Appendix for: "Political Shocks, Public Debt and the Design of Monetary and Fiscal Institutions"

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Appendix for “Political Shocks, Public Debt and the Design of Monetary and Fiscal Institutions” by Beetsma and Bovenberg

Notation: for a generic variable $y$, we define $y_t = E_{t-1} [y]$ and $y_t^d = y_t - y_t^e$.

A Derivation of infinite-horizon commitment solution.

The central bank selects $\pi_t$ so as to minimize:

$$\frac{1}{2} \left[ \alpha_M (\pi_t - \pi_t^g) + \left[ \nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t \right]^2 \right] + \theta_t \left[ E_{t-1} (\pi_t - \pi_t^g) + \beta E_t \left[ L^{CB}_{t+1} \right] \right]. \tag{1}$$

where $\pi_t^g$ depends only on the shocks $\mu_t$ and $\eta_t$, so that $E_{t-1} (\pi_t^g) = 0$. The CB’s first-order conditions for $\pi_t$ and $\pi_t^g$ can be written as, respectively:

$$\alpha_M (\pi_t - \pi_t^g) + \nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t + \theta_t = 0,$$

$$E_{t-1} [\nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t] + \theta_t = 0,$$

which can be combined to give:

$$\alpha_M (\pi_t - \pi_t^g) + \nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t - E_{t-1} [\nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t] = 0. \tag{2}$$

The government, which is of the type $\eta_t$ in period $t$ selects $\tau_t$ and $d_t$ so as to minimise:

$$V_t^{G_t} = \frac{1}{2} \sum_{\xi = t}^{\infty} \beta^{\xi-t} E_{\xi} \left[ \frac{\alpha_g \pi_{\xi}^2 + \left[ \nu (\pi_{\xi} - \pi_{\xi}^g - \tau_{\xi}) - \mu_{\xi} - \tilde{x}_{\xi} \right]^2 + \alpha_g \left[ - (1 + \rho) \delta_{\xi-1} + \tau_{\xi} + d_{\xi} - (\tilde{g}_{\xi} + \eta_{\xi}) \right]^2 \right], \tag{3}$$

where superscript “$G_t$” indicates that losses are evaluated according to the loss function of the party that is in power in period $t$. The first-order conditions for $\tau_t$ and $d_t$ are:

$$-\nu \left[ \nu (\pi_t - \pi_t^g - \tau_t) - \mu_t - \tilde{x}_t \right] + \alpha_g \left[ g_t - (\tilde{g}_t + \eta_t) \right] = 0, \tag{4}$$
\[ \alpha_g \left[ (\tilde{g}_t + \eta_t) - g_t \right] = \beta \left[ \partial E_t \left( V^\tau_{t+1} \right) / \partial d_t \right], \tag{5} \]

\[ g_t + (1 + \rho) d_{t-1} = \tau_t + d_t, \tag{6} \]

and the transversality condition that:

\[ \lim_{\xi \to \infty} \left( \frac{1}{1+\rho} \right)^{\xi-t} d_{\xi+1} = 0, \tag{7} \]

with probability 1. The complete system of equations to be used to solve for the outcomes is (2), (4), (5), (6) and (7). We solve the system first for given debt policies, after which we also solve for the debt policy.

### A.1 Derivation of outcomes for given debt policies

We first derive the deterministic components of the outcomes (step 1). Then, we derive the responses to the shocks (step 2).

**Step 1**: take as-of-the-start-of-period-\(t\) expectations of the system (2), (4) and (6) to give:

\[ \alpha_{\pi M} \pi_t^e = 0, \tag{8} \]

\[ \nu^2 \left( \pi_t^e + \hat{\pi}_t \right) + \alpha_g \left( g_t^e - \tilde{g}_t \right) = 0, \tag{9} \]

\[ g_t^e + (1 + \rho) d_{t-1} = \tau_t^e + d_t^e. \tag{10} \]

The solution is:

\[ \pi_t^e = 0, \tag{11} \]

\[ \tilde{x}_t - x_t^e = \left[ 1/\nu \right] \left[ K_t + (1 + \rho) d_{t-1} - d_t^e \right], \tag{12} \]

\[ \tilde{g}_t - g_t^e = \left[ 1/1+\alpha_g \right] \left[ K_t + (1 + \rho) d_{t-1} - d_t^e \right], \tag{13} \]

where, as defined in the main text,

\[ P = 1/\nu^2 + 1/\alpha_g. \tag{14} \]

**Step 2**: Subtract (8), (9) and (10) from (2), (4) and (6), respectively, to obtain:
\[(\alpha_{\pi M} + \nu^2) \pi_t^d - \alpha_{\pi M} \pi_t^* - \nu^2 (\pi_t^d + \frac{\mu_t}{\nu}) = 0, \] (15)

\[-\nu^2 \pi_t^d + \nu^2 (\pi_t^d + \frac{\mu_t}{\nu}) + \alpha_g (g_t^d - \eta_t) = 0, \] (16)

\[g_t^d = \pi_t^d + \alpha_g. \] (17)

Combine (15) and (16) to eliminate \(\pi_t^d\) and obtain:

\[\pi_t^d + \frac{\mu_t}{\nu} = \pi_t^* + \left(\frac{1}{\alpha_{\pi M}} + 1/\nu^2\right) \alpha_g (\eta_t - g_t^d), \]

which can be combined with (17) to give:

\[g_t^d - \eta_t = \left[\frac{1/\alpha_{\pi M}}{1/\nu^2}\right] \pi_t^* + \left[\frac{1/\alpha_{\pi M}}{1/\nu^2}\right] \left[\pi_t^* + \alpha_g (\eta_t - g_t^d)\right], \] (18)

where, as defined in the main text,

\[P_M^* = \frac{1}{\alpha_{\pi M} + 1/\nu^2 + 1/\alpha_g}. \] (19)

Hence,

\[\pi_t^d + \frac{\mu_t}{\nu} = \pi_t^* - \left[\frac{1/\alpha_{\pi M} + 1/\nu^2}{1/\nu^2}\right] \left[\pi_t^* + \alpha_g (\eta_t - g_t^d)\right]. \] (20)

Hence,

\[x_t^d = \left[\frac{1/\nu}{1/\nu^2}\right] \left[\pi_t^* + \alpha_g (\eta_t - g_t^d)\right]. \] (21)

Using (11), (12), (13), (18), (20) and (21) we obtain:

\[\pi_t = \pi_t^* + \pi_t^d = \pi_t^* + \left[\frac{1/\alpha_{\pi M}}{1/\nu^2}\right] \left[\pi_t^* + \alpha_g (\eta_t - g_t^d - \pi_t^*)\right], \] (22)

\[\tilde{x}_t - x_t = \left[\frac{1/\nu}{1/\nu^2}\right] \left[K_t + (1 + \rho) d_t - \pi_t^*\right] + \left[\frac{1/\nu}{1/\nu^2}\right] \left[(\frac{\mu_t}{\nu} + \eta_t) - d_t^d - \pi_t^*\right], \] (23)

\[(\tilde{\eta}_t + \eta_t) - \eta_t = \left[\frac{1/\alpha_{\pi M}}{1/\nu^2}\right] \left[K_t + (1 + \rho) d_t - \pi_t^*\right] + \left[\frac{1/\alpha_{\pi M}}{1/\nu^2}\right] \left[(\frac{\mu_t}{\nu} + \eta_t) - d_t^d - \pi_t^*\right]. \] (24)
A.2 Derivation of the solution for public debt

We are now in a position to characterize debt policy. From now on in Appendix A, we set \( \pi_t^* = 0 \). To evaluate \( \partial E_t (V_{t+1}^{G_t}) / \partial d_t \), forward (22), (23) and (24) by one period and substitute the resulting expressions into government \( G_t \)'s expected loss in period \( t + 1 \),

\[
\frac{1}{2} E_t \left[ \alpha_x \pi_{t+1}^2 + (x_{t+1} - \tilde{x}_{t+1})^2 + \alpha_g (g_{t+1} - (\tilde{g}_{t+1} + \eta_t))^2 \right] .
\]

The derivative with respect to \( d_t \) of the expression thus obtained is:

\[
E_t \left[ (\tilde{x}_{t+1} - x_{t+1}) (1 + \rho) \left[ \frac{1}{ \beta_1 } \right] + (\tilde{g}_{t+1} + \eta_t - g_{t+1}) (1 + \rho) \left[ \frac{1}{ \beta_2 } \right] \right] .
\]

Hence, combining this with (5), we obtain:

\[
\alpha_g (\tilde{g}_t + \eta_t - g_t) = \beta^a E_t \left[ (\tilde{x}_{t+1} - x_{t+1}) \left[ \frac{1}{ \beta_1 } \right] + (\tilde{g}_{t+1} + \eta_t - g_{t+1}) \left[ \frac{1}{ \beta_2 } \right] \right] .
\]

Combine this with (24) and the one-period forwarded expressions of (23) and (24), to give:

\[
K_t + (1 + \rho) d_{t-1} - d_t^f = \beta^a \left\{ \left[ K_{t+1} + (1 + \rho) d_t - d_t^f \right] - \left[ \frac{1}{ \beta_2 } \right] \left[ \frac{1}{ \beta_2 } \right] \right\} .
\]

Hence,

\[
K_t + (1 + \rho) d_{t-1} - d_t^f = \beta^a \left\{ \left[ K_{t+1} + (1 + \rho) d_t - d_t^f \right] - \left[ \frac{1}{ \beta_2 } \right] \left[ \frac{1}{ \beta_2 } \right] \right\} ,
\]

where we have used that \( E_t [\eta_{t+1}] = 0 \). We solve (25) in two steps. 

**Step 1**: take expectations of (25) as of end of period \( t - 1 \):

\[
K_t + (1 + \rho) d_{t-1} - d_t^f = \beta^a [K_{t+1} + (1 + \rho) d_t^f - E_{t-1} d_{t+1}] .
\]

The solution is:

\[
d_t^f = \frac{K_t + (1 + \rho) d_{t-1} - \beta^a [K_{t+1} - E_{t-1} (d_{t+1})]}{1 + \beta^a (1 + \rho)} .
\]

An explicit solution for \( d_t^f \) will be derived later on.
Step 2: Subtract (26) from (25):

\[
\left[ \frac{P}{P_{M}^*} \right] \left[ (\frac{\mu}{\nu} + \eta_t) - d_t^d \right] = \beta^* (1 + \rho) d_t^d + \beta^* (E_{t-1}d_{t+1} - E_t d_{t+1}) + \beta^* \eta_t. \tag{28}
\]

Hence,

\[
d_t^d = \left[ \frac{1}{1 + \beta^*(1 + \rho) (P_{M}^*/P)} \right] \left( \frac{\mu}{\nu} + \eta_t \right) + \left[ \frac{\beta^* (P_{M}^*/P)}{1 + \beta^*(1 + \rho) (P_{M}^*/P)} \right] \left[ E_t (d_{t+1}) - E_{t-1} (d_{t+1}) - \eta_t \right]. \tag{29}
\]

We can find the final solution for \( d_t^d \) as follows. Forward (27) by \( \xi \geq 1 \) periods. Next, take expectations as of end-of-periods \( t \) and \( t - 1 \) and subtract the latter from the former, to obtain:

\[
E_t (d_{t+\xi}) - E_{t-1} (d_{t+\xi}) = \frac{(1 + \rho) [E_t (d_{t+\xi-1}) - E_{t-1} (d_{t+\xi-1})] + \beta^* [E_t (d_{t+\xi+1}) - E_{t-1} (d_{t+\xi+1})]}{1 + \beta^*(1 + \rho)}. \tag{30}
\]

Guess that \( E_t (d_{t+\xi+1}) - E_{t-1} (d_{t+\xi+1}) = \varphi [E_t (d_{t+\xi}) - E_{t-1} (d_{t+\xi})] \), \( \forall \xi \geq 1 \), where \( \varphi \) is a constant to be solved for. Substitute this into (30) and rewrite the result to yield the following equation in \( \varphi \):

\[
\beta^* \varphi^2 - [1 + \beta^* (1 + \rho)] \varphi + (1 + \rho) = 0.
\]

This equation yields two solutions: \( \varphi = 1 + \rho \), which is excluded, because it violates (7),\(^1\) and \( \varphi = 1/\beta^* \). Using this solution, for \( \xi = 1 \) we can write (30) as:

\[
E_t (d_{t+1}) - E_{t-1} (d_{t+1}) = \frac{1}{\beta^*} d_t^d.
\]

Substitute this back into expression (29) for \( d_t^d \) and rewrite to give:

\[
d_t^d = \left[ \frac{1}{1 + (P_{M}^*/P) [\beta^*(1 + \rho) - 1]} \right] \left[ \frac{\mu}{\nu} + (1 - \beta^* (P_{M}^*/P)) \eta_t \right]. \tag{31}
\]

\(^1\)We have that \( E_t (d_{t+\xi}) - E_{t-1} (d_{t+\xi}) = (1 + \rho)^\xi [E_t (d_t) - E_{t-1} (d_t)] \), so that the effect of a shock in period \( t \) on public debt “explodes” over time. Taking \( \lim_{\xi \to \infty} \left( \frac{1}{1 + \rho} \right)^\xi [E_t (d_{t+\xi}) - E_{t-1} (d_{t+\xi})] = (1 + \rho) [E_t (d_t) - E_{t-1} (d_t)] \), which is generally non-zero.
A.2.1 Derivation of an explicit solution for $E_{t-1}d_t$

Using (12), we have that:

\[
\ddot{x}_{t+\xi} - E_{t-1}d_{t+\xi} = \left[ \frac{1}{\beta} \right] [K_t + \xi + (1 + \rho) E_{t-1}d_{t+\xi-1} - E_{t-1}d_{t+\xi}] \Rightarrow
\]

\[
\ddot{x}_{t+\xi+1} - E_{t-1}d_{t+\xi+1} = \left[ \frac{1}{\beta} \right] [K_{t+\xi+1} + (1 + \rho) E_{t-1}d_{t+\xi} - E_{t-1}d_{t+\xi+1}],
\]

where $\xi \geq 0$. Note that

\[
K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1} - E_{t-1}d_{t+\xi} = \frac{[K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1}] - [K_{t+\xi+1} + (1 + \rho) E_{t-1}d_{t+\xi-1} + \beta^\pi (K_{t+\xi+1} - E_{t-1}d_{t+\xi+1})]}{1 + \beta^\pi (1 + \rho)}
\]

\[
= \beta^\pi \left[ \frac{(1 + \rho) [K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1}] + (K_{t+\xi+1} - E_{t-1}d_{t+\xi+1})}{1 + \beta^\pi (1 + \rho)} \right],
\]

where we have used (27) (with forwarding) for $E_{t-1}d_{t+\xi}$. Again using (27),

\[
K_{t+\xi+1} + (1 + \rho) E_{t-1}d_{t+\xi} - E_{t-1}d_{t+\xi+1} = \frac{[K_{t+\xi+1} - E_{t-1}d_{t+\xi+1}] + (1 + \rho) [K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1}] - \beta^\pi (1 + \rho) (K_{t+\xi+1} - E_{t-1}d_{t+\xi+1})}{1 + \beta^\pi (1 + \rho)}
\]

\[
= \frac{(1 + \rho) [K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1}] + (K_{t+\xi+1} - E_{t-1}d_{t+\xi+1})}{1 + \beta^\pi (1 + \rho)}
\]

\[
= \frac{1}{\beta} [K_{t+\xi} + (1 + \rho) E_{t-1}d_{t+\xi-1} - E_{t-1}d_{t+\xi}] \cdot
\]

Hence, combining this with (32) and (33), one has:

\[
\ddot{x}_{t+\xi+1} - E_{t-1}d_{t+\xi+1} = \frac{1}{\beta} \left[ \ddot{x}_{t+\xi} - E_{t-1}d_{t+\xi} \right].
\]

Similarly, we find that

\[
\ddot{g}_{t+\xi+1} - E_{t-1}g_{t+\xi+1} = \frac{1}{\beta} \left[ \ddot{g}_{t+\xi} - E_{t-1}g_{t+\xi} \right].
\]

Taking expectations of the intertemporal government financing requirement as of the end of period $t - 1$ and combining the result with (34) and (35), one obtains:
\[ F_t = \sum_{\xi=0}^{\infty} (1 + \rho)^{-\xi} \left[ (\bar{x}_{t+\xi} - E_{t-1}x_{t+\xi}) / \nu + (\bar{y}_{t+\xi} - E_{t-1}y_{t+\xi}) \right] \]  \hspace{1cm} (36) \\
= \sum_{\xi=0}^{\infty} \left[ \frac{1}{\beta^{t+\xi}} \right] \left[ (\bar{x}_t - E_{t-1}x_t) / \nu + (\bar{y}_t - E_{t-1}y_t) \right]. \\
Hence,

\[ \implies \left( \bar{x}_t - E_{t-1}x_t \right) / \nu + \left( \bar{y}_t - E_{t-1}y_t \right) = \psi_0^C F_t, \] \\
where

\[ \psi_0^C = \frac{\beta^t(1+\rho)^{-1}}{\beta^t(1+\rho)}. \]  \hspace{1cm} (37) \\
Using (12) and (13), we have that \( (\bar{x}_t - E_{t-1}x_t) / \nu = (\alpha_2/\nu^2) (\bar{y}_t - E_{t-1}y_t) \) and, hence,

\[ \left( \bar{x}_t - E_{t-1}x_t \right) / \nu = \left[ \frac{1/\nu^2}{\beta^t} \right] \psi_0^C F_t, \]  \hspace{1cm} (38) \\
\[ \bar{y}_t - E_{t-1}y_t = \left[ \frac{1/\alpha_2}{\beta^t} \right] \psi_0^C F_t, \]  \hspace{1cm} (39)

We use (38) and (39) to obtain an explicit solution for \( E_{t-1}(d_t) \). Take expectations of the government financing requirement to give

\[ K_t + (1 + \rho) d_{t-1} - E_{t-1}d_t = \left[ (\bar{x}_t - E_{t-1}x_t) / \nu \right] + (\bar{y}_t - E_{t-1}y_t). \]  \hspace{1cm} (40) \\
Hence, using (38) and (39) and the definition of \( F_t \) one has:

\[ K_t + (1 + \rho) d_{t-1} - E_{t-1}d_t = \psi_0^C \left[ (1 + \rho) d_{t-1} + G_t \right]. \]

Hence, using the definition of \( G_t \):

\[ E_{t-1}d_t = \frac{1}{\beta^t} d_{t-1} + K_t - \psi_0^C \sum_{\xi=t}^{\infty} (1 + \rho)^{-\xi} K_{\xi} \]

\[ = \frac{1}{\beta^t} d_{t-1} + \frac{1}{\beta^t(1+\rho)} K_t + \left[ \frac{1}{\beta^t(1+\rho)} - 1 \right] \frac{1}{\beta^t} \sum_{\xi=t+1}^{\infty} (1 + \rho)^{-\xi} K_{\xi} \]

\[ = \frac{1}{\beta^t} d_{t-1} + \frac{1}{\beta^t(1+\rho)} G_t - \frac{1}{1+\rho} G_{t+1} \]

\[ = \frac{1}{\beta^t} d_{t-1} + \frac{(G_t - G_{t+1}) + (1 - \beta^t) G_{t+1}}{\beta^t(1+\rho)}. \]  \hspace{1cm} (41)
Using (38), (39), (22), (23), (24) and the solution (31) for $d_t^4$, one obtains the complete solutions for inflation, the output shortfall, the spending shortfall and public debt:

$$\pi_t = \left[\frac{1/\alpha_s M}{P_t^{\#}}\right] \left[q_2 \left(\frac{\mu_n}{\nu} + \eta_t\right) + g_3 \eta_t\right],$$  \hspace{1cm} (42)

$$\tilde{x}_t - x_t = \left[\frac{1/\nu}{\nu}\right] \psi^C_0 F_t + \left[\frac{1/\nu}{\nu}\right] \left[q_2 \left(\frac{\mu_n}{\nu} + \eta_t\right) + g_3 \eta_t\right],$$  \hspace{1cm} (43)

$$(\tilde{g}_t + \eta_t) - g_t = \left[\frac{1/\alpha_s}{\nu}\right] \psi^C_0 F_t + \left[\frac{1/\alpha_s}{\nu}\right] \left[q_2 \left(\frac{\mu_n}{\nu} + \eta_t\right) + g_3 \eta_t\right],$$  \hspace{1cm} (44)

$$d_t = \frac{1}{\beta} d_{t-1} + \frac{(G_1 - G_{t+1}) + (1 - \beta^*) G_{t+1}}{\beta(1 + \rho)} + q_1 \left[\frac{\mu_n}{\nu} + \left(1 - \beta^* \frac{F^*}{P^*}\right) \eta_t\right],$$  \hspace{1cm} (45)

where

$$q_1 = \frac{1}{1 + (P_{M_t}/P)(\beta^*(1 + \rho) - 1)}, \hspace{0.5cm} q_2 = \frac{(P_{M_t}/P)(\beta^*(1 + \rho) - 1)}{1 + (P_{M_t}/P)(\beta^*(1 + \rho) - 1)}, \hspace{0.5cm} q_3 = \frac{\beta^* (P_{M_t}/P)}{1 + (P_{M_t}/P)(\beta^*(1 + \rho) - 1)}.$$

B Solutions for the two-period model with commitment

B.1 The case of no targets

B.1.1 Outcomes conditional on debt

Setting $\pi_1^* = \pi_2^* = 0$ and realizing that $d_2^* = d_2^\# = 0$, we obtain the first- and second period outcomes for inflation and the output and spending shortfalls directly from (22), (23) and (24):

$$\pi_1 = \left[\frac{1/\alpha_s M}{P_t^{\#}}\right] \left[\left(\frac{\mu_n}{\nu} + \eta_1\right) - d_1^\#\right],$$  \hspace{1cm} (46)

$$\tilde{x}_1 - x_1 = \left[\frac{1/\nu}{\nu}\right] \left[K_1 + (1 + \rho) d_0 - d_1^\#\right] + \left[\frac{1/\nu}{\nu}\right] \left[\left(\frac{\mu_n}{\nu} + \eta_1\right) - d_1^\#\right],$$  \hspace{1cm} (47)

$$(\tilde{g}_1 + \eta_1) - g_1 = \left[\frac{1/\alpha_s}{\nu}\right] \left[K_1 + (1 + \rho) d_0 - d_1^\#\right] + \left[\frac{1/\alpha_s}{\nu}\right] \left[\left(\frac{\mu_n}{\nu} + \eta_1\right) - d_1^\#\right],$$  \hspace{1cm} (48)

and
\[ \pi_2 = \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right), \]  
(49)

\[ \tilde{x}_2 - x_2 = \left[ \frac{1/\nu}{P_{M}^*} \right] [K_2 + (1 + \rho) d_1] + \left[ \frac{1/\nu}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right), \]  
(50)

\[ (\tilde{g}_2 + \eta_2) - g_2 = \left[ \frac{1/\nu}{P_{M}^*} \right] [K_2 + (1 + \rho) d_1] + \left[ \frac{1/\nu}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right). \]  
(51)

### B.1.2 Solution for debt

The first-order condition for public debt is given by (25) for \( t = 1 \) with \( d_2^t = 0 \) imposed:

\[ [K_1 + (1 + \rho) d_0 - d_1^t] + \left[ \frac{P_1}{P_{M}^*} \right] \left( \frac{\mu_1}{\nu} + \eta_1 \right) - d_1^t \]

\[ = \beta^* \{ [K_2 + (1 + \rho) d_1] + \eta_1 \}, \]  
(52)

We solve (52) in two steps. In the first step, we take expectations \( E_0 [\cdot] \) on both sides of (52) and solve to give:

\[ d_1^t = \frac{[K_1 + (1 + \rho) d_0] - \beta^* K_2}{1 + \beta^* (1 + \rho)}. \]  
(53)

In the second step we take difference of (52) and its expectation and solve to give:

\[ d_1^t = \left[ \frac{1}{1 + \beta^* (1 + \rho)(P_{M}^*/P)} \right] \frac{\mu_1}{\nu} + \left[ \frac{1 - \beta^* (P_{M}^*/P)}{1 + \beta^* (1 + \rho)(P_{M}^*/P)} \right] \eta_1. \]  
(54)

### B.1.3 Final solution and society’s expected loss

We substitute the solutions for the public debt components \( d_1^t \) and \( d_1^t \) back into (46)-(51), to obtain the final solutions for inflation and the output and spending gaps in the two periods:

\[ \pi_1 = \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \psi_1 \left( \frac{\mu_1}{\nu} \right) + \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \left( \frac{2 + \rho}{1 + \rho} \right) \psi_1 \eta_1, \]  
(55)

\[ \tilde{x}_1 - x_1 = \left[ \frac{1/\nu}{P_{M}^*} \right] \left[ \frac{\beta^* (1 + \rho)}{1 + \beta^* (1 + \rho)} \right] F_1 + \left[ \frac{1/\nu}{P_{M}^*} \right] \psi_1 \left( \frac{\mu_1}{\nu} \right) + \left[ \frac{1/\nu}{P_{M}^*} \right] \left( \frac{2 + \rho}{1 + \rho} \right) \psi_1 \eta_1, \]  
(56)

\[ (\tilde{g}_1 + \eta_1) - g_1 = \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \left[ \frac{\beta^* (1 + \rho)}{1 + \beta^* (1 + \rho)} \right] F_1 + \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \psi_1 \left( \frac{\mu_1}{\nu} \right) + \left[ \frac{1/\alpha_M}{P_{M}^*} \right] \left( \frac{2 + \rho}{1 + \rho} \right) \psi_1 \eta_1, \]  
(57)
\[
\pi_2 = \left[ \frac{1/\alpha_2 M}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right), \quad (58)
\]

\[
\tilde{x}_2 - x_2 = \left[ \frac{1/\nu}{P} \right] \left[ \frac{1}{1 + \beta^*(1 + \rho)} F_1 \right] + \left[ \frac{1/\nu}{P^*} \right] \frac{1}{\beta^*(P_{M}^*/P)} \psi_1 \left( \frac{\mu_1}{\nu} \right) + \frac{1}{P^*} \left[ \frac{1}{1 - \beta^*(P_{M}^*/P)} \right] \psi_1 \eta_1 + \left[ \frac{1/\nu}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right), \quad (59)
\]

\[
(\tilde{g}_2 + \eta_2) - g_2 = \left[ \frac{1/\alpha_1}{P} \right] \left[ \frac{1}{1 + \beta^*(1 + \rho)} \right] F_1 + \left[ \frac{1/\alpha_2}{P^*} \right] \frac{1}{\beta^*(P_{M}^*/P)} \psi_1 \left( \frac{\mu_1}{\nu} \right) + \frac{1}{P^*} \left[ \frac{1}{1 - \beta^*(P_{M}^*/P)} \right] \psi_1 \eta_1 + \left[ \frac{1/\alpha_2}{P_{M}^*} \right] \left( \frac{\mu_2}{\nu} + \eta_2 \right), \quad (60)
\]

where, as defined in the main text,

\[
\psi_1 \equiv \frac{\beta^*(1 + \rho)(P_{M}^*/P)}{1 + \beta^*(1 + \rho)(P_{M}^*/P)}. \quad (61)
\]

We can now use (55)-(60) to compute society’s expected loss:

\[
E_0 \left[ V_1^S \right] = \frac{1}{2} E_0 \left[ \alpha_x \pi_1^2 + (x_1 - \tilde{x}_1)^2 + \alpha_g \left( g_1 - \tilde{g}_1 \right)^2 \right] + \frac{1}{2} E_0 \left[ \alpha_x \pi_2^2 + (x_2 - \tilde{x}_2)^2 + \alpha_g \left( g_2 - \tilde{g}_2 \right)^2 \right]
\]

\[
= T_1 + T_2 + T_3 + T_4 + T_5, \quad (62)
\]

where \( T_1, \ldots, T_5 \) are defined in the main text.

**B.2 Debt target combined with inflation targets**

Substituting

\[
\pi_t^* = \lambda_0 \left( \frac{\mu_t}{\nu} \right) + \lambda_{1t} \eta_t, \quad t = 1, 2, \quad (63)
\]

\[
d_1^F = \gamma_0 + \gamma_1 \frac{\mu_1}{\nu} + \gamma_2 \eta_1, \quad (64)
\]

into (22), (23) and (24) yield the following outcomes in periods 1 and 2:

\[
\pi_1 = \left[ \lambda_{01} \left( \frac{\mu_1}{\nu} \right) + \lambda_{11} \eta_1 \right] + \left[ \frac{1/\alpha_2 M}{P_{M}^*} \right] \left[ (1 - \gamma_1 - \lambda_{01}) \frac{\mu_1}{\nu} + (1 - \gamma_2 - \lambda_{11}) \eta_1 \right],
\]
\[ \ddot{x}_1 - x_1 = \left[ \frac{1}{\alpha_x} \right] \left[ K_1 + (1 + \rho) \frac{\mu}{\nu} - \gamma_0 \right] \\
+ \left[ \frac{1}{\alpha_x} \right] \left[ (1 - \gamma_1 - \lambda_0) \frac{\mu}{\nu} + (1 - \gamma_2 - \lambda_{11}) \eta_1 \right], \]

\[(\ddot{g}_1 + \eta_1) - g_1 = \left[ \frac{1}{\alpha_x} \right] \left[ K_1 + (1 + \rho) \frac{\mu}{\nu} - \gamma_0 \right] \\
+ \left[ \frac{1}{\alpha_x} \right] \left[ (1 - \gamma_1 - \lambda_0) \frac{\mu}{\nu} + (1 - \gamma_2 - \lambda_{11}) \eta_1 \right], \]

and

\[
\pi_2 = \left[ \lambda_{02} \left( \frac{\mu}{\nu} \right) + \lambda_{12} \eta_2 \right] + \left[ \frac{1}{\alpha_x} \right] \left[ (1 - \lambda_{02}) \frac{\mu}{\nu} + (1 - \lambda_{12}) \eta_2 \right],
\]

\[ \ddot{x}_2 - x_2 = \left[ \frac{1}{\alpha_x} \right] \left[ K_2 + (1 + \rho) \left( \gamma_0 + \gamma_1 \frac{\mu}{\nu} + \gamma_2 \eta_1 \right) \right] \\
+ \left[ \frac{1}{\alpha_x} \right] \left[ (1 - \lambda_{02}) \frac{\mu}{\nu} + (1 - \lambda_{12}) \eta_2 \right], \]

\[(\ddot{g}_2 + \eta_2) - g_2 = \left[ \frac{1}{\alpha_x} \right] \left[ K_2 + (1 + \rho) \left( \gamma_0 + \gamma_1 \frac{\mu}{\nu} + \gamma_2 \eta_1 \right) \right] \\
+ \left[ \frac{1}{\alpha_x} \right] \left[ (1 - \lambda_{02}) \frac{\mu}{\nu} + (1 - \lambda_{12}) \eta_2 \right], \]

where we realize that \( d_2^x = d_2^s = 0 \). Substitute these outcomes into the expected social loss, to yield:

\[
E_0 \left[ V_1^S \right] = \frac{1}{2} E_0 \left[ \alpha_x \pi_1^2 + (x_1 - \ddot{x}_1)^2 + \alpha_y \left( g_1 - \ddot{g}_1 \right)^2 \right] \\
+ \frac{1}{2} E_0 \left[ \alpha_x \pi_2^2 + (x_2 - \ddot{x}_2)^2 + \alpha_y \left( g_2 - \ddot{g}_2 \right)^2 \right] \\
= C_1 + C_2 + C_3 + C_4 + (C_{51} + C_{52}) + C_6 + (C_{71} + C_{72}), \quad (65)
\]

where
\[C_1 = \frac{1}{2} \frac{1}{\gamma^2} \left[ \frac{\beta^* (1+\rho)}{1+\beta^*(1+\rho)} \right] F_{11}^2,\]  
(66)

\[C_2 \equiv \frac{1}{2} \alpha \left[ \lambda_0 + \frac{1/\alpha_m}{P_M} (1 - \gamma_1 - \lambda_0) \right]^2 \frac{\sigma^2}{\gamma^2} + \frac{1}{2} \frac{P}{(P_M)^2} (1 - \gamma_1 - \lambda_0)^2 \frac{\sigma^2}{\gamma^2},\]  
(67)

\[C_3 = \frac{1}{2} \beta^* (1+\rho) \frac{1}{\gamma^2} \frac{\sigma^2}{\gamma^2},\]  
(68)

\[C_4 \equiv \frac{1}{2} \alpha \left[ \lambda_0 + \frac{1/\alpha_m}{P_M} (1 - \lambda_0) \right]^2 \frac{\sigma^2}{\gamma^2} + \frac{1}{2} \frac{P}{(P_M)^2} (1 - \lambda_0)^2 \frac{\sigma^2}{\gamma^2},\]  
(69)

\[C_{51} \equiv \frac{1}{2} \alpha \left[ \lambda_1 + \frac{1/\alpha_m}{P_M} (1 - \gamma_2 - \lambda_1) \right]^2 \sigma^2_{\eta} + \frac{1}{2} \frac{P}{(P_M)^2} (1 - \gamma_2 - \lambda_1)^2 \sigma^2_{\eta},\]  
(70)

\[C_{52} \equiv \frac{1}{2} \alpha \left[ 1 - 2 \left( \frac{1/\alpha_m}{P_M} \right) (1 - \gamma_2 - \lambda_1) \right] \sigma^2_{\eta},\]  
(71)

\[C_6 = \frac{1}{2} \beta^* (1+\rho) \frac{1}{\gamma^2} \sigma^2_{\eta},\]  
(72)

\[C_{71} \equiv \frac{1}{2} \alpha \left[ \lambda_2 + \frac{1/\alpha_m}{P_M} (1 - \lambda_1) \right]^2 \sigma^2_{\eta} + \frac{1}{2} \frac{P}{(P_M)^2} (1 - \lambda_1)^2 \sigma^2_{\eta},\]  
(73)

\[C_{72} \equiv \frac{1}{2} \alpha \left[ 1 - 2 \left( \frac{1/\alpha_m}{P_M} \right) (1 - \lambda_1) \right] \sigma^2_{\eta},\]  
(74)

Here, \(C_1\) is the expected loss associated with the deterministic component of the intertemporal government financing requirement,\(^2\) \(C_2\) and \(C_3\) are the expected losses in the first, respectively second, period from imperfect stabilization of \(\mu_1\), \(C_4\) is the expected loss from imperfect stabilization of \(\mu_2\), \(C_{51} + C_{52}\) and \(C_6\) are, respectively, the first- and second-period expected losses from suboptimal stabilization of \(\eta_1\), and \(C_{71} + C_{72}\) is the expected loss from imperfect stabilization of \(\eta_2\).

### B.3 Inflation targets only

Substituting (63) into (22), (23) and (24) yield the following outcomes in periods 1 and 2:

\[\pi_1 = \left[ \lambda_{01} \left( \frac{\mu_{01}}{\gamma} \right) + \lambda_{11} \eta_1 \right] + \left[ \frac{1/\alpha_m}{P_M} \right] \left[ (1 - \lambda_{01}) \frac{\mu_{01}}{\gamma} + (1 - \lambda_{11}) \eta_1 - d_1^T \right],\]

\[\tilde{x}_1 - x_1 = \left[ \frac{1/\nu}{P_T} \right] \left[ K_1 + (1+\rho) d_0 - \alpha_1^T \right] + \left[ \frac{1/\nu}{P_T} \right] \left[ (1 - \lambda_{01}) \frac{\mu_{01}}{\gamma} + (1 - \lambda_{11}) \eta_1 - d_1^T \right],\]

\(^2\)Here, we have already made use of the fact that society’s optimal \(\gamma_0\) is (53).
\[(\tilde{g}_1 + \eta_1) - g_1 = \left[ \frac{1}{\alpha_0} \right] [K_1 + (1 + \rho) d_0 - d_1^t] + \left[ \frac{1}{\alpha_0} \frac{\mu_0}{\nu} \right] [(1 - \lambda_{01}) \frac{\mu_0}{\nu} + (1 - \lambda_{11}) \eta_1 - d_1^t], \]

and

\[
\pi_2 = \left[ \lambda_{02} \left( \frac{\mu_2}{\nu} \right) + \lambda_{12} \eta_2 \right] + \left[ \frac{1}{\alpha_0} \frac{\mu_0}{\nu} \right] [(1 - \lambda_{02}) \frac{\mu_2}{\nu} + (1 - \lambda_{12}) \eta_2],
\]

\[
\tilde{x}_2 - x_2 = \left[ \frac{1}{\nu} \right] [K_2 + (1 + \rho) d_1] + \left[ \frac{1}{\nu} \right] [(1 - \lambda_{02}) \frac{\mu_2}{\nu} + (1 - \lambda_{12}) \eta_2],
\]

\[
(\tilde{g}_2 + \eta_2) - g_2 = \left[ \frac{1}{\alpha_0} \right] [K_2 + (1 + \rho) d_1] + \left[ \frac{1}{\alpha_0} \frac{\mu_0}{\nu} \right] [(1 - \lambda_{02}) \frac{\mu_0}{\nu} + (1 - \lambda_{12}) \eta_2].
\]

As in the case without targets, the first-order condition for public debt can be written as:

\[
\alpha_{\beta} (\tilde{g}_1 + \eta_1 - g_1) = \beta^* \left[ \frac{1}{\nu} \right] [K_2 + (1 + \rho) d_1 + \eta_1].
\]

Hence,

\[
[K_1 + (1 + \rho) d_0 - d_1^t] + \left[ \frac{P}{P_M} \right] [(1 - \lambda_{01}) \frac{\mu_0}{\nu} + (1 - \lambda_{11}) \eta_1 - d_1^t] = \beta^* [K_2 + (1 + \rho) d_1 + \eta_1].
\]

As before, taking expectations of this equation, we can solve for \(d_1^t\). Next, subtracting from this equation its expected version, we can solve for \(d_1\). The complete solution for public debt is:\footnote{Substituting the optimal values for \(\lambda_{01}\) and \(\lambda_{11}\) that we find in Appendix C below, we obtain expression (4.3) in the main text.}

\[
d_1 = \frac{[K_1 + (1 + \rho) d_0] - \beta^* K_2}{1 + \beta^* (1 + \rho)} + \left[ \frac{1 - \lambda_{01}}{1 + \beta^* (1 + \rho) (P_{M}/P)} \right] \frac{\mu_0}{\nu} + \left[ \frac{1 - \lambda_{11} - \beta^* (P_{M}/P)}{1 + \beta^* (1 + \rho) (P_{M}/P)} \right] \eta_1. \tag{75}
\]

Using this expression for \(d_1\) in the above expressions for inflation, the output shortfall and the spending shortfall, we obtain:

\[
\pi_1 = \left[ \lambda_{01} \left( \frac{\mu_0}{\nu} \right) + \lambda_{11} \eta_1 \right] + \left[ \frac{1}{\alpha_0} \frac{\mu_0}{\nu} \right] (1 - \lambda_{01}) \psi \frac{\mu_0}{\nu}
\]

\[
+ \left[ \frac{1}{\alpha_0} \frac{\mu_0}{\nu} \right] [1 + (1 - \lambda_{11}) (1 + \rho)] \psi \frac{\mu_0}{1 + \rho} \eta_1,
\]

\[
12
\]
\[ \hat{x}_1 - x_1 = \left[ \frac{1}{\mu} \right] \beta^* (1 + \rho) F_1 + \left[ \frac{1}{\mu} \right] (1 - \lambda_{01}) \psi_1 \mu \psi \]

\[ + \left[ \frac{1}{\mu} \right] \psi \left( 1 + (1 - \lambda_{11}) (1 + \rho) \right) \frac{\psi_1}{1 + \rho} \eta_1, \]

\[ (\tilde{g}_1 + \eta_1) - g_1 = \left[ \frac{1}{\mu} \right] \beta^* (1 + \rho) F_1 + \left[ \frac{1}{\mu} \right] (1 - \lambda_{01}) \psi_1 \mu \psi \]

\[ + \left[ \frac{1}{\mu} \right] \psi \left( 1 + (1 - \lambda_{11}) (1 + \rho) \right) \frac{\psi_1}{1 + \rho} \eta_1, \]

and

\[ \pi_2 = \left[ \lambda_{02} \left( \frac{\mu}{\nu} \right) + \lambda_{12} \eta_2 \right] + \left[ \frac{1}{\mu} \right] \left( 1 - \lambda_{02} \right) \frac{\mu}{\nu} + \left( 1 - \lambda_{12} \right) \eta_2, \]

\[ \hat{x}_2 - x_2 = \left[ \frac{1}{\mu} \right] \left[ \frac{1}{\mu} \beta (1 + \rho) \right] F_1 + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] (1 - \lambda_{01}) \psi_1 \mu \psi \]

\[ + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] \left( 1 - \lambda_{11} \right) - \beta^* (P^*_{M}/P) \right] \psi_1 \eta_1 \]

\[ + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] \left( 1 - \lambda_{02} \right) \frac{\mu}{\nu} + \left( 1 - \lambda_{12} \right) \eta_2, \]

\[ (\tilde{g}_2 + \eta_2) - g_2 = \left[ \frac{1}{\mu} \right] \left[ \frac{1}{\mu} \beta (1 + \rho) \right] F_1 + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] (1 - \lambda_{01}) \psi_1 \mu \psi \]

\[ + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] \left( 1 - \lambda_{11} \right) - \beta^* (P^*_{M}/P) \right] \psi_1 \eta_1 \]

\[ + \left[ \frac{1}{\mu} \beta^* (P^*_{M}/P) \right] \left( 1 - \lambda_{02} \right) \frac{\mu}{\nu} + \left( 1 - \lambda_{12} \right) \eta_2. \]

As before, substitute these outcomes into the expected social loss, to yield:

\[ E_0 [V_1^{S}] = C_1 + \tilde{C}_2 + \tilde{C}_3 + C_4 + \left( \tilde{C}_{51} + \tilde{C}_{52} \right) + \tilde{C}_6 + (C_{71} + C_{72}), \]

where \( C_1, C_4, C_{71} \) and \( C_{72} \) are given by (66), (69), (73) and (74), respectively, and
\[ \tilde{C}_2 \equiv \frac{1}{2} \alpha_\pi \left[ \lambda_{11} + \frac{1}{\alpha_{x M}} \left( \frac{1 + (1 - \lambda_{11})(1 + \rho)}{1 + \rho} \right) \psi_1 \right]^2 \frac{\sigma^2}{\psi^2} + \frac{1}{2} \frac{P}{(P_{M}^s)} \left( 1 - \lambda_{11} \right)^2 \psi_1 \frac{\sigma^2}{\psi^2}, \]

\[ \tilde{C}_3 = \frac{1}{2} \frac{1}{\beta P} \left[ \frac{1}{\beta^s (P_{M}^s/P)} \right]^2 \left( 1 - \lambda_{11} \right)^2 \psi_1 \frac{\sigma^2}{\psi^2}, \]

\[ \tilde{C}_{51} \equiv \frac{1}{2} \lambda_{11} + \frac{1}{\alpha_{x M}} \left( \frac{1 + (1 - \lambda_{11})(1 + \rho)}{1 + \rho} \right) \psi_1 \right]^2 \frac{\sigma^2}{\psi^2} + \frac{1}{2} \frac{P}{(P_{M}^s)} \left( 1 + (1 - \lambda_{11})(1 + \rho) \right)^2 \psi_1 \frac{\sigma^2}{\psi^2}, \]

\[ \tilde{C}_{52} \equiv \frac{1}{2} \alpha_\pi \left[ 1 - 2 \left( \frac{1}{\alpha_{x M}} \right) \left( \frac{1 + (1 - \lambda_{11})(1 + \rho)}{1 + \rho} \right) \psi_1 \right]^2 \frac{\sigma^2}{\psi^2}, \]

\[ \tilde{C}_6 \equiv \frac{1}{2} \beta^s \frac{1}{(P_{M}^s/P)} \left[ \frac{1 - \lambda_{11} - \beta^s (P_{M}^s/P)}{\beta^s (P_{M}^s/P)} \right]^2 \psi_1 \frac{\sigma^2}{\psi^2}. \]

The components of (76) stand for: \( C_1 \) is the expected loss associated with the deterministic component of the intertemporal government financing requirement, \( \tilde{C}_2 \) and \( \tilde{C}_3 \) are the expected losses in the first, respectively second, period from imperfect stabilization of \( \mu_1 \); \( C_4 \) is the expected loss from imperfect stabilization of \( \mu_2 \); \( \tilde{C}_{51} \) and \( \tilde{C}_{52} \) are, respectively, the first- and second-period expected losses from suboptimal stabilization of \( \eta_1 \), and \( C_{71} + C_{72} \) is the expected loss from imperfect stabilization of \( \eta_2 \).

C Proofs of Propositions 1, 2 and 3:

C.1 Proof of Proposition 1

We have to minimize (65) with respect to \( \alpha_{x M}, \lambda_{01}, \gamma_1, \lambda_{11}, \gamma_2, \lambda_{02} \) and \( \lambda_{12} \). Hold \( \alpha_{x M} \) constant until further notice. All expressions in the other parameters are quadratic and the second-order derivative of (65) in each of these other parameters is always strictly positive. Hence, by solving the first-order conditions, we obtain the optimum.

Taking the first-order condition with respect to \( \lambda_{01} \), and rewriting, yields:

\[ \alpha_\pi \left[ P \lambda_{01} + \left( 1/\alpha_{x M} \right) \left( 1 - \gamma_1 \right) \right] = \left( 1 - \gamma_1 - \lambda_{01} \right) \]

\[ \Rightarrow \left( 1 + \alpha_\pi P \right) \lambda_{01} = \left( 1 - \alpha_\pi/\alpha_{x M} \right) \left( 1 - \gamma_1 \right). \]

Next, take the first-order condition with respect to \( \gamma_1 \) and rewrite to yield:

\[ \beta^s \left( 1 + \rho \right) \frac{(P_{M}^s)^2}{P} \gamma_1 = \frac{\alpha_{x M}}{\alpha_{x}} \left[ P \lambda_{01} + \left( \frac{1}{\alpha_{x M}} \right) \left( 1 - \gamma_1 \right) \right] + P \left( 1 - \gamma_1 - \lambda_{01} \right). \]
Combine this expression with (77) and solve to give:

\[ \gamma_1 = \frac{(\alpha_{\pi}/\alpha_{\pi M}) + \alpha_{\pi} P \lambda_{01}}{(\alpha_{\pi}/\alpha_{\pi M}) + \beta^*(1+p) \left( P^* / P \right)}. \]

Next, combine this expression with (78) and solve to give the solution for \( \lambda_{01} \):

\[ \lambda_{01} = \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) \left[ \frac{\beta^*(1+p) / P}{1 + \beta^*(1+p) \left( P^* / P \right)} \right]. \tag{79} \]

Substitute this back into the expression of \( \gamma_1 \) and solve to give:

\[
\gamma_1 = \frac{\alpha_{\pi} + (1 - \alpha_{\pi}) \left[ \frac{\beta^*(1+p)}{1 + \beta^*(1+p) \left( P^* / P \right)} \right]}{\alpha_{\pi} / \alpha_{\pi M} + \beta^*(1+p) \left( P^* / P \right)}
= \left[ \frac{1}{1 + \beta^*(1+p) \left( P^* / P \right)} \right] \left[ \frac{\alpha_{\pi} \beta^*(1+p) + \beta^*(1+p) \left( \frac{P^*}{P} \right)}{\alpha_{\pi} + \beta^*(1+p) \left( \frac{1}{\alpha_{\pi M}} \right) \left( \frac{P^*}{P} \right)} \right]
= \frac{1}{1 + \beta^*(1+p) \left( P^* / P \right)}. \tag{80} \]

Further, note that if \( \alpha_{\pi M} = \alpha_{\pi} \), then \( \lambda_{01} = 0 \).

Take the first-order condition with respect to \( \lambda_{11} \), and rewrite to give:

\[ P^* \lambda_{11} = \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) (1 - \gamma_1) - \frac{1}{\alpha_{\pi}} P_{M}. \tag{81} \]

Next, take the first-order condition with respect to \( \gamma_2 \), and rewrite to give:

\[
\left[ \frac{\alpha_{\pi}}{\alpha_{\pi M}} + P + \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} - 1 \right) \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) \frac{P}{P^*} + \beta^* (1 + \rho) \left( \frac{P_P}{P} \right)^2 \right] \gamma_2
= \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} + P \right) + \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} - 1 \right) \left[ \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) \frac{P}{P^*} - \frac{1}{\alpha_{\pi}} \frac{P_{M}}{P}\right] - P^*. \]

Working out the coefficient of \( \gamma_2 \) as well as its right-hand side, this expression reduces to:

\[ \gamma_2 = \frac{1}{\alpha_{\pi}} \frac{1}{\alpha_{\pi} \left( \frac{1}{\alpha_{\pi M}} - 1 \right) \frac{1}{P^*}} P_{M}. \tag{82} \]

Substituting this into (81), we can solve for \( \lambda_{11} \). Finally, we observe that, if \( \alpha_{\pi M} = \alpha_{\pi} \), then \( \lambda_{11} = -\frac{1}{\alpha_{\pi} P} \) and \( \gamma_2 = 0 \).

Taking the first-order condition with respect to \( \lambda_{02} \), and rewriting, yields:

\[ \lambda_{02} = \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) \frac{1}{P^*}. \tag{83} \]
Next, take the first-order condition with respect to \( \lambda_{12} \) and rewrite to give:

\[
\lambda_{12} = -\frac{1}{\alpha_{\pi M}} \frac{1}{P}.
\]  

(84)

Finally, we solve for the optimal value of \( \alpha_{\pi M} \). To this end, we inspect each of the terms \( C_1, \ldots, C_{72} \) that together form (65). The term \( C_1 \) does not depend on \( \alpha_{\pi M} \).

As regards to the term \( C_2 \), we observe from (78) that we can write:

\[
1 - \gamma_1 - \lambda_{01} = \left[ \frac{\alpha_{\pi}}{1-\alpha_{\pi}/\alpha_{\pi M}} \right] P^*_M \lambda_{01}.
\]

Hence, we can write:

\[
C_2 = \frac{1}{2} \alpha_{\pi} \left[ \left( \frac{1}{1-\alpha_{\pi}/\alpha_{\pi M}} \right) \lambda_{01} \right]^2 \frac{\sigma^2}{v^2} + \frac{1}{2} P \left[ \left( \frac{\alpha_{\pi}}{1-\alpha_{\pi}/\alpha_{\pi M}} \right) \lambda_{01} \right]^2 \frac{\sigma^2}{v^2}
\]

\[
= \frac{1}{2} \alpha_{\pi}^2 P^* \left[ \left( \frac{1}{1-\alpha_{\pi}/\alpha_{\pi M}} \right) \lambda_{01} \right]^2 \frac{\sigma^2}{v^2}
\]

\[
= \frac{1}{2} \left[ 3 \beta (1 + \rho) \left( \frac{P^*}{P} \right) \right] \frac{\sigma^2}{v^2},
\]

(85)

where we have used (79). This expression does not depend on \( \alpha_{\pi M} \).

Next, we observe that \( C_3 \) does not depend on \( \alpha_{\pi M} \), because \( \gamma_1 \) does not depend on \( \alpha_{\pi M} \).

We turn now to the term \( C_4 \). With the help of (83) we can write this term as:

\[
C_4 = \frac{1}{2} \alpha_{\pi} \left[ \frac{\alpha_{\pi M}}{P^*_M} + \frac{P \left( \frac{1}{\alpha_{\pi M}} - 1 \right)}{P^* \frac{P^*_M}{P^*}} \right]^2 \frac{\sigma^2}{v^2} + \frac{1}{2} P \left[ \frac{P^* \left( \frac{1}{\alpha_{\pi M}} - 1 \right)}{P^* \frac{P^*_M}{P^*}} \right]^2 \frac{\sigma^2}{v^2}
\]

\[
= \frac{1}{2} \alpha_{\pi} \left[ \frac{\alpha_{\pi M}}{P^*_M} \left( 1 + \frac{P}{P^*} \right) + \frac{P \left( \frac{1}{\alpha_{\pi M}} - 1 \right)}{P^* \frac{P^*_M}{P^*}} \right]^2 \frac{\sigma^2}{v^2}
\]

\[
= \frac{1}{2} \alpha_{\pi} \left[ \frac{\sigma^2}{v^2} \left( \frac{1}{P^*} \right)^2 + \frac{P}{(P^*)^2} \right] \frac{\sigma^2}{v^2}
\]

\[
= \frac{1}{2} \left[ 3 \beta \frac{\sigma^2}{v^2} \right],
\]

which does not depend on \( \alpha_{\pi M} \).

Now, turn to the terms \( C_{51} \) and \( C_{52} \). First, using (81), work out:

\[
1 - \gamma_2 - \lambda_{11} = \left( 1 - \gamma_2 \right) - \left( \frac{1}{\alpha_{\pi}} - \frac{1}{\alpha_{\pi M}} \right) \frac{1}{P} \left( 1 - \gamma_2 \right) + \frac{1}{\alpha_{\pi}} \frac{P^*_M}{P^*} \left( \frac{P^*}{P} - \gamma_2 \right).
\]

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Also, using (81),

\[
\lambda_{11} + \left(\frac{1}{\alpha_{\pi M} P_M} \right) (1 - \gamma_2 - \lambda_{11}) \\
= \frac{P}{P_M} \lambda_{11} + \left(\frac{1}{\alpha_{\pi M} P_M} \right) (1 - \gamma_2) \\
= \frac{P}{P_M} \lambda_{11} + \left(\frac{1}{\alpha_{\pi} \alpha_{\pi M} P_M} \right) \left(1 - \gamma_2 \right) - \frac{1}{\alpha_{\pi}} \left(1 - \gamma_2 \right) + \left(\frac{1}{\alpha_{\pi M} P_M} \right) (1 - \gamma_2) \\
= \frac{1}{\alpha_{\pi}} (1 - \gamma_2) - \frac{1}{\alpha_{\pi}} \\
= -\frac{1}{\alpha_{\pi}} \gamma_2.
\]

Hence, the first component of \( C_{51} \) is minimized by setting \( \alpha_{\pi M} = \alpha_{\pi} \), because in that case \( \gamma_2 = 0 \), by (82). Next, take the sum of the second term of \( C_{51} \) and \( C_{52} \):

\[
\frac{1}{2} \frac{P}{(P_M)^2} \left( \frac{P}{P_M} - \gamma_2 \right)^2 \sigma_\eta^2 + \frac{1}{2} \alpha_{\pi} \sigma_\eta^2 + \frac{1}{2} \frac{P}{P_M} (\gamma_2 - \frac{P}{P_M}) \sigma_\eta^2 \\
= \frac{1}{2} \frac{P}{(P_M)^2} (\frac{P}{P_M} - \gamma_2)^2 \sigma_\eta^2 + \frac{1}{2} \alpha_{\pi} \sigma_\eta^2 + \frac{1}{2} \frac{P}{P_M} (\gamma_2 - \frac{P}{P_M}) \sigma_\eta^2 \\
= \left[ \frac{1}{2} \frac{P}{P_M} - \frac{1}{2} \alpha_{\pi} \gamma_2 + \frac{1}{2} \frac{P}{(P_M)^2} \gamma_2 + \frac{1}{2} \frac{P}{P_M} (\gamma_2 - \frac{P}{P_M}) \right] \sigma_\eta^2 + \frac{1}{2} \alpha_{\pi} \sigma_\eta^2 \\
= \left[ -\frac{1}{2} \frac{P}{P_M} + \frac{1}{2} \frac{P}{(P_M)^2} \gamma_2 \right] \sigma_\eta^2 + \frac{1}{2} \alpha_{\pi} \sigma_\eta^2.
\]

This is minimized by setting \( \alpha_{\pi M} = \alpha_{\pi} \), so that \( \gamma_2 = 0 \), by (82).

The term \( C_6 \) is minimized by setting \( \alpha_{\pi M} = \alpha_{\pi} \), so that \( \gamma_2 = 0 \).

Finally, we turn to minimizing \( C_{71} \) and \( C_{72} \). Substituting (84) into the expression for \( C_{71} \), we obtain:

\[
C_{71} \equiv \frac{1}{2} \alpha_{\pi} \left( \left( \frac{P}{P_M} \right) \lambda_{12} + \left( \frac{1}{\alpha_{\pi M} P_M} \right) \right)^2 \sigma_\eta^2 + \frac{1}{2} \frac{P}{(P_M)^2} \left( \frac{P}{P_M} \right)^2 \sigma_\eta^2 = \frac{1}{2} \frac{P}{P_M} \sigma_\eta^2,
\]

which does not depend on \( \alpha_{\pi M} \). Finally,

\[
C_{72} \equiv \frac{1}{2} \alpha_{\pi} \sigma_\eta^2 - \frac{1}{P_M} (1 - \lambda_{12}) \sigma_\eta^2 = \frac{1}{2} \alpha_{\pi} \sigma_\eta^2 - \frac{1}{P_M} \sigma_\eta^2,
\]

which also does not depend on \( \alpha_{\pi M} \).

To summarize, we have shown that each component of (65) does not depend on \( \alpha_{\pi M} \), after the other parameters have been chosen optimally, or is minimized at \( \alpha_{\pi M} = \alpha_{\pi} \).
C.2 Proof of Proposition 2

We have to minimize (76) with respect to \( \alpha_{\pi M}, \lambda_{01}, \lambda_{11}, \lambda_{02} \) and \( \lambda_{12} \). The optimal values for \( \lambda_{02} \) and \( \lambda_{12} \) follow directly from the proof of Proposition 1 and are thus given by (83) and (84). Hold \( \alpha_{\pi M} \) constant until further notice. The expressions in \( \lambda_{01} \) and \( \lambda_{11} \) are quadratic and the second-order derivative of (76) in either of these two parameters is always strictly positive. Hence, by solving the first-order conditions, we obtain the optimum.

Differentiating (76) with respect to \( \lambda_{01} \) and using a substantial amount of straightforward algebra yields, as before, the solution (79) for \( \lambda_{01} \). One can check that this must be the optimal value, because (as we show below) it implies that \( \tilde{C}_2 \) and \( \tilde{C}_3 \) equal, respectively, \( C_2 \) given by (85) and \( C_3 \) given by (86) with \( \lambda_{01} \) substituted. Because the latter two are the corresponding expressions obtained with both optimal debt and inflation targets, they must lead to the lowest stabilization loss associated with \( \mu_1 \).

The first-order condition for \( \lambda_{11} \) is:

\[
\alpha_{\pi} \left[ \lambda_{11} + \frac{1 - \alpha_{\pi M}}{P_{M}} \left( \frac{1}{(1 - \beta)} \right) \psi_1 \right] - \frac{P}{(P_{M})^{2}} \left[ \frac{1 - \alpha_{\pi M}}{P_{M}} \right] \psi_1^{2} - \beta \left[ \frac{1 - \lambda_{11} \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \right] \psi_1^{3} = 0
\]

\[
\Leftrightarrow \alpha_{\pi} \left[ 1 - \frac{1 - \alpha_{\pi M}}{P_{M}} \right] \lambda_{11} - \frac{\alpha_{\pi M}}{P_{M}} \psi_1 \left[ 1 - \frac{1 - \alpha_{\pi M}}{P_{M}} \right] \psi_1^{2} + \frac{P}{(P_{M})^{3}} \left[ \frac{1}{\beta \left( P_{M}/P \right)^{3}} \right] \psi_1^{3} = 0
\]

\[
\Leftrightarrow \lambda_{11} \left[ \alpha_{\pi} - 2 \left( \frac{\alpha_{\pi M}}{P_{M}} \right) \psi_1 + \left( \frac{\alpha_{\pi M}}{P_{M}} \right) \psi_1^{2} \right] - \frac{Q_{M}}{(P_{M})^{2}} \left( \frac{2 + \beta}{1 + \beta} \right) \psi_1^{3} + \frac{P}{(P_{M})^{3}} \left[ \frac{1 - \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \right] \psi_1^{4} + \frac{P}{(P_{M})^{3}} \frac{1 - \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \psi_1^{5} = 0.
\]

Hence,

\[
\Leftrightarrow \lambda_{11} \left[ \alpha_{\pi} - 2 \left( \frac{\alpha_{\pi M}}{P_{M}} \right) \psi_1 + \left( \frac{Q_{M}}{(P_{M})^{2}} + \frac{P}{(P_{M})^{3}} \right) \beta \left( \frac{1 - \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \right) \psi_1^{2} \right] = \frac{Q_{M}}{(P_{M})^{2}} \left( \frac{2 + \beta}{1 + \beta} \right) \psi_1^{3} + \frac{P}{(P_{M})^{3}} \frac{1 - \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \psi_1^{4} + \frac{P}{(P_{M})^{3}} \frac{1 - \beta \left( P_{M}/P \right)}{\beta \left( P_{M}/P \right)^{3}} \psi_1^{5} \psi_1^{1}.
\]
With quite a bit of straightforward algebra, we can write:

$$\alpha_{\pi} - 2 \left( \frac{\alpha_{\pi} / \alpha_{\pi M}}{P_{M}} \right) \psi_1 + \left( \frac{Q_M}{(P_M)^2} + \frac{P}{(P_M)^2} \beta^2 (1 + \rho) \left( \frac{2 \beta P}{(\beta (P_M)^2)} \right) \right) \psi_1^2 = \frac{\alpha_{\pi} (1 + \beta^2 (1 + \rho)^2)}{[1 + \beta^2 (P_M)^2 (1 + \rho)]^2}. $$

Hence,

$$\lambda_{11} = \frac{Q_M}{(P_M)^2} \left( \frac{\alpha_{\pi} / \alpha_{\pi M}}{1 + \rho} \right) + \beta \frac{1 - \beta^2 (P_M)^2}{(\beta (P_M)^2)} \left[ \beta^2 (P_M)^2 (1 + \rho) \right] - \left[ \frac{\alpha_{\pi} (2 + \rho)}{1 + \beta^2 (1 + \rho)} \right] \left[ \beta^2 (P_M)^2 (1 + \rho) \right].$$

Some algebra shows that the numerator of this expression can be written as

$$- \left[ \beta^2 (2 + \rho) / P \right] \frac{\alpha_{\pi} M}{\alpha_{\pi}} \left[ 1 + \beta^2 (1 + \rho) (P_M)^2 / P \right],$$

so that

$$\lambda_{11} = - \frac{1}{\alpha_{\pi M} P} \frac{\beta^2 (2 + \rho)}{1 + \beta^2 (1 + \rho)}. \quad (86)$$

Finally, we solve for the optimal value of $\alpha_{\pi M}$. To this end, we inspect each of the terms that together form (76). The term $C_1$ does not depend on $\alpha_{\pi M}$. As regards to the terms $\tilde{C}_2$ and $\tilde{C}_3$, we observe that:

$$(1 - \lambda_{01}) \psi_1 = \frac{\beta^2 (1 + \rho) (P_M^2 / P)}{1 + \beta^2 (1 + \rho) (P_M^2 / P)},$$

and

$$\lambda_{01} + \frac{1}{\alpha_{\pi M} P} (1 - \lambda_{01}) \psi_1 = \frac{\beta^2 (1 + \rho) \frac{1}{\alpha_{\pi M} P}}{1 + \beta^2 (1 + \rho) (P_M^2 / P)},$$

so that

$$\tilde{C}_2 = \frac{1}{2} \frac{1}{\alpha_{\pi M}} \left[ \frac{\beta^2 (1 + \rho) / P}{1 + \beta^2 (1 + \rho) (P_M^2 / P)} \right]^2 \sigma_{\psi}^2 + \frac{1}{2} \frac{P}{(P_M)^2} \left[ \frac{\beta^2 (1 + \rho) (P_M^2 / P)}{1 + \beta^2 (1 + \rho) (P_M^2 / P)} \right]^2 \sigma_{\psi}^2$$

and

$$\tilde{C}_3 = \frac{1}{2} \left[ \beta^2 (1 + \rho) / P \right] \left[ \frac{1}{1 + \beta^2 (1 + \rho) (P_M^2 / P)} \right]^2 \sigma_{\psi}^2,$$

both of which do not depend on $\alpha_{\pi M}$.

The optimal value of $\lambda_{02}$ is the same as in the proof of Proposition 2. Hence, $C_4$ does not depend on $\alpha_{\pi M}$.
Next, we turn to \( \tilde{C}_{51} \) and substitute (86) and (61) for \( \lambda_{11} \) and \( \psi_1 \), respectively:

\[
\tilde{C}_{51} \equiv \frac{1}{2} \alpha_\eta \left[ \lambda_{11} + \frac{1}{\alpha_{\pi M}} \frac{2+\rho}{1+\rho} \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right) \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
+ \frac{1}{2} \left( \frac{P}{P_{M^*}} \right) \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right)^2 \left[ \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
= \frac{1}{2} \alpha_\eta \left[ \lambda_{11} + \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right) \frac{1}{\alpha_{\pi M}} \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
+ \frac{1}{2} \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right)^2 \left[ \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
= \frac{1}{2} \alpha_\eta \left[ -\frac{1}{\alpha_{\pi M}} \frac{1}{P} \frac{\beta^* (2+\rho)}{1+\beta^* (1+\rho)} + \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right) \frac{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
+ \frac{1}{2} \left( \frac{2+\rho}{1+\rho} - \lambda_{11} \right)^2 \left[ \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \right]^2 \sigma_\eta^2 \\
= \frac{1}{2} \alpha_\eta \left[ -\frac{1}{\alpha_{\pi M}} \frac{1}{P} \frac{\beta^* (2+\rho)}{1+\beta^* (1+\rho)} \right]^2 \sigma_\eta^2,
\]

which is independent of \( \alpha_{\pi M} \).

Now, we consider \( \tilde{C}_{52} \), which we can write, after substituting (86) and (61) for \( \lambda_{11} \) and \( \psi_1 \), respectively:

\[
\tilde{C}_{52} = \frac{1}{2} \alpha_\eta \sigma_\eta^2 - \left( \frac{1}{P_{M^*}} \right) \left( \frac{2+\rho}{1+\rho} \right) \frac{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho)} \frac{\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)} \sigma_\eta^2 \\
= \frac{1}{2} \alpha_\eta \sigma_\eta^2 - \beta^* \frac{(2+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho)} \sigma_\eta^2,
\]

which is independent of \( \alpha_{\pi M} \).

Next, turn to \( \tilde{C}_{6} \). We observe that

\[
1 - \lambda_{11} - \beta^* \left( \frac{P_{M^*}}{P} \right) = 1 + \frac{1}{\alpha_{\pi M}} \frac{1}{P} \frac{\beta^* (2+\rho)}{1+\beta^* (1+\rho)} - \beta^* \frac{(2+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho)} \\
= 1 + \frac{1}{\alpha_{\pi M}} \frac{1}{P} \beta^* \left[ \frac{2+\rho}{1+\beta^* (1+\rho)} - 1 \right] - \beta^* \\
= (1 - \beta^*) \left[ 1 + \frac{1}{\alpha_{\pi M}} \frac{1}{P} \beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right) \right] \\
= (1 - \beta^*) \left[ \frac{1+\beta^* (1+\rho) \left( \frac{P_{M^*}}{P} \right)}{1+\beta^* (1+\rho)} \right].
\]

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Substitute this into the expression for \( \tilde{\mathcal{C}}_6 \) and rewrite, using (61) for \( \psi_1 \), to yield:

\[
\tilde{\mathcal{C}}_6 = \frac{1}{2} \beta \left( 1 - \beta^* \right)^2 \left[ \frac{1}{\beta^* (P_M^*/P)} \right] \left[ \frac{1 + \beta^* (1 + \rho) (P_M^*/P)}{1 + \beta^* (1 + \rho)} \right]^2 \left[ \frac{\beta^* (1 + \rho) (P_M^*/P)}{1 + \beta^* (1 + \rho) (P_M^*/P)} \right] \sigma_\eta^2
\]

which does not depend on \( \alpha_{\pi M} \) either.

The optimal value of \( \lambda_{12} \) is the same as in the proof of Proposition 2. Hence, \( C_{\tau 1} \) and \( C_{\tau 2} \) do not depend on \( \alpha_{\pi M} \).

To summarize, we have shown that none of the components of (76) depends on \( \alpha_{\pi M} \), after the other parameters have been chosen optimally.

### C.3 Proof of Proposition 3

The term \( T_1 \) does not depend on \( \alpha_{\pi M} \). We can write the term \( T_2 \) as:

\[
\frac{1}{2} \left[ \frac{Q_M}{(P_M^*)} + \frac{1}{\beta^* (1 + \rho) (P_M^*)} \right] \psi_1^2 \left( \frac{\sigma_{\mu}^2}{\sigma_\eta^2} + \sigma_\eta^2 \right)
\]

\[
= \frac{1}{2} \left[ \frac{Q_M}{P_M} + \frac{1}{\beta^* (1 + \rho)} \right] \left[ \frac{1 + \beta^* (1 + \rho)}{1 + \beta^* (1 + \rho) (P_M^*/P)} \right]^2 \left( \frac{\sigma_{\mu}^2}{\sigma_\eta^2} + \sigma_\eta^2 \right)
\]

\[
= \frac{1}{2} \beta^* (1 + \rho) \left( \frac{\sigma_{\mu}^2}{\sigma_\eta^2} + \sigma_\eta^2 \right) \frac{P + \beta^* (1 + \rho) Q_M}{P + \beta^* (1 + \rho) (P_M^*/P)}
\]

We only need to consider the final factor. The sign of its derivative with respect to \( \alpha_{\pi M} \) is given by the expression:

\[
\left[ P + \beta^* (1 + \rho) P_M^* \right] \ast \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} \right) \ast \beta^* (1 + \rho) -
\]

\[
2 \left[ P + \beta^* (1 + \rho) Q_M \right] \ast \beta^* (1 + \rho) \ast - \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} \right)
\]

\[
= \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} \right) \beta^* (1 + \rho) \left[ - \frac{P + \beta^* (1 + \rho) Q_M}{\alpha_{\pi M} (P + \beta^* (1 + \rho) P_M^*)} \right]
\]

\[
= \left( \frac{\alpha_{\pi}}{\alpha_{\pi M}} \right) \beta^* (1 + \rho) \left[ 1 + \beta^* (1 + \rho) \right] \left[ 1 - \frac{\alpha_{\pi}}{\alpha_{\pi M}} \right] P,
\]

which is negative if \( \alpha_{\pi M} < \alpha_{\pi} \), zero if \( \alpha_{\pi M} = \alpha_{\pi} \) and positive if \( \alpha_{\pi M} > \alpha_{\pi} \).

The sign of the derivative of the term \( T_3 \) is given by:
\[
P_M^* \left( \frac{\partial Q_M}{\partial \alpha M} \right) - 2Q_M \left( \frac{\partial P_M^*}{\partial \alpha M} \right) \\
= 2 \frac{1}{\alpha M} \left[ Q_M - \alpha_M \right] P_M^* \\
= 2 \frac{1}{\alpha M} P \left[ 1 - \frac{\alpha_M}{\alpha M} \right],
\]
which is negative if \(\alpha_M < \alpha\), zero if \(\alpha_M = \alpha\) and positive if \(\alpha_M > \alpha\).

We write the term \(T_4\) as follows:

\[T_4 = \frac{1}{2} (T_{41} + T_{42} + T_{43}) \psi_1^2 \sigma_\eta^2,\]

where

\[T_{41} = \frac{1}{\alpha_M} \left( \frac{\alpha_M}{\alpha M} - 1 \right), \quad T_{42} = \frac{Q_M}{(1+\rho)(P_M^*)^2}, \quad T_{43} = \beta \frac{1}{\alpha M}.,\]

Hence,

\[\frac{\partial T_4}{\partial \alpha M} = \frac{1}{2} \left( \frac{\partial T_{41}}{\partial \alpha M} + \frac{\partial T_{42}}{\partial \alpha_M} + \frac{\partial T_{43}}{\partial \alpha_M} \right) \psi_1^2 \sigma_\eta^2 + (T_{41} + T_{42} + T_{43}) \psi_1 \left( \frac{\partial T_4}{\partial \alpha M} \right) \sigma_\eta^2.\]

We observe that \(\partial \psi_1 / \partial \alpha M < 0\) and that \(\partial T_{43} / \partial \alpha M = 0\). In addition,

\[\frac{\partial T_{41}}{\partial \alpha M} = \frac{1}{\alpha_M} \left( 1 + \frac{\alpha_M}{\alpha M} - 1 \right) \left( \frac{1}{1+\rho} \right) \frac{1}{(P_M^*)^2} \left[ - \frac{1}{\alpha_M} + \left( 1 - \frac{2}{\alpha M} \right) \frac{1}{(1+\rho)^2} \frac{P}{P_M^*} \right],\]

which is negative for \(\alpha_M \leq \alpha\). Further,

\[\frac{\partial T_{42}}{\partial \alpha M} = \frac{2}{\alpha_M} \left( 1 - \frac{\alpha_M}{\alpha M} \right) \frac{P}{(P_M^*)^2},\]

which is negative if \(\alpha_M < \alpha\), zero if \(\alpha_M = \alpha\) and positive if \(\alpha_M > \alpha\). Hence, \(T_4\) is minimized by making \(\alpha_M > \alpha\).

Let us now turn to the term \(T_5\). We have that

\[\frac{\partial T_5}{\partial \alpha M} = - \frac{1}{\alpha_M} \left[ \frac{\beta^*(1/P)^2(1+\rho)(2+\rho)}{[1+\beta^*(P_M^*/P)(1+\rho)]^2} \right] \sigma_\eta^2 < 0.\]

Further, we have that:

\[\frac{\partial T_6}{\partial \alpha M} = - \frac{1}{\alpha_M^2} \beta \frac{1}{(P_M^*)^2} \sigma_\eta^2 < 0.\]

This confirms Lemma 3.
D Derivation of infinite-horizon discretionary equilibrium.

D.1 General part of the derivation

In period \( t \) the CB minimizes over \( \pi_t \):

\[
V_t^{CB} = \frac{1}{2} \{ \alpha_{\pi M} \pi_t^2 + [\nu (\pi_t - \pi_t^e - \tau_t) - \mu_t - \tilde{x}_t]^2 \} + \beta E_t \left[ V_{t+1}^{CB} \right].
\]

Because \( E_t \left[ V_{t+1}^{CB} \right] \) does not depend on \( \pi_t \) (the state variables are the countries’ debt levels), the CB’s first-order condition is:

\[
\alpha_{\pi M} \pi_t + \nu [\pi_t - \pi_t^e - \tau_t] - \mu_t - \tilde{x}_t = 0. \tag{88}
\]

The government selects \( \tau_t \) and \( d_t \) so as to minimize (3). Again, the first-order conditions are (4), (5), (6) and (7).

We solve first for the intratemporal allocation. That is, we solve for inflation, taxes and spending as functions of public debt. The system to be solved at this stage is thus (88), (4) and (6).

D.1.1 Derivation of outcomes for given debt policies

We first derive the deterministic components of the outcomes (step 1). Then, we derive the responses to the shocks (step 2).

\textbf{Step 1}: Take as-of-the-start-of-period-\( t \) expectations of the system (88), (4) and (6) to give:

\[
\alpha_{\pi M} \pi_t^e - \nu^2 (\pi_t^e + \frac{\tilde{x}_t}{\nu}) = 0, \tag{89}
\]

(9) and (10). The solution of the resulting system is:

\[
\pi_t^e = \left[ \frac{1/\alpha_{\pi M}}{p} \right] \left[ K_t + (1 + \rho) d_{t-1} - d_t^f \right], \tag{90}
\]

(12) and (13).

\textbf{Step 2}: Subtract (89), (9) and (10) from (88), (4) and (6), respectively, to give (15), (16) and (17). The solution of this system has been computed before and is given by (18), (20) and (21). Combining these last three equations with (90), (12) and (13), we obtain:

\[
\pi_t = \left[ \frac{1/\alpha_{\pi M}}{p} \right] [K_t + (1 + \rho) d_{t-1} - d_t^f] + \left[ \frac{1/\alpha_{\pi M}}{p}\frac{\mu}{\nu} \right] [(\frac{\nu}{\mu} + \eta_t) - d_t^f], \tag{91}
\]
\[
\tilde{x}_t - x_t = \left[ \frac{1}{\nu^2} \right] [K_t + (1 + \rho) d_{t-1} - d^*_t] + \left[ \frac{1}{\nu^2} \right] \left[ \left( \frac{\nu}{\nu^2} + \eta_t \right) - d^*_t \right], \tag{92}
\]

\[
(\tilde{g}_k + \eta_t) - g_t = \left[ \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right] [K_t + (1 + \rho) d_{t-1} - d^*_t] + \left[ \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right] \left[ \left( \frac{\nu}{\nu^2} + \eta_t \right) - d^*_t \right]. \tag{93}
\]

### D.1.2 Derivation of solution for public debt

We are now in a position to characterize debt policy. To evaluate \( \partial E_t \left( V_{t+1}^{G_1} \right) / \partial d_t \), we forward (91), (92) and (93) by one period and substitute the resulting expressions into:

\[
\frac{1}{2} \mathbb{E}_t \left[ \alpha_\pi \pi_{t+1}^2 + (x_{t+1} - \tilde{x}_{t+1})^2 + \alpha_g (g_{t+1} - (\tilde{g}_{t+1} + \eta_t))^2 \right].
\]

The derivative with respect to \( d_t \) of the expression thus obtained is:

\[
\mathbb{E}_t \left[ \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right] (1 + \rho) \left[ \frac{1}{\nu} \right] + \left[ \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right] \left[ \left( \frac{\nu}{\nu^2} + \eta_t \right) - d^*_t \right].
\]

Hence, combining this with (5), the first-order condition for \( d_t \) is:

\[
\alpha_g (\tilde{g}_k + \eta_t - g_t) = \beta^* \mathbb{E}_t \left[ \pi_{t+1} \left( \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right) + (\tilde{x}_{t+1} - x_{t+1}) \left[ \frac{1}{\nu} \right] \right].
\]

Combine this with the expressions for \( \pi_{t+1}, \tilde{x}_{t+1} - x_{t+1} \) and \( \tilde{g}_{t+1} - g_{t+1} \) obtained from (91), (92) and (93) to give,

\[
\left[ \frac{1}{\nu} \right] [K_t + (1 + \rho) d_{t-1} - d^*_t] + \left[ \frac{1}{\nu} \right] \left[ \left( \frac{\nu}{\nu^2} + \eta_t \right) - d^*_t \right] \]

\[
= \beta^* \left\{ \frac{\alpha_\pi \pi_{t+1}^2}{\nu} \right\} \left( K_{t+1} + (1 + \rho) d_t - d^*_{t+1} \right) + \left[ \frac{1}{\nu} \right] \left( \frac{\nu}{\nu^2} + \eta_t \right),
\]

because \( \mathbb{E}_t [\eta_{t+1}] = 0 \). Here (as in the main text):

\[
Q_M = \alpha_\pi / \alpha_\pi^2 + 1/\nu^2 + 1/\alpha_g. \tag{94}
\]

Hence,
\[ [K_t + (1 + \rho) d_{t-1} - d_t^c] + \left[ \frac{P}{P_{M_t}} \right] \left[ \left( \frac{\mu^*_t}{\nu} + \eta_t \right) - d_t^f \right] = \\
\beta^* \left[ \left( \frac{Q_{M_t}}{P_t} \right) (K_{t+1} + (1 + \rho) d_t - d_{t+1}^c) + \eta_t \right]. \quad (95) \]

We can solve for public debt in two steps.

**Step 1:** take expectations of (95) as of (end of) \( t - 1 \):

\[ K_t + (1 + \rho) d_{t-1} - d_t^c = \\
\beta^* \left[ \left( \frac{Q_{M_t}}{P_t} \right) (K_{t+1} + (1 + \rho) d_t - d_{t+1}^c) + \eta_t \right]. \quad (96) \]

The solution is:

\[ E_{t-1} d_t = \frac{[K_t + (1 + \rho) d_{t-1} - d_t^c] - \beta^* (Q_{M_t}/P_t) (K_{t+1} - E_{t-1} d_{t+1})}{1 + \beta^* (Q_{M_t}/P_t)}. \quad (97) \]

**Step 2:** Subtract (96) from (95):

\[ \left[ \frac{P}{P_{M_t}} \right] \left[ \left( \frac{\mu^*_t}{\nu} + \eta_t \right) - d_t^f \right] = \beta^* (1 + \rho) \left( \frac{Q_{M_t}}{P_t} \right) d_t^f + \\
\beta^* \left( \frac{Q_{M_t}}{P_t} \right) [E_{t-1} (d_{t+1}) - E_t (d_{t+1})] + \beta^* \eta_t, \quad (98) \]

and solve this to yield:

\[ d_t^f = \left[ \frac{1}{1 + \beta^* (1 + \rho) (P_{M_t}/P_t)} \right] \left( \frac{\mu^*_t}{\nu} + \eta_t \right) - \left[ \frac{\beta^* (P_{M_t}/P_t)}{1 + \beta^* (1 + \rho) (P_{M_t}/P_t)} \right] \eta_t + \\
\left[ \frac{\beta^* (P_{M_t}/P_t) (Q_{M_t}/P_t)}{1 + \beta^* (1 + \rho) (P_{M_t}/P_t)} \right] \left[ E_t (d_{t+1}) - E_{t-1} (d_{t+1}) \right]. \quad (99) \]

Through the final term the shock will be spread out over the entire future horizon.

We can find the final solution for \( d_t^f \) as follows. Use (97) forwarded by \( \xi - t \) \( (\xi \geq t + 1) \) periods to obtain:

\[ E\left( d_{t+\xi} \right) - E_{t-1} \left( d_{t+\xi} \right) = \frac{(1 + \rho) \left[ E_t (d_{t+\xi-1}) - E_{t-1} (d_{t+\xi-1}) \right] + \beta^* (Q_{M_t}/P_t) \left[ E_t (d_{t+\xi+1}) - E_{t-1} (d_{t+\xi+1}) \right]}{1 + \beta^* (1 + \rho) (Q_{M_t}/P_t)}. \quad (100) \]

The non-explosive solution is found in the same way as before and is given by:

\[ E_t (d_{t+1}) - E_{t-1} (d_{t+1}) = \frac{1}{\beta^* (Q_{M_t}/P_t)} d_t^f. \]
Substitute this back into (99) and rewrite to give:

\[
d_t^d = \left[ \frac{1}{1 + (P_M^* / P)[\beta^* (1 + \rho) (Q_M / P) - 1]} \right] \left[ \frac{\mu}{\phi^*} + (1 - \beta^* (P_M^* / P)) \eta_t \right]. \tag{101}
\]

### D.1.3 Derivation deterministic components of \( \tilde{x}_{it} - x_{it} \), \( \tilde{y}_{it} - y_{it} \) and \( \pi_t \)

The derivation is similar to that in the case of the second best. Using (92), one has

\[
\tilde{x}_{t+1} - E_{t-1} x_{t+1} = \frac{1}{1 + \rho} [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi - 1} - E_{t-1} d_{t+1, \xi}]
\]

\[
\tilde{x}_{t+1} - E_{t-1} x_{t+1} = \frac{1}{1 + \rho} [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1} - E_{t-1} d_{t+1, \xi + 1}],
\]

where \( \xi \geq t \). Note that

\[
K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi - 1} - E_{t-1} d_{t+1, \xi} = \frac{1 + \beta^* (1 + \rho) (Q_M / P)}{1 + \beta^* (1 + \rho) (Q_M / P)} [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1}] + (1 + \rho) (Q_M / P) [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1}]
\]

where we have used (97) (with forwarding) for \( E_{t-1} d_{t+1, \xi} \). Further,

\[
K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1} - E_{t-1} d_{t+1, \xi + 1} = \frac{1 + \beta^* (1 + \rho) (Q_M / P)}{1 + \beta^* (1 + \rho) (Q_M / P)} [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1}] + (1 + \rho) (Q_M / P) [K_{t+1} + (1 + \rho) E_{t-1} d_{t+1, \xi + 1}]
\]

Hence,

\[
\tilde{x}_{t+1} - E_{t-1} x_{t+1} = \left[ \frac{1}{\beta^* (Q_M / P)} \right] [\tilde{x}_{t+1} - E_{t-1} x_{t+1}]
\]

For public spending we derive similarly:

\[
\tilde{y}_{t+1} - E_{t-1} y_{t+1} = \left[ \frac{1}{\beta^* (Q_M / P)} \right] [\tilde{y}_{t+1} - E_{t-1} y_{t+1}]
\]

Having derived these recursions, we use (36) to obtain the deterministic components of the final outcomes:

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\[
\sum_{\xi=0}^{\infty} (1 + \rho)^{-\xi} \left[ (\tilde{x}_{t+\xi} - E_{t-1}x_{t+\xi})/\nu + (\tilde{g}_{t+\xi} - E_{t-1}g_{t+\xi}) \right] = F_t \\
\implies \sum_{\xi=0}^{\infty} \left[ \frac{1}{\beta^*(1 + \rho)(Q_M/P)} \right]^\xi \left[ (\tilde{x}_{t} - E_{t-1}x_t) /\nu + (\tilde{g}_{t} - E_{t-1}g_t) \right] = F_t \\
\implies (\tilde{x}_{t} - E_{t-1}x_t) /\nu + (\tilde{g}_{t} - E_{t-1}g_t) = \left[ \frac{\beta^*(1 + \rho)(Q_M/P) - 1}{\beta^*(1 + \rho)(Q_M/P)} \right] F_t.
\]

Hence,

\[
(\tilde{x}_{t} - E_{t-1}x_t) /\nu = \frac{1}{\beta^*(1 + \rho)(Q_M/P)} \left[ \frac{\beta^*(1 + \rho)(Q_M/P) - 1}{\beta^*(1 + \rho)(Q_M/P)} \right] F_t, 
\]

\[
\tilde{g}_{t} - E_{t-1}g_t = \frac{1}{\beta^*(1 + \rho)(Q_M/P)} \left[ \frac{\beta^*(1 + \rho)(Q_M/P) - 1}{\beta^*(1 + \rho)(Q_M/P)} \right] F_t. 
\]

(102) \hspace{2cm} (103)

D.1.4 Computation of \( E_{t-1}(d_t) \)

We compute now \( E_{t-1}(d_t) \). Combining the expectation of the government financing requirement, (40), with (102) and (103) and replacing \( F_t \), one has:

\[
K_t + (1 + \rho) d_{t-1} - E_{t-1}d_t = \left[ \frac{\beta^*(1 + \rho)(Q_M/P) - 1}{\beta^*(1 + \rho)(Q_M/P)} \right] [(1 + \rho) d_{t-1} + G_t].
\]

Hence,

\[
E_{t-1}d_t = \frac{1}{\beta^*(Q_M/P)} d_{t-1} + K_t - \left[ \frac{\beta^*(1 + \rho)(Q_M/P) - 1}{\beta^*(1 + \rho)(Q_M/P)} \right] \sum_{\xi=0}^{\infty} (1 + \rho)^{-\xi-1} K_{\xi} \\
= \frac{1}{\beta^*(Q_M/P)} d_{t-1} + \frac{1}{\beta^*(1 + \rho)(Q_M/P)} K_t + \left[ \frac{\frac{1}{\beta^*(1 + \rho)(Q_M/P)} - 1}{1 + \rho} \right] \sum_{\xi=0}^{\infty} (1 + \rho)^{-\xi-1} K_{\xi} \\
= \frac{1}{\beta^*(Q_M/P)} d_{t-1} + \frac{1}{\beta^*(1 + \rho)(Q_M/P)} G_t - \frac{1}{1 + \rho} G_{t+1} \\
= \frac{1}{\beta^*(Q_M/P)} d_{t-1} + \frac{(G_t - G_{t+1}) + [1 - \beta^*(Q_M/P)] G_{t+1}}{\beta^*(1 + \rho)(Q_M/P)}.
\]

D.1.5 The complete outcomes

Using (91), (92), (93), (102), (103) and (101), we have:
\[ \pi_t = \left[ \frac{1/\alpha M}{\beta P} \right] \psi_0^P F_t + \left[ \frac{1/\alpha M}{\beta P} \right] \left[ \psi_1^P \left( \frac{\mu}{v} + \eta_t \right) + p_2 \eta_t \right], \quad (104) \]

\[ \bar{x}_t = \left[ \frac{1/\alpha M}{\beta P} \right] \psi_0^P F_t + \left[ \frac{1/\alpha M}{\beta P} \right] \left[ \psi_1^P \left( \frac{\mu}{v} + \eta_t \right) + p_2 \eta_t \right], \quad (105) \]

\[ \bar{\eta}_t - \eta_t = \left[ \frac{1/\alpha M}{\beta P} \right] \psi_0^P F_t + \left[ \frac{1/\alpha M}{\beta P} \right] \left[ \psi_1^P \left( \frac{\mu}{v} + \eta_t \right) + p_2 \eta_t \right], \quad (106) \]

\[ d_t = \frac{1}{\beta (Q_M/P)} d_{t-1} + \frac{(G_t-x_{t-1})[1-\beta^*(Q_M/P)]G_{t-1} + p_1 \left( \frac{\mu}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_t \right]}{1 + (F_{t-1}/P)[\beta^*(1+\rho)(Q_M/P)-1]} \]

\[ \psi_0^P \equiv \frac{\beta^*(1+\rho)(Q_M/P)-1}{\beta^*(1+\rho)(Q_M/P)}, \quad \psi_1^P \equiv \frac{(F_{t-1}/P)[\beta^*(1+\rho)(Q_M/P)-1]}{1 + (F_{t-1}/P)[\beta^*(1+\rho)(Q_M/P)-1]}, \]

\[ p_1 \equiv \frac{1}{1 + (F_{t-1}/P)[\beta^*(1+\rho)(Q_M/P)-1]}, \quad p_2 \equiv \frac{\beta^*(P_t/P)}{1 + (F_{t-1}/P)[\beta^*(1+\rho)(Q_M/P)-1]} \]

**E Derivation of expected social loss in infinite-horizon model**

**E.1 Commitment**

Using (45) and its lags, we can write:

\[ F_t = (1 + \rho) d_{t-1} + G_t \]

\[ = (1 + \rho) \left\{ \frac{1}{\beta^*} d_{t-2} + \frac{G_{t-1}}{\beta^*(1+\rho)} + q_1 \left[ \frac{\mu_0}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-1} \right] \right\} + G_t \]

\[ = (1 + \rho) \frac{1}{\beta^*} d_{t-2} + (1 + \rho) q_1 \left[ \frac{\mu_1}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-1} \right] + \frac{1}{\beta^*} G_{t-1} \]

\[ = (1 + \rho) \frac{1}{\beta^*} \left\{ \frac{1}{\beta^*} d_{t-3} + \frac{G_{t-2}}{\beta^*(1+\rho)} + q_1 \left[ \frac{\mu_2}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-1} \right] \right\} \]

\[ + (1 + \rho) q_1 \left[ \frac{\mu_2}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-1} \right] + \frac{1}{\beta^*} G_{t-1} \]

\[ = (1 + \rho) \left( \frac{1}{\beta^*} \right)^2 d_{t-3} + \frac{1}{\beta^*} (1 + \rho) q_1 \left[ \frac{\mu_2}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-2} \right] \]

\[ + (1 + \rho) q_1 \left[ \frac{\mu_2}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-2} \right] + \left( \frac{1}{\beta^*} \right)^2 G_{t-2} \]

\[ = \left( \frac{1}{\beta^*} \right)^{t-1} [(1 + \rho) d_0 + G_1] + (1 + \rho) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta^*} \right)^{\xi-1} q_1 \left[ \frac{\mu_{t-\xi}}{v} + \left( 1 - \beta^* \frac{P_t}{P} \right) \eta_{t-\xi} \right]. \]
We can now combine this with (42), (43) and (44) to give:

\[
\pi_t = \left[ \frac{1}{\alpha_s P_M} \right] [q_2 \left( \frac{\mu}{\nu} + \eta_t \right) + q_3 \eta_t],
\]

\[
\tilde{x}_t - x_t = \left[ \frac{1}{\beta^s} \right] \psi^C (1 + \rho) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta^s} \right)^{\xi-1} q_1 \left[ \frac{\mu}{\nu} + \left( 1 - \beta^s P_M \frac{\beta^s}{\beta^s} \right) \eta_{t-\xi} \right] + \left[ \frac{1}{\nu P_M} \right] [q_2 \left( \frac{\mu}{\nu} + \eta_t \right) + q_3 \eta_t] + \left[ \frac{1}{\beta^s} \right] \psi^C \left( \frac{1}{\beta^s} \right)^{t-1} [(1 + \rho) d_0 + G_1],
\]

\[
\tilde{g}_t - g_t = \left[ \frac{1}{\alpha_s P_M} \right] \psi^C (1 + \rho) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta^s} \right)^{\xi-1} q_1 \left[ \frac{\mu}{\nu} + \left( 1 - \beta^s P_M \frac{\beta^s}{\beta^s} \right) \eta_{t-\xi} \right] + \left[ \frac{1}{\alpha_s P_M} \right] \psi^C \left( \frac{1}{\beta^s} \right)^{t-1} [(1 + \rho) d_0 + G_1] - \eta_t.
\]

Let us now compute society’s expected loss:

\[
E_0 \left[ \alpha_s \pi_t^2 + (x_t - \tilde{x}_t)^2 + \alpha_g (g_t - \tilde{g}_t)^2 \right] = \left[ \frac{1}{P} \right] \left\{ \psi^C \left( \frac{1}{\beta^s} \right)^{t-1} [(1 + \rho) d_0 + G_1] \right\}^2 + \left[ \frac{\psi^C}{P} \right] (1 + \rho)^2 \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta^s} \right)^{2(\xi-1)} q_1^2 \left[ \frac{\sigma^2_\pi}{\sigma^2_\pi} + \left( 1 - \beta^s P_M \frac{\beta^s}{\beta^s} \right)^2 \sigma^2_\eta \right] + \left[ \frac{Q}{P_M} \right] \left[ q_2^2 \frac{\sigma^2_\pi}{\sigma^2_\pi} + (q_2 + q_3)^2 \sigma^2_\eta \right] + \alpha_g \left[ 1 - 2 \left( \frac{\alpha_s P_M}{\sigma^2_\pi} \right) (q_2 + q_3) \right] \sigma^2_\eta.
\]

\[
= \left[ \frac{1}{P} \right] \left\{ \psi^C \left( \frac{1}{\beta^s} \right)^{t-1} [(1 + \rho) d_0 + G_1] \right\}^2 + \left[ \frac{\psi^C}{P} \right] (1 + \rho)^2 \left[ \frac{1 - \alpha_s \beta^s (\beta^s - 1)}{1 - (1/\beta^s)} \right] q_1^2 \left[ \frac{\sigma^2_\pi}{\sigma^2_\pi} + \left( 1 - \beta^s P_M \frac{\beta^s}{\beta^s} \right)^2 \sigma^2_\eta \right] + \left[ \frac{Q}{P_M} \right] \left[ q_2^2 \frac{\sigma^2_\pi}{\sigma^2_\pi} + (q_2 + q_3)^2 \sigma^2_\eta \right] + \alpha_g \left[ 1 - 2 \left( \frac{\alpha_s P_M}{\sigma^2_\pi} \right) (q_2 + q_3) \right] \sigma^2_\eta.
\]

Hence,
\[ \frac{1}{\pi} \sum_{t=1}^{\infty} \beta^{t-1} E_0 \left[ \alpha \pi_t^2 + \left( x_t - \bar{x}_t \right)^2 + \alpha_g (g_t - \bar{g}_t)^2 \right] \\
\frac{1}{\pi} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \left( \frac{\psi}{P} \right)^2 \right] (1 + \rho)^2 \left[ \frac{1 - (1/\beta)^{2(t-1)}}{1 - (1/\beta)^{2t}} \right] q_1^2 \left[ \frac{\sigma^2}{\sigma_Y^2} + \left( 1 - \beta^* \frac{P_{y_t}}{P} \right)^2 \sigma^2_n \right]
\]
\[
+ \frac{1}{\pi} \frac{1}{1-\beta} \left[ \frac{Q_{m_t}}{(P_{y_t})} \right] \left[ q_2 \frac{\sigma^2}{\sigma_Y^2} + (q_2 + q_3)^2 \sigma^2_n \right] + \frac{1}{2} \frac{1}{1-\beta} \alpha_g \left[ 1 - 2 \left( \frac{1}{1/\beta} \right) (q_2 + q_3) \right] \sigma^2_n \]

where “*” is used to denote that terms not containing \( \alpha_{\pi M} \) have been dropped.

Note that

\[ \frac{1}{\pi} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \frac{1}{1 - (1/\beta)^{2t}} \right] \left[ \frac{1}{1 - (1/\beta)^t} \right] = \frac{1}{\pi} \frac{1}{1 -(1/\beta)^t} \sum_{t=1}^{\infty} \left[ \frac{1}{1 - (1/\beta)^t} \right] = \frac{1}{\pi} \frac{(\beta^{t-1})}{1 -(1/\beta)^t}. \]  

(108)

Hence,

\[ \frac{1}{\pi} \frac{1}{1-\beta} \left[ \frac{1}{1 - (1/\beta)^{t-1}} \right] \left[ \frac{1}{1 - (1/\beta)^t} \right] q_1^2 \left[ \frac{\sigma^2}{\sigma_Y^2} + \left( 1 - \beta^* \frac{P_{y_t}}{P} \right)^2 \sigma^2_n \right]
\]
\[
- \frac{1}{2} \frac{1}{1-\beta} \left[ \frac{1}{1 - (1/\beta)^{t-1}} \right] \left[ \frac{1}{1 - (1/\beta)^t} \right] q_1^2 \left[ \frac{\sigma^2}{\sigma_Y^2} + \left( 1 - \beta^* \frac{P_{y_t}}{P} \right)^2 \sigma^2_n \right]
\]
\[
+ \frac{1}{2} \frac{1}{1-\beta} \left[ \frac{Q_{m_t}}{(P_{y_t})} \right] \left[ q_2 \frac{\sigma^2}{\sigma_Y^2} + (q_2 + q_3)^2 \sigma^2_n \right] + \frac{1}{2} \frac{1}{1-\beta} \alpha_g \left[ 1 - 2 \left( \frac{1}{1/\beta} \right) (q_2 + q_3) \right] \sigma^2_n \]

= \frac{1}{\pi} \frac{1}{1-\beta} \left[ \frac{1}{1 - (1/\beta)^{t-1}} \right] \left[ \frac{1}{1 - (1/\beta)^t} \right] q_1^2 \left[ \frac{\sigma^2}{\sigma_Y^2} + \left( 1 - \beta^* \frac{P_{y_t}}{P} \right)^2 \sigma^2_n \right]
\]
\[
- \frac{1}{2} \frac{1}{1-\beta} \left[ \frac{1}{1 - (1/\beta)^{t-1}} \right] \left[ \frac{1}{1 - (1/\beta)^t} \right] q_1^2 \left[ \frac{\sigma^2}{\sigma_Y^2} + \left( 1 - \beta^* \frac{P_{y_t}}{P} \right)^2 \sigma^2_n \right]
\]
\[
+ \frac{1}{2} \frac{1}{1-\beta} \left[ \frac{Q_{m_t}}{(P_{y_t})} \right] \left[ q_2 \frac{\sigma^2}{\sigma_Y^2} + (q_2 + q_3)^2 \sigma^2_n \right] + \frac{1}{2} \frac{1}{1-\beta} \alpha_g \left[ 1 - 2 \left( \frac{1}{1/\beta} \right) (q_2 + q_3) \right] \sigma^2_n,
\]

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which equals

\[
\begin{align*}
\frac{1}{2} \left[ \frac{1}{P} \right] q_1^2 \left[ \frac{\sigma_2^2}{P^2} + \left( 1 - \beta^* \frac{P_{3n}}{P} \right)^2 \right] \sigma_\eta^2 & \left[ \frac{\beta^* (1 + \rho) - 1}{P} - \frac{(1 - \beta^* \frac{P_{3n}}{P})^2}{(1 - \beta^*) \beta^* (1 + \rho) - 1} \right] \\
+ \frac{1}{2} \left[ \frac{1}{P} \right] \frac{Q_M}{P_{3n}} \left[ \frac{\sigma_2^2}{P^2} + (q_2 + q_3)^2 \sigma_\eta^2 \right] & \left[ 1 - 2 \left( \frac{1}{\alpha_n} \right) (q_2 + q_3) \right] \sigma_\eta^2 \\
= \frac{1}{2} \left[ \frac{1}{P} \right] q_1^2 \left[ \frac{\sigma_2^2}{P^2} + \left( 1 - \beta^* \frac{P_{3n}}{P} \right)^2 \right] \sigma_\eta^2 & \left[ \frac{\beta^* (1 + \rho) - 1}{P} \right] \\
+ \frac{1}{2} \left[ \frac{1}{P} \right] \frac{Q_M}{P_{3n}} \left[ \frac{\sigma_2^2}{P^2} + (q_2 + q_3)^2 \sigma_\eta^2 \right] & \left[ 1 - 2 \left( \frac{1}{\alpha_n} \right) (q_2 + q_3) \right] \sigma_\eta^2
\end{align*}
\]

\[
= \frac{1}{2} \frac{1}{1 - \beta^*} \frac{\sigma_2^2}{P^2} \left[ \frac{\beta^* (1 + \rho) - 1}{P} \right] q_1^2 + \frac{Q_M}{P_{3n}} \left[ \beta^* (1 + \rho) - 1 \right]^2 \sigma_\eta^2
\]

\[
+ \frac{1}{2} \frac{1}{1 - \beta^*} \sigma_\eta^2 \left[ \frac{\beta^* (1 + \rho) - 1}{P} \right] \left( 1 - \beta^* \frac{P_{3n}}{P} \right)^2 \sigma_\eta^2 + \frac{Q_M}{P_{3n}} \left( \frac{P_{3n}}{P} \right)^2 \left[ \beta^* (2 + \rho) - 1 \right]^2 \sigma_\eta^2
\]

\[
+ \frac{1}{2} \frac{1}{1 - \beta^*} \sigma_\eta^2 \left[ 1 - 2 \left( \frac{1}{\alpha_n} \right) (q_2 + q_3) \right] \sigma_\eta^2
\]

\[
= \frac{1}{2} \frac{1}{1 - \beta^*} \frac{\sigma_2^2}{P^2} \frac{\beta^* (1 + \rho) - 1}{P} U_1 + \frac{1}{2} \frac{1}{1 - \beta^*} \sigma_\eta^2 U_2 + \frac{1}{2} \frac{1}{1 - \beta^*} \sigma_\eta^2 U_3.
\]

where

\[
U_1 \equiv q_1^2 \left[ 1 + \left( \frac{Q_M}{P} \right) \left[ \beta^* (1 + \rho) - 1 \right] \right] = \frac{P^2 \left[ \beta^* (1 + \rho) - 1 \right] Q_M}{P \left[ 1 + (P_{3n}/P) \beta^* (1 + \rho) - 1 \right]^2},
\]

\[
U_2 \equiv q_1^2 \left\{ \frac{\beta^* (1 + \rho) - 1}{P} \left[ 1 - \beta^* \left( \frac{P_{3n}}{P} \right) \right] \right\} \left[ \frac{Q_M}{P_{3n}} \right] \left[ \beta^* (2 + \rho) - 1 \right]^2
\]

\[
= \frac{P^2 \beta^* (1 + \rho) - 1 \left[ 1 - \beta^* \left( \frac{P_{3n}}{P} \right) \right]^2 + \beta^* (2 + \rho) - 1^2 \sigma_\eta^2 Q_M}{P \left[ 1 + (P_{3n}/P) (\beta^* (1 + \rho) - 1) \right]^2},
\]

\[
U_3 \equiv 1 - 2 \left( \frac{1}{\alpha_n} \right) (q_2 + q_3) = 1 - 2 \left( \frac{1}{\alpha_n} \right) \left( \frac{P_{3n}}{P} \right) \frac{\beta^* (2 + \rho) - 1}{1 + (P_{3n}/P) \beta^* (1 + \rho) - 1}
\]

\[
= 1 - 2 \left( \frac{1}{\alpha_n} \right) \left[ \frac{\beta^* (2 + \rho) - 1}{1 + (P_{3n}/P) \beta^* (1 + \rho) - 1} \right].
\]

We will now investigate each of the terms \(U_1\), \(U_2\) and \(U_3\). For \(U_1\), we have:
\[ \text{sgn} \left( \frac{\partial U_1}{\partial \alpha_{\pi M}} \right) = \text{sgn} \left\{ \frac{-2\alpha_{\pi M}[1 + \frac{P_t^*}{P(t^*)/(1+\rho) - 1]} + \frac{Q_{\pi M}}{P(t^*)/(1+\rho) - 1]} {\left[1 + \left( \frac{P_t^*}{P(t^*)/(1+\rho) - 1} \right)^2 \right]^5} \right\} \]

where \( \text{sgn}(x) \) stands for the sign of expression \( x \). Hence, \( U_1 \) is decreasing in \( \alpha_{\pi M} \) for \( \alpha_{\pi M} < \alpha_{\pi} \), increasing in \( \alpha_{\pi M} \) for \( \alpha_{\pi M} > \alpha_{\pi} \) and reaches its strict global minimum at \( \alpha_{\pi M} = \alpha_{\pi} \). For \( U_2 \), we find, after quite a substantial amount of straightforward algebra, that:

\[ \text{sgn} \left( \frac{\partial U_2}{\partial \alpha_{\pi M}} \right) = \text{sgn} \left\{ \frac{\frac{2}{\alpha_{\pi M}} \left[ \frac{P(t^*)/(1+\rho) - 1] - (\beta^*)^2 (1+\rho) P(t^*)/(1+\rho) \right]} {\left[1 + \left( \frac{P_t^*}{P(t^*)/(1+\rho) - 1} \right)^2 \right]^5} \right\} \]

Hence, \( U_2 \) is strictly decreasing for \( \alpha_{\pi M} < \alpha_{\pi} \). Finally, we see immediately that \( U_3 \) is decreasing in \( \alpha_{\pi M} \).

### E.2 Discretion

Using (107) and its lags, we can write:

\[
U_t = (1 + \rho) d_{t-1} + G_t \\
= (1 + \rho) \left\{ \frac{1}{\beta^*(Q_{\pi M}/P)} d_{t-2} + \frac{G_{t-1} - \beta^*(Q_{\pi M}/P)G_{t-1}}{\beta^*(1+\rho)(Q_{\pi M}/P)} + p_1 \left[ \frac{\mu_{t-1}}{\nu} + (1 - \beta^*P_t G_{t-1}) \eta_{t-1} \right] \right\} + G_t \\
= (1 + \rho) \left\{ \frac{1}{\beta^*(Q_{\pi M}/P)} d_{t-3} + \frac{G_{t-2} - \beta^*(Q_{\pi M}/P)G_{t-2}}{\beta^*(1+\rho)(Q_{\pi M}/P)} + p_1 \left[ \frac{\mu_{t-2}}{\nu} + (1 - \beta^*P_t G_{t-2}) \eta_{t-2} \right] \right\} + (1 + \rho) p_1 \left[ \frac{\mu_{t-1}}{\nu} + (1 - \beta^*P_t G_{t-1}) \eta_{t-1} \right] + \frac{1}{\beta^*(Q_{\pi M}/P)} G_{t-1}
\]
\[
\begin{align*}
&= (1 + \rho) \left( \frac{1}{\beta (Q_{M/P})} \right)^2 \left( \frac{\mu_{t-3}}{\nu} + \frac{1}{\beta (Q_{M/P})} (1 + \rho) p_1 \left[ \frac{\mu_{t-2}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{t-2} \right] 
+ (1 + \rho) p_1 \left[ \frac{\mu_{t-1}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{t-1} \right] + \left( \frac{1}{\beta (Q_{M/P})} \right)^2 G_{t-2} \right) \\
&= \left( \frac{1}{\beta (Q_{M/P})} \right)^{t-1} \left[ (1 + \rho) d_0 + G_1 \right] + \\
&(1 + \rho) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta (Q_{M/P})} \right)^{\xi-1} p_1 \left[ \frac{\mu_{\xi-\xi}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{\xi-\xi} \right].
\end{align*}
\]

We can now combine this with (104), (105) and (106) to give:

\[
\begin{align*}
\pi_t &= \left[ \frac{1}{\alpha P_t^M} \right] \psi_0^P \left( 1 + \rho \right) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta (Q_{M/P})} \right)^{\xi-1} p_1 \left[ \frac{\mu_{\xi-\xi}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{\xi-\xi} \right] \\
&\quad + \left[ \frac{1}{\alpha P_t^M} \right] \left[ \psi_1^P \left( \frac{\mu_{1-\xi}}{\nu} + \eta_{t-1} \right) + p_2 \eta_t \right] + \left[ \frac{1}{\alpha P_t^M} \right] \psi_0^P \left( \frac{1}{\beta (Q_{M/P})} \right)^{t-1} \left[ (1 + \rho) d_0 + G_1 \right],
\end{align*}
\]

\[
\begin{align*}
\tilde{x}_t - x_t &= \left[ \frac{1}{\nu P_t^M} \right] \psi_0^P \left( 1 + \rho \right) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta (Q_{M/P})} \right)^{\xi-1} p_1 \left[ \frac{\mu_{\xi-\xi}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{\xi-\xi} \right] \\
&\quad + \left[ \frac{1}{\nu P_t^M} \right] \left[ \psi_1^P \left( \frac{\mu_{1-\xi}}{\nu} + \eta_{t-1} \right) + p_2 \eta_t \right] + \left[ \frac{1}{\nu P_t^M} \right] \psi_0^P \left( \frac{1}{\beta (Q_{M/P})} \right)^{t-1} \left[ (1 + \rho) d_0 + G_1 \right],
\end{align*}
\]

\[
\begin{align*}
\tilde{g}_t - g_t &= \left[ \frac{1}{\gamma P_t^M} \right] \psi_0^P \left( 1 + \rho \right) \sum_{\xi=1}^{t-1} \left( \frac{1}{\beta (Q_{M/P})} \right)^{\xi-1} p_1 \left[ \frac{\mu_{\xi-\xi}}{\nu} + \left( 1 - \beta^* \frac{P_t}{P_{t-1}} \right) \eta_{\xi-\xi} \right] \\
&\quad + \left[ \frac{1}{\gamma P_t^M} \right] \left[ \psi_1^P \left( \frac{\mu_{1-\xi}}{\nu} + \eta_{t-1} \right) + p_2 \eta_t \right] + \left[ \frac{1}{\gamma P_t^M} \right] \psi_0^P \left( \frac{1}{\beta (Q_{M/P})} \right)^{t-1} \left[ (1 + \rho) d_0 + G_1 \right] \\
&\quad - \eta_h,
\end{align*}
\]

Let us now compute society’s expected loss:
\[
E_0 \left[ \alpha_x \pi^2_t + (x_t - \bar{x}_t)^2 + \alpha_g (g_t - \bar{g}_t)^2 \right]
\]

\[
= \left[ \frac{Q_M}{P^2} \right] \left\{ \psi_0^P \left( \frac{1}{\beta^*(Q_M/P)} \right)^{l-1} \left[ (1 + \rho) d_0 + G_1 \right] \right\}^2 + \\
\left[ \frac{Q_M}{P^2} \right] (1 + \rho)^2 \left( \psi_0^P \right)^2 \sum_{\xi=1}^{l-1} \left( \frac{1}{\beta^*(Q_M/P)} \right)^{2(\xi-1)} P_1^2 \left[ \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( 1 - \beta^* \frac{P_{M^*}}{P} \right)^2 \sigma^2_g \right] + \\
\left[ \frac{Q_M}{P_{M^*}^2} \right] \left[ \left( \psi_1^P \right)^2 \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( \psi_1^P + p_2 \right)^2 \sigma^2_g \right] + \alpha_g \left[ 1 - 2 \left( \frac{1/\alpha_x}{P_{M^*}} \right) \left( \psi_1^P + p_2 \right) \sigma^2_g \right]
\]

\[
= \left[ \frac{Q_M}{P^2} \right] \left\{ \psi_0^P \left( \frac{1}{\beta^*(Q_M/P)} \right)^{l-1} \left[ (1 + \rho) d_0 + G_1 \right] \right\}^2 + \\
\left[ \frac{Q_M}{P^2} \right] (1 + \rho)^2 \left( \psi_0^P \right)^2 \left[ \frac{1-(1/\beta^* Q_M/P)^{\beta/(1-1)}}{1-(1/\beta^* Q_M/P)^{\beta/(1-1)}} \right] P_1^2 \left[ \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( 1 - \beta^* \frac{P_{M^*}}{P} \right)^2 \sigma^2_g \right] + \\
\left[ \frac{Q_M}{P_{M^*}^2} \right] \left[ \left( \psi_1^P \right)^2 \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( \psi_1^P + p_2 \right)^2 \sigma^2_g \right] + \alpha_g \left[ 1 - 2 \left( \frac{1/\alpha_x}{P_{M^*}} \right) \left( \psi_1^P + p_2 \right) \sigma^2_g \right]
\]

Hence,

\[
\frac{1}{2} \sum_{t=1}^{\infty} \beta^t \left[ \alpha_x \pi^2_t + (x_t - \bar{x}_t)^2 + \alpha_g (g_t - \bar{g}_t)^2 \right]
\]

\[
= \frac{1}{2} \beta^*(1+\rho) \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{Q_M}{P^2} \right] \left[ \frac{(\beta^*(1+\rho)Q_M/P^{\beta/(1-1)})}{\beta^*(1+\rho)(Q_M/P)^{\beta/(1-1)}} \right] + \\
\frac{1}{2} \sum_{t=1}^{\infty} \beta^t \left[ \frac{Q_M}{P^2} \right] \left[ (1 + \rho)^2 \left( \psi_0^P \right)^2 \left[ \frac{1-(1/\beta^* Q_M/P)^{\beta/(1-1)}}{1-(1/\beta^* Q_M/P)^{\beta/(1-1)}} \right] P_1^2 \left[ \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( 1 - \beta^* \frac{P_{M^*}}{P} \right)^2 \sigma^2_g \right] \right] + \\
\frac{1}{2} \sum_{t=1}^{\infty} \beta^t \left[ \frac{Q_M}{P_{M^*}^2} \right] \left[ \left( \psi_1^P \right)^2 \frac{\sigma^2_{\pi^2}}{\beta^2} + \left( \psi_1^P + p_2 \right)^2 \sigma^2_g \right] + \alpha_g \left[ 1 - 2 \left( \frac{1/\alpha_x}{P_{M^*}} \right) \left( \psi_1^P + p_2 \right) \sigma^2_g \right]
\]

where we have used that:
\[
\frac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \frac{Q_M}{P_t^2} \right] \left\{ \psi_0^P \left( \frac{1}{\beta(1+\rho)(Q_M/P_t)^t} \right)^{t-1} \left[ (1 + \rho) d_0 + G_1 \right] \right\}^2
\]

\[
= \frac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \frac{Q_M}{P_t^2} \right] \left( \psi_0^P \right)^2 \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{1}{\beta(1+\rho)(Q_M/P_t)^t} \right]^{2(t-1)}
\]

\[
= \frac{1}{2} \left( \psi_0^P \right)^2 \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{Q_M}{P_t^2} \right] \sum_{t=1}^{\infty} \beta^t \left( \frac{1}{\beta(1+\rho)(Q_M/P_t)^t} \right)^{t-1}
\]

\[
= \frac{1}{2} \left( \psi_0^P \right)^2 \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{Q_M}{P_t^2} \right] \beta^* \left( 1+\rho \right) (Q_M/P_t)^2 \beta^*(1+\rho)(Q_M/P_t)^{-2}
\]

\[
= \frac{1}{2} \left( \psi_0^P \right)^2 \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{Q_M}{P_t^2} \right] \beta^*(1+\rho)(Q_M/P_t)^{-2}
\]

and

\[
\frac{1}{2} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \frac{1}{1-(1/\beta^*(Q_M/P_t)^2)^2} \right] - \beta^*(1+\rho)(Q_M/P_t)^2 \left[ \frac{1}{1-(1/\beta^*(Q_M/P_t)^2)^2} \right]
\]

\[
= \frac{1}{2} \left[ \frac{1}{1-(1/\beta^*(Q_M/P_t)^2)^2} \right] \sum_{t=1}^{\infty} \left( \frac{\beta^*(1+\rho)(Q_M/P_t)^2}{\beta^* (1+\rho)(Q_M/P_t)^{2} - 1} - \beta^*(1+\rho)(Q_M/P_t)^2 (1-\beta) \right)
\]

\[
= \frac{1}{2} \left[ \frac{1}{1-(1/\beta^*(Q_M/P_t)^2)^2} \right] \left( \frac{\beta^*(1+\rho)(Q_M/P_t)^2}{\beta^*(1+\rho)(Q_M/P_t)^{2} - 1} \right)
\]

\[
= \frac{1}{2} \left[ \frac{1}{1-(1/\beta^*(Q_M/P_t)^2)^2} \right] \left( \beta^*(Q_M/P_t)^2 \right).
\]

We can rewrite the expression for society’s expected loss further as:

\[
\frac{1}{2} \beta^*(1+\rho) \left[ (1 + \rho) d_0 + G_1 \right]^2 \left[ \frac{Q_M}{P_t^2} \right] \left[ \frac{\beta^*(1+\rho)(Q_M/P_t)^2}{\beta^*(1+\rho)(Q_M/P_t)^{2} - 1} \right] +
\]

\[
\frac{1}{2} \frac{1}{\beta^*(1+\rho)(Q_M/P_t)^2} \left[ 1 + \left( \frac{Q_M}{P_t^2} \right) \beta^*(1+\rho)(Q_M/P_t)^{2} - 1 \right] \left[ \frac{\beta^*(1+\rho)(Q_M/P_t)^2}{\beta^*(1+\rho)(Q_M/P_t)^{2} - 1} \right]
\]

\[
+ \frac{1}{2} \frac{1}{\beta^*(1+\rho)(Q_M/P_t)^2} \left( \left( \psi_1^P \right)^2 \sigma_\eta^2 + \left( \psi_2^P + p_2 \right)^2 \sigma_\eta^2 \right) + \frac{1}{2} \frac{1}{\beta^*(1+\rho)(Q_M/P_t)^2} \sigma_\eta^2,
\]

where we have used that:

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\[
\frac{(\beta^* Q_M/P)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \left[ \frac{Q_M}{P_M} \right] (1 + \rho)^2 \left( \psi_0^P \right)^2 \rho_1^2 \\
= \frac{(\beta^* Q_M/P)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \left[ \frac{Q_M}{P_M} \right] (1 + \rho)^2 \left[ \frac{\beta^* (1 + \rho)(Q_M/P) - 1}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right]^2 \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \\
= \left[ \frac{Q_M/P^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P)}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 .
\]

Hence, society’s expected loss can be written as:

\[
V_1 + V_2 \sigma_\theta^2 + V_3 \sigma_\eta^2,
\]

\[
V_1 \equiv \frac{1}{2} \frac{1}{\beta^* (1 + \rho)} \left[ (1 + \rho) d_0 + G_1 \right] \left( \frac{Q_M}{P_M} \right) \left[ \frac{\beta^* (1 + \rho)(Q_M/P - 1)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right],
\]

\[
V_2 \equiv \frac{1}{2} \frac{1}{1 - \beta} \left\{ \left[ \frac{Q_M}{P_M} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P - 1)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P) - 1}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \right\} \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \\
= \frac{1}{2} \frac{1}{1 - \beta} \left\{ \left[ \frac{Q_M}{P_M} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P) - 1}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \right\} \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \\
= \frac{1}{2} \frac{1}{1 - \beta} \left[ \frac{Q_M}{P_M} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \\
= \frac{1}{2} \frac{1}{1 - \beta} \left[ \frac{Q_M}{P_M} \right] \left[ \frac{\beta^* (1 + \rho)(Q_M/P)^2}{\beta^* (1 + \rho)(Q_M/P)^2 - 1} \right] \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2 \left[ \frac{1}{1 + (P_M^*/P_M)\beta^* (1 + \rho)(Q_M/P)} \right]^2.
\]

These expressions are programmed, so that we can explore society’s expected loss as a function of \(\alpha_x M\).