigma_\eta^2 U_3.
\end{aligned}
$$

where

$$
\begin{aligned}
U_1 &\equiv q_1^2\left\{1+\left[\tfrac{Q_M}{P}\right]\left[\beta^*(1+\rho)-1\right]\right\} = \tfrac{P+[\beta^*(1+\rho)-1]Q_M}{P\left[1+\left(P_M^*/P\right)(\beta^*(1+\rho)-1)\right]^2}, \\
U_2 &\equiv q_1^2\left\{\tfrac{\beta^*(1+\rho)-1}{P}\left[1-\beta^*\left(\tfrac{P_M^*}{P}\right)\right]^2 + \left[\tfrac{Q_M}{P^2}\right]\left[\beta^*(2+\rho)-1\right]^2\right\} \\
&= \tfrac{P[\beta^*(1+\rho)-1]\left[1-\beta^*\left(P_M^*/P\right)\right]^2+[\beta^*(2+\rho)-1]^2 Q_M}{P^2\left[1+\left(P_M^*/P\right)(\beta^*(1+\rho)-1)\right]^2}, \\
U_3 &\equiv 1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)(q_2+q_3) = 1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\tfrac{\left(P_M^*/P\right)[\beta^*(2+\rho)-1]}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)-1]} \\
&= 1-2\left(\tfrac{1/\alpha_g}{P}\right)\left[\tfrac{\beta^*(2+\rho)-1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)-1]}\right].
\end{aligned}
$$

We will now investigate each of the terms U_1, U_2 and U_3. For U_1, we have:

$$
\begin{aligned}
sgn\left(\frac{\partial U_1}{\partial \alpha_{\pi M}}\right) &= sgn\left\{\frac{-\frac{2\alpha_\pi}{\alpha_{\pi M}^3}\left[1+\frac{P_M^*}{P}\left[\beta^*(1+\rho)-1\right]\right]+\frac{2}{\alpha_{\pi M}^2}\left[1+\frac{Q_M}{P}\left[\beta^*(1+\rho)-1\right]\right]}{\left[1+\left(P_M^*/P\right)\left(\beta^*(1+\rho)-1\right)\right]^3}\right\} \\
&= sgn\left\{\frac{\frac{1}{\alpha_{\pi M}^2}-\frac{\alpha_\pi}{\alpha_{\pi M}^3}}{\left[1+\left(P_M^*/P\right)\left(\beta^*\left(1+\rho\right)-1\right)\right]^3}\right\},
\end{aligned}
$$

where $sgn\,(x)$ stands for the sign of expression x. Hence, U_1 is decreasing in $\alpha_{\pi M}$ for $\alpha_{\pi M} < \alpha_\pi$, increasing in $\alpha_{\pi M}$ for $\alpha_{\pi M} > \alpha_\pi$ and reaches its strict global minimum at $\alpha_{\pi M} = \alpha_\pi$. For U_2, we find, after quite a substantial amount of straightforward algebra, that:

$$
\begin{aligned}
& sgn\left(\frac{\partial U_2}{\partial \alpha_{\pi M}}\right) \\
= \;& sgn\left\{\frac{2}{\alpha_{\pi M}^2}\frac{P\left[\beta^*(2+\rho)-1\right]\left\{\begin{array}{c}-\frac{\beta^*}{\alpha_{\pi M}}\left[\beta^*\left(1+\rho\right)-1\right]-\left(\beta^*\right)^2\left(1+\rho\right)P \\ +P\left(1-\frac{\alpha_\pi}{\alpha_{\pi M}}\right)\beta^*\left(1+\rho\right)\left[\beta^*\left(2+\rho\right)-1\right]\end{array}\right\}}{\left[1+\left(P_M^*/P\right)\left(\beta^*(1+\rho)-1\right)\right]^3}\right\}.
\end{aligned}
$$

Hence, U_2 is strictly decreasing for $\alpha_{\pi M} < \alpha_\pi$. Finally, we see immediately that U_3 is decreasing in $\alpha_{\pi M}$.

E.2 Discretion

Using (107) and its lags, we can write:

$$
\begin{aligned}
F_t &= \left(1+\rho\right)d_{t-1}+G_t \\
&= \left(1+\rho\right)\left\{\frac{1}{\beta^*(Q_M/P)}d_{t-2}+\frac{G_{t-1}-\beta^*(Q_M/P)G_t}{\beta^*(1+\rho)(Q_M/P)}+p_1\left[\frac{\mu_{t-1}}{\nu}+\left(1-\beta^*\frac{P_M^*}{P}\right)\eta_{t-1}\right]\right\}+G_t \\
&= \left(1+\rho\right)\frac{1}{\beta^*(Q_M/P)}d_{t-2}+\left(1+\rho\right)p_1\left[\frac{\mu_{t-1}}{\nu}+\left(1-\beta^*\frac{P_M^*}{P}\right)\eta_{t-1}\right]+\frac{1}{\beta^*(Q_M/P)}G_{t-1} \\
&= \left(1+\rho\right)\frac{1}{\beta^*(Q_M/P)}\left\{\frac{1}{\beta^*(Q_M/P)}d_{t-3}+\frac{G_{t-2}-\beta^*(Q_M/P)G_{t-1}}{\beta^*(1+\rho)(Q_M/P)}+p_1\left[\frac{\mu_{t-2}}{\nu}+\left(1-\beta^*\frac{P_M^*}{P}\right)\eta_{t-2}\right]\right\} \\
&\quad +\left(1+\rho\right)p_1\left[\frac{\mu_{t-1}}{\nu}+\left(1-\beta^*\frac{P_M^*}{P}\right)\eta_{t-1}\right]+\frac{1}{\beta^*(Q_M/P)}G_{t-1}
\end{aligned}
$$

$$
\begin{aligned}
&= (1+\rho)\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^2 d_{t-3} + \tfrac{1}{\beta^*(Q_M/P)}(1+\rho)\,p_1\left[\tfrac{\mu_{t-2}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-2}\right] \\
&\quad + (1+\rho)\,p_1\left[\tfrac{\mu_{t-1}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-1}\right] + \left(\tfrac{1}{\beta^*(Q_M/P)}\right)^2 G_{t-2} \\
&= \left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{t-1}\left[(1+\rho)\,d_0 + G_1\right] + \\
&\quad (1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{\xi-1} p_1\left[\tfrac{\mu_{t-\xi}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right].
\end{aligned}
$$

We can now combine this with (104), (105) and (106) to give:

$$
\begin{aligned}
\pi_t &= \left[\tfrac{1/\alpha_{\pi M}}{P}\right]\psi_0^P(1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{\xi-1} p_1\left[\tfrac{\mu_{t-\xi}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right] \\
&\quad + \left[\tfrac{1/\alpha_{\pi M}}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right) + p_2\eta_t\right] + \left[\tfrac{1/\alpha_{\pi M}}{P}\right]\psi_0^P\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{t-1}\left[(1+\rho)\,d_0 + G_1\right], \\
\tilde{x}_t - x_t &= \left[\tfrac{1/\nu}{P}\right]\psi_0^P(1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{\xi-1} p_1\left[\tfrac{\mu_{t-\xi}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right] \\
&\quad + \left[\tfrac{1/\nu}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right) + p_2\eta_t\right] + \left[\tfrac{1/\nu}{P}\right]\psi_0^P\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{t-1}\left[(1+\rho)\,d_0 + G_1\right], \\
\tilde{g}_t - g_t &= \left[\tfrac{1/\alpha_g}{P}\right]\psi_0^P(1+\rho)\sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{\xi-1} p_1\left[\tfrac{\mu_{t-\xi}}{\nu} + \left(1-\beta^*\tfrac{P_M^*}{P}\right)\eta_{t-\xi}\right] \\
&\quad + \left[\tfrac{1/\alpha_g}{P_M^*}\right]\left[\psi_1^P\left(\tfrac{\mu_t}{\nu}+\eta_t\right) + p_2\eta_t\right] + \left[\tfrac{1/\alpha_g}{P}\right]\psi_0^P\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{t-1}\left[(1+\rho)\,d_0 + G_1\right] \\
&\quad -\eta_t,
\end{aligned}
$$

Let us now compute society's expected loss:

$$
\begin{aligned}
& \mathrm{E}_0\left[\alpha_\pi \pi_t^2+\left(x_t-\tilde{x}_t\right)^2+\alpha_g\left(g_t-\tilde{g}_t\right)^2\right] \\
= & \left[\tfrac{Q_M}{P^2}\right]\left\{\psi_0^P\left(\tfrac{1}{\beta^*\left(Q_M/P\right)}\right)^{t-1}\left[(1+\rho)\, d_0+G_1\right]\right\}^2+ \\
& \left[\tfrac{Q_M}{P^2}\right](1+\rho)^2\left(\psi_0^P\right)^2 \sum_{\xi=1}^{t-1}\left(\tfrac{1}{\beta^*\left(Q_M/P\right)}\right)^{2(\xi-1)} p_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right]+ \\
& \left[\tfrac{Q_M}{\left(P_M^*\right)^2}\right]\left[\left(\psi_1^P\right)^2 \tfrac{\sigma_\mu^2}{\nu^2}+\left(\psi_1^P+p_2\right)^2 \sigma_\eta^2\right]+\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right] \sigma_\eta^2 \\
= & \left[\tfrac{Q_M}{P^2}\right]\left\{\psi_0^P\left(\tfrac{1}{\beta^*\left(Q_M/P\right)}\right)^{t-1}\left[(1+\rho)\, d_0+G_1\right]\right\}^2+ \\
& \left[\tfrac{Q_M}{P^2}\right](1+\rho)^2\left(\psi_0^P\right)^2\left[\tfrac{1-\left(1/\left(\beta^* Q_M/P\right)\right)^{2(t-1)}}{1-\left(1/\left(\beta^* Q_M/P\right)\right)^2}\right] p_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right]+ \\
& \left[\tfrac{Q_M}{\left(P_M^*\right)^2}\right]\left[\left(\psi_1^P\right)^2 \tfrac{\sigma_\mu^2}{\nu^2}+\left(\psi_1^P+p_2\right)^2 \sigma_\eta^2\right]+\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right] \sigma_\eta^2 .
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \tfrac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_0\left[\alpha_\pi \pi_t^2+\left(x_t-\tilde{x}_t\right)^2+\alpha_g\left(g_t-\tilde{g}_t\right)^2\right] \\
= & \tfrac{1}{2} \tfrac{1}{\beta^*(1+\rho)}\left[(1+\rho)\, d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\left[\tfrac{\left(\beta^*(1+\rho) Q_M/P-1\right)^2}{\beta^*(1+\rho)\left(Q_M/P\right)^2-1}\right]+ \\
& \tfrac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1}\left[\tfrac{Q_M}{P^2}\right](1+\rho)^2\left(\psi_0^P\right)^2\left[\tfrac{1-\left(1/\left(\beta^* Q_M/P\right)\right)^{2(t-1)}}{1-\left(1/\left(\beta^* Q_M/P\right)\right)^2}\right] p_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right] \\
& +\tfrac{1}{2} \tfrac{1}{1-\beta}\left[\tfrac{Q_M}{\left(P_M^*\right)^2}\right]\left[\left(\psi_1^P\right)^2 \tfrac{\sigma_\mu^2}{\nu^2}+\left(\psi_1^P+p_2\right)^2 \sigma_\eta^2\right]+\tfrac{1}{2} \tfrac{1}{1-\beta} \alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right] \sigma_\eta^2 \\
= & \tfrac{1}{2} \tfrac{1}{\beta^*(1+\rho)}\left[(1+\rho)\, d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\left[\tfrac{\left(\beta^*(1+\rho) Q_M/P-1\right)^2}{\beta^*(1+\rho)\left(Q_M/P\right)^2-1}\right]+ \\
& \tfrac{1}{2} \tfrac{1}{1-\beta} \tfrac{\left(\beta^* Q_M/P\right)^2}{\beta^*(1+\rho)\left(Q_M/P\right)^2-1}\left[\tfrac{Q_M}{P^2}\right](1+\rho)^2\left(\psi_0^P\right)^2 p_1^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^* \tfrac{P_M^*}{P}\right)^2 \sigma_\eta^2\right] \\
& +\tfrac{1}{2} \tfrac{1}{1-\beta}\left[\tfrac{Q_M}{\left(P_M^*\right)^2}\right]\left[\left(\psi_1^P\right)^2 \tfrac{\sigma_\mu^2}{\nu^2}+\left(\psi_1^P+p_2\right)^2 \sigma_\eta^2\right]+\tfrac{1}{2} \tfrac{1}{1-\beta} \alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right] \sigma_\eta^2,
\end{aligned}
$$

where we have used that:

$$
\begin{aligned}
& \tfrac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\tfrac{Q_M}{P^2}\right]\left\{\psi_0^P\left(\tfrac{1}{\beta^*(Q_M/P)}\right)^{t-1}\left[(1+\rho)\,d_0+G_1\right]\right\}^2 \\
= \; & \tfrac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\tfrac{Q_M}{P^2}\right]\left(\psi_0^P\right)^2\left[(1+\rho)\,d_0+G_1\right]^2\left[\tfrac{1}{\beta^*(Q_M/P)}\right]^{2(t-1)} \\
= \; & \tfrac{1}{2}\left(\psi_0^P\right)^2\left[(1+\rho)\,d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\sum_{t=1}^{\infty}\left[\tfrac{\beta}{(\beta^* Q_M/P)^2}\right]^{t-1} \\
= \; & \tfrac{1}{2}\left(\psi_0^P\right)^2\left[(1+\rho)\,d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\tfrac{\beta^*(1+\rho)(Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1} \\
= \; & \tfrac{1}{2}\left[(1+\rho)\,d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\left[\tfrac{1}{\beta^*(1+\rho)}\right]\tfrac{[\beta^*(1+\rho)(Q_M/P)-1]^2}{\beta^*(1+\rho)(Q_M/P)^2-1},
\end{aligned}
$$

and

$$
\begin{aligned}
& \tfrac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left[\tfrac{1-(1/(\beta^* Q_M/P))^{2(t-1)}}{1-(1/(\beta^* Q_M/P))^2}\right] \\
= \; & \tfrac{1}{2}\left[\tfrac{1}{1-(P/(\beta^* Q_M))^2}\right]\sum_{t=1}^{\infty}\left\{\beta^{t-1}-\left[\tfrac{1}{\beta^*(1+\rho)(Q_M/P)^2}\right]^{t-1}\right\} \\
= \; & \tfrac{1}{2}\left[\tfrac{1}{1-(P/(\beta^* Q_M))^2}\right]\left\{\tfrac{1}{1-\beta}-\left[\tfrac{\beta^*(1+\rho)(Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\right\} \\
= \; & \tfrac{1}{2}\left[\tfrac{1}{1-(P/(\beta^* Q_M))^2}\right]\tfrac{\left[\beta^*(1+\rho)(Q_M/P)^2-1\right]-\beta^*(1+\rho)(Q_M/P)^2(1-\beta)}{(1-\beta)\left[\beta^*(1+\rho)(Q_M/P)^2-1\right]} \\
= \; & \tfrac{1}{2}\left[\tfrac{1}{1-(P/(\beta^* Q_M))^2}\right]\tfrac{(\beta^* Q_M/P)^2-1}{(1-\beta)\left[\beta^*(1+\rho)(Q_M/P)^2-1\right]} \\
= \; & \tfrac{1}{2}\left[\tfrac{(\beta^* Q_M/P)^2}{(\beta^* Q_M/P)^2-1}\right]\tfrac{(\beta^* Q_M/P)^2-1}{(1-\beta)\left[\beta^*(1+\rho)(Q_M/P)^2-1\right]} \\
= \; & \tfrac{1}{2}\tfrac{1}{1-\beta}\tfrac{(\beta^* Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}.
\end{aligned}
$$

We can rewrite the expression for society's expected loss further as:

$$
\begin{aligned}
& \tfrac{1}{2}\tfrac{1}{\beta^*(1+\rho)}\left[(1+\rho)\,d_0+G_1\right]^2\left[\tfrac{Q_M}{P^2}\right]\left[\tfrac{(\beta^*(1+\rho)Q_M/P-1)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]+ \\
& \tfrac{1}{2}\tfrac{1}{1-\beta}\left[\tfrac{Q_M/P^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\left[\tfrac{\beta^*(1+\rho)(Q_M/P)-1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]^2\left[\tfrac{\sigma_\mu^2}{\nu^2}+\left(1-\beta^*\tfrac{P_M^*}{P}\right)^2\sigma_\eta^2\right] \\
& +\tfrac{1}{2}\tfrac{1}{1-\beta}\left[\tfrac{Q_M}{\left(P_M^*\right)^2}\right]\left[\left(\psi_1^P\right)^2\tfrac{\sigma_\mu^2}{\nu^2}+\left(\psi_1^P+p_2\right)^2\sigma_\eta^2\right]+\tfrac{1}{2}\tfrac{1}{1-\beta}\alpha_g\left[1-2\left(\tfrac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right]\sigma_\eta^2,
\end{aligned}
$$

where we have used that:

$$
\begin{aligned}
& \frac{(\beta^* Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\left[\frac{Q_M}{P^2}\right](1+\rho)^2\left(\psi_0^P\right)^2 p_1^2 \\
= \;& \frac{(\beta^* Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\left[\frac{Q_M}{P^2}\right](1+\rho)^2\left[\frac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}\right]^2\left[\frac{1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]^2 \\
= \;& \left[\frac{Q_M/P^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\left[\frac{\beta^*(1+\rho)(Q_M/P)-1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]^2.
\end{aligned}
$$

Hence, society's expected loss can be written as:

$$
V_1 + V_2\frac{\sigma_\mu^2}{\nu^2} + V_3\sigma_\eta^2,
$$

$$
\begin{aligned}
V_1 \;&\equiv\; \frac{1}{2}\frac{1}{\beta^*(1+\rho)}\left[(1+\rho)\,d_0+G_1\right]^2\left[\frac{Q_M}{P^2}\right]\left[\frac{(\beta^*(1+\rho)Q_M/P-1)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right], \\
V_2 \;&\equiv\; \frac{1}{2}\frac{1}{1-\beta}\left\{\left[\frac{Q_M/P^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\left[\frac{\beta^*(1+\rho)(Q_M/P)-1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]^2+\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left(\psi_1^P\right)^2\right\} \\
&=\; \frac{1}{2}\frac{1}{1-\beta}\left\{\left[\frac{Q_M/P^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]+\left[\frac{Q_M}{P^2}\right]\right\}\left[\frac{\beta^*(1+\rho)(Q_M/P)-1}{1+\left(P_M^*/P\right)[\beta^*(1+\rho)(Q_M/P)-1]}\right]^2 \\
&=\; \frac{1}{2}\frac{1}{1-\beta}\frac{Q_M}{\left(P_M^*\right)^2}\left[\frac{\beta^*(1+\rho)(Q_M/P)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\left(\psi_1^P\right)^2, \\
V_3 \;&\equiv\; \frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M/\left(P_M^*\right)^2}{\beta^*(1+\rho)(Q_M/P)^2-1}\right]\left(\psi_1^P\right)^2\left(1-\beta^*\frac{P_M^*}{P}\right)^2 \\
&\quad +\frac{1}{2}\frac{1}{1-\beta}\left[\frac{Q_M}{\left(P_M^*\right)^2}\right]\left(\psi_1^P+p_2\right)^2+\frac{1}{2}\frac{1}{1-\beta}\alpha_g\left[1-2\left(\frac{1/\alpha_g}{P_M^*}\right)\left(\psi_1^P+p_2\right)\right].
\end{aligned}
$$

These expressions are programmed, so that we can explore society's expected loss as a function of $\alpha_{\pi M}$.