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On the Performance of Different Control Charting Rules

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In the literature a number of control charting rules are proposed to decide whether a process is in control or out of control. Some issues with these rules will be highlighted in this article. By redefining and listing a set of rules we will evaluate their performance on the $\bar{X}$, R, S and S^2 charts. Also we will compare the performance of these rules using their power curves to figure out the superior ones. Application of a few of these rules with real data sets will show their detection ability and use for practitioners. Copyright © 2011 John Wiley & Sons, Ltd.

Keywords: biasedness; control charts; monotonicity; power; runs rules; statistical process control

1. Introduction

Shewhart-type control charts are used to monitor the location or/and spread parameter of the distribution of a quality characteristic. To improve the performance of Shewhart-type charts for detecting small shifts, runs rules are used along with these charts. The standard set of extra runs rules was suggested by the *Western Electric Statistical Quality Control Handbook* of 1956 (cf. Western Electric Company[1], see also Nelson[2]). To test for unnatural patterns it was suggested to use several runs rules simultaneously. The cost we have to pay for this gain is an increase in the false alarm rate. Another performance characteristic is the run length. In Does and Schriever[3] the effect of combining runs rules is studied. They show that the combination of runs rules decreases the run length percentiles considerably. By combining all rules the run length will be too low for practical purposes if the process is in control.

The effect of using runs rules with the $\bar{X}$ chart has been studied by Champ and Woodall[4], among others. They denoted by $T(k-m,k,a,b)$ the runs rule that signals if $k-m$ out of the last k sample means fall in the interval (a, b). Some of the runs rules in the *Western Electric Handbook* (cf. Western Electric Company[1]) are special cases of the types $T(k,k,a,b)$ and $T(k,k+1,a,b)$. Weiler[5] proposes the use of runs rules of the type $T(k,k,a,b)$. Page[6] suggests to combine rule $T(1,1,a,b)$ with rule $T(k,k,a,b)$. Other suggestions are made by the authors of References [4, 7–18], among others.

However, there is a serious issue with the $T(k-m,k,a,b)$ runs rules because these rules are not able to work independently to address different magnitudes of shifts. We will illustrate this by comparing three runs rules for the upper sided S^2 chart. The three runs rules are:

Rule 1: $T(1,1,a,\infty)$.
Rule 2: $T(2,3,b,\infty)$.
Rule 3: $T(4,5,c,\infty)$.

In the abovementioned rules, *Rule 1* can address large shifts, whereas *Rule 2* and *Rule 3* are good at detecting smaller shifts. Note that to simultaneously address both types of shifts we have to combine these rules (e.g. *Rule 1* with *Rule 2* or *Rule 1* with *Rule 3*). Imposing more rules simultaneously complicates the application of control chart and also inflates the false alarm rate. The problem of inflated false alarm rate may be handled by appropriately adjusting the control limit coefficients used with each rule as is done by Klein[12] and Khoo[13]. To overcome the complications of applying more rules at a time we suggest a separate use of each rule. However, an independent and separate application of these rules and similar other rules with a control chart structure may cause an abnormal power pattern as will be shown in our example with the upper sided S^2 chart. When we fix

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Table I. Power of *Rules* 1, 2 and 3 for the S^2 chart (Upper Sided) at $n=5$

Rules δ^2	*Rule 1*: 1/1 $a=4.0628$	*Rule2*: 2/3 $b=2.6688$	*Rule 3*: 4/5 $c=1.64755$
1	0.002700	0.002700	0.002700
1.05	0.005271	0.006088	0.006501
1.1	0.009352	0.012096	0.013495
1.15	0.015332	0.021637	0.024809
1.2	0.023535	0.035451	0.041254
1.25	0.034192	0.053949	0.063118
1.3	0.047416	0.077128	0.090080
1.4	0.081468	0.135464	0.155421
1.5	0.124575	0.203437	0.226720
2	0.397574	0.436760	0.409600
4	0.907406	0.122351	0.085876
10	0.996872	0.004113	0.002650
30	0.999959	0.000053	0.000033

Table II. Different sensitizing rules

$T(k-m, k, h_{k-m,k}, \infty)$	An out-of-control signal is received if at least $k-m$ points of k consecutive points fall above the control (signaling) line $h_{k-m,k}$ of the sampling distribution of the control charting statistic, where $0 \leq m \leq k-1$.
$T(k-m, k, -\infty, h^{\wedge}_{k-m,k})$	An out-of-control signal is received if $k-m$ of k consecutive points are below the control (signaling) line $h^{\wedge}_{k-m,k}$ of the sampling distribution of the control charting statistic, where $0 \leq m \leq k-1$.

the false alarm rate (α) equal to 0.0027 for the S^2 chart (upper sided) we obtain the following control lines in the three rules: $a=4.0628; b=2.6688$ and $c=1.64755$. Now an out-of-control signal is given by *Rule 1* if 1 point is above 4.0628, by *Rule 2* if 2 out of 3 consecutive points fall above 2.6688 and by *Rule 3* if 4 out of 5 consecutive points fall above 1.64755. In this setup we have used samples of size 5 coming from $N(\mu, \delta\sigma)$, with mean 0 and standard deviation 1, where δ represents the amount of shift. Here $\delta=1$ implies that there is no shift in σ (hence the process is in control) and $\delta>1$ means that there is an increase in σ (hence the process is out of control). Discriminatory powers are computed out of the abovementioned three rules and are provided in Table I.

It can be seen from Table I that for *Rule 1* the power keeps increasing with the increase in δ^2, does not get smaller than α and ultimately converges to 1. But in case of *Rules 2* and 3 the power increases with the increase in δ^2 for $\delta^2 \leq 2$ while in case of $\delta^2>2$ the power starts decreasing with the increase in δ^2, eventually becomes smaller than the pre-specified value of α value and converges to 0 instead of 1 which is quite abnormal and unexpected. We term this issue as biasedness and non-monotonicity which are not desirable features for a control chart (cf. Acosta-Mejia and Pignatiello[18]). The results of our Table I clearly indicate that *Rules 2* and *3* (and similar other rules) may not be used in their independent capacities. However, *Rule 1* does not face these types of issues as can be seen from Table I. It means that *Rule 1* can be used independently with a control chart structure but *Rules 2* and *3* have no independent identity.

These problems in the form of the unusual performance of different rules may be overcome by combining the rules (e.g. *Rules 2* and/or *3* with *Rule 1*) at the cost of increased false alarm rate. As mentioned earlier we can manage this issue of inflated false alarm rates by appropriately adjusting the control limits for each rule but simultaneous use of more rules at a time may complicate the application and also makes the control chart structure unattractive for practitioners. Ideally, each rule should have its own independent characteristics and should be able to work with a control chart structure in its true spirit.

In the following section we formulate the runs rules schemes for independent use. Then we evaluate the performance of the different runs rules schemes in terms of the power. An illustrative example shows its use in practice. Finally, conclusions and recommendations are given.

2. Giving the sensitizing rules an independent identity

In order to overcome the problems with *Rules 2* and *3* as shown in Table I and to make these rules workable separately (without connecting their use with any other rule) we redefine the rules and generalize them to some more similar rules to be used with control chart structures to enhance their performance. Table II consists of a set of different sensitizing rules which may be used with a control chart in their own autonomous capacities.

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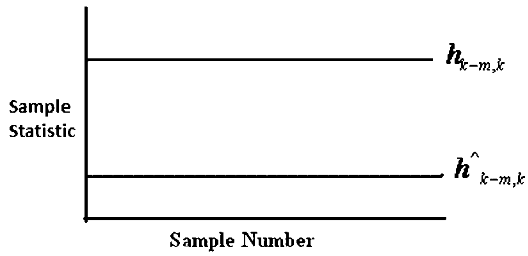
The control lines ($h^{\wedge}_{k-m,k}$, $h_{k-m,k}$) for $0 \le m \le k-1$ defined in Table II can be determined from the sampling distribution of the control charting statistic, depending upon the pre-specified value of α. Graphically, it may be shown in the form of the following figure in general:



The lines ($h^{\wedge}_{k-m,k}$, $h_{k-m,k}$), for $0 \le m \le k-1$ may be defined by using the overall α on the right tail of the sampling distribution for only the upper sided control limit (i.e. we have no lower limit and only $h_{k-m,k}$ is on the chart), on the left tail of the sampling distribution for only the lower sided control limit (i.e. we have no upper limit and only $h^{\wedge}_{k-m,k}$ is on the chart) and half of the α on both the tails for the two-sided control limits on a chart (i.e. both limits are on the chart like in the figure).

It has to be noted that the phrase 'at least' in the rules of Table II have a significant meaning which gives the rule an independent identification for each of the other rules when we change the values of k and/or m (which was in fact missing in the example in Section 1, cf. Table I). The inclusion of these phrases is in fact the suggested redefining for these runs rules schemes which help in overcoming the problems mentioned above in Section 1. It gives an attractive independence to each rule. The signaling limits $h^{\wedge}_{k-m,k}$ and $h_{k-m,k}$ are set according to the pre-specified value of α and the choices of m and k in the runs rules schemes of Table II. This is done using the mathematical expression

$$\alpha = \sum_{k-m \le k} \frac{k!}{(k-m)!m!} p^{(k-m)}(1-p)^m \quad \text{where } 0 \le m \le k-1$$

where α is the pre-specified false alarm rate and p is the probability of a single point falling outside the respective signaling limits depending upon $k-m$ and k. This equation may be solved for p using any choice of α, m and k. The resulting value of p may help in picking up the appropriate quantile point on the sampling distribution of a control charting statistics for different $(k-m)$ out of k runs rules schemes and hence computing the respective control limits followed by evaluating the power of detecting out-of-control signals.

Particularly, we will consider the commonly used Shewhart-type control charts, namely, $\bar{X}$, R, S and S^2 charts in this study, but the application of these schemes may be generalized for any type of chart.

3. Performance evaluation and comparisons

In this section we evaluate the performance of different $(k-m)$ out of k runs rules schemes using the power as the performance criterion. It is quite popular among practitioners to prefer a statistical technique (e.g. a testing procedure) which has higher power (cf. Mahoney and Magel[19]). The literature supporting power evaluation criterion with respect to control charts may be also seen in References[20–22].

Now we evaluate the performance of the schemes defined in Table II by computing their powers for several values of m and k. We show that these schemes are able to overcome the problems indicated in Section 1 and behave according to the expectations attached to them in Section 2. Based on this power evaluation we also compare the performance of each scheme with the other schemes and observe their rank order with respect to each other. We have evaluated the performance of these $\bar{X}$, R, S and S^2 charts in terms of the power using 24 different schemes from Table II which can be obtained by choosing $k=1,2,\ldots,9$ and $m=0,1,2$. Of course other values of k and m are possible but we think that the values chosen cover the practical and commonly used situations. We have also assumed normality of the quality characteristic of interest say X (i.e. $X \sim N(\mu,\sigma)$).

For the said purposes we consider the upper sided control limits of, $\bar{X}$, R, S and S^2 charts for the sake of brevity and provide their power curves at $\alpha=0.0027$ in Figures 1–4 (for ease in comparison) in which the power is plotted versus different amounts of shifts in the process parameter. These shifts are considered in terms of $\delta\sigma$ for all the charts except S^2 where shifts are considered in terms of $\delta^2\sigma^2$. It is to be mentioned that for power computation we have developed an efficient code in R language and used it in our study.

We categorize the curves into three types referring to different rules as: some with $(k-m)=2$ (i.e. $1/3, 2/4, 3/5, 4/6, 5/7, 6/8, 7/9$) some with $(k-m)=1$ (i.e. $1/2, 2/3, 3/4, 4/5, 5/6, 6/7, 7/8, 8/9$) and some with $(k-m)=0$ (i.e. $1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7, 8/8, 9/9$). It would provide an ease in display and discussion as well. As showing all 24 curves on a graph would complicate the display, we have chosen to show a few representative curves using the $(k-m)$ categorization of these rules. In the following Figures 1–4 only six rules are given for display and the rest would be covered in the discussion. The corresponding power values for some selective shifts are also provided in the Appendix A Tables AI–AIV for the four charts in the respective order of Figures 1–4.

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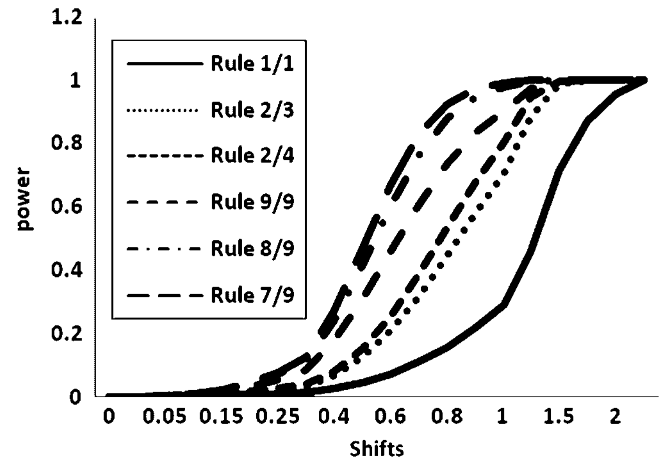

Figure 1. Power curves of different rules at $n=5$ for the $\bar{X}$ chart (upper sided)

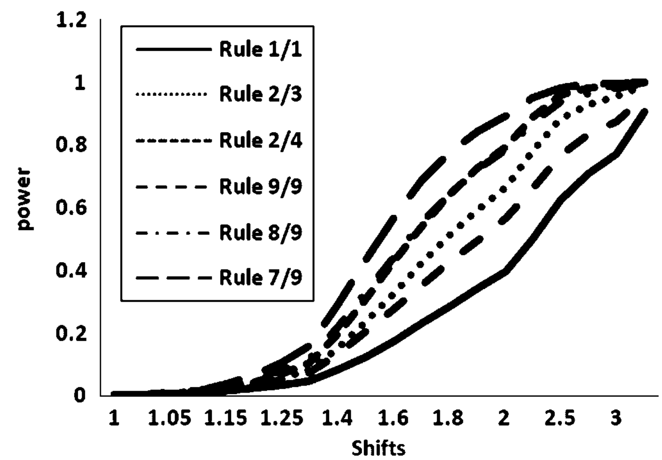

Figure 2. Power curve2 of different rules at $n=5$ for the R chart (upper sided)

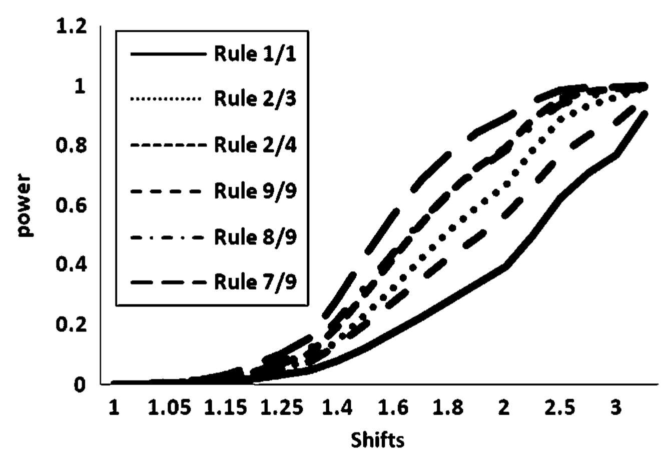

Figure 3. Power curves of different rules at $n=5$ for the S chart (upper sided)

The respective $h_{k-m,k}$'s used in the power curves of the four charts given above in Figures 1–4 are respectively provided in Tables III–VI and these are given as:

A similar behavior was observed for other values of n and α. The power curve analysis of all the 24 rules advocated that all the schemes are unbiased and monotonic. Moreover, the schemes where $(k-m)=2$ generally perform better followed by those

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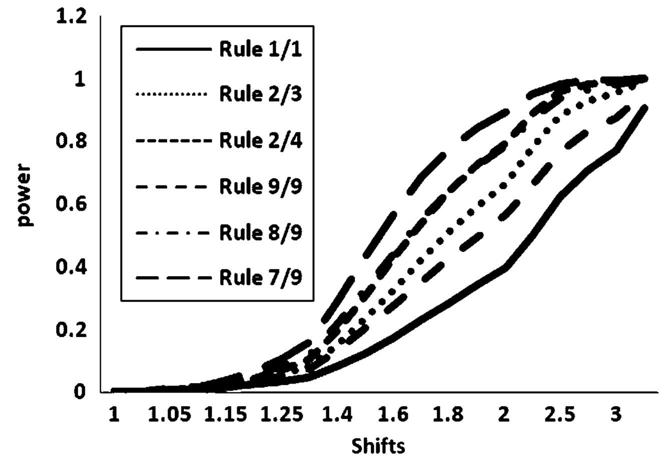

Figure 4. Power curves of different rules at $n=5$ for the S^2 chart (upper sided)

Table III. Control lines of different sensitizing rules at $n=5$ for the $\bar{X}$ chart (upper sided)

Rule	1/1	2/2	1/2	3/3	2/3	1/3	4/4	3/4
$h_{k-m,k}$	1.244216	0.727225	1.34154	0.484648	0.839103	1.395809	0.333448	0.600203
Rule	2/4	5/5	4/5	3/5	6/6	5/6	4/6	7/7
$h_{k-m,k}$	0.904842	0.226341	0.449046	0.670552	0.144673	0.340664	0.521217	0.079346
Rule	6/7	5/7	8/8	7/8	6/8	9/9	8/9	7/9
$h_{k-m,k}$	0.257255	0.41337	0.025301	0.190059	0.329854	−0.02054	0.134161	0.262224

Table IV. Control lines of different sensitizing rules at=5 for the R chart (upper sided)

Rule	1/1	2/2	1/2	3/3	2/3	1/3	4/4	3/4
$h_{k-m,k}$	5.1201	3.839592	5.376479	3.277863	4.106906	5.518022	2.943846	3.541697
Rule	2/4	5/5	4/5	3/5	6/6	5/6	4/6	7/7
$h_{k-m,k}$	4.266604	2.716187	3.19796	3.706131	2.545364	2.959533	3.360344	2.413317
Rule	6/7	5/7	8/8	7/8	6/8	9/9	8/9	7/9
$h_{k-m,k}$	2.781399	3.118335	2.306364	2.640111	2.936175	2.216399	2.523959	2.792012

Table V. Control lines of different sensitizing rules at $n=5$ for the S chart (upper sided)

Rule	1/1	2/2	1/2	3/3	2/3	1/3	4/4	3/4
$h_{k-m,k}$	2.015639	1.532514	2.109351	1.316924	1.634601	2.162075	1.187114	1.41858
Rule	2/4	5/5	4/5	3/5	6/6	5/6	4/6	7/7
$h_{k-m,k}$	1.695258	1.097602	1.286017	1.481415	1.030844	1.193218	1.348877	0.978443
Rule	6/7	5/7	8/8	7/8	6/8	9/9	8/9	7/9
$h_{k-m,k}$	1.123214	1.255253	0.935804	1.067778	1.184078	0.900156	1.02235	1.127351

Table VI. Control lines of different sensitizing rules at=5 for the S^2 chart (upper sided)

Rule	1/1	2/2	1/2	3/3	2/3	1/3	4/4	3/4
$h_{k-m,k}$	4.062793	2.348611	4.449728	1.734285	2.671919	4.674558	3.450777	2.012298
Rule	2/4	5/5	4/5	3/5	6/6	5/6	4/6	7/7
$h_{k-m,k}$	2.873965	1.204726	1.653841	2.194586	1.062636	1.423775	1.819466	0.957347
Rule	6/7	5/7	8/8	7/8	6/8	9/9	8/9	7/9
$h_{k-m,k}$	1.261611	1.575657	0.875718	1.09825	1.402035	0.810276	1.045198	1.270917

where $(k-m)=1$ and the least efficient are those where $(k-m)=0$. Hence, higher values of k, e.g. using more consecutive points have in general a better performance. Furthermore, we have observed that the S chart generally performs better as compared to the R chart for different $(k-m)$ out of k runs rules scheme.

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4. Illustrative example and application

Besides evidence in terms of statistical efficiency it is always a good approach to test a technique on some real data for its practical implications. While working with practical data sets in the industry, practitioners look for a direct application of these rules so that out-of-control signals may be received timely. For this purpose we consider here a data set taken from Alwan[23] which refers back to the classical book by W. A. Shewhart of 1931 (cf. Shewhart[24]), containing the data on 204 consecutive measurements of the electrical resistance of insulation in megohms. The data set may be found on page 380 of Alwan[23]. To illustrate the application of the schemes given in Table II we apply two of them on the same data set to see what output they show. The other rules may also be applied very easily and for this purpose we have written an application code in *R* language for the practitioner's convenience to use these rules on the real data sets (the code used for this purpose is available from the authors and may be provided on request). The final control chart display along with a full summary using our code for 1 out of 1 and 1 out of 2 schemes are given in Figures 5 and 6. It is evident from these figures that the same out-of-control signals (10 in total) are received for 1 out of 1 rule (cf. Figure 5) as those given by Alwan[23] at $\alpha=0.0027$. The application of 1 out of 2 rule has given more (13 in total) out-of-control signals (cf. Figure 6) as compared to those of 1 out of 1 keeping the false alarm rate fixed α at 0.0027. Similarly, practitioners may easily apply the other rules on the real data sets.

This application of the runs rules schemes exhibits that their implementation is quite efficient and also simple for the practitioner's use on the real data sets.

Summary for out-of-control signals (for Figure 5)

Control chart:	*Shewhart Xbar*
Control limits (T):	*4977.989 4018.364*
False alarm rate:	*0.0027*
Sample size:	*4*
Subgroup size:	*51*
Out-of-control signals received at subgroups# : 3 4 5 15 16 22 31 36 44 51	
Total # of out-of-control signals using 1/1:10	

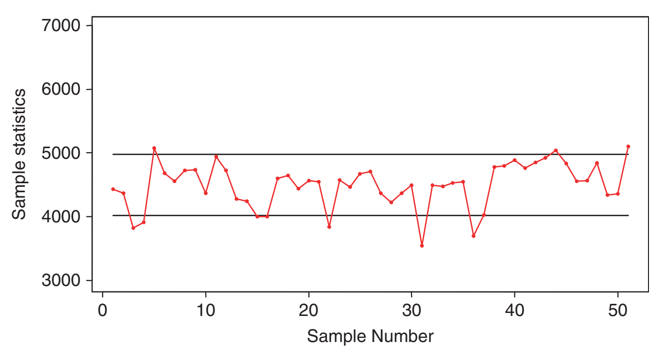

Figure 5. $\bar{X}$ Control chart for megaohm data using 1/1 Rule

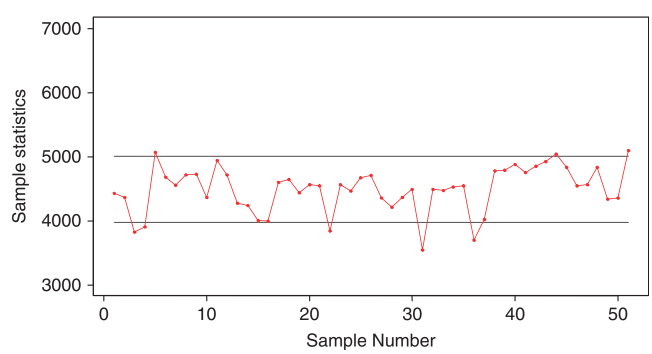

Figure 6. $\bar{X}$ Control chart for megaohm data using 1/2 Rule

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Summary for out-of-control signal (for Figure 6)

Control chart:	*Shewhart Xbar*
Control limits (T):	*5010.461 3985.892*
False alarm rate:	*0.0027*
Sample size:	*4*
Subgroup size:	*51*
Out-of-control signals received at subgroups# : 3 4 5 6 22 23 31 32 36 37 44 45 51	
Total # of out-of-control signals using 1/2:13	

5. Conclusions and recommendations

Control charting is quite popular in SPC to identify problematic subgroups over a given sequence of time points. A timely signaling is a natural desire of practitioners and for this purpose we have different types of control charts. Shewhart-type control charts are efficient at detecting large shifts and extra sensitizing runs rules schemes provide good support to their design structures but at the same time they introduce some problems with their properties. These problems include: (i) biasedness and non-monotonicity in case of separate use of each rule in general and hence no independent identity of any rule for different types of shifts; (ii) need of simultaneous application of more rules at a time which results into an inflated false alarm rate and unattractive structures for practitioners's use.

By appropriately redefining these runs rules we have shown that each rule may have its own independent identification and hence may be used in its own capacity separately giving simplicity to the design structures of control charts. It also overcomes the issues of inflated false alarm rate, biasedness and non-monotonicity.

The application of some of these rules has been implemented on EWMA control charts by Abbas *et al*.[25] and CUSUM control charts by Riaz *et al*.[26].

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Appendix A

The corresponding power values for some selective shifts are provided in Tables AI–AIV.

Table AI. Powers of different sensitizing rules for $\bar{X}$ chart (upper sided) at $n=5$.

	Rule					
Shifts	1/1	2/3	2/4	9/9	8/9	7/9
0	0.0027	0.0028	0.0026	0.0027	0.0026	0.0027
0.5	0.048	0.1283	0.155	0.3098	0.4171	0.4692
1	0.2925	0.7062	0.8025	0.9033	0.9776	0.9918
1.5	0.7164	0.9859	0.9971	0.997	1	1
2	0.9545	0.9999	1	1	1	1
3	1	1	1	1	1	1

Table AII. Powers of different sensitizing rules for R chart (upper sided) at $n=5$

	Rule					
Shifts	1/1	2/3	2/4	9/9	8/9	7/9
1	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
1.05	0.0049	0.0061	0.0063	0.006	0.0069	0.0074
1.25	0.0307	0.0514	0.0643	0.0475	0.0791	0.1008
2	0.3663	0.6387	0.7678	0.5221	0.774	0.8819
3	0.7463	0.9536	0.9886	0.8603	0.9813	0.997
20	0.9998	1	1	0.9999	1	1

Table AIII. Powers of different sensitizing rules for S chart (upper sided) at $n=5$

	Rule					
Shifts	1/1	2/3	2/4	9/9	8/9	7/9
1	0.0027	0.0027	0.0027	0.0026	0.0026	0.0028
1.05	0.008	0.0089	0.0092	0.006	0.0071	0.0077
1.25	0.0343	0.0568	0.0716	0.0537	0.0819	0.1068
2	0.3971	0.7962	0.7962	0.5564	0.7846	0.892
3	0.7713	0.9604	0.991	0.8775	0.9827	1
20	1	1	1	1	1	1

Table AIV. Powers of different sensitizing rules for S^2 chart (upper sided) at $n=5$

	Rule					
Shifts	1/1	2/3	2/4	9/9	8/9	7/9
1	0.0027	0.0026	0.0027	0.0027	0.0027	0.0027
1.05	0.0053	0.0089	0.0094	0.009	0.0097	0.0104
1.25	0.0342	0.0567	0.071	0.0538	0.0822	0.1066
2	0.3976	0.6679	0.7954	0.5681	0.7849	0.8922
3	0.7713	0.9602	0.9912	0.8778	0.9825	0.9974
20	1	1	1	1	1	1

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